Neutrosophy in Situation Analysis

Anne-Laure Jousselme and Patrick Maupin
Defence R & D Canada - Valcartier
Decision Support Systems
2459, Pie XI Blvd North
Val-Bélair, QC, G3J 1X5, CANADA
{Anne-Laure.Jousselme,Patrick.Maupin}@drdc-rddc.gc.ca

Abstract – *In situation analysis (SA), an agent observing a scene receives information from heterogeneous sources of information including for example remote sensing devices, human reports and databases. The aim of this agent is to reach a certain level of awareness of the situation in order to make decisions. For the purpose of applications, this state of awareness can be conceived as a state of knowledge in the classical epistemic logic sense. Considering the logical connection between belief and knowledge, the challenge for the designer is to transform the raw, imprecise, conflictual and often paradoxical information received from the different sources into statements understandable by both man and machines. Hence, quantitative (i.e. measuring the world) and qualitative (i.e. reasoning about the structure of the world) information processing coexist in SA. A great challenge in SA is the conciliation of both aspects in mathematical and logical frameworks. As a consequence, SA applications need frameworks general enough to take into account the different types of uncertainty and information present in the SA context, doubled with a semantics allowing meaningful reasoning on situations. The aim of this paper is to evaluate the capacity of neutrosophic logic and Dezert- Smarandache theory (DSmT) to cope with the ontological and epistemological problems of SA.*

Keywords: Situation analysis, possible worlds, neutrosophic logic, Dezert-Smarandache theory.

1 Introduction

In Situation Analysis (SA), an agent observing a scene receives information from heterogeneous sources of information including for example remote sensing devices, human reports and databases. The aim of this agent is to reach a certain awareness of the situation in order to take decisions. For the purpose of applications, this state of awareness can be conceived as a state of knowledge in the classical epistemic logic sense. Considering the logical connection between belief and knowledge, the challenge for the designer is to transform the raw, imprecise, conflictual and often paradoxical information received from the different sources into statements understandable by both man and machines. Hence, quantitative (*i.e. measuring the world*) and qualitative (*i.e. reasoning about the structure of the world*) information processing coexist in SA. A great challenge in SA is the conciliation of both aspects in mathematical and logical frameworks. As a consequence, SA applications need frameworks general enough to take into account the different types of uncertainty and information present in the SA context, doubled with a semantics allowing meaningful reasoning on belief, knowledge and situations.

The aim of this paper is to evaluate the potential of neutrosophic logic and Dezert-Smarandache theory (DSmT) to cope with the ontological and epistemological obstacles in SA (section 3), *i.e.* problems due to the nature of things and to cognitive limitations of the agents, human or artificial. A particularity of SA is that most of the time it is impossible to list every possible situation that can occur. The elements of the corresponding frame of discernment cannot, thus, be considered as an exhaustive list of situations. Furthermore, in SA situations are not clearcut elements of the frame of discernment (section 4). Considering these particular aspects, especially the richer ontology on which it is based, DSmT appears as an appropriate modeling tool for uncertainty in SA (section 5.2). On the other hand, we assess the ability of neutrosophic logic to process symbolic and numerical statements on belief and knowledge using the possible worlds semantics (5.3). Moreover, we investigate the representation of neutrosophic concepts of neutrality and opposite in the possible worlds semantics for situation modelization.

2 Situation analysis

The term *situation* appears in the mid-fourteenth century derived from medieval Latin *situatio* meaning *being placed into a certain location*. By the middle of the seventeenth century situation is used to discuss the moral dispositions of a person, more specifically the set of circumstances a person lies in, the relations linking this person to its *milieu* or surrounding *environment*. As will be shown below, the latter definition is close to what is meant today in the field of High-Level Data Fusion, where the mental state of *situation awareness* is studied in interaction with the surrounding environment. Common synonyms of situation with a corresponding meaning are *setting, case, circumstances, condition, plight, scenario, state, picture, state of affairs*.

Although the notion of situation is used informally in everyday language to designate a given state of affairs, a simplified view of the world, and even the position of certain objects, situation is nowadays a central concept in High-Level Data Fusion where it has been given more or less formal definitions. For Pew [1], a situation is "*a set of envi-*

ronmental conditions and system states with which the participant is interacting that can be characterized uniquely by a set of information, knowledge, and response options".

