Warp Drive Basic Science Written For "Aficionados". Chapter I - Miguel Alcubierre.

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Abstract

Alcubierre Warp Drive is one of the most exciting Spacetimes of General Relativity.It was the first Spacetime Metric able to develop Superluminal Velocities.However some physical problems associated to the Alcubierre Warp Drive seemed to deny the Superluminal Behaviour.We demonstrate in this work that some of these problems can be overcomed and we arrive at some interesting results although we used two different Shape Functions one continuous $g(rs)$ as an alternative to the original Alcubierre $f(rs)$ and a Piecewise Shape Function $f_{pc}(rs)$ as an alternative to the Ford-Pfenning Piecewise Shape Function with a behaviour similar to the Natario Warp Drive producing effectively an Alcubierre Warp Drive without Expansion/Contraction of the Spacetime. Horizons will exists and cannot be avoided however we found a way to "overcome" this problem.We also introduce here the Casimir Warp Drive.

1 The Beginning-Faster Than Light Space Travel

The Challenge of Faster Than Light Space Travel started in 1905 when Einstein published his Special Theory of Relativity(SR).Einstein established a Universe speed limit:Light Speed c.Nothing can travel Faster Than the Speed Of Light(at least locally in SR Frames of Reference) or as is also known:"Thou Shall Not Travel Faster Than The Speed Of Light". This can be better pictured by the equations given below:

$$m = \frac{m_0}{\sqrt{1 - (\frac{v^2}{c^2})}} \tag{1}$$

$$E_0 = m_0 c^2 \tag{2}$$

$$E = mc^2 \tag{3}$$

$$K = E - E_0 \tag{4}$$

From the set of equations above if we accelerate a body we give it Kinetic energy K but since due to the equivalence between mass and energy this Kinetic energy also have mass..so a body in motion have Kinetic energy but have a mass m that is much heavier than the same body at the rest with mass m_0

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because the Kinetic energy accounts for a mass increase.As faster the body moves the body possesses more Kinetic energy and more mass....it becomes more heavier...as it becomes more heavier it will require a stronger force to accelerate the body giving even more Kinetic energy which means to say even more mass and the body becomes even more heavier.....Ad Infinitum..... In order to reach the Speed Of Light an infinite amount of energy and an infinite force is needed.So its impossible to reach The Speed Of Light and if we cannot reach it we cannot surpass it.This is the reason why we cannot travel Faster Than Light.This line of reason started by Einstein was scientifically correct from 1905 to 1993. Until 1994 Faster Than Light Space Travel was regarded a Province of the Realm of Science Fiction but in that year something happened:one of the major shifts in the line of reason of Modern Science and perhaps the Greatest Of All.1994 was the year when the concept of Faster Than Light Space Travel changed radically from the "Complete Impossible And Solely In Science Fiction" to a "Maybe Possible But We Don't Have The Technology To Afford It".In the year of 1994 a Revolutionary paper appeared bringing by the first time to Modern Science the possibility of Faster Than Light Space Travel.Now we know that the idea behind the 1994 paper will not work at least in a $3+1$ Einstein Universe[1] and we will demonstrate in this section that we really need Extra Dimensions to achieve Faster Than Light Space Travel[2].But the 1994 paper will ever be considered a Historical paper and a Landmark because perhaps this paper changed forever the course of Modern Science.It is not exaggerated to say that the Humanity started is first steps in the Route To The Stars in the year of 1994[3].The Historical Revolutionary paper of 1994 was written by the Mexican mathematician Miguel Alcubierre from Universidad Nacional Autonona de Mexico(UNAM) and is entitled "The Warp Drive Hyper-Fast Travel Within General Relativity".This paper launched the foundations of the so-called Alcubierre Warp Drive.The main idea behind the Alcubierre Warp Drive is to create a local Spacetime Distortion surrounding a spaceship that will generates an expansion of Spacetime behind the spaceship and a contraction in front of the spaceship(see abstract and pg 3 and 6 of [4]).Since the spaceship is at the rest inside the local Spacetime distortion(known as Warp Bubble) and the Warp Bubble moves away from the spaceship the departure point (expansion) while at the same time brings to the front of the spaceship the destination point(contraction) the spaceship do not moves at all inside the Warp Bubble so while remaining at the rest inside the Warp Bubble the spaceship will not have the mass increase of Special Relativity(see pg 6 of [4],pg 2 in [2])[4] and can afford Faster Than Light Space Travel(see pg 8 of [4],pg 2 of [1],pg 3 of [3],pg 2 in [2],pg 1 in [9],pg 1 in [10],pg 2 in [12]).

2 Alcubierre Warp Drive Basics

We will now examine in details some of the features of the Alcubierre Warp Drive. The Alcubierre Warp Drive can be defined by the following Spacetime ansatz in $3+1$ Spacetime Dimensions(see pg 4 and eqs 6 and 8 in [4],pg 3 and eqs 1 to 3 in [1],pg 2 and eqs 1 to 2 in [2],pg 4 to 5 and eqs 1 to 3 in [3],pg 2 to 3 eqs 4 to 6 in [9],pg 2 eqs 2.1,2.2 and pg 3 eq 2.11 in [10],pg 2 eqs 2 to 3 in [12]) [5][6]:

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[1]this is not entirely true and we will demonstrate that the Alcubierre Warp Drive is still a valid solution of the Einstein Field Equations of General Relativity that allows Faster Than Light Space Travel but of course an impact with a Large Black Hole would change the picture

[2]Extra Dimensions are a safer way to circumvent Large Black Holes

[3]interview of Miguel Alcubierre to Sergio de Regules of the magazine "Como Ves" of Universidad Nacional Autonoma de Mexico(UNAM).We can provide the original PDF Acrobat Reader of the interview in Spanish with an English translation for those interested

[4]if the spaceship don't suffer time dilatation then according to Lorentz Transformations it will not suffer mass increase

[5]Alcubierre,Ford,Pfenning,Broeck,Clark Hiscock and Larson used a signature (-,+,+,+),we use a signature (+,-,-,-)

[6]Alcubierre Warp Drive is not a diagonalized metric because it contains the Spacetime Metric Tensor components g_{01} and g_{10} and $g_{01} = g_{10}$.This can be easily seen from the factor $[dx - vsf(rs)dt]^2$.Alcubierre Warp Drive is a particular case of a

$$ds^2 = dt^2 - [dx - vsf(rs)dt]^2 - dy^2 - dz^2 \tag{5}$$

where

$$vs = \frac{dxs}{dt} \tag{6}$$

$$rs(t) = \sqrt{[x - xs(t)]^2 + y^2 + z^2} \tag{7}$$

$$f(rs) = \frac{tanh[\delta(rs + R)] - tanh[\delta(rs - R)]}{2tanh(\delta R)} \tag{8}$$

Now a little bit of Warp Drive Basics:xs is the center of the Warp Bubble where the spaceship remains,x is any position inside the Warp Bubble.For the spaceship $x = xs$(see pg 5 and 6 in [4],pg 3 in [1],pg 2 in [2])[7].$vs = \frac{dxs}{dt}$ is the speed of the center of the Warp Bubble where the spaceship resides with respect to a distant observer at the rest with respect to a SR Local Frame(see pg 3 in [1],pg 4 in [3]).To put it simply the ship is as the rest inside the Warp Bubble and feels no acceleration and no g-forces but the Warp Bubble can move itself with a arbitrary large speed $vs = \frac{dxs}{dt}$ with respect to a distant observer(see pg 2 in [2],see pg 2 in [9],see pg 2 in [12]).rs is the distance between a given point and the center of the Warp Bubble at $x = xs$(see pg 3 in [1]).$f(rs)$ is the so-called Shape Function that makes the Alcubierre Warp Drive works.The parameters of $f(rs)$ are these:δ is the thickness of the Warp Bubble and can be given arbitrarily,R is the radius of the Warp Bubble and rs was already described(see pg 3 in [1]).We have three possible values for the Shape Function:

- 1)Shape Function inside the Warp Bubble $rs \leq R$ (Flat Spacetime) $f(rs) = 1$ everywhere
- 2)Shape Function in the walls of the Warp Bubble(Warped Region) $0 < f(rs) < 1$
- 3)Shape Function outside the Warp Bubble $rs \gg R$ (Flat Spacetime) $f(rs) = 0$

According to pg 3 in [1],eq 7 pg 4 in [4],eq 7 pg 3 in [9],pg 2 in [10] and pg 2 in [12] the Shape Function $f(rs) = 1$ inside the Warp Bubble where $rs \leq R$ and $f(rs) = 0$ outside the Warp Bubble where $rs \gg R$.Assuming a continuous $f(rs)$ there must exists a region where $f(rs)$ decreases from 1 to 0 and this region is the Warped Region associated to the Warp Bubble Walls[8].

Considering that δ the thickness of the Warp Bubble Walls can have a significant value we will redefine the values of the Shape Function as follows[9]:

- 1)Shape Function inside the Warp Bubble $rs \leq R - \delta$ (Flat Spacetime) $f(rs) = 1$ everywhere
- 2)Shape Function in the walls of the Warp Bubble(Warped Region) $R-\delta < rs < R+\delta$ $0 < f(rs) < 1$
- 3)Shape Function outside the Warp Bubble $rs \geq R + \delta$ (Flat Spacetime) $f(rs) = 0$

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family of Spacetime ansatz as we shall see in this section

[7]pg 5 in [4] the center of the Warp Bubble is defined by $[xs(t), 0, 0]$.look also to eq 6 pg 4 in [1] and consider the Spatial Components only.remember that inside the Warp Bubble $f(rs) = 1$ so Ford-Pfenning are taking time derivatives of the Alcubierre center

[8]δ defines the "size" of the Warped Region

[9]combining eqs 3 and 4 pg 3 in [1]

We have also three possible locations to place observers:

- 1)Observer in the spaceship in the center of the Warp Bubble $f(rs) = 1$
- 2)Observer in the middle of the Warped Region $0 < f(rs) < 1$
- 3)Observer outside the Warped Bubble at a faraway distance $f(rs) = 0$

According to pg 6 in [4] the Warped Region is the worst location to place an observer because Spacetime is not flat and the Tidal Forces are very large.Then we are left with two locations only:

- 1)Observer in the spaceship in the center of the Warp Bubble $f(rs) = 1$
- 2)Observer outside the Warped Bubble at a faraway distance $f(rs) = 0$

We will now demonstrate how Alcubierre in 1994 bypassed the limitations posed by Einstein from 1905 to 1993. Imagine two observers A and B at the rest initially with clocks synchronized with respect to a SR Local Frame C.Both have with respect to C the same $ds^2 = dt^2$.

- 1)Observer A enters in a Warp Bubble that instantaneously achieves a large $vs >> 1$[10].Faster Than Light $(c = 1)$.In the center of the Warp Bubble $x = xs$ and $f(rs) = 1$
- 2)Observer B remains outside the Warp Bubble $f(rs) = 0$

Although y and z cannot be null[11] the motion will occurs only in the $x - axis$ so we can write the Alcubierre Warp Drive as follows:

$$ds^2 = dt^2 - [dx - vsf(rs)dt]^2 \tag{9}$$

- 1)Situation seen by A:$x = xs$:$f(rs) = 1$:$vs = \frac{dxs}{dt}$:$dx = dxs$

$$ds^2 = dt^2 - [dxs - vsdt]^2 \tag{10}$$

$$ds^2 = dt^2 - [dxs - \frac{dxs}{dt}dt]^2 \tag{11}$$

$$ds^2 = dt^2 - [dxs - dxs]^2 \tag{12}$$

$$ds^2 = dt^2 \tag{13}$$

- 2)Situation seen by B while observing A moving away:$f(rs) = 0$ for B but B can observe A moving away from him with speed vs:$x = xs$ for A:$dx = dxs$ for A

$$ds^2 = dt^2 - dxs^2 \tag{14}$$

[10]the action is being taken by an external observer
[11]we will see why y and z cannot be null

Remember that observer B is still synchronized to the Frame C so with respect to C B possesses a $ds^2 = dt^2$ as a proper time.But A passed instantaneously from the rest to a $vs >> 1$ and A measuring its proper time measures also a $ds^2 = dt^2$ and remains in a Timelike Geodesics and at the rest with respect to a Local Frame inside the Warp Bubble.A felt no accelerations when passing from the rest to a $vs >> 1$.So A remains synchronized to C because A passed instantaneously from the rest to $vs >> 1$ while synchronized to C.On the other hand B measures A with a speed $vs >> 1$ and in a Spacelike Geodesics.Both have the same proper time with respect to C.No time dilatation.No Special Relativity and A is moving away from B Faster Than Light.

