

DECELERATION PARAMETER Q(Z) AND THE ROLE OF NUCLEATED GW ‘GRAVITON GAS’ IN THE DEVELOPMENT OF DE ALTERNATIVES

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The case for a four dimensional graviton mass (non zero) influencing reacceleration of the universe in five dimensions is stated, with particular emphasis upon if five dimensional geometries as given below give us new physical insight as to cosmological evolution. A comparison with the quantum gas hypothesis of Glinka shows how stochastic GW/ gravitons may emerge in vacuum nucleated space, with emphasis upon comparing their number in phase space, as compared with different strain values

1 Introduction

1.1 What can be said about gravitational wave density value detection?

We will start with a first-principle introduction to detection of gravitational wave density using the definition given by Maggiore [1]

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(v) \Rightarrow h_0^2 \Omega_{gw}(v) \cong 3.6 \cdot \left[\frac{n_f}{10^{37}} \right] \cdot \left(\frac{v}{1kHz} \right)^4 \quad (1.1)$$

where n_f is the frequency-based numerical count of gravitons per unit phase space. The author suggests that n_f may also depend upon the interaction of gravitons with neutrinos in plasma during early-universe nucleation, as modeled by M. Marklund *et al*². Ω_{gw} has the following relic universe candidate values as given by Figure 1 below

Combining experimental confirmation of Eq. (1.1) with observations and use of different choices for $H = \frac{\dot{a}}{a}$ and $\Omega \equiv \rho(t)/\rho_{critical}$ will be tied in, with analysis of the diagram of Figure 1 below

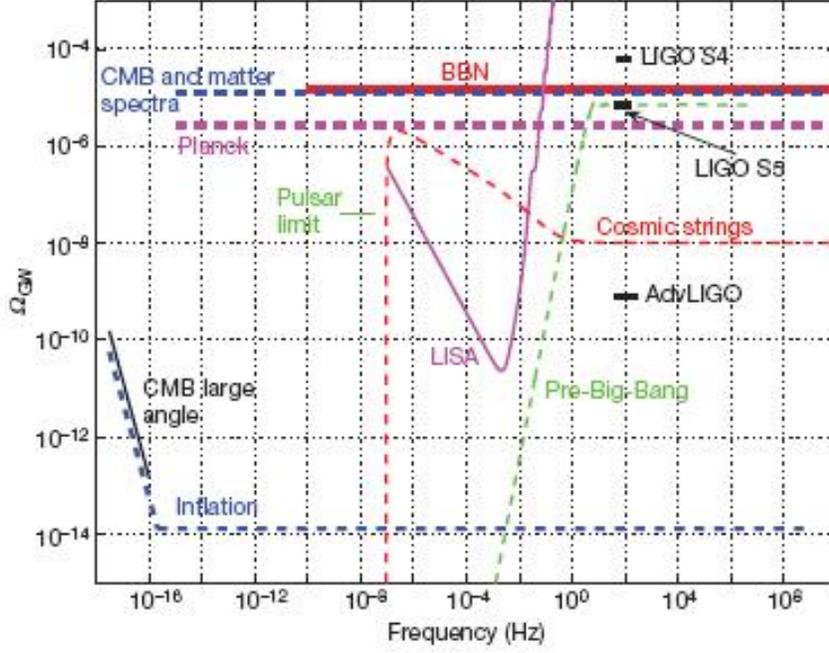


Figure 1. B. P. Abbott et al. [2] (2009) shows the relation between Ω_g and frequency.

The relation between Ω_g and the spectrum $h(v_s, \tau)$ is written by Grishchuk, [3] , as

$$\Omega_g \approx \frac{\pi^2}{3} \left(\frac{v}{v_H} \right)^2 h^2(v, \tau), \quad (1.2)$$

We will be using Eq. (1.2) with the range of values as presented in Figure 1, and also prepare for a candidate discriminating criteria for a number count for Ω_g based upon Glinka's [4] quantum gas work, namely

$$n_f = [1/4] \cdot \left[\sqrt{\frac{v(a_{initial})}{v(a)}} - \sqrt{\frac{v(a)}{v(a_{final})}} \right] \quad (1.3)$$

As well as, if $h_0 \sim .75$

$$\Omega_{gw}(v) \cong \frac{3.6}{h_0^2} \cdot \left[\frac{n_f}{10^{37}} \right] \cdot \left(\frac{v}{1kHz} \right)^4 \quad (1.4)$$