2.1 Situation awareness as a mental state

For Endsley and Garland [2] *Situation awareness* (SAW) is "*the perception of the elements in the environment within a volume of time and space, the comprehension of their meaning and the projection of their status in the near future*" (Fig. 1). SAW is also defined in [3] as "the active mental *representation* of the status of current cognitive functions activated in the cognitive system in the context of achieving the goals of a specific task". In particular, SAW involves three key tasks: (1) Perception, (2) Comprehension and (3) Projection, in a general multiagent context.

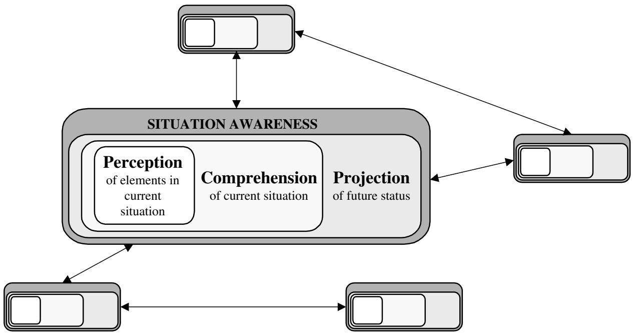

Fig. 1: The three basic processes of situation awareness according to Endlsey and Garland (modified from [2]), in a multi-agent context.

In contemporary cognitive science the concept of *mental representation* is used to study the interface between the external world and mind. Mental states are seen as relations between agents and mental representations. Formally, and following Pitt's formulation [4], for an agent to be in a psychological state Ψ with semantic property Γ is for that agent to be in a Ψ-appropriate relation to a mental representation of an appropriate kind with semantic property Γ. As far as mental states are concerned, purely syntactic approaches are not adequate for representation since semantic concepts need to be modeled.

2.2 Situation Analysis as a process

For Roy [5] "*Situation Analysis is a process, the examination of a situation, its elements, and their relations, to provide and maintain a product, i.e. a state of* Situation Awareness *(SAW) for the decision maker*". For a given situation the SA process creates and maintains a mental representation of the situation. Situation analysis corresponds to the levels 2, 3 and 4 of the JDL data fusion model [6, 7], hence to higher-levels of data fusion. A revisited version of the well-known model is presented on figure 2, with classical applications associated to the different levels. A complete situation model must take into account the following tasks of: A. Situation *perception* composed of Situation Element

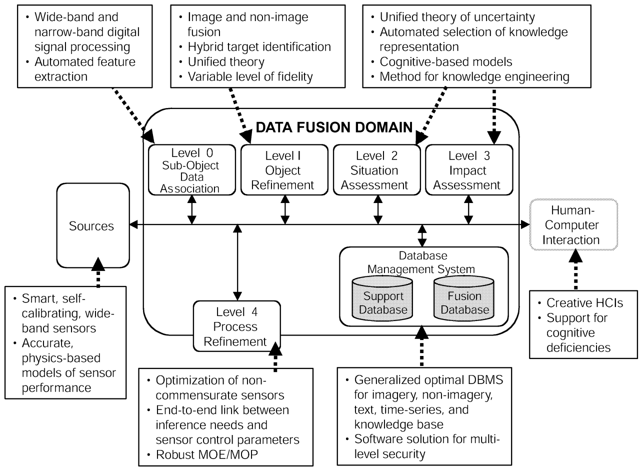

Fig. 2: Revisited JDL data fusion model and applications [8].

Acquisition, Common Referencing, Perception Origin Uncertainty Management, and Situation Element Perception Refinement as subtasks. B. Situation *comprehension* composed of Situation Element Contextual Analysis, Situation Element Interpretation, Situation Classification, Situation Recognition, and Situation Assessment as subtasks. C. Situation *projection* composed of Situation Element Projection, Impact Assessment, Situation Monitoring. Situation Watch, and Process Refinement [5].

The conception of a system for SA must rely on a mathematical and/or logical formalism capable of translating the mechanisms of the SAW process at the human level. The formalism should also allow the possibility to encompass the case of multi-agent systems in which the state of awareness can be distributed over several agents rather than localized. A logical approach based on a possible worlds semantics for reasoning on belief and knowledge is proposed in [9]. This work by Halpern can be used as a blueprint considering that it allows to handle numerical evaluations of probabilities, thus treating separately but nevertheless linking belief, knowledge and uncertainty.