This is the Revolutionary concept of the Warp Drive as a Dynamical Spacetime introduced by Alcubierre in 1994

- 1)Motion Faster Than Light
- 2)No limitations from Special Relativity.No time dilatation.No mass increase
- 3)Both observers remains synchronized between themselves

3 Problems Raised Against Alcubierre Warp Drive

However it was discovered that the Alcubierre Warp Drive have serious problems and drawbacks that from a realistic point of view can poses serious obstacles to its physical feasibility

- 1)Horizons-Causally Disconnected portions of Spacetime
- 2)Doppler Blueshifts and Impacts with hazardous objects
- 3)Enormous energy densities required to create it

We will now examine these in details.While negative energy problem can be ameliorated and the levels of energy can be lowered [2] and [8] the two firsts are serious obstacles [3],[5] and [12].If we keep the Alcubierre Warp Drive at subluminal speeds the Horizon problem disappears and perhaps with a unknown form of Quantum Gravity that encompasses non-local Quantum Entanglements[12] would be possible to solve Horizons at superluminal speeds but the Doppler Blueshifts and the collision with large objects remains the most serious problem whether subluminal or superluminal. It is easy to see that an Alcubierre Warp Drive even with the negative energy or Horizons problem solved by a still unknown spacetime metric would even face the Doppler Blueshifts of incoming photons from Cosmic Background Radiation with wavelengths shifted towards synchrotron radiation[13] impacting the Warp Bubble making the Alcubierre Warp Drive unstable.Plus impact with hazardous objects such as protons,electrons,Clouds of Space Dust or Debris,Asteroids,Meteors,Comets,Supernovas,Neutron Stars or Black Holes that would appear in front of an Alcubierre Warp Drive in a realistic travel across Outer Space in our Galaxy[14] would disrupt and destroy any kind of Alcubierre Warp Drive.[15]

[12]EPR-Einstein Podolsky Rosen paradox,Bell Inequalities etcetera

[13]one of the most lethal forms or radiation with even more penetrating capability than gamma radiation.Think about how many photons of COBE we have per cubit centimeter of space and think in how many cubic centimeters of space exists between Earth and Proxima Centauri at 4 light-years away to better understand how synchrotron radiation is hazardous for the Alcubierre Warp Drive.

[14]Clark,Hiscock,Larson mentions in pg 4 of [3] "Warp Drive Starships Plying The Galaxy".However Space is not empty and a collision with a Black Hole at 1000 times the speed of light is not for fun.

[15]"Travelling in Hyperspace is not dusting crops boy.Without precise calculations we will impact a Neutron Star or a Supernova or even a Meteor Shower".Harrison Ford as Captain Han Solo talking to Mark Hamill as Luke Skywalker in the

4 First Objection:Horizons-Causally Disconnected portions of Space-time

Imagine a observer in a spaceship in the center of the Warp Bubble and a Warp Bubble speed vs.For a while we assume the Warp Bubble moves at subluminal speed $vs << 1$ and the motion occurs in the $x - axis$.The Alcubierre Warp Drive can be written in the following form(see pg 2 eq 1 in [12] and pg 2 eq 2.3 in [10][16] :

$$ds^2 = [1 - (vsf(rs))^2]dt^2 + 2vsf(rs)dxdt - dx^2 \tag{15}$$

Lets imagine that the observer in the spaceship at the Warp Bubble center is sending photons to the front or the rear of the Warp Bubble in order to accelerate it or to drive its control.We know that photons moves in a Null-Like Geodesics $ds^2 = 0$.Then we have:

$$0 = [1 - (vsf(rs))^2]dt^2 + 2vsf(rs)dxdt - dx^2 \tag{16}$$

$$0 = [1 - (vsf(rs))^2] + 2vsf(rs)\frac{dx}{dt} - (\frac{dx}{dt})^2 \tag{17}$$

but $v = \frac{dx}{dx}$ is the speed of the photon so we are left with[17]:

$$0 = [1 - (vsf(rs))^2] + 2vsf(rs)v - v^2 \tag{18}$$

$$A = vsf(rs) \tag{19}$$

$$0 = 1 - A^2 + 2Av - v^2 \tag{20}$$

Pay attention in the multiplication by -1

$$0 = -1 + A^2 - 2Av + v^2 \tag{21}$$

$$v^2 - 2Av + A^2 - 1 = 0 \tag{22}$$

$$B = -2A \tag{23}$$

$$C = A^2 - 1 \tag{24}$$

$$v^2 + Bv + C = 0 \tag{25}$$

$$v^2 + Bv + C = 0 \tag{26}$$

George Lucas movie Star Wars Episode IV A New Hope

[16] Everett,Roman and Gonzalez-Diaz used a signature (-,+,+,+) we use a signature (+,-,-,-)

[17] Alcubierre Warp Drive do not obey Lorentz Tansformations

$$v = \frac{-B \pm \sqrt{B^2 - 4C}}{2} \tag{27}$$

$$v = \frac{2A \pm \sqrt{4A^2 - 4A^2 + 4}}{2} \tag{28}$$

$$v = \frac{2A \pm \sqrt{4}}{2} \dashrightarrow v = \frac{2A \pm 2}{2} \dashrightarrow v = A \pm 1 \dashrightarrow v = vsf(rs) \pm 1 \tag{29}$$

The expression above have two solutions:one is for the photon sent towards the rear of the Warp Bubble and the other is for the photon sent towards the front of the Warp Bubble

- 1.1)-photon sent towards the rear of the Warp Bubble

$$v2 = vsf(rs) + 1 \tag{30}$$

Nothing special to say in this case except that the photon will reach the rear Warp Bubble Wall and outside the Warp Bubble the Shape Function $f(rs) = 0$ and the photon will leave the Warp Bubble with a speed $v2 = 1$(the speed is really -1)[18].

- 1.2)-photon sent towards the front of the Warp Bubble

$$v1 = vsf(rs) - 1 \tag{31}$$

This case is more complicated because inside the Warp Bubble the Shape Function $f(rs) = 1$.The expression becomes

$$v1 = vs - 1 \tag{32}$$

For a subluminal $vs < 1$ ok but assuming we are accelerating the Warp Bubble to achieve a Faster Than Light speed $vs > 1$ in a given moment we must pass by $vs = 1$ and $v1 = 0$.The photon stops.Any photon sent by the observer to the front of the Warp Bubble will never reach it.The observer looses the capability to signal the front of the Warp Bubble that becomes Causally Disconnected from the observer.Et Voila Le Horizon(see abs and pg 3 of [12],fig 1 and pg 7 of [5]).An observer inside the Warp Bubble cannot accelerate the Warp Bubble to a Faster Than Light Speed.It must be made from outside by an external observer (see the comment on pg 3 of [12] about the actions to create or change the Warp Bubble trajectory or speed being taken by an external observer whose light cone contains all the trajectory of the Warp Bubble). Assuming that we have an Alcubierre Warp Drive on Earth Orbit with some crew members ready to go to the Messier-1 the Crab Nebula at 6000 light-years from Earth.Earth can accelerate the Alcubierre Warp Drive to a Faster Than Light speed and sent it.But when arriving at Crab Nebula the astronauts will want of course to de-accelerate the Warp Bubble and stop to explore the object.Passing from a $vs >> 1$ to a $vs = 0$ to explore the object will require a continuous de-acceleration and in a given time $vs = 1$ and $v1 = 0$ and the crew will loose contact with the front of the Warp Bubble.Unless is somebody out there in he Crab Nebula

[18] actually the speed is -1.the photon moves in a direction opposite to the direction of the Warp Bubble motion.we multiplied by -1 between eqs 210 and 211

to help our astronauts they will never de-accelerate.The Horizon problem is one of the most serious faced by the Alcubierre Warp Drive in its present form. In pg 5 of [10] Gonzalez-Diaz propose the use of the Alcubierre Warp Drive as a Time-Machine to overcome the Horizons Problem.In our attempt to "save and rescue back" the Alcubierre Warp Drive we will propose another way to overcome the Horizons problem that seems to be more realistic than Time Travel.

- 1)-According to Everett-Roman an external observer contains all the Warp Bubble trajectory.[19] Hence our mothership to Crab Nebula would have many Space Probes eg Warp Drive Drones.The mothership would eject one of these Warp Drones and the Drone would "engineer" the Spacetime Metric around the motherhip creating the Alcubierre Warp Drive and accelerate it to a $vs >> 1$ and send it to Crab Nebula.The mothership do not create the Warp Bubble.It would be created by the Warp Drone that would performs as the Everett-Roman external observer whose light cone contains all the Warp Bubble . If NASA uses the gravitational field of Venus as a "slingshot" to accelerate Space Probes to Jupiter and NASA also used the Command Module,the Service Module and a Lunar Module(a spaceship divided in 3 parts) to land on the Moon and only the Command Module returned to Earth while the Lunar Module was abandoned on the Moon and the Service Module on Earth orbit so why not an external Warp Drone generating the Warp Bubble???.This is more reasonable and more affordable than Time Machines.
- 2)-Once at Crab Nebula the crew could perhaps de-accelerate from a $vs >> 1$ to a $vs = 1$. They would loose contact with the front part of the Warp Bubble but the crew members can still signal the rear part of the Warp Bubble.They could send a signal to destroy the rear part of the Warp Bubble from behind.The front part of the Warp Bubble would go on since it cannot be signalized.We still don't know the consequences of the destruction or disruption of the Warp Bubble from behind.The crew would have to de-accelerate by conventional propulsion eg Bussard Ramscoops or Bussard Ramjets.
- 3)-In order to go back to Earth the mothership would eject another Warp Drone to create another Alcubierre Warp Drive and accelerate it to a $vs >> 1$ and send the Warp Bubble back to Earth

This approach seems to be more reasonable than Time Machines[20] and would solve the Horizons Problem eliminating the first obstacle against the Alcubierre Warp Drive.However we can point also 4 drawbacks:

- 1)-The Warp Drone on Earth orbit could perhaps be re-utilized by another mothership but the one at Crab Nebula could be used only one time.After sending the Warp Bubble back to Earth it would remain abandoned[21].
- 2)-Each Alcubierre Warp Drive could only be used for a "one-way" trip.
- 3)-We don't know what would happens to the mothership if the Warp Bubble is destroyed from behind.
- 4)-We don't know how to "engineer" the Spacetime to create the Alcubierre Warp Drive.

[19] see the comment on pg 3 of [12] about the actions to create or change the Warp Bubble trajectory or speed being taken by an external observer whose light cone contains all the trajectory of the Warp Bubble

[20] more of this on the negative energy section

[21] we assume there are no "advanced civilizations" at Crab Nebula

5 Second Objection:Doppler Blueshifts and Impacts with hazardous objects:

This is the most terrible and formidable obstacle against the physical feasibility of the Alcubierre Warp Drive. While the mothership remains at the rest inside the Warp Bubble the Warp Bubble moves itself with a great velocity vs with respect to the rest of the Universe and will impact hazardous objects as we pointed out before.We will use the Clark-Hiscock-Larson-Natario approach to demonstrate how terrible are impacts against the Warp Bubble.Considering light particles eg photons of COBE and we have too many per cubic centimeter of space.Applying the non-relativistic Doppler-Fizeau expression [22]

$$f = f_0 \frac{c + va}{c - vb} \tag{33}$$

Where we have:

- 1)-f is the photon frequency seen by an observer
- 2)-f_0 is the original frequency of the emitted photon
- 3)-c is the light speed.in our case $c = 1$
- 4)-va is the speed of the light source approaching the observer.in our case is vs
- 5)-vb in the speed of the light source moving away from the observer.in our case $vb = 0$

Rewriting the Doppler-Fizeau expression for an incoming photon approaching the Warp Bubble from the front we should expect for(see eq 26 pg 9 and pg 11 in [3],pg 8 in [5]):

$$f = f_0(1 + vs) \tag{34}$$

Energy E is Planck Constsnt $\hbar$ multiplied by frequency so for the energy we would have:

$$E = E_0(1 + vs) \tag{35}$$

Now we can see how bad is the Doppler Blueshift for the Alcubierre Warp Drive.The energy of the photon impacting the Warp Bubble E is much greater than the original photon energy $E_0.E >> E_0$ and as far as vs increases to Faster Than Light speeds the problem becomes worst.In order to achieve an affordable and reasonable time for an Interstellar Travel in our Galaxy from the point of view of the crew members the ship would perhaps needs to attain a $vs = 200$.Two hundred times Faster Than Light but see again pg 11 in [3].A photon of COBE would impact the Warp Bubble at two hundred times Faster Than Light with the energy of an entire Solar Photosphere!!!!!.And how many photons of COBE we have per cubic centimeter of space????.Each one impacting the Warp Bubble with the energy of a Solar Photosphere?????.Plus how many cubic centimeters of space we have between the Sun and Proxima Centauri???.The approach of Clark-Hiscock-Larson and Natario is the most terrible and formidable obstacle against the Alcubierre Warp Drive. Perhaps the Doppler Blueshift problem can be overcomed and the second obstacle against the Alcubierre Warp Drive can be removed and the Alcubierre Warp Drive can still be "saved and rescued back" in this way:

[22]remember again the fact that the Alcubierre Warp Drive do not obey Lorentz Transformations

- 1)-According to eq 19 pg 8 in [4],eq 8 pg 6 in [1] and eq 5.8 pg 70 in [15] the energy density of the Alcubierre Warp Drive is negative.It violates all the energy conditions(WEC,NEC,SEC).We are not concerned and not worried about with this due to [2],[13] and [8] where the energy density can be lowered to affordable levels and also due to abs of [11] where macroscopic amounts of negative energy densities can be created.We also know that the Casimir Effect can create the negative energy densities.(see pg 9 in [4]).Still according to eq 25 pg 9 in [1] if the energy density is negative then the total energy of the Alcubierre Warp Drive is negative too.But if the energy E of the Alcubierre Warp Drive is negative then the total mass M of the Alcubierre Warp Drive is also negative.