If we take into consideration having $a \sim a_{final}$, then Eq. (1.3) above will, in most cases be approximately

$$n_f = [1/4] \cdot \left[\sqrt{\frac{v(a_{initial})}{v(a)}} - 1 \right] \sim [1/4] \cdot \left[\sqrt{\frac{v(a_{initial})}{v(a)}} \right] \quad (1.5)$$

For looking at $\Omega_g \approx 10^{-5} - 10^{-14}$, with $\Omega_g \approx 10^{-5}$ in pre big bang scenarios, with initial values of frequency set for $v(a_{initial}) \approx 10^8 - 10^{10}$ Hz, as specified by Grishkuk[5] $v(a_{final}) \approx 10^0 - 10^2$ Hz near the present era, and $a \sim [a_{final} = 1] - \delta^+$, i.e. close to the final value of today's scale value, we can obtain the following table of would be n density values in the regime for which $a \sim [a_{final} = 1] - \delta^+$ represents

Table 1: If one assumes $\Omega_g \approx 10^{-5}$

$v(a) \approx v(a_{final}) \approx 10 - 10^2$	$v(a_{initial})$	n_f (from Eq. 1.5)
***	10^3	10^{32}
***	10^8	10^{12}
***	10^{10}	10^3

Table 2: If one assumes $\Omega_g \approx 10^{-10}$

$v(a) \approx v(a_{final}) \approx 10 - 10^2$	$v(a_{initial})$	n_f (from Eq. 1.5)
***	10^3	$\sim 10^{27}$
***	10^8	$\sim 10^7$
***	10^{10}	$\sim 10^{-2}$ (not measurable)

Table 3: If one assumes $\Omega_g \approx 10^{-14}$

$v(a) \approx v(a_{final}) \approx 10 - 10^2$	$v(a_{initial})$	n_f (from Eq. 1.5)
***	10^3	$\sim 10^{23}$
***	10^8	$\sim 10^3$
***	10^{10}	$\sim 10^{-6}$ (not measurable)

As will be explained in Appendix A, there is a way to make a relation between graviton count and entropy, so then the numbers associated with n_f are a de facto counting algorithm for entropy per unit phase space. Note that the highest counting numbers for entropy are associated with $\Omega_g \approx 10^{-5}$, which according to Fig 1 above is associated with pre big bang GW/ graviton production. Having $\Omega_g \approx 10^{-14}$ is associated with usual inflation, as given in Fig 1 above.

I.e. if one is looking for standard creation of entropy paradigms associated with the early universe, a typical phase transition argument for early entropy production is given by A. Tawfik [5], in 2008, which for QCD regimes

$$S_{total} \equiv V_3 \cdot T^3 \sim 2.05 \cdot 10^{58} \quad (1.6)$$

We assume that here, $S_{total} \sim 10^{58}$ may be associated with Graviton/ GW and with frequencies initially of the order of about 10^8 to 10^{10} in the beginning of cosmological evolution. Such a huge burst of graviton production would lead to measurable consequences. This is for temperatures of the order of $T \sim 174 \cdot MeV$, and if one factors in the volume of space time one is obtaining very likely values in between Tables 1 and 2 above

Consider if there is, then also a small graviton mass, i.e. as factored in, in

$$m_n(Graviton) = \frac{n}{L} + 10^{-65} \text{ grams} \quad (1.7)$$

Note that Rubakov [11] writes KK graviton representation as, after using the following normalization $\int \frac{dz}{a(z)} \cdot [h_m(z) \cdot h_{\tilde{m}}(z)] \equiv \delta(m - \tilde{m})$ where J_1, J_2, N_1, N_2 are different forms of Bessel functions, to obtain the KK graviton/ DM candidate representation along RS dS brane world

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \quad (1.8)$$

This Eq. (1.8) is for KK gravitons having a TeV magnitude mass $M_z \sim k$ (i.e. for mass values at .5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK graviton, which is moving with a speed proportional to H^{-1} with regards to the negative tension brane with $h \equiv h_m(z \rightarrow 0) = const \cdot \sqrt{\frac{m}{k}}$ as an initial starting value for the KK graviton mass, before the KK graviton, as a ‘massive’ graviton moves with velocity H^{-1} along the RS

dS brane. If so, and if $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$ represents an initial state, then one may relate the mass of the KK graviton, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass lower in value, i.e. of $m_{\text{graviton}} (4\text{-Dim GR}) \sim 10^{-48} eV$, as opposed to $M_X \sim M_{\text{KK-Graviton}} \sim .5 \times 10^9 eV$. Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky et.al. [12] who argue for graviton mass using CMBR measurements, of $M_{\text{KK-Graviton}} \sim 10^{-20} eV$. Also Eq. (1.9) will be the starting point used for a KK tower version of Eq. (1.9) below. So from Maarten's [14] per,

$$\dot{a}^2 = \left[\left(\frac{\tilde{\kappa}^2}{3} \left[\rho + \frac{\rho^2}{2\lambda} \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \right] \quad (1.9)$$