Furthermore, mathematical and logical frameworks used to model mental states should be able to represent and process autoreference such as beliefs about one's own beliefs, beliefs about beliefs about . . . and so on.

3 Obstacles to estimation and prediction in Situation Analysis

There are two kinds of limits to state estimation and prediction in Situation Analysis. *Ontological limits* due to the nature of things and *epistemic limits* due to cognitive limitations of the agents, human or artificial.

Typical obstacles [10] are *anarchy* and *instability* when the situation is not governed by an identifiable law or in the absence of nomic stability. *Chance* and *chaos*, are serious obstacles to state evaluation and prediction as far as an exact estimation is sought for although regularities and determinism are observed. Another typical obstacle is the *vagueness* of concepts. Natural language concepts are inherently vague, meaning that their definition is approximate and borderline cases arise. This is true as well for properties but also for concepts.

Indeterminacy is another unavoidable obstacle. It may arise from paradoxical conclusions to a given inference (*i.e.* Russell's paradox), from impossible physical measurements (*i.e.* position *and* speed of an atomic particle) or for practical reasons (*i.e.* NP-complete problems). Indeterminacy may also be proposed as a conclusion to specific unanswerable questions in order to nevertheless allow reasoning using the remaining information.

Ignorance of the underlying laws governing the situation is a major cause of uncertainty. For example not knowing that a given tactical maneuver is possible precludes the possibility to predict its occurrence. Especially present in human affairs *innovation* can be a major obstacle in SA. New kinds of objects (weapons), processes (courses of action) or ideas (doctrines) arise and one has no choice but to deal with it and adapt.

Myopia or data ignorance, is also a typical problem in SA. Data must be available on time in order to assess a situation, meaning that even if the information sources exist circumstances can prevent their delivery. Another case of myopia occurs when data is not available in sufficient detail, as in pattern recognition when classes are only coarsely defined or when sensors have limited spatial resolution. Data is thus accessible through estimations obtained by sampling as in surveys, by the computation of aggregates as in Data Fusion or by the modelization of rough estimates. As a consequence the available data is only imprecise and incomplete and leads most of the time to conflicting choices of decision.

Any attempt in the conception of a system is be bounded by *inferential incapacity* of human or artificial agents. Limitations in agents can arise because of a lack of awareness. As far as knowledge is concerned, an agent cannot always give a value to a proposition, for example if it is not even aware of the existence of the concept denoted by the proposition at hand. Agents are *resource bounded* meaning that agents have only limited memorization capabilities, in some cases they have power supply limitations, etc. or have only limited cognitive and computational capabilities. Agents may also have limited visual or auditory acuity. Sometimes, these limitations come from the outside and are situation driven: electronic countermeasures, only a limited amount of time or money is available to do the job, etc. Furthermore agents cannot focus on all issues simultaneously. As Fagin and Halpern puts it in [11] "[...] Even if $\mathcal{A}$ does perfect reasoning with respect to the limited number of issues on which he is focusing in any given frame of mind, he may not put his conclusions together. Indeed, although in each frame of mind agent $\mathcal{A}$ may be consistent, the conclusions $\mathcal{A}$ draws in different frames of mind may be inconsistent." Finally, agents must work with an inconsistent set of beliefs. For example, we know that lying is amoral, but in some case we admit it could be a good alternative to a crisis.

4 Basic principles in Situation Analysis

Given the causes of uncertainty and the constraints linked to it, we identify three basic principles which should guide SA problem-solving: (1) allowing statements about uncertainty to be made, (2) enrichment of the universe of discourse, (3) allowing autoreference.