- 2)-A negative mass M would generate a negative Gravitational Bending Of Light and a negative Gravitational Field that would repeal objects instead of attract.Consider a Asteroid of mass M_A positive of course and an Alcubierre Warp Drive of mass M_W negative.The Gravitational Force would be negative and given by $F = G\frac{M_A M_W}{d^2}$.So the Asteroid would be naturally shifted from the Warp Bubble.Of course this is also due to our Shape Function $f(rs)$ that allow a different distribution of the negative energy density T^{00}.While in original Alcubierre Shape Function the negative energy is concentrated in a region toroidal perpendicular to the direction of the motion of the spaceship(see pg 6 and fig 3 pg 7 in [1] and pg 70 and fig 5.3 pg 71 in [15]) while the spaceship remains on empty space vulnerable to Doppler Blueshifted photons[23] our Warp Bubble involves the spaceship with a sphere of negative energy protecting it from impacts.Due to the negative Bending of Light the photons would be shifted too[24]. The Spacetime Curvature of an object of negative mass is opposite to the similar one of a positive mass so while the Sun bends photons in a inwards direction a negative mass objects would bend photons in a outwards direction. While other autors considers the negative energy a pathology we consider it a bless because a positive energy density would attract objects disrupting effectively the Warp Bubble.

- 3)-The Warp Bubble cannot be signalized from outside with photons.Our Warp Drone while "engineering" the Alcubierre Warp Drive of negative mass M_W would send packets of "Casimir matter" of negative mass M_C to the Warp Bubble by a still unknown process.The gravitational force between two negative masses would cancel the minus sign in both masses making the gravitational force attractive so the Warp Bubble would only be signalized by negative masses.Also the crew members while signalizing the rear part of the Warp Bubble must uses negative matter to disrupt the Warp Bubble from behind.Remember that the negative matter moves at subluminal speeds but at least we know that the Warp Bubble lies entirely in the light-cone of the Warp Drone and we also know that the crew members can signalize the rear part of the Warp Bubble.

This could perhaps solve the Doppler Blueshift and Impacts with Hazardous Objects Problem as the second obstacle against the Alcubierre Warp Drive but again we must point at least 3 drawbacks:

- 1)-It could perhaps shift COBE photons or Asteroids but what if a Large Black Hole appears...???

- 2)-A very massive positive object would generate a strong gravitational repulsive force repealing the Warp Bubble.What the consequences???Could the Warp Bubble be disrupted or destroyed by a large repulsive gravitational force?????

- 3)-We don't know how to "engineer" the Spacetime to create the Alcubierre Warp Drive.

[23]the ship remains in the center of the Warp Bubble and this means the black regions on fig 3 or fig 5.3 leaving the ship "unprotected" while the negative energy regions are the white toroidal regions

[24]more of this on the negative energy section

6 Third Objection:Enormous energy densities required to create it

This item is the most easily to be overcomed:See [2] and [8] but instead of other Warp Drive geometries [2],[5] more complicated [25] we will demonstrate how the negative energy can be "lowered" in the original Alcubierre Warp Drive.The solution will come from the work of Ford and Pfenning [1],[15]. All we have to do is to choose the "correct" Shape Function.The energy density in the Alcubierre Warp Drive is given by(see eq 19 pg 8 in [4],eq 8 pg 6 in [1] and eq 5.8 pg 70 in [15]):

$$T^{00} = \frac{1}{8\pi}G^{00} = -\frac{1}{8\pi}\frac{vs^2\rho^2}{4rs^2}[\frac{df(rs)}{drs}]^2 = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df(rs)}{drs}]^2 \tag{36}$$

With [26] :

$$\rho^2 = y^2 + z^2 \tag{37}$$

As far as the Warp Bubble accelerates to $vs >> 1$[27] the energy density becomes bigger...and more negative.Assuming that the Alcubierre Warp Drive have motion only in the $x - axis$ then ρ^2 is a constant.The "trick" to lower the energy density lies in the term $\frac{df(rs)}{drs}$.As Ford and Pfenning pointed out correctly in pg 3 of [1] and in pg 68 of [15] we dont need to choose a particular form of $f(rs)$:Any function $f(rs) = 1$ inside the Warp Bubble where $rs < R$ or $rs < R - \delta$ in the original Alcubierre Warp Bubble thickness δ and $f(rs) = 0$ where $rs > R$ or $rs > R + \delta$ while decreasing from 1 to 0 in the Warped Region $R - \delta < rs < R + \delta$ $0 < f(rs) < 1$ is a valid Shape Function.So everything depends on the form of the Shape Function.A "good" Shape Function will perform better and lowers the energy density more than a "bad" or a "evil" Shape Function.The original Alcubierre Shape Function is complicated due to the hyperbolic terms so in order to simplify the energy density calculations Ford and Pfenning introduced the Piecewise Shape Function.By manipulating the Piecewise Shape Function we can lower the energy density requirements without strange topologies.The Piecewise Shape Function is defined by(eq 4 pg 3 in [1] and eq 5.4 pg 68 in [15]):

- 1) $f_{pf} = 1 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $f_{pf} = -\frac{1}{\Delta}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $f_{pf} = 0 \dashrightarrow rs > R + \frac{\Delta}{2}$

The parameter Δ is the thickness of the Warped Region in the Piecewise Shape Function and is related to the thickness of the Warped Region δ in the original Alcubierre Shape Function by the following expression(eq 5 pg 4 in [1] and eq 5.5 pg 68 in [15]):

$$\Delta = \frac{[1 + tanh^2(\delta R)]^2}{2\delta tanh(\delta R)} \tag{38}$$

The Piecewise Shape Function gives results similar to the Alcubierre Shape Function(see comments before eq 8 pg 6 in [1] and before eq 5.8 pg 70 in [15])[28]

[25] The Warp Drives of Broeck and Natario are more complicated than the original Alcubierre one. It will be difficult to generate the Alcubierre Warp Drive not to mention these ones

[26] now we know why y and z cannot be zero.the T^{00} would be null leading to a unphysical result

[27] action taken by an external observer eg Warp Drone

[28] fortunately this is not entirely true.The Ford-Pfenning Piecewise Shape Function can give results different than the original Alcubierre Shape Function as Ford-Pfenning mentioned in the comments.We will demonstrate this during the calculations of the total energy

Ford and Pfenning computed the total energy for an Alcubierre Warp Drive with a constant Faster Than Light Speed vs in pg 9 eqs 25 and 26 of [1] and in pg 73 eqs 5.25 and 5.26 of [15] as follows

$$T^{00} = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df_{pf}(rs)}{drs}]^2 \quad (39)$$

$$E = \int [T^{00}]dV_{volume} \quad (40)$$

$$E = -\frac{vs^2}{32\pi}\int [\frac{\rho^2}{rs^2}[\frac{df_{pf}(rs)}{drs}]^2]dV_{volume} \quad (41)$$

passing to spherical coordinates

$$E = -\frac{vs^2}{12}\int [rs^2[\frac{df_{pf}(rs)}{drs}]^2]drs \quad (42)$$

Note again that the "trick" to lower the integral of the total energy is the term $[\frac{df_{pf}(rs)}{drs}]^2$

Ford and Pfenning uses the so-called Quantum Inequalities(QI) to place limits on the thickness of the Warp Bubble Walls.Although we do not worry too much about the QI due to Krasnikov in [13] and [8] we will comment how the QI can easily be overcomed and QI can also be used to demonstrate how to lower the energy density to affordable levels.

Inserting the QI eqs 9 and 10 of [1] and eqs 5.9 and 5.10 of [15][29]

$$\frac{\tau_0}{\pi}\int_{-\infty}^{+\infty}\frac{T_{\mu\nu}U^{\mu}U^{\nu}}{\tau^2+\tau_0^2}d\tau = \frac{\tau_0}{\pi}\int_{-\infty}^{+\infty}\frac{T^{\mu\nu}U_{\mu}U_{\nu}}{\tau^2+\tau_0^2}d\tau = \frac{\tau_0}{\pi}\int_{-\infty}^{+\infty}\frac{T^{00}}{\tau^2+\tau_0^2}d\tau \geq -\frac{3}{32\pi^2\tau_0^4} \quad (43)$$

Where τ is an inertial observer proper time and τ_0 is an arbitrary sampling time(see pg 7 of [1] and pg 70 of [15])

$$\frac{\tau_0}{\pi}\int_{-\infty}^{+\infty}\frac{-\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df_{pf}(rs)}{drs}]^2}{\tau^2+\tau_0^2}d\tau \geq -\frac{3}{32\pi^2\tau_0^4} \quad (44)$$

$$-\frac{\tau_0}{\pi}\int_{-\infty}^{+\infty}\frac{vs^2\rho^2[\frac{df_{pf}(rs)}{drs}]^2}{32\pi rs^2(\tau^2+\tau_0^2)}d\tau \geq -\frac{3}{32\pi^2\tau_0^4} \quad (45)$$

$$-\frac{\tau_0}{\pi}\int_{-\infty}^{+\infty}[\frac{df_{pf}(rs)}{drs}]^2\frac{vs^2\rho^2}{32\pi rs^2(\tau^2+\tau_0^2)}d\tau \geq -\frac{3}{32\pi^2\tau_0^4} \quad (46)$$

And we arrived at the eq 10 of [1] and eq 5.10 of [15]

$$\tau_0\int_{-\infty}^{+\infty}[\frac{df_{pf}(rs)}{drs}]^2\frac{vs^2}{rs^2(\tau^2+\tau_0^2)}d\tau \leq \frac{3}{\rho^2\tau_0^4} \quad (47)$$

$$t_0\int_{-\infty}^{+\infty}[\frac{df_{pf}(rs)}{drs}]^2\frac{vs^2}{rs^2(t^2+t_0^2)}dt \leq \frac{3}{\rho^2 t_0^4} \quad (48)$$

The sampling time $\tau_0 = t_0$ according to Ford and Pfenning is very small compared to the time t the Warp Bubble is changing the speed vs so they consider a Warp Bubble with constant speed vs.(see pg 7 of [1] and pg 71 of [15]).

[29] see eqs 8 and 10 of [1] and eqs 5.8 and 5.10 of [15]

$$(t_0)(vs^2)\int_{-\infty}^{+\infty}[\frac{df_{pf}(rs)}{drs}]^2\frac{1}{rs^2(t^2+t_0^2)}dt \leq \frac{3}{\rho^2 t_0^4} \tag{49}$$

see the right side of eq 14 pg 8 in [1] and eq 5.14 pg 71 in [15]

$$t_0\int_{-\infty}^{+\infty}[\frac{df_{pf}(rs)}{drs}]^2\frac{1}{rs^2(t^2+t_0^2)}dt \leq \frac{3}{vs^2\rho^2 t_0^4} \tag{50}$$

We know that inside the Warp Bubble $f_{pf} = 1$ and outside the Warp Bubble $f_{pf} = 0$ so for both cases f_{pf} is a constant and $[\frac{df_{pf}(rs)}{drs}]^2 = 0$.We are interested in the behavior of the QI only in the Warped Region.Hence we can write the QI as:

$$t_0\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[\frac{df_{pf}(rs)}{drs}]^2\frac{1}{rs^2(t^2+t_0^2)}dt \leq \frac{3}{vs^2\rho^2 t_0^4} \tag{51}$$

But we know from the definition of the Piecewise Shape Function that[30]:

- 1) $[\frac{df_{pf}(rs)}{drs}]^2 = 0 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $[\frac{df_{pf}(rs)}{drs}]^2 = [-\frac{1}{\Delta}]^2 \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $[\frac{df_{pf}(rs)}{drs}]^2 = 0 \dashrightarrow rs > R + \frac{\Delta}{2}$

$$t_0\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[-\frac{1}{\Delta}]^2\frac{1}{rs^2(t^2+t_0^2)}dt \leq \frac{3}{vs^2\rho^2 t_0^4} \tag{52}$$

$$t_0\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}\frac{1}{\Delta^2}\frac{1}{rs^2(t^2+t_0^2)}dt \leq \frac{3}{vs^2\rho^2 t_0^4} \tag{53}$$

Our expression differs a little bit from eq 14 pg 8 in [1] and eq 5.14 pg 71 in [15] but the right side is the same.

$$t_0\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}\frac{1}{rs^2(t^2+t_0^2)}dt \leq \frac{3\Delta^2}{vs^2\rho^2 t_0^4} \tag{54}$$

As Ford and Pfenning says the sampling time $\tau_0 = t_0$ is arbitrary and very small compared to the time t the Warp Bubble is changing the speed vs so we can easily see that:(see pg 7 of [1] and pg 71 of [15]).

$$t_0 << \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}\frac{1}{rs^2(t^2+t_0^2)}dt \tag{55}$$

Hence we can make the sampling time very very small for a Warp Bubble of almost constant speed vs since the time needed for change of the speed in the Warp Bubble would always be greater than the sampling time and for the energy calculations Ford and Pfenning considered a Warp Bubble of constant velocity vs although for a Warp Bubble of variable speed in a short period of time the sampling time would perhaps be not so small at all.We will address this later in this section.