Maartens [14] gives a 2nd Friedman equation, as

$$\dot{H}^2 = \left[- \left(\frac{\tilde{\kappa}^2}{2} \cdot [p + \rho] \cdot \left[1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2 \frac{m}{a^4} + \frac{K}{a^2} \right] \quad (1.10)$$

Also, if we are in the regime for which $\rho \cong -P$, for red shift values z between zero to 1.0-1.5 with exact equality, $\rho = -P$, for z between zero to .5. The net effect will be to obtain, due to Eq. (1.10), and use $a \equiv [a_0 = 1] / (1 + z)$. As given by Beckwith [8]

$$q = - \frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{2}{1 + \tilde{\kappa}^2 [\rho/m] \cdot (1+z)^4 \cdot (1 + \rho/2\lambda)} \approx -1 + \frac{2}{2 + \delta(z)} \quad (1.11)$$

Eq. (1.10) assumes $\Lambda = 0 = K$, and the net effect is to obtain, a substitute for DE, by presenting how gravitons with a small mass done with $\Lambda \neq 0$, even if curvature $\mathbf{K} = 0$

2 Consequences of small graviton mass for reacceleration of the universe

In a revision of Alves *et. al.* [13] Beckwith [8] used a higher-dimensional model of the brane world and Marsden [10] KK graviton towers. The density ρ of the brane world in the Friedman equation as used by Alves *et. al.* [13] is use by Beckwith [8] for a non-zero graviton

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g \cdot (c=1)^6}{8\pi G (\hbar=1)^2} \right] \cdot \left(\frac{1}{14 \cdot (1+z)^3} + \frac{2}{5 \cdot (1+z)^2} - \frac{1}{2} \right) \quad (1.12)$$

I.e. Eq. (1.12) above is making a joint DM and DE model, with all of Eq. (1.12) being for KK gravitons and DM, and 10^{-65} grams being a 4 dimensional DE. Beckwith [15] found at $z \sim .4$, a billion years ago, that acceleration of the universe increased, as shown in Fig. 1. This would be a very good verification of Ng. hypothesis [15], and would check work done by Buonnano [16]

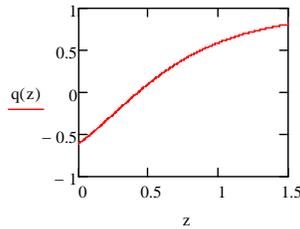


Fig. 2: Reacceleration of the universe based on Beckwith³ (note that $q < 0$ if $z < .423$)

Comparisons with axion flux results from the sun, i.e. what would be needed to measure DM flux for a new model of DM/ DE

The following plot, is a re do by the author , using Dimpoulou's et al's [17] value of axion flux, derived in 1986, which has some similarities with similar results cited by Lazaruth, et al. [18] , 1992, as to the flux of axions from the sun. It would be appropriate to do the same with the DM implied by Eq. (1.7) . Fig 2 below is plotted from a formula [17] which is one which eventually should be duplicated in some fashion for DM searches

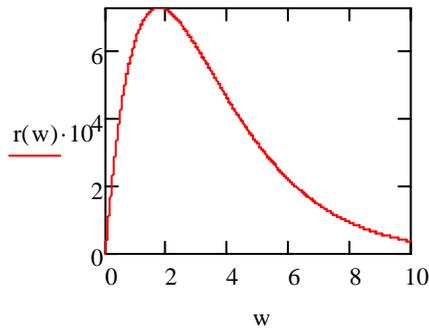


Fig 3 : Beckwith's re do of axion flux from the sun, in terms of (frequency=energy , if $\hbar = 1$), with plot in terms of KeV values of solar axions.