4.1 Statements about uncertainty

Uncertainty has two main meanings in most of the classical dictionaries [12]: Meaning I - Uncertainty as a state of mind; Meaning II - Uncertainty as a physical property of information. The first meaning refers to the state of mind of an agent, which does not possess the needed information or knowledge to make a decision; the agent is in a state of uncertainty: "I'm not sure that this object is a table". The second meaning refers to a physical property, representing the limitation of perception systems: "The length of this table is uncertain". In theories of uncertain reasoning, uncertainty is often described as imperfection of information, as errors on measures for example, and does not depend on any kind of state of mind. As a sociologist, Gérald Bronner [13] considers uncertainty as a state of mind, this state depending on our power on the uncertainty, and our capacity to avoid it. He distinguishes two types of uncertainty: *uncertainty in finality* (or material uncertainty) and *uncertainty of sense*. Uncertainty in finality is "the state of an individual that, wanting to fulfill a desire, is confronted with the open field of the possibles" (ex.: Will my car start?). Whereas uncertainty of sense is "the state of an individual when a part, or the whole of its systems of representation is deteriorated or can be". Uncertainty in finality corresponds to the uncertainty in which lies our understanding of the world, while uncertainty of sense bears on the representation of the world. Bronner classifies uncertainty in finality into three types, according to one's power on uncertainty, and the capacity to avoid it: Situation of type I: Uncertainty does not depend on the agent and can not be avoided; Situation of type II: Uncertainty does not depend on the agent but can be avoided; Situation of type III: Uncertainty is generated by the agent and can be avoided.

In situation analysis, agents are confronted to uncertainty of sense (data driven) from the bottom-up perspective and to uncertainty in finality (goal driven) from the top-down perspective.

Propositional Calculus (PC) relies on the principle of bivalence expressing the fact that a proposition is either TRUE or FALSE. Hence, only two truth values are allowed leaving no way to express uncertainty. There are many ways go beyond bivalence. The most common is to introduce supplementary truth values in the PC framework, and then reject the principle of bivalence. The signification of the supplementary truth value differs from one author to another, from one logic to another. However, it is common to denote truth, falsity and indeterminacy by 1, 0 and $\frac{1}{2}$ respectively. A three-valued logic can be generalized to an n-valued logic, and by extension to fuzzy logic, with an infinite number of truth-values ranging on the real set interval $[0; 1]$.

In neutrosophic logic, another approach is adopted to represent uncertainty. An indeterminacy assignment is explicitly defined, conjointly and independently with truth and falsity assignments. Instead of manipulating a single value (as it is the case even in three-valued logics), a triplet of values is considered simultaneously.

4.2 Enrichment of the universe of discourse

An ontology with a fixed number of objects is often insufficient to describe the complete situation. That is why ontology needs to be enriched, and this task can be achieved in different manners.

Instead of assigning measures to a given set S, one can consider its power set (*i.e.* the set of all subsets of S). This leads to an enrichment of the ontology (the set of measurable propositions is augmented artificially) allowing ignorance to be best represented, as well as a supplementary types of conflict to be taken into account. If probability theory is based on the classical set notion, the notion of power set is the basis of Dempster-Shafer theory, possibility theory and rough sets theory.

Hence, yet another extension in this direction is the construction of the hyper-power set constituted of all the combinations of the union and intersection operators applied to the elements of S. Besides enriching the ontology, this structure allows one to account for vague concepts as their intersection is considered. Extending the definition of the probability measure the hyper-power set is the principle of Dezert-Smarandache theory.

Fuzzy sets are another means to represent vague (fuzzy) concepts by allowing elements to belong more or less to a given set. This is accomplished by associating to each of its elements a membership degree to this set, *i.e.* a value varying between 0 and 1, defining a membership function for this set.

Rough sets, allowing the representation of indiscernability between elements is another way to deal with vagueness. This notion presupposes a definition of a frame of discernment, a set of distinct, exhaustive and exclusive objects. This frame of discernment is supposed to be the finer accessible refining. A partition of indiscernible objects representing our limited knowledge is built from this frame of discernment. A vague concept is then represented by its lower and upper bounds being unions of elements of the partition. The set is vague since its indiscernible elements belong and not belong to this set.

4.3 Autoreference

The notion of hyperset has been introduced by Aczel [14] and Barwise and Etchemendy [15] to overcome Russell's paradox. A recursive definition extends the notion of classical set, leading to infinitely deep sets (for example, $x = 1 + 1/x$). Hence, the ontology is enriched. With the notion of hyperset comes the graph metaphor which replaces the "container" metaphor. According to Barwise and Moss [16], the notion of hypersets "*transposes the limitation of the size of sets doctrine (separation axiom) to the domain of semantics*". They are for example used in situation theory for autoreferential sentences like "I lie". Since hypersets theory encompasses classical sets as a special case and since DSmT is built upon a notion of classical set, it could be possible to enrich DSmT with hypersets thus allowing self-referential statements.