[30] we consider a Warp Bubble of constant radius R and a constant thickness Δ

$$\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} \frac{1}{rs^2(t^2+t_0^2)}dt \leq \frac{3\Delta^2}{vs^2\rho^2 t_0^5} \quad (56)$$

A large Warp Bubble thickness Δ coupled to a very small sampling time t_0 would make the term $\frac{3\Delta^2}{vs^2\rho^2 t_0^5}$ larger than the integral making the QI hold as shown in the expression above and not restraining the size of the Warp Bubble thickness.This is the reason why we agree with Krasnikov in [13] and [8].Note that for a large Warp Bubble speed vs the things becomes more difficult to make the QI hold because for a large speed vs the term $\frac{3\Delta^2}{vs^2\rho^2 t_0^5}$ becomes smaller.We will see the same on the total energy integral calculations but we will present a way to overcome this.

Back to total energy integral of the Warp Bubble we should expect for(see eq 27 and 28 pg 10 in [1] and eq 5.27 pg 73 in [15]):

$$E = -\frac{1}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(vs^2)(rs^2)[\frac{df_{pf}(rs)}{drs}]^2]drs = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)[\frac{df_{pf}(rs)}{drs}]^2]drs \quad (57)$$

$$E = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)[-\frac{1}{\Delta}]^2]drs = -\frac{vs^2}{12}[-\frac{1}{\Delta}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}(rs^2)drs \quad (58)$$

$$E = -\frac{1}{12}[\frac{vs^2}{\Delta^2}]\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}(rs^2)drs = -\frac{1}{12}[\frac{vs}{\Delta}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}(rs^2)drs = -\frac{1}{12}(vs^2)(\frac{R^2}{\Delta}+\frac{\Delta}{12}) \quad (59)$$

From the expression above we can see that a large Warp Bubble speed vs will raise the amount of negative energy needed.Ford and Pfenning restricted the thickness Δ of the Warp Bubble to the same scale of the Planck size(see eq 23 pg 9 in [1] and eq 5.23 pg 72 in [15]) and the term $\frac{R^2}{\Delta}$ results in a huge number for the total energy integral.Dividing a Warp Bubble radius $R = 100m$ according to Ford-Pfenning by a number close to the Planck size results in a number roughly ten times the mass of the Universe of negative energy needed to sustain the Warp Bubble[31]. But note that Ford-Pfenning says also in pg 10 of [1] and in pg 73 of [15] that a Warp Bubble thickness $\Delta = 1$ and Ford-Pfenning mentions explicitly a violation of the QI integral would lower the magnitude of negative energy needed to sustain a Warp Bubble of radius $R = 100m$ from 10 times the mass of the Universe to a $\frac{1}{4}$ of a Solar Mass.An improvement without shadows of doubt but not too much

We have two choices to low the negative energy needed to sustain the Warp Bubble

- 1)-a Warp Bubble of a thickness $\Delta < R \dashrightarrow \Delta \simeq R$ would make the term $\frac{R^2}{\Delta} \simeq R$ but this would be similar to the situation $\Delta = 1$ due to the term $\frac{\Delta}{12} \simeq \frac{R}{12}$

- 2)-Introducing a new Shape Function

While the Alcubierre choice for a continuous Shape Function involved a toroidal geometry that would not protect the spaceship from impacts of Doppler Blueshifted photons(making valid the obstacle raised by Clark-Hiscock-Larson and Natario) and Ford-Pfenning introduced a Piecewise Shape Function that divides 1 by a Warp Bubble thickness Δ of the magnitude of the Planck Length and dividing 1 by a Planck Length of 10^{-35} would result in the huge number of 10 times the mass of the Universe in negative energy to sustain a Warp Bubble.Ford and Pfenning created a Piecewise Shape Function that really demands 10 times the mass of the Universe in negative energy to sustain a Warp Bubble.This don't means to say that the Warp

[31] see pg 10 after eq 31 of [1] and see pg 73 after eq 5.30 of [15]

Drive is impossible.Its impossible with the Shape Function choosed by Ford and Pfenning and impossible with the Shape Function choosed by Alcubierre.But as Ford-Pfenning pointed out any Shape Function that gives 1 inside the Warp Bubble 0 far from it and $0 < f(rs) < 1$ in the Warped Region is a equally valid Shape Function.We will now present three Shape Functions inspired on the Ford-Pfenning Piecewise Shape Function.Our Piecewise Shape Functions will low the energy density requirements of the Alcubierre Warp Drive to low and affordable levels and still have a Warp Bubble topology that will protect the ship against incoming Blueshifted photons or impacts with small objects eg Asteroids.[32]

The reason why Ford and Pfenning arrived at the huge number of 10 times the mass of the Universe or at least $\frac{1}{4}$ of a Solar mass of negative energy needed to sustain a Warp Bubble was due to the term they choosed for the Warped Region

$$f_{pf} = -\frac{1}{\Delta}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2} \tag{60}$$

It is easy to see that if $\frac{1}{\Delta} = \frac{1}{10^{-35}}$ and the shape of the total energy integral is $E = -\frac{1}{12}[\frac{vs}{\Delta}]^2 \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} (rs^2) drs$

then dividing 1 by 10^{-35} we would get a big number and to make the things even worst we are dividing $[\frac{vs}{\Delta}]^2$.We are dividing a high velocity vs by 10^{-35} and raise to a power of 2.Of course the result would then be a physical unattainable amount of negative energy.

Our Piecewise Shape Functions were designed to enter in the total energy integral

$E = -\frac{vs^2}{12} \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)[\frac{df_{pf}(rs)}{drs}]^2] drs$

Lowering the energy density requirements due to our different factor $\frac{df_{pf}(rs)}{drs}$

Our Piecewise Shape Functions are in total agreement with the Ford-Pfenning definitions resembling the original Ford-Pfenning Piecewise Shape Function(eq 4 pg 3 in [1] and eq 5.4 pg 68 in [15]) and our Piecewise Shape Functions also gives 1 inside the Warp Bubble 0 far from it and $0 < f_{pf} < 1$ in the Warped Region.

We will introduce now a factor $h(rs)$ that is 1 in the region where the ship is located,starts to grow when entering in the Warped Region reaches its maximum value in the center of the Warped Region decreases when we approach the end of the Warped Region and is again 1 outside the Warped Region.

The continuous expression for $h(rs)$ is given by:

$$h(rs) = [\frac{1 + tanh[\delta(rs - R)]^2}{2}]^{-\frac{R}{\Delta}\frac{vs}{c}} \tag{61}$$

In the expression above $c = 1$ and we divide in the power factor the Radius R by the Thickness Δ and the ship speed vs by the light speed c.This enable ourselves to attain an enormous value for $h(rs)$ in the center of the Warped Region.[33]

Our continuous Shape Function would then be given by:

$$g(rs) = \frac{f(rs)}{h(rs)} \tag{62}$$

This is an alternative to the Alcubierre continuous Shape Function that provides $0 < f(rs) < 1$ in the Warped Region.Our factor $h(rs)$ gives us a $h(rs) >> 1$ in the Warped Region and dividing $\frac{f(rs)}{h(rs)}$ would enable ourselves to get a $0 < g(rs) < 1$ but closer to 0 than the original Alcubierre Shape Function.

[32]an impact with a Large Black Hole would perhaps destroy the Warp Bubble.

[33]Consider for example a Radius $R = 20meters$,$\delta = 2meters$,Thickness $\Delta = 10meters$,speed $vs = 100timeslightspeed$,light speed $c = 1$ and a rs that varies from 0 to $40meters$.We can provide a Microsoft Excel simulation for those interested.

Our Shape Function is somewhat more complicated than the Alcubierre one and we will use the Ford-Pfenning Piecewise behavior to study the factor $h(rs)$.

A Piecewise expression for our factor $h(rs)$ would be given by:

- 1) $hrs = 1 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $h(rs) = |R|^2 or |R|^4 or |R|^6 \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $h_{rs} = 1 \dashrightarrow rs > R + \frac{\Delta}{2}$

To reproduce the enormous value attained by the continuous expressions for $h(rs)$ in the Warped Region[34] we use the modulus of the Radius.[35]

Our Piecewise Shape Functions resembles the original Ford-Pfenning Piecewise Shape Function but like our continuous Shape Function is also divided by the factor $h(rs)$.

Here are our three Piecewise Shape Functions:

- 1) $f_{pf} = 1 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $f_{pf} = -\frac{1}{\Delta |R|^2}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $f_{pf} = 0 \dashrightarrow rs > R + \frac{\Delta}{2}$
- 1) $f_{pf} = 1 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $f_{pf} = -\frac{1}{\Delta |R|^4}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $f_{pf} = 0 \dashrightarrow rs > R + \frac{\Delta}{2}$
- 1) $f_{pf} = 1 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $f_{pf} = -\frac{1}{\Delta |R|^6}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $f_{pf} = 0 \dashrightarrow rs > R + \frac{\Delta}{2}$

It is easy to see that in the Warped Region our Shape Functions will also give the Ford-Pfenning result $0 < f_{pf} < 1$ due to the following expressions:[36]

$$f_{pf} = -\frac{1}{\Delta |R|^2}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2} \tag{63}$$

$$f_{pf} = -\frac{1}{\Delta |R|^4}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2} \tag{64}$$

$$f_{pf} = -\frac{1}{\Delta |R|^6}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2} \tag{65}$$

[34] Our Microsoft Excel simulation with the values provided by a previous footnote gives a value of $1,61 \times 10^{60}$

[35] in the previous version of this paper we got the idea of the modulus of the Radius however in the equations we placed the radius.This is a correction

[36] consider a Warp Bubble Radius $R = 100 meters$ and a Warp Bubble Thickness $\Delta = 10 meters$ and compute the values of the rs inside the Warped Region or example for a $rs = 100 meters$.Anyone can see that our expressions also obeys $0 < f_{pf} < 1$

Our Piecewise Shape Functions also use the Warp Bubble thickness Δ but we don't care about QI due to our previous QI calculations and note that Ford-Pfenning considered also a thickness $\Delta = 1$ in a clear QI violation.But we introduced the Warp Bubble modulus of the Radius R in the definitions of our Shape Functions.The Warp Bubble modulus of the Radius R is the key ingredient to lower the energy density requirements to low and affordable levels eliminating the third obstacle against the Alcubierre Warp Drive[37].We will see the calculations right now:

The Total Energy Integral is given by:(see eq 27 and 28 pg 10 in [1] and eq 5.27 pg 73 in [15])

$$E = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)[\frac{df_{pf}(rs)}{drs}]^2]drs \tag{66}$$

Our factors $\frac{df_{pf}(rs)}{drs}$ when inserted in the Total Energy Integral will give the results shown below:

$$[\frac{df_{pf}(rs)}{drs}]^2 = [-\frac{1}{\Delta|R|^2}]^2 = [\frac{1}{\Delta|R|^2}]^2 \tag{67}$$

$$[\frac{df_{pf}(rs)}{drs}]^2 = [-\frac{1}{\Delta|R|^4}]^2 = [\frac{1}{\Delta|R|^4}]^2 \tag{68}$$

$$[\frac{df_{pf}(rs)}{drs}]^2 = [-\frac{1}{\Delta|R|^6}]^2 = [\frac{1}{\Delta|R|^6}]^2 \tag{69}$$

$$E = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)[-\frac{1}{\Delta|R|^2}]^2]drs = -\frac{vs^2}{12}[\frac{1}{\Delta|R|^2}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)]drs \tag{70}$$

$$E = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)[-\frac{1}{\Delta|R|^4}]^2]drs = -\frac{vs^2}{12}[\frac{1}{\Delta|R|^4}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)]drs \tag{71}$$

$$E = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)[-\frac{1}{\Delta|R|^6}]^2]drs = -\frac{vs^2}{12}[\frac{1}{\Delta|R|^6}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)]drs \tag{72}$$

Note that this negative energy would mean a negative mass and a negative gravitational force that would repeal objects instead of attract.This will be useful to protect the ship from incoming blueshifted photons and hazardous objects.We will show that our Piecewise Shape Functions don't have the toroidal geometry of the Alcubierre Shape Function and we can protect the ship against incoming blueshifted photons.This is the reason why we must have the negative energy in front of the ship and not in a toroidal distribution.