The rate equation, as used by Beckwith, is dominated by terms looking like $\omega \cdot \exp[-\omega + \varpi]$ with ϖ a to be determined plasma interaction effect, and ω part of the expression for permitted solar axion energy values. $\omega \gg \varpi$ in most cases. What the author thinks is appropriate, is to find grounds for a similar energy plot of DM value

from a suitably modified version of Eq. (1.7) above, for KK dark matter candidates. Doing so would mean understanding how a rate equation for DM production could commence using a model as to KK DM production/ evolution. The author thinks that an early universe counter part to the known axion production in stars model would be appropriate, with n_Z the number density of Z atoms with an ionized K shell, n_e the number density of free electrons, ρ a general density of states for the axion producing background, and σ the axio-recombination (free-bound) cross-section [17]

$$\Gamma_{\text{axion-production-in-stars}} = \sum_Z \frac{n_Z n_e \langle \sigma \cdot v_e \rangle}{\tilde{\rho}} \quad (1.13)$$

In the early universe, for a KK dark matter counter part we would still have a density value $\tilde{\rho}$, to consider, a possible σ for some interaction of KK DM production, but due to early universe conditions, there would be no counter part to n_Z , or n_e . Numerous authors, including [R. Durrer](#), [G. Marozzi](#), [M. Rinaldi](#) ;,[19] have used very early universe plasmas, going back to about the electro weak transition, and turbulence as a model for early universe GW production. One would need to specify how to obtain σ for some interaction of KK DM production which is why observation of the mass and width (or cross section) of one or more KK graviton, as part of a DM candidate, at the LHC, as remarked by Grzadkowski, et al, [20] may be the only way to obtain experimental inputs into a graviton production / KK DM version of Eq. (1.13) above. However, this leaves open the question if the cross section σ for the LHC values of massive gravitons, etc, would be the same as what would occur for early universe conditions. So far, the only possible known theoretical calculations of the above are along the lines of σ arising from photon and neutrino annihilation rates [21] lack of specific mechanisms for DM/ graviton productions are one of the many reasons why the author turned to Y. Ng's counting algorithm, for an equivalence between DM/Gravitons and entropy production, as given in Appendix A below [15]

Conclusion. We need to determine if GW/ Gravitons can do double duty as DM / DE candidates in cosmic evolution.

Beckwith [22,23], investigated if gravitons could be a graviton gas for a substitute for a vacuum energy, as well as considered a suggestion by Yurov [24], of double inflation which if verified would justify Fig 1 above. He looks forward to presenting elaborations of these ideas in fore coming conferences in 2010. It would be highly significant if semi classical treatments of the graviton can be shown to be consistent with Fig 2 above. And if there is a way to come up with a DM/DE phenomenology similar to Fig 3, via significant modification of the rate equation of axion in stars idea, to reflect early universe conditions.

Appendix A ENTROPY GENERATION VIA NG'S INFINITE QUANTUM STATISTICS

Information counting ties in with information packing as brought up in the use of small graviton creation volume, V ; for relic gravitons of a high frequency (short wave length) right after the big bang would be consistent Graviton volume V for nucleation is tiny, well inside inflation. So the log factor drops out of entropy S if V is chosen properly for both Eq. (A.1) and Eq. (A.2). Ng's [15] result begins with modification of the entropy/partition function Ng used in an approximation of temperature, starting with early temperature $T \approx R_H^{-1}$ (R_H can be thought of as a representation of the region of space of the particles in question). Furthermore, assume that the volume of space is of the form $V \approx R_H^3$ and look at a numerical factor $N \sim (R_H/l_p)^2$, where the denominator is Planck's length (on the order of 10^{-35} centimeters). We also specify a "wavelength" $\lambda \approx T^{-1}$. So the value of $\lambda \approx T^{-1}$ and of R_H are the same order of magnitude. Note Ng [15] changed conventional statistics: he outlined how to get $S \approx N$, or $S \approx \langle n \rangle$ (where $\langle n \rangle$ is graviton density). Begin with a partition function

$$Z_N \sim \left(\frac{1}{N!} \right) \cdot \left(\frac{V}{\lambda^3} \right)^N \quad (\text{A.1})$$

This, according to Ng, leads to entropy of the limiting value of, if $S = (\log[Z_N])$ will be modified by

$$S \approx N \cdot (\log[V/N\lambda^3] + 5/2) \xrightarrow{\text{Ng-inf inite-Quantum-Statistics}} N \cdot (\log[V/\lambda^3] + 5/2) \approx N \quad (\text{A.2})$$

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