5 Neutrosophic frameworks for Situation Analysis

The possible world semantics provides an intuitive means for reasoning about situations. It delivers a general approach to providing semantics to logical approaches with applicability to neutrosophic logic (section 5.2). However, possible worlds semantics is often borrowed from logical approaches to fill the lack of semantics of numerical approaches, as it will be detailed below.

5.1 Possible worlds semantics

A Kripke model [17] is a mathematical structure that can be viewed as a *directed labeled graph*. The graph's nodes are the possible worlds s belonging to a set S of possible worlds, labeled by truth assignments π. A world s is considered *possible* with respect to another world s' whenever there is an edge linking s and s'. This link is defined by an arbitrary binary relation, technically called the *accessibility relation*. More formally,

> Assume a set Φ of propositional atoms. A *Kripke model* is a triple structure $\mathcal{M}_K$ of the form $\langle S, \pi, \mathsf{R}\rangle$ where
>
> - S is a non-empty set (the set of possible worlds);
> - $\pi : S \longrightarrow (\Phi \longrightarrow \{0;1\})$ is a truth assignment to the atoms per possible world;
> - $\mathsf{R} \subseteq S \times S$ is the accessibility relation.
>
> where $\{0;1\}$ states for $\{\text{TRUE}; \text{FALSE}\}$.

Hence, for each $s \in S$, there is an associated truth assignment $\pi(s)$ defined from Φ to $\{0;1\}$ such that:

$$\pi(s)(\phi) = \begin{cases} 1 \text{ if } s \vDash \phi \\ 0 \text{ if } s \nvDash \phi \end{cases} \tag{1}$$

where ϕ is a propositional atom of Φ. $s \vDash \phi$ means that the world s entails the proposition ϕ, or in other words, that ϕ is TRUE in s.

Truth set - To each ϕ of Φ, there is an associated truth set A_ϕ of all the elements of S for which $\pi(s)(\phi)$ is TRUE:

$$A_\phi = \{s \in S | \pi(s)(\phi) = 1\} \tag{2}$$

A_ϕ is then the set of possible worlds in which ϕ is TRUE, and can also be noted $A_\phi = \{s \in S | s \vDash \phi\}$.

5.2 Probability assignations and structures

Let S be the frame of discernment, s a singleton of S and A any subset of S. In probability theory, measurable objects are singletons s of S. The measures assigned to any subsets A of S are guided by the additivity axiom. Hence, measurable elements belong to a σ-algebra χ of 2^S. In Dempster-Shafer theory [18, 19], any element of the power set of S, 2^S are measurable. Finally, Dezert-Smarandache theory allows any element of the hyper-power set of S, D^S, to be measured. Apart these extensions to probability theory that rely on the definition set of the probability measure, there exists a clear interest for giving a better semantics to these numerical approaches. For its probabilistic logic, Nilsson uses the possible worlds semantics to build a "semantical generalization of logic", combining logic with probability theory [20]. Later on, Fagin and Halpern [21] and also Bundy [22] extend Nilsson's structure for probabilities allowing all elements of the power set to be measurable, leading to a general structure just as Dempster-Shafer theory generalizes probability theory.

5.2.1 Nilsson structure

A *Nilsson structure* is a tuple $\mathcal{S}_N = \langle S, \chi, p, \Phi, \pi \rangle$ where

- $S = \{s_1, s_2, s_3, \ldots\}$, the set of all possible worlds;
- χ, a σ-algebra of subsets of S;
- p, a probability measure on S;
- Φ, the set of propositions;
- π, a mapping $\pi : \Phi \to 2^S$, characterizing for each $\phi \in \Phi$ the set of possible worlds $A_\phi = \{s \in S$ in which ϕ is TRUE$\}$.

In a Nilsson structure, p is defined on χ (the set of measurable subsets) but not on 2^S. In other words, χ_π (the image of χ by π) is assumed to be a sub-algebra of χ to ensure that $p(\phi) = p(A_\phi)$. Giving up this condition is a means to extend p to 2^S (hence Nilsson structure) and leads to Dempster-Shafer (DS) structure as formalized in [21][1].