Remember that the Gravitational Force in units $G = c = 1$ for an Alcubierre Warp Drive of mass $M_w = -\frac{vs^2}{12}[\frac{1}{\Delta|R|^6}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)]drs$ approaching a positive object(eg a small Asteroid) of mass M_a would be given by $F = \frac{M_w M_a}{d}$ being d the separation distance and as far as the object approaches the Alcubierre Warp Drive the distance d becomes smaller making the repulsive force bigger repealing the object.The negative force comes from the minus sign in $M_w = -\frac{vs^2}{12}[\frac{1}{\Delta|R|^6}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)]drs$

From the expressions above it is easy to see that our Piecewise Shape Functions are able to low the energy density requirements in the original Alcubierre Warp Drive Geometry without recurring to

[37]if Ford and Pfenning can use the Warp Bubble thickness Δ in the definition of the Shape Function then why not use the Warp Bubble modulus of the Radius R?.it will low the energy density and will produce the desired result $0 < f_{pf} < 1$

other more complicated solutions(Broeck [2],Natario [5])[38].With the third obstacle solved we can say with confidence that the Alcubierre Warp Drive is "still alive"[39] as a fully functional Superluminal and Faster Than Light Spacetime Ansatz of General Relativity.Miguel Alcubierre was right after all these years[40].The Alcubierre Warp Drive is a Wonderful Idea.The Historical Paper of 1994 will ever be considered as a Revolutionary Paper for the Human Science because it launched the First Foundations of the Faster Than Light Space Travel.

While in original Alcubierre Shape Function the negative energy is concentrated in a region toroidal perpendicular to the direction of the motion of the spaceship(see pg 6 and fig 3 pg 7 in [1] and pg 70 and fig 5.3 pg 71 in [15]) while the spaceship remains on empty space vulnerable to Doppler Blueshifted photons[41] our Warp Bubble involves the spaceship with a sphere of negative energy protecting it from impacts.Due to the negative Bending of Light the photons would be shifted too

The energy density T^{00} of the Stress-Energy-Momentum Tensor for our Piecewise Shape Functions is given by:

$$T^{00} = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df_{pf}(rs)}{drs}]^2 = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{1}{\Delta|R|^2}]^2 \tag{73}$$

$$T^{00} = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df_{pf}(rs)}{drs}]^2 = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{1}{\Delta|R|^4}]^2 \tag{74}$$

$$T^{00} = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df_{pf}(rs)}{drs}]^2 = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{1}{\Delta|R|^6}]^2 \tag{75}$$

Note that in the region where the negative energy resides T^{00} cannot be zero and this region is the Warped Region where $0 < f_{pf} < 1$ and $\frac{df_{pf}(rs)}{drs} \neq 0$.This means to say the region where rs approaches the Warp Bubble Radius R or better the region :$R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$

$$T^{00} = -\frac{vs^2}{32\pi}\frac{y^2+z^2}{[x-xs(t)]^2+y^2+z^2}[\frac{1}{\Delta|R|^2}]^2 \tag{76}$$

$$T^{00} = -\frac{vs^2}{32\pi}\frac{y^2+z^2}{[x-xs(t)]^2+y^2+z^2}[\frac{1}{\Delta|R|^4}]^2 \tag{77}$$

$$T^{00} = -\frac{vs^2}{32\pi}\frac{y^2+z^2}{[x-xs(t)]^2+y^2+z^2}[\frac{1}{\Delta|R|^6}]^2 \tag{78}$$

A Warp Bubble Sphere of Radius $R = x^2+y^2+z^2$ surrounding the spaceship is the ideal way.Note that if we keep fixed y and z and move ourselves in the $x-axis$ starting from the spaceship position xs towards

[38]the funny thing is the fact that as large as the Warp Bubble modulus of the Radius R is greater as low the energy density becomes.This can be a math curiosity but anyone can see that our calculations are right due to the form of our Piecewise Shape Functions.

[39]if an Alcubierre Warp Drive collides with a Large Black Hole it would be destroyed.This don't means to say the Alcubierre Warp Drive is impossible because NASA Space Shuttle in the neighborhoods of the same Black Hole would be destroyd too and we know that the Space Shuttle is possible

[40]although the original Shape Function choosed by Alcubierre with the toroidal distribution of negative energy up and below the spaceship would not protect the ship against blueshifted photons making valid the objections raised by Clark-Hiscock-Larson and Natario

[41]the ship remains in the center of the Warp Bubble and this means the black regions on fig 3 or fig 5.3 leaving the ship "unprotected" while the negative energy regions are the white toroidal regions

the Warp Bubble Radius and towards the Warped Region in the front of the spaceship then according to the definition of rs when rs approaches R and we would have $R = [x - xs(t)]^2 + y^2 + z^2$

Note that in this case T^{00} do no vanish and we have negative energy in front of the spaceship protecting it from incoming objects.

Suppose now we are in the position of the spaceship $x = xs$ and we move ourselves in the plane $y - z$ to the Warp Bubble Radius R and the Warped Region upside(or downside the ship).The expressions for T^{00} would then be:

$$T^{00} = -\frac{vs^2}{32\pi}[\frac{1}{\Delta|R|^2}]^2 \tag{79}$$

$$T^{00} = -\frac{vs^2}{32\pi}[\frac{1}{\Delta|R|^4}]^2 \tag{80}$$

$$T^{00} = -\frac{vs^2}{32\pi}[\frac{1}{\Delta|R|^6}]^2 \tag{81}$$

Note that we can move backwards or forwards or upstairs or downstairs but the ship will always have a non-null T^{00} involving the ship as a protective "cocoon" a real Warp Bubble.

While a toroidal distribution of the negative energy upside and downside with the front of the ship in empty space leaves the ship unprotected we closed the front of the ship in order to protect it.The original Ford-Pfenning Piecewise Shape Function could also be used to close the front of the ship.This is the main difference between the Alcubierre Shape Function and the Ford-Pfenning Piecewise Shape Function.

The analogous expressions for the Ford-Pfenning Piecewise would then be:

$$T^{00} = -\frac{vs^2}{32\pi}\frac{y^2 + z^2}{[x - xs(t)]^2 + y^2 + z^2}[\frac{1}{\Delta}]^2 \tag{82}$$

$$T^{00} = -\frac{vs^2}{32\pi}[\frac{1}{\Delta}]^2 \tag{83}$$

Note that Ford-Pfenning also describes a Warp Bubble as a "cocoon" surrounding and protecting the ship although this "cocoon" requires more energy than ours because they did not included the Warp Bubble modulus of the Radius R in the definition of the Piecewise Shape Function.

Another remarkable thing is the fact that the Alcubierre Warp Drive defines a expansion of the Spacetime behind the spaceship and a contraction in front of it due to the nature of the Alcubierre Shape Function.

7 An Alcubierre Warp Drive without Expansion/Contraction of the Spacetime

Our Piecewise Shape Function and the one of Ford-Pfenning does not defines expansion or contraction.Our Warp Drive and Ford-Pfenning one involves the ship in a "cocoon" that is being carried out by the Spacetime "stream" just like a fish in the stream of a river while the ship remains at the rest with respect to the "cocoon" center.Imagine a fish a salmon for example being carried out by a river stream and imagine our "cocoon" being carried out by the Spacetime "stream".Our "cocoon" is the fish the river is the Spacetime "stream" and inside the "coccon" similar to the fish the ship remains at the rest free from g-forces.In a way our Warp Drive and Ford-Pfenning one are very similar to the Natario Warp Drive that also don't suffer expansion or contraction.(see abstract and pg 1 of [5])

We will demonstrate this right now:

Starting with the original Alcubierre expression for the expansion of the volume elements given as follows but using our Piecewise Shape Functions:(see eq 12 pg 5 in [4],eq 9 pg 3 in [9] and pg 4 of [5])

$$\theta = vs \frac{x - xs}{rs} \frac{df_{pf}(rs)}{drs} \tag{84}$$

$$\theta = vs \frac{x - xs}{\sqrt{[x - xs(t)]^2 + y^2 + z^2}} [-\frac{1}{\Delta|R|^2}] \tag{85}$$

$$\theta = vs \frac{x - xs}{\sqrt{[x - xs(t)]^2 + y^2 + z^2}} [-\frac{1}{\Delta|R|^4}] \tag{86}$$

$$\theta = vs \frac{x - xs}{\sqrt{[x - xs(t)]^2 + y^2 + z^2}} [-\frac{1}{\Delta|R|^6}] \tag{87}$$

Assuming that the ship lies at the rest in the center of the Warp Bubble and for a Frame placed in the ship with the ship in the origin then $xs = 0$ and $rs = \sqrt{x^2 + y^2 + z^2}$(see eq 24 pg 9 in [1] and eq 5.24 pg 73 in [15])[42]

$$\theta = vs \frac{x}{\sqrt{x^2 + y^2 + z^2}} [-\frac{1}{\Delta|R|^2}] \tag{88}$$

$$\theta = vs \frac{x}{\sqrt{x^2 + y^2 + z^2}} [-\frac{1}{\Delta|R|^4}] \tag{89}$$

$$\theta = vs \frac{x}{\sqrt{x^2 + y^2 + z^2}} [-\frac{1}{\Delta|R|^6}] \tag{90}$$

If we define the following coordinate(a comoving ship Frame)$x^l = x - xs$

$$\theta = vs \frac{x^l}{\sqrt{(x^l)^2 + y^2 + z^2}} [-\frac{1}{\Delta|R|^2}] \tag{91}$$

$$\theta = vs \frac{x^l}{\sqrt{(x^l)^2 + y^2 + z^2}} [-\frac{1}{\Delta|R|^4}] \tag{92}$$

$$\theta = vs \frac{x^l}{\sqrt{(x^l)^2 + y^2 + z^2}} [-\frac{1}{\Delta|R|^6}] \tag{93}$$

It is easy to see that for example the expansion when $x^l = -5$ is balanced by the contraction when $x^l = 5$ but remember that we have division by powers of the Warp Bubble Radius R[43] multiplied by the Warp Bubble Thickness Δ to reduce the effect to almost zero.

In order to reduce the expansion/contraction of the Spacetime volume elements would be desirable to remove the speed of the Warp Bubble vs from the expressions above.As a matter of fact would be desirable

[42] see Ford and Pfenning observation of a constant speed vs and observation taken in the initial time $t = 0$

[43] at least in the case R^6.Consider a Warp Bubble Radius of $R = 100 meters$ and a speed 100 times the light speed and we will have an expansion/contraction factor of 10^{-2}

to removes vs from the Total Energy Integral and from the energy density to low even more the energy requirements.Mathematically we can do this although we dont know if this could be attained in the physical reality.This can be done by redefining the Piecewise Shape Functions including also the modulus of the Warp Bubble speed vs together with the Warp Bubble modulus of the Radius R and the Warp Bubble Thickness Δ to allow the remotion of the term vs[44] in the expansion/contraction expression as follows[45][46]:

Here are the new Piecewise Shape Functions:

- 1) $f_{pf} = 1 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $f_{pf} = -\frac{1}{\Delta|R|^2|vs|}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $f_{pf} = 0 \dashrightarrow rs > R + \frac{\Delta}{2}$
- 1) $f_{pf} = 1 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $f_{pf} = -\frac{1}{\Delta|R|^4|vs|}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $f_{pf} = 0 \dashrightarrow rs > R + \frac{\Delta}{2}$
- 1) $f_{pf} = 1 \dashrightarrow rs < R - \frac{\Delta}{2}$
- 2) $f_{pf} = -\frac{1}{\Delta|R|^6!vs|}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2}$
- 3) $f_{pf} = 0 \dashrightarrow rs > R + \frac{\Delta}{2}$

We are interested in the terms inside the Warped Region which means to say:

$$f_{pf} = -\frac{1}{\Delta|R|^2|vs|}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2} \tag{94}$$

$$f_{pf} = -\frac{1}{\Delta|R|^4|vs|}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2} \tag{95}$$

$$f_{pf} = -\frac{1}{\Delta|R|^6|vs|}(rs - R - \frac{\Delta}{2}) \dashrightarrow R - \frac{\Delta}{2} < rs < R + \frac{\Delta}{2} \tag{96}$$

Their derivatives(consider the division by vs as a division by $|vs|$ giving an unit vector):

$$[\frac{df_{pf}(rs)}{drs}]^2 = [-\frac{1}{\Delta|R|^2 vs}]^2 = [\frac{1}{\Delta|R|^2 vs}]^2 \tag{97}$$

$$[\frac{df_{pf}(rs)}{drs}]^2 = [-\frac{1}{\Delta|R|^4 vs}]^2 = [\frac{1}{\Delta|R|^4 vs}]^2 \tag{98}$$

$$[\frac{df_{pf}(rs)}{drs}]^2 = [-\frac{1}{\Delta|R|^6 vs}]^2 = [\frac{1}{\Delta|R|^6 vs}]^2 \tag{99}$$

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[44] replacing this by a unit vector

[45] like Ford and Pfenning we consider a constant Faster Than Light Speed vs in pg 9 eqs 25 and 26 of [1] and in pg 73 eqs 5.25 and 5.26 of [15]

[46] note that inside the Warped Region any one of these new Piecewise Shape Functions defined in function of the Warp Bubble Radius R Thickness Δ and speed vs satisfies the Ford-Pfenning requirement of $0 < f_{pf} < 1$.Consider a Radius $R = 100 meters$ a Thickness $\Delta = 10 meters$ and a speed $vs = 100 timeslightspeed$

Total Energy Integral:

$$E = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)[-\frac{1}{\Delta|R|^2 vs}]^2]drs = -\frac{1}{12}[\frac{1}{\Delta|R|^2}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)]drs \quad (100)$$

$$E = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)[-\frac{1}{\Delta|R|^4 vs}]^2]drs = -\frac{1}{12}[\frac{1}{\Delta|R|^4}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)]drs \quad (101)$$

$$E = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)[-\frac{1}{\Delta|R|^6 vs}]^2]drs = -\frac{1}{12}[\frac{1}{\Delta|R|^6}]^2\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)]drs \quad (102)$$

