The New Prime theorem（24）

Hardy-Littlewood conjecture K: x^3+k

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Abstract

Using Jiang function we prove Hardy-Littlewood conjecture K: x^3+k [4].

Theorem 1. Let m be an even number which is not a cube.

$$P_1 = P^3 + m \quad (m \neq a^3)\,. \qquad (1)$$

There exist infinitely many primes P such that P_1 is a prime.

Proof. We have Jiang function [1,2]

$$J_2(\omega) = \prod_P [P-1-\chi(P)]\,, \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$q^3 + m \equiv 0 \ (\mathrm{mod}\, P),\ q = 1, \cdots, P-1\,. \qquad (3)$$

We have

$$m^{\frac{P-1}{3}} \equiv 1 \quad (\mathrm{mod}\, P)\,. \qquad (4)$$

If (4) has a solution then $\chi(P)=3$. If (4) has no solutions then $\chi(P)=0$. $\chi(P)=1$ otherwise.

For every even number m we have

$$J_2(\omega) \neq 0\,. \qquad (5)$$

We prove that in (1) there are infinitely many prime soultions.
We have asymptotic formula [1,2]

$$\pi_2(N,2) = \left|\{P \le N : P_1 = prime\}\right| \sim \frac{J_2(\omega)\omega}{3\phi^2(\omega)} \frac{N}{\log^2 N}\,, \qquad (6)$$

where $\phi(\omega) = \prod_P (P-1)$.

In the same way we are able to prove $P_1 = P^3 - m$.

Theorem 2. Let n be an odd number which is not a cube

$$P_1 = (2P)^3 + n (n \neq a^3). \qquad (7)$$

There exist infinitely many primes P such that P_1 is a prime.

Proof. we have Jiang function [1,2]

$$J_2(\omega)=\prod_P [P-1-\chi(P)], \quad (8)$$

where $\chi(P)$ is the number of solutions of congruence.

$$(2q)^3+n\equiv 0 \pmod P, \ q=1,\cdots,P-1. \quad (9)$$

We have

$$n^{\frac{P-1}{3}}\equiv 1 \pmod P. \quad (10)$$

If (10) has a solution then $\chi(P)=3$. If (10) has no solutions then $\chi(P)=0$, $\chi(P)=1$ otherwise. For every odd number we have

$$J_2(\omega)\neq 0. \quad (11)$$

We prove that there are infinitely many prime solutions in (7).
We have asymptotic formula [1,2]

$$\pi_2(N,2)=\left|\{P\le N: P_1=prime\}\right| \sim \frac{J_2(\omega)\omega}{3\phi^2(\omega)}\frac{N}{\log^2 N}. \quad (12)$$

In the same way we are able to prove $P_1=(2P)^3-n$.

Remark. The prime number theory is basically to count the Jiang function $J_{n+1}(\omega)$ and Jiang prime k-tuple singular series $\sigma(J)=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1,2], which can count the number of prime number. The prime distribution is not random. But Hardy prime k-tuple singular series $\sigma(H)=\prod_P\left(1-\frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false [3-8], which can not count the number of prime numbers.

References

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Szemerédi's theorem does not directly to the primes, because it can not count the number of primes. It is unusable. Cramér's random model can not prove prime problems. It is incorrect. The probability of $1/\log N$ of being prime is false.

Assuming that the events "P is prime", "$P+2$ is prime" and "$P+4$ is prime" are independent, we conclude that P, $P+2$, $P+4$ are simultaneously prime with probability about $1/\log^3 N$. There are about $N/\log^3 N$ primes less than N. Letting $N\to\infty$ we obtain the prime conjecture, which is false.

The tool of additive prime number theory is basically the Hardy-Littlewood prime tuple conjecture, but can not prove and count any prime problems[6].