5.2.2 Dempster-Shafer structure

A *Dempster-Shafer structure* [21] is a tuple $\mathcal{S}_{DS} = \langle S, \chi, p, \Phi, \pi \rangle$ in which χ and π are not required to be related in any sense. Hence, instead of a single probability measure p from χ to $[0, 1]$, a Dempster-Shafer structure gives a pair of probability measures p_* and p^*, known respectively as *inner* and *outer extensions* ($p_*(A) = \sup\{p(B)|B \subseteq A, B \in \chi\}$ and $p^*(A) = \inf\{p(B)|B \supseteq A, B \in \chi\}$), and the value $p(A_\phi)$ is replaced by the interval:

$$p_*(A_\phi) \leq p(A_\phi) \leq p^*(A_\phi) \qquad (3)$$

A Nilsson structure is then a special case of Dempster-Shafer structures, in which

$$p_*(A_\phi) = p^*(A_\phi) = p(A_\phi) \qquad (4)$$

for any $\phi \in \Phi$.

[1] Another way is to consider a partial mapping π, leading to Bundy's structure of incidence calculus [22].

5.2.3 Dezert-Smarandache structure

In an equivalent manner to the extension of Nilsson's structure to DS structure, the definition of p can be extended to D^S, allowing all elements of the hyper-power set to have non-null probability. We obtain then what we can call a *Dezert-Smarandache structure* (DSm structure), an extension of the DS structure in an equivalent way as DSmT is an extension of Dempster-Shafer theory.

One benefit of the resulting structure for situation analysis, is that it provides an interesting framework for dealing with both vagueness and conflict, combining the logical, semantical and reasoning aspect through the possible worlds semantics, and the measuring, combination aspect through the DSmT.

5.2.4 Example: Ron suits

This example is proposed in [21] as *Example 2.4*:

> "*Ron has two blue suits and two gray suits. He has a very simple method for deciding what color suit to wear on any particular day: he simply tosses a (fair) coin. If it lands heads, he wears a blue suit and if it lands tails, he wears a gray suit. Once he's decided what color suit to wear, he just chooses the rightmost suit of that color on the rack. Both of Ron's blue suits are single-breasted, while one of Ron's gray suit is single-breasted and the other is double-breasted. Ron's wife, Susan, is (fortunately for Ron) a little more fashion-conscious than he is. She also knows how Ron makes his sartorial choices. So, from time to time, she makes sure that the gray suit she considers preferable is to the right (which depends on current fashions and perhaps on other whims of Susan). Suppose we don't know about the current fashion (or about Susan's current whims). What can we say about the probability of Ron's wearing a single-breasted suit on Monday?* [21]"

Let P be a set of primitive propositions, $P = \{p_1, p_2\}$. Let p_1="The suit is gray" and let p_2="The suit is double-breasted". S is the set of possible states of the world, *i.e.* the set of possible worlds, where a state corresponds in this example to a selection of a particular suit by Ron. To fix the ideas, let number the suits from 1 to 4. Hence, $S = \{s_1, s_2, s_3, s_4\}$, s_i being the world in which Ron chooses the suit i. Table 1 give some sets of worlds of interest and their associated formula. To describe the state of a world (*i.e.* the truth values of each propositions in P) we use π, the truth assignment. For each s in S, we have a truth assignment $\pi(s)$ defined from P to $\{0; 1\}$, such that $\pi(s)(p) = 0$ if p is false in s, and $\pi(s)(p) = 1$ if p is true in s.

Here, we have only 4 measurable events: $\mu(s_1, s_2) = \mu(s_3, s_4) = \frac{1}{2}$, $\mu(\varnothing) = 0$ and $\mu(S) = 1$. The question of interest here (What is the probability of Ron's wearing a single-breasted suit?) concerns another non-measurable event, *i.e.* (s_1, s_2, s_3). In [21], the authors gave this example to illustrate the utility of attributing values to non-measurable events, and then introduce Demspter-Shafer

Table 1: Some subsets of possible worlds of interest and their associated formula.

World(s)	Meaning	Formula
(s_1, s_2)	A blue suit	$\neg p_1$
(s_3, s_4)	A gray suit	p_1
(s_1, s_2, s_3)	A single-breasted suit	$\neg p_2$

structures. Their conclusion for this example is then that the best we can say is that $\frac{1}{2} \leq \mu(s_1, s_2, s_3) \leq 1$, based on the inner and outer measures.