Energy Density:

$$T^{00} = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df_{pf}(rs)}{drs}]^2 = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{1}{\Delta|R|^2 vs}]^2 = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^2}]^2 \quad (103)$$

$$T^{00} = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df_{pf}(rs)}{drs}]^2 = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{1}{\Delta|R|^4 vs}]^2 = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^4}]^2 \quad (104)$$

$$T^{00} = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df_{pf}(rs)}{drs}]^2 = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{1}{\Delta|R|^6 vs}]^2 = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^6}]^2 \quad (105)$$

$$T^{00} = -\frac{1}{32\pi}\frac{y^2+z^2}{[x-xs(t)]^2+y^2+z^2}[\frac{1}{\Delta|R|^2}]^2 \quad (106)$$

$$T^{00} = -\frac{1}{32\pi}\frac{y^2+z^2}{[x-xs(t)]^2+y^2+z^2}[\frac{1}{\Delta|R|^4}]^2 \quad (107)$$

$$T^{00} = -\frac{1}{32\pi}\frac{y^2+z^2}{[x-xs(t)]^2+y^2+z^2}[\frac{1}{\Delta|R|^6}]^2 \quad (108)$$

Expansion of the volume elements:[47]

$$\theta = vs\frac{x-xs}{\sqrt{[x-xs(t)]^2+y^2+z^2}}[-\frac{1}{\Delta|R|^2 vs}] = \frac{x-xs}{\sqrt{[x-xs(t)]^2+y^2+z^2}}[-\frac{1}{\Delta|R|^2}] \quad (109)$$

$$\theta = vs\frac{x-xs}{\sqrt{[x-xs(t)]^2+y^2+z^2}}[-\frac{1}{\Delta|R|^4 vs}] = \frac{x-xs}{\sqrt{[x-xs(t)]^2+y^2+z^2}}[-\frac{1}{\Delta|R|^4}] \quad (110)$$

$$\theta = vs\frac{x-xs}{\sqrt{[x-xs(t)]^2+y^2+z^2}}[-\frac{1}{\Delta|R|^6 vs}] = \frac{x-xs}{\sqrt{[x-xs(t)]^2+y^2+z^2}}[-\frac{1}{\Delta|R|^6}] \quad (111)$$

Note that the inclusion of the Warp Bubble speed vs in the definition of the Piecewise Shape Functions reduces the energy density even more and makes the expansion/contraction of the volume elements almost close to zero resembling the Natario Warp Drive.An Alcubierre Warp Drive practically without expansion or contraction just like the Natario one.Our Piecewise Shape Functions fits well in the Alcubierre Warp Drive Spacetime to restore its physical feasibility as a valid and fully functional Superluminal and Faster Than Light Spacetime Ansatz of General Relativity.All the pathologies can be solved after all.Again we want

[47]even for the first case R^2 consider a Warp Bubble Radius $R = 100meters$ and a Warp Bubble Thickness $\Delta = 10meters$.anyone can see that the expansion/contraction is almost close to zero

to say that Miguel Alcubierre had a Wonderful Idea.By removing the original Alcubierre Shape Function and by modifying the Ford-Pfenning Piecewise Shape Function to satisfy our needs we can overcome the unphysical features and retain the Geometrical Beauty of the original Alcubierre Warp Drive without recurring to more complicated topologies(Broeck [2],Natario [5]).

8 More Restrictions that can be solved

Our Piecewise Shape Function can even overcome the restrictions raised by Lobo-Visser([9]) and we will demonstrate this for the WEC but in order to do that we will compute the mass M_w of the Alcubierre Warp Drive as follows:

$$M_w = -\frac{1}{12}[\frac{1}{\Delta|R|^2}]^2 \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)]drs = -\frac{1}{12}[\frac{1}{\Delta|R|^2}]^2|_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[\frac{1}{3}rs^3] + C \tag{112}$$

$$M_w == -\frac{1}{12}[\frac{1}{\Delta|R|^4}]^2 \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)]drs = -\frac{1}{12}[\frac{1}{\Delta|R|^4}]^2|_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[\frac{1}{3}rs^3] + C \tag{113}$$

$$M_w = -\frac{1}{12}[\frac{1}{\Delta|R|^6}]^2 \int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}} [(rs^2)]drs = -\frac{1}{12}[\frac{1}{\Delta|R|^6}]^2|_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[\frac{1}{3}rs^3] + C \tag{114}$$

$$M_w = -\frac{1}{36}[\frac{1}{\Delta|R|^2}]^2[(R+\frac{\Delta}{2})^3 - (R-\frac{\Delta}{2})^3] + C = -\frac{1}{18}[\frac{1}{\Delta|R|^2}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] + C \tag{115}$$

$$M_w = -\frac{1}{36}[\frac{1}{\Delta|R|^4}]^2[(R+\frac{\Delta}{2})^3 - (R-\frac{\Delta}{2})^3] + C = -\frac{1}{18}[\frac{1}{\Delta|R|^4}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] + C \tag{116}$$

$$M_w = -\frac{1}{36}[\frac{1}{\Delta|R|^6}]^2[(R+\frac{\Delta}{2})^3 - (R-\frac{\Delta}{2})^3] + C = -\frac{1}{18}[\frac{1}{\Delta|R|^6}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] + C \tag{117}$$

In the last set of eqs above we consider a Warp Bubble Radius $R = 100meters$ and a Thickness $\Delta = 10meters$.

Our Piecewise Shape Function can satisfy the restrictions posed by Lobo and Visser([9]) concerning the relations between the mass of the spaceship and the negative mass of the Alcubierre Warp Drive. While Lobo and Visser considered subluminal velocities in order to use the linearized gravity applied to the weak-field limit(not possible at Superluminal velocities) applied to the Alcubierre original Shape Function where the Warp Bubble velocity vs appears in the energy density equation affecting the energy conditions(WEC in our case[48]) and the relations between the mass of the spaceship and the effective Warp Drive mass(see abstract of [9] eq 14 and eqs 91 to 95 for the WEC)[49][50] constraining the Warp Bubble

[48] BEC,NEC,SEC,DEC,PEC,REC or whatever will appear in a future work.Our goal is to convince the maximum nmber of readers that the Alcubierre Warp Drive is "still alive" as a fully functional Superluminal and Faster Than Light ansatz of General Relativity and WEC is enough

[49] see also pg 3 between eqs 8 and 9 where Lobo-Visser says that the total ADM mass of the spaceship and Warp Field Generators must be exactly compensated by the ADM mass due to the Stress Energy Momentum Tensor of the Warp Field itself

[50] in our approach:what spaceship????the mothership or the Warp Drone????

velocity to subluminal levels but this constriction is a particular consequence of the original Alcubierre Shape Function.We demonstrate that by eliminating the velocity from the energy density equation due to our Piecewise Shape Function we can also eliminate the Warp Bubble velocity from the energy conditions violations equations.We do not restrict the speed of the Alcubierre Warp Drive to subluminal speeds allowing our version of the Alcubierre Warp Drive to attain Superluminal and Faster Than Light velocities while maintaining the equilibrium of the WEC energy condition considering the relations between the mass of the spaceship and the effective mass of the Alcubierre Warp Drive.

We will demonstrate this right now:

Lobo and Visser defines the WEC equilubrium condition as $T_{\mu\nu}V^{\mu}V^{\nu} \geq 0$ where V^{μ} is any Timelike Vector or $T_{\mu\nu}U^{\mu}U^{\nu} \geq 0$ where U^{μ} is the four-velocity of an Eulerian Observer.The physical interpretation of the WEC is the fact that the local energy density is always positive for a non-massless systems.(see pg 3 before eq 10 in [9])

The WEC energy condition violation is given by Lobo-Visser as follows:(see eqs 11 and 12 pg 4 of [9])

$$T_{\mu\nu}U^{\mu}U^{\nu} < 0 = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df(rs)}{drs}]^2 < 0 = T_{00} < 0 \tag{118}$$

From above we can reckognize the expression for the energy density of the Alcubierre Warp Drive and this means to say that the energy density of the Alcubierre Warp Drive is always negative in a clear WEC violation.(see comments on pg 4 between eqs 12 and 13 of [9] and comment on page 8 after eq 19 in [4])

Lobo-Visser defines the mass of the Alcubierre Warp Drive as follows:(see eq 13 pg 4 of [9])

$$M_w = -\frac{vs^2}{12}\int_{R-\frac{\Delta}{2}}^{R+\frac{\Delta}{2}}[(rs^2)[\frac{df(rs)}{drs}]^2]drs \tag{119}$$

The integral above can be solved exactly but due to the hyperbolic nature of the original Alcubierre Shape Function it will result in difficult and useless polylog functions[51] as mentioned by Lobo-Visser.

The mass of the Alcubierre Warp Drive is computed by Lobo-Visser for the particular case of the Alcubierre Shape Function as the following estimative(see eq 14 pg 4 in [9]):

$$M_w = -vs^2R^2\delta = \frac{-vs^2R^2}{\Delta} \tag{120}$$

Note that the mass of the Alcubierre Warp Drive and hence the negative energy density requirements raises quadratically with the Warp Bubble Radius R and velocity vs and inversely to the Warp Bubble Thickness Δ.(see comment on pg 4 after eq 14 in [9])
As fastest the Warp Bubble moves or as bigger the Warp Bubble becomes the negative energy density requirements and hence WEC violations becomes more worst.
This is a consequence of the original Alcubierre Shape Function.

Lobo and Visser in pg 6 section III of [9] discuss linearized gravity applied to a subluminal version of the Alcubierre Warp Drive $vs << 1$.We concern ourselves with section $III.A$ pg 6 where WEC is analyzed.According to the linearized gravity in weak-field limit the WEC is given by the following equations:(see eqs 33,38 and 39 pg 7 of [9])

$$T_{\mu\nu}U^{\mu}U^{\nu} = T_{00} = \frac{1}{8\pi}G_{00} \tag{121}$$

[51] Mathematica,Mathlab,Maple etc.see comment on pg 4 after eq 14 in [9]

$$G_{00} = O(vs^2) \tag{122}$$

$$T_{\mu\nu}U^{\mu}U^{\nu} = T_{00} = O(vs^2) \tag{123}$$

$$\int T_{00}dx^3 = \int O(vs^2)dx^3 = M_w = -vs^2R^2\delta = \frac{-vs^2R^2}{\Delta} \tag{124}$$

Remember that the eqs above are exclusively for the mass,Stress Energy Momentum Tensor and energy density of the Alcubierre Warp Drive.Specially the last equation from the set above relates the Alcubierre Warp Drive energy density to its mass.This equation will be used later in this section.

Lobo and Visser applies linearized gravity not only to the Alcubierre Warp Drive as above but also to a static source (eg the spaceship) in section $III.D$ pg 8 of [9].

The metric is given by:(see eq 52 pg 8 in [9])[52][53]

$$ds^2 = dt^2 - dx^2 - dy^2 - dz^2 + \Phi(x, y, z)(dt^2 + dx^2 + dy^2 + dz^2) \tag{125}$$

This equation cames from the formalism of the linearized theory given below with $h_{\mu\nu} << 1$ and $|\Phi| << 1$.(see top and between eqs 55 and 56 of pg 9 in [9])

$$ds^2 = (n_{\mu\nu} + h_{\mu\nu})dx^{\mu}dx^{\nu} \tag{126}$$

From the Einstein Tensor Component G_{00} applied to $h_{\mu\nu}$ we arrive at the classical Poisson equation(see appendix C pg 16 and 17 and eq 55 pg 9 in [9])

$$\nabla^2\Phi = 4\pi\rho \tag{127}$$

In the equation above ρ is the mass density of the spaceship.Hence we can say that the "mass-energy" density (eg the Stress Energy Momentum Tensor) of the spaceship is:

$$T_{\mu\nu}U^{\mu}U^{\nu} = T_{00} = \rho > 0 \tag{128}$$

$$M_{ship} = \int \rho dx^3 > 0 \tag{129}$$

Note that the mass-energy density of the spaceship satisfies the WEC because the mass of the ship is positive.

Now the reader can understand the point of view of Lobo and Visser:we have one Stress Energy Momentum Tensor for the spaceship that do not violates the WEC due to the positive mass of the spaceship and another Stress Energy Momentum Tensor for the mass of the Alcubierre Warp Drive Bubble where the ship is immersed and this Stress Energy Momentum Tensor violates the WEC due to the negative mass of the Alcubierre Warp Drive.Both spaceship and Alcubierre Warp Drive forms a combined system by addition of both masses and why?:Because the ship is immersed inside the Warp Drive Bubble so both masses must be treated as a single one combined system and this combined system would be desirable to do not violate the WEC.If we want to keep the WEC valid for the combined system then the

[52]Lobo and Visser used the signature (-+,+,+).we will use the signature (+,-,-,-).by inverting the signature we invert the signs of the matrix elements of eq 53

[53]here Φ is the weak gravitational field unable to produce velocities close to that of light.see comment on pg 9 of [9]

modulus of the mass of the ship must always be greater than the modulus of the mass of the Alcubierre Warp Drive where the ship is immersed because the ship mass is positive and the Alcubierre Warp Drive mass is negative.This is the main idea behind the work of Lobo and Visser.

$$M_{combined_system} = M_{ship} + M_w > 0 \tag{130}$$

$$|M_{ship}| > |M_w| \tag{131}$$

The equations above are the fundamental conditions for a combined system spaceship-Alcubierre Warp Bubble that do not violates the WEC according to Lobo-Visser

We don't need to analyze the rest of the calculations of Lobo and Visser to demonstrate that the mass of the combined system is what they call "ADM mass" because we already got the idea behind their work and this is the most important thing.Omitting the details of linearized gravity in the weak-field limit with first order or second order terms etcetera and keeping in mind that the ship when immersed inside an Alcubierre Warp Drive both forms a combined system spaceship-Alcubierre Warp Drive by adding both masses because both masses are integrated into a single piece interacting gravitationally (and negatively) between each other and must be treated together as a combined system.We will compare two combined systems:

- 1)-Combined system between a spaceship of mass M_{ship} immersed inside an Alcubierre Warp Drive Bubble of mass $M_{f(rs)}$ with the original Alcubierre Shape Function $f(rs)$
- 2)-Combined system between a spaceship of mass M_{ship} immersed inside an Alcubierre Warp Drive Bubble of mass $M_{f_{pc}(rs)}$ with our Piecewise Shape Functions $f_{pc}(rs)$

This is meant to demonstrate quickly to the reader that whether subluminal or Superluminal our Piecewise Shape Function satisfies the ADM requirements of Lobo-Visser for a combined system and don't suffer WEC violations.This eliminates the last constriction against the Original and Wonderful idea of the Alcubierre Warp Drive.Although the original Alcubierre Shape Function needs to be replaced by a continuous Function with the behavior of our Piecewise Shape Function to avoid all these pathologies in order to make the Alcubierre Warp Drive a valid and fully-functional Superluminal and Faster Than Light ansatz of General Relativity(and we are working on a continuous Shape Function with this behavior that will appear in a future work) we want to say that Miguel Alcubierre was Brilliant and the 1994 paper will ever be regarded as a Historical and Revolutionary paper.

- 1)-Combined system between a spaceship of mass M_{ship} immersed inside an Alcubierre Warp Drive Bubble of mass $M_{f(rs)}$ with the original Alcubierre Shape Function $f(rs)$ that satisfies the WEC

$$M_{combined_system} = M_{ship} + M_{f(rs)} > 0 \tag{132}$$

$$M_{combined_system} = M_{ship} - \frac{vs^2 R^2}{\Delta} > 0 \tag{133}$$

$$M_{ship} > \frac{vs^2 R^2}{\Delta} \tag{134}$$

Compare the expression above with eqs 94 and 95 pg12 of [9] and see the comment between these expressions.The net energy of the Alcubierre Warp Drive cannot exceed the net energy of the spaceship itself if we want to satisfy the WEC.This places severe restrictions to the radius R and speed vs of the Alcubierre Warp Drive.

- 2)-Combined system between a spaceship of mass M_{ship} immersed inside an Alcubierre Warp Drive Bubble of mass $M_{f_{pc}(rs)}$ with our Piecewise Shape Functions $f_{pc}(rs)$ that satisfies the WEC

$$M_{combined_system} = M_{ship} + M_{f_{pc}(rs)} > 0 \tag{135}$$

$$M_{combined_system} = M_{ship} - \frac{1}{18}[\frac{1}{\Delta|R|^2}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] > 0 \tag{136}$$

$$M_{combined_system} = M_{ship} - \frac{1}{18}[\frac{1}{\Delta|R|^4}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] > 0 \tag{137}$$

$$M_{combined_system} = M_{ship} - \frac{1}{18}[\frac{1}{\Delta|R|^6}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] > 0 \tag{138}$$

$$M_{ship} > \frac{1}{18}[\frac{1}{\Delta|R|^2}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] \tag{139}$$

$$M_{ship} > \frac{1}{18}[\frac{1}{\Delta|R|^4}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] \tag{140}$$

$$M_{ship} > \frac{1}{18}[\frac{1}{\Delta|R|^6}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] \tag{141}$$

Our Piecewise Shape Function allows the possibility of an Alcubierre Warp Drive that whether subluminal or Superluminal will always satisfy the Lobo-Visser WEC energy conditions.Consider a Warp Bubble of Radius $R = 100meters$ a Thick of $\Delta = 10meters$ and a speed $vs = 100timesfasterthanlight$.And this is due to the fact that by including the speed of the Warp Bubble vs in the definition of the Piecewise Shape Function $f_{pc}(rs)$ we were able to remove the speed from the energy density and from the energy conditions.

While we lowered the energy density requirements of the Alcubierre Warp Drive to acceptable and physically reasonable levels we would like to comment the work of Ford-Roman.[11]

We know that the Alcubierre Warp Drive violates the WEC energy condition $T_{\mu\nu}V^\mu V^\nu \geq 0$(cite pg 2 of [11]) but we also know that our Piecewise Shape Function allows the reduction of the energy density requirements to sustain a Warp Bubble.

In abstract,pages 2,3,5 after eq 15 and pg 18 in the beginning of section 5 of [11]) Ford-Roman mentions the possibility to produce large amounts of negative energy.This sounds good for the Warp Drive of course because if we have no restrictions of negative energy production then the physical feasibility of the Warp Drive is reinforced.But of course while we welcome the large outputs of negative energy density we want to use low and affordable energy density levels for the Warp Drive in order to sustain its credibility.

According to Ford-Roman the Stress Energy Momentum Tensor for a generically coupled scalar field would be given by:

$$T_{\mu\nu} = \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} \nabla_\mu \Phi \nabla^\mu \Phi + g_{\mu\nu} V(\Phi) \tag{142}$$

$$T_{\mu\nu} = \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} \nabla_\mu \Phi g^{\mu\mu} \nabla_\mu \Phi + g_{\mu\nu} V(\Phi) \tag{143}$$

The following expressions are taken from eq 3 pg 3 in [11] without the parameter ξ

$$T_{\mu\nu} = \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} \nabla_\mu \Phi \nabla^\mu \Phi - g_{\mu\nu} V(\Phi) \tag{144}$$

$$T_{\mu\nu} = \nabla_\mu \Phi \nabla_\nu \Phi - \frac{1}{2} g_{\mu\nu} \nabla_\mu \Phi g^{\mu\mu} \nabla_\mu \Phi - g_{\mu\nu} V(\Phi) \tag{145}$$

The energy density for the Warp Drive is given by T_{00} from the following equation:(see eqs 11 and 12 pg 4 and eqs 33,38 and 39 pg 7 of [9]:

$$T_{\mu\nu} U^\mu U^\nu < 0 = -\frac{1}{32\pi} \frac{vs^2 \rho^2}{rs^2} [\frac{df(rs)}{drs}]^2 < 0 = T_{00} < 0 \tag{146}$$

Note that Ford-Roman mentions explicitly in pg 2 of [11] the solutions of Visser-Barcelo wormhole in which a large flux of negative energy is injected into a wormhole "throat" in order to keep its stability.

The positive mass of of the Black Hole coupled together with the negative energy flux is also a combined system just like the spaceship and the Warp Drive Bubble.The WEC according to Ford-Roman can be violated(see pg 2 of [11] the mention to the works of Bekenstein,Deser,Flanagan and Wald).Note that the energy flux although great will not exceed the total rest mass M of the Black Hole.

But since our energy density for the Warp Drive is low and affordable we don't need to violate the WEC

Writing the energy density for the Alcubierre Warp Drive in function of the scalar field for the Basini-Capozziello Stress Energy Momentum Tensor and using our Piecewise Shape Function we would have:

$$T_{00} = \nabla_0 \Phi \nabla_0 \Phi - \frac{1}{2} g_{00} \nabla_0 \Phi g^{00} \nabla_0 \Phi + g_{00} V(\Phi) \tag{147}$$

We know that $g_{00} \times g^{00} = 1$

$$T_{00} = \nabla_0 \Phi \nabla_0 \Phi - \frac{1}{2} \nabla_0 \Phi \nabla_0 \Phi + V(\Phi) \tag{148}$$

$$T_{00} = \frac{1}{2} \nabla_0 \Phi \nabla_0 \Phi + V(\Phi) \tag{149}$$

$$T_{00} = \frac{1}{2}\nabla_0^2\Phi + V(\Phi) \tag{150}$$

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 + V(\Phi) \tag{151}$$

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 + V(\Phi) = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^2}]^2 \tag{152}$$

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 + V(\Phi) = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^4}]^2 \tag{153}$$

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 + V(\Phi) = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^6}]^2 \tag{154}$$

Using the Ford-Roman expression:

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 - V(\Phi) \tag{155}$$

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 - V(\Phi) = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^2}]^2 \tag{156}$$

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 - V(\Phi) = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^4}]^2 \tag{157}$$

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 - V(\Phi) = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^6}]^2 \tag{158}$$

In pg 4 of [11] Ford-Roman makes $V(\Phi) = 0$.Then we would have:

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^2}]^2 \tag{159}$$

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^4}]^2 \tag{160}$$

$$T_{00} = \frac{1}{2}[\frac{\partial\Phi}{\partial t}]^2 = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^6}]^2 \tag{161}$$

Although we have a WEC violation for the negative mass-energy of the Alcubierre Warp Drive the partial derivative of the scalar field have low values.

9 Casimir Warp Drive??? Why Not???

The negative energy density needed to create the Alcubierre Warp Drive could perhaps be obtained by the Casimir Effect as mentioned by Miguel Alcubierre in pg 9 of [4].

Combining the negative energy density of the Casimir Effect with the negative energy of the Alcubierre Warp Drive we will obtain a new set of equations.Although these equations are only a theoreticsl analysis of a possible "mini Backreaction" between Alcubierre and Casimir we call it simply:The Casimir Warp Drive

A detailed explanation of the Casimir Effect can be found in pg 6 to 9 of [7],pg 2 to 9 in [14]

According to pg 4 of [14],pg 6 of [7] and pg 2 of [6] the negative energy of the Casimir Effect between parallel conducting and reflecting plates is given by:(see eq 14 pg 9 of [7])[54]

$$\frac{1}{A}\xi_0 = -\frac{\pi^2}{720}\frac{\hbar c}{L^3} \tag{162}$$

Where $A = L_z \times L_y$ is the area of each parallel plate in the plane yOz and L is the distance between the plates.(see fig 1 pg 6 in [7])

$$\xi_0 = -\frac{\pi^2}{720}\frac{\hbar c}{L^3}A \tag{163}$$

The energy density between planes inclined by an angle α is given by:(see eqs 4.2,4.3 and 4.4 pg 38 in [14]).

$$T^{00} = -\frac{f(\alpha)}{720\pi^2 L^4} \tag{164}$$

Conformal scalar with Dirichlet boundary conditions:

$$f(\alpha) = \frac{\pi^2}{2\alpha^2}(\frac{\pi^2}{\alpha^2} - \frac{\alpha^2}{\pi^2}) \tag{165}$$

Electromagnetism with perfect coundutor boundary conditions:

$$f(\alpha) = (\frac{\pi^2}{\alpha^2} + 11)(\frac{\pi^2}{\alpha^2} - 1) \tag{166}$$

Ford and Sopova in pg 2 of [6] presents the energy density as:

$$T_{00} = -\frac{\pi^2}{720}\frac{1}{L^4} \tag{167}$$

Ford and Sopova also mentions in pg 2 the WEC violation of the Casimir Effect and the need of exotic matter for transversable wormholes.

Equalizing the negative energy of the Casimir Effect with the one of the Alcubierre Warp Drive with our Piecewise Shape Function we should expect for:

[54] equations 353 and 354 written with the conventional values for G,c and $\hbar$.for the rest we use $G = c = \hbar = 1$.

$$T_{00} = -\frac{1}{32\pi}\frac{vs^2\rho^2}{rs^2}[\frac{df(rs)}{drs}]^2 = -\frac{\pi^2}{720}\frac{1}{L^4} \tag{168}$$

$$T_{00} = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^2}]^2 = -\frac{\pi^2}{720}\frac{1}{L^4} \tag{169}$$

$$T_{00} = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^4}]^2 = -\frac{\pi^2}{720}\frac{1}{L^4} \tag{170}$$

$$T_{00} = -\frac{1}{32\pi}\frac{\rho^2}{rs^2}[\frac{1}{\Delta|R|^6}]^2 = -\frac{\pi^2}{720}\frac{1}{L^4} \tag{171}$$

Now we can compute the distance between the parallel plates of our Casimir Warp Drive to generate the Alcubierre Spacetime Geometry(at least in theory)

$$L^4 = \frac{32\pi^3}{720}\frac{rs^2}{\rho^2}[\Delta|R|^2]^2 \tag{172}$$

$$L^4 = \frac{32\pi^3}{720}\frac{rs^2}{\rho^2}[\Delta|R|^4]^2 \tag{173}$$

$$L^4 = \frac{32\pi^3}{720}\frac{rs^2}{\rho^2}[\Delta|R|^6]^2 \tag{174}$$

If the distance between the plates grows then the negative energy density becomes smaller.However we know that the plates must stay close to each other

Computing the negative energy E_w of the Alcubierre Warp Drive and relating it with the Casimir negative energy we should expect for:

$$E_w = -\frac{c^2}{18}[\frac{1}{\Delta|R|^2}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] = -\frac{\pi^2}{720}\frac{\hbar c}{L^3}A \tag{175}$$

$$E_w = -\frac{c^2}{18}[\frac{1}{\Delta|R|^4}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] = -\frac{\pi^2}{720}\frac{\hbar c}{L^3}A \tag{176}$$

$$E_w = -\frac{c^2}{18}[\frac{1}{\Delta|R|^6}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] = -\frac{\pi^2}{720}\frac{\hbar c}{L^3}A \tag{177}$$

Now the distance between the Casimir plates to generate the exotic matter for the Alcubierre Warp Drive is given by:

$$\frac{40c}{\hbar A\pi^2}[\frac{1}{\Delta|R|^2}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] = \frac{1}{L^3} \tag{178}$$

$$\frac{40c}{\hbar A\pi^2}[\frac{1}{\Delta|R|^4}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] = \frac{1}{L^3} \tag{179}$$

$$\frac{40c}{\hbar A\pi^2}[\frac{1}{\Delta|R|^6}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] = \frac{1}{L^3} \tag{180}$$

Note that the magnitude of the value of the Planck Constant 10^{-34} combined with the magnitude of the light speed value 10^8 reduces the distance between the plates.

10 Conclusions:

We know that our approach of the Alcubierre Warp Drive is only theoretical but at least we tried to solve the unphysical pathologies.Only three obstacle remains:

- 1)- We don't know what would happen to the spaceship if the Warp Bubble is destroyed from behind.
- 2)- An impact with a large positive mass object would generate a strong gravitational repulsive force that could disrupt or destroy the front part of the Warp Buble.Again we don't know the consequences for the spaceship if the Warp Bubble is destroyed from the front.
- 3)-We don't know to generate the Shape Function in order to "engineer" the Alcubierre Warp Drive Spacetime.

In order to terminate the discussion about the subject of the Alcubierre Warp Drive we will compute the Gravitational Bending Of Light in the neighborhoods of the Warp Bubble negative mass.

The classical $4D$ formula of General Relativity for the Bending Of Light is given by(see pg 1781 eq 18 in [3] and pg 70 eq 157 in [**?**]):[55]

$$\Delta\omega = \frac{4M}{r_0} \tag{181}$$

Where M is the mass of the body that generates the Bending Of Light and r_0 is the distance between the body and the Light photons.The Bending Of Light is positive for a body of positive mass.

Computing for the Alcubierre Warp Drive using the results of our Piecewise Shape Function we should expect for:

$$\Delta\omega = -\frac{4}{18r_0}[\frac{1}{\Delta|R|^2}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] \tag{182}$$

$$\Delta\omega = -\frac{4}{18r_0}[\frac{1}{\Delta|R|^4}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] \tag{183}$$

$$\Delta\omega = -\frac{4}{18r_0}[\frac{1}{\Delta|R|^6}]^2[R^2\Delta + R^2\frac{\Delta}{2} + \frac{\Delta^3}{8}] \tag{184}$$

We can clearly see from above that the Bending Of Light although very small is negative for the Alcubierre Warp Drive.

The negative Gravitational Force between an incoming particle of a given positive mass M_p approaching the Warp Bubble at a distance d considering our Piecewise Shape Function is given by:

[55] equations written with the $5D$ Extra Dimensional term removed.

$$F = -GM_p \frac{1}{18d^2} [\frac{1}{\Delta |R|^2}]^2 [R^2 \Delta + R^2 \frac{\Delta}{2} + \frac{\Delta^3}{8}] \quad (185)$$

$$F = -GM_p \frac{1}{18d^2} [\frac{1}{\Delta |R|^2}]^2 [R^2 \Delta + R^2 \frac{\Delta}{2} + \frac{\Delta^3}{8}] \quad (186)$$

$$F = -GM_p \frac{1}{18d^2} [\frac{1}{\Delta |R|^2}]^2 [R^2 \Delta + R^2 \frac{\Delta}{2} + \frac{\Delta^3}{8}] \quad (187)$$

Note that if the distance d becomes small the negative force grows proportionally.Also note the fact that we mentioned the Warp Bubble being destroyed from behind and the front part going on this would mean that the pieces of the small negative energy density from the front part would be repealed by the negative gravitational force between these pieces and positive mass objects.The reminiscent of the front part would never be a threat for ships or planets.

This could deflect small incoming particles and protect the Warp Bubble from impacts although a giant Black Hole or a Neutron Star would pose a very serious threat.

In our example of Horizons we used the Crab Nebula but for distances of 6000 Light-Years the probability of an impact with a large positive mass object would be of a great concern.

Alcubierre Warp Drive would be suitable perhaps for interstellar travels of "short" distance eg Sun and Proxima Centauri or Sirius because we know that(we hope) there are no Neutron Stars or Black Holes between Earth and these Stars.This terminates our discussion on the Alcubierre Warp Drive..Also we tried to demonstrate that Superluminal and Faster Than Light Space Travel is possible in $3 + 1$ Spacetime although with some limitations.

11 The End

In this work we tried to restore the physical feasibility of the original and wonderful idea of the Alcubierre Warp Drive.Our new Piecewise Shape Functions solved the pathologies of negative energy and Doppler Blueshifts using the original Alcubierre geometry but without expansion or contraction imitating the behavior of the Natario Warp Drive.The expansion/contraction is a consequence of the Alcubierre choice for the Shape Function. For the problem of the Horizons we took inspiration from the NASA Apollo spacecrafts eg the Command Module and the Service Module.The Alcubierre Warp Drive spaceship(the mothership) is like the Command Module:the mothership must eject the Warp Drone that contains the entire light cone of the mothership as the external observer that will create the Warp Bubble following the work of the American physicists Allen Everett and Thomas Roman from Tufts University Medford Massachussets United States and the Warp Drone will create the Warp Bubble with the mothership inside and will trigger the motion[56]. The Warp Drone is like the Service Module. When arriving at the destination point the mothership will destroy the rear part of the Warp Bubble.The front will go on but since we use low negative energy and the gravitational force is repulsive the front will not be harmful to planets for example. Once triggered the motion the Warp Drone will be abandoned in Space just like the Apollo Service Module. Each Alcubierre Warp Drive can only be used for a "one-way" trip. Impact with large Black Holes are serious threats but considering a Warp Bubble of large radius the impact would destroy the front of the Bubble and the ship would perhaps stop but at a safe distance from the Events Horizon allowing the ship to escape.The ship would then eject another Warp Drone to continue the journey however

[56]see the comment on pg 3 of [12] about the actions to create or change the Warp Bubble trajectory or speed being taken by an external observer whose light cone contains all the trajectory of the Warp Bubble

this is an expensive way to travel.The Alcubierre Warp Drive would be suitable to interstellar travel of "short" distances eg Sun-Sirius,Sun-Tau Ceti,Sun-Proxima Centauri where no Black Holes are expected to be found.For "long" interstellar travel the Chung-Freese Superluminal Braneworld would be better. We still don't know how to generate the Alcubierre Warp Drive but once generated it would be of Immense value. Even a subluminal version of the Alcubierre Warp Drive would be able to reach the region of Space where the Pioneers were lost in a matter of days(or hours) avoiding the need to wait more 40 years. Lastly we will comment the prediction of 3 American physicists Chad Clark,William Hiscock and Shane Larson from Montana State University Bozeman Montana United States in the paper "Null Geodesics In The Alcubierre Warp Drive:The View From The Bridge" about the "Warp Drive Starships Plying the Galaxy"[57].We are confident that this prediction will one day become true and the Space Pilots of the Distant Future will look backwards to their Remote Past to Salute the year of 1994 when the Mexican Mathematician Miguel Alcubierre from Universidad Nacional Autonoma de Mexico(UNAM) appeared with a paper called "The Warp Drive:Hyper Fast Travel Within General Relativity".This paper will ever be regarded as the paper that started it all.Miguel Alcubierre was the first to overcome the limitations of Einstein Special Relativity. If this Legend of Space Pioneers will ever become true then Miguel Alcubierre already have a place in the Human History:He will always be remembered from Here to the Eternity and with the most Profound Feeling of Gratitude by the Future Generations as the Father of the Warp Drive

[57]Clark,Hiscock,Larson mentions in pg 4 of [3] "Warp Drive Starships Plying The Galaxy".

12 Epilogue

- "The only way of discovering the limits of the possible is to venture a little way past them into the impossible."-Arthur C.Clarke[58]
- "We need someone to shake our views. Otherwise we are doomed to live in the past."-Edward Halerewicz Jr.[59]
- "Only once we leave our tiny precious homeworld and venture out in the void of space will we truly find our destiny out among the stars. Its up to us each and every one of us to fight hard and work to make it happen"-Simon Jenks[60]
- "Earth was the cradle of Humanity but Humans will not stay on the cradle forever"-Konstantin Edvardovitch Tsiolkovsky[61]
- "The Victory Belongs to the ones that Believe it the Most.And Believe it the Longest:We gonna make Believe"-Alec Baldwin[62]
- "The supreme task of the physicist is to arrive at those universal elementary laws from which the cosmos can be built up by pure deduction. There is no logical path to these laws; only intuition, resting on sympathetic understanding of experience, can reach them"-Albert Einstein[63][64]

[58]special thanks to Maria Matreno from Residencia de Estudantes Universitas Lisboa Portugal for providing the Second Law Of Arthur C.Clarke

[59]American Physicist from Sprinngfield Illinois and author of one of the most beautiful webpages on the Internet about the Physics of the Alcubierre Warp Drive

[60]British Web designer living in San Antonio Texas United States of America and also author of one of the most beautiful webpages on the Internet about the Physics of the Alcubierre Warp Drive

[61]The Father of the Russia Space Program. Mentor of Serguei Pavlovitch Koroliev and Yuri Alekseyevitch Gagarin

[62]Alec Baldwin as Lieutenant-Colonel James B.Doolitle-April 1942. From the Jerry Bruckheimer and Michael Bay movie:Pearl Harbor

[63]"Ideas And Opinions" Einstein compilation, ISBN $0-517-88440-2$, on page 226."Principles of Research" ([Ideas and Opinions],pp.224-227), described as "Address delivered in celebration of Max Planck's sixtieth birthday (1918) before the Physical Society in Berlin"

[64]appears also in the Eric Baird book Relativity in Curved Spacetime ISBN $978-0-9557068-0-6$

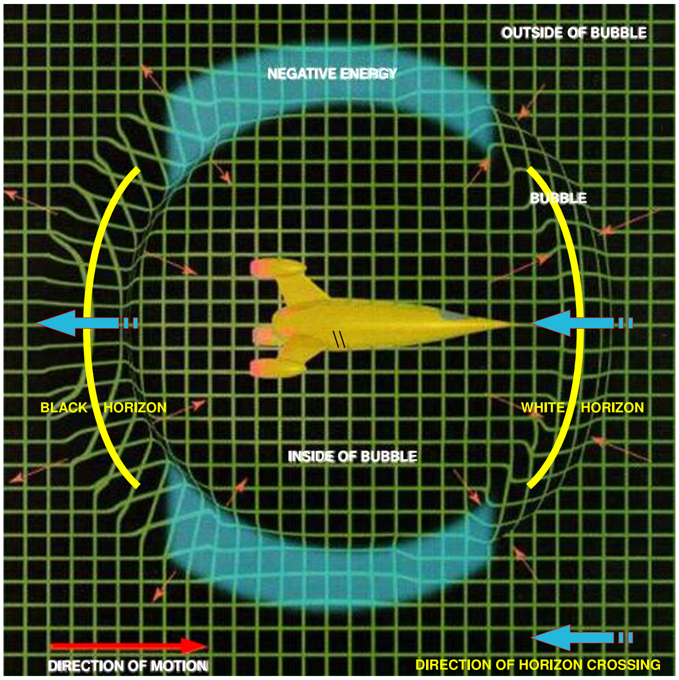

Figure 1: Artistic representation of the Alcubierre Warp Drive Bubble . From the point of view of an observer inside the bubble, the front (back) wall looks like the horizon of a white (black) hole (*yellow solid lines*). Large amounts of exotic matter are concentrated in the walls on a plane orthogonal to the direction of motion. (Source: pg 4 in [16])

13 An image worths more than 1000 words

From above we can see that the front of the ship is not involved by negative energy.Then there are no protection for the ship when impacting hazardous objects or photons highly Doppler Blueshifted. Our idea using a different Piecewise Shape Function is to place negative energy also in the rear and in the front of the ship in a Warp Drive Bubble without Expansion/Contraction resembling the Natario Warp Drive.The repulsive behaviour of the negative energy in front of the ship would protect the ship against impacts. The Warp Drive Bubble would then be carried away by the Spacetime Stream just like a fish in the stream of a river.

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