Modeling the problem with 4 states means that given our prior knowledge, these states correspond to the only possible situations after Ron's selection: He will select one and only one suit among the 4 available. However, suppose that the two parts of the suits may have been mixed so we have two pieces (trousers and jacket) on the same coat-hanger. The 4 possible worlds correspond then to the 4 coat-hangers, and no longer to the 4 distinct suits. Imagining that the trousers is inside the jacket, Ron will select his suit only on the basis of the color of the jacket. Suppose for example, that the coat-hanger he selects supports a blue jacket and gray trousers. Then, waht is the corresponding state of the world? Clearly, this situation has not been considered in the modelisation of the problem, based on a DS structure. However, using a DSm structure allow the elements of the hyperpower set of S to be measurable. Hence, the state resulting of a selection of a mixed suit corresponds to $s_i \wedge s_j$, with $i \neq j$. This means that we are in both worlds s_i and s_j, and that with a single selection, Ron selected in fact two suits. So, we allow other events than those forecast to overcome.

5.3 Possible worlds semantics for neutrosophic logic

Neutrosophic logic is presented as a general framework for logical approaches [23], as it is extended in three distinct directions:

1. With ϕ, are considered Non-ϕ (what is not ϕ), Anti-ϕ (the opposite of ϕ), Neut-ϕ (what is neither ϕ nor Anti-ϕ) and ϕ' (a version of ϕ);

2. The semantics is based on **three** assignments, not a single one as it is commonly used in the other logics;

3. These three "truth" assignments take their values as **subsets** of the **hyperreal** interval $]^-0, 1^+[$, instead in $[0, 1]$.

While in a Kripke model, ϕ can only be TRUE, *i.e.* $\pi(s)(\phi) = 1$ or FALSE *i.e.* $\pi(s)(\phi) = 0$, ϕ is allowed to be $T\%$ TRUE and $F\%$ FALSE, and $I\%$ INDETERMINATE in neutrosophic logic. ϕ is thus characterized by a triplet of truth-values, called the *neutrosophical value*:

$$\text{NL}(\phi) = (T(\phi), I(\phi), F(\phi)) \tag{5}$$

where $(T(\phi), I(\phi), F(\phi)) \subset]^-0, 1^+[^3$, $]^-0, 1^+[$ being an interval of hyperreals.

The "truth" assignment π becomes then $\pi = (\pi_T, \pi_F, \pi_I)$, a three-dimensional assignment, where π_T is the truth assignment, π_F is the falsity assignment and π_I is the indeterminacy assignment. Hence, in each possible world s of S, a proposition ϕ can be evaluated as $\pi_T(s)(\phi)$ TRUE, $\pi_F(s)(\phi)$ FALSE and $\pi_I(s)(\phi)$ INDETERMINATE. It follows that to ϕ is associated a truth-set A_ϕ^T, a falsity-set A_ϕ^F and an indetermincay-set A_ϕ^I:

$$A_\phi^T = \{s \in S | \pi_T(s)(\phi) \neq 0\}$$
$$A_\phi^F = \{s \in S | \pi_F(s)(\phi) \neq 0\}$$
$$A_\phi^I = \{s \in S | \pi_I(s)(\phi) \neq 0\}$$

Note that A_ϕ^T, A_ϕ^T and A_ϕ^T are fuzzy sets and may overlap.

Knowledge and belief - Halpern in [24] gives the following definitions for knowledge and belief in PWS:

- ϕ is **known** if it is TRUE in **all** the possible worlds s of S

- ϕ is **believed** if it is TRUE in **at least one** possible world s of S

On the other hand, Smarandache [25] uses the notion of world and states that $T(\phi) = 1^+$ if ϕ is TRUE in **all** the possible worlds s of S (absolute truth) and $T(\phi) = 1$ if ϕ is TRUE in **at least one** possible world s of S (relative truth) (see Tab. 5.3). Hence, in the neutrosophical framework, we can state the following definitions for knowledge and belief: ϕ is **known** if $T(\phi) = 1^+ \equiv F(\phi) =^- 0$ and ϕ is **believed** if $T(\phi) = 1 \equiv F(\phi) = 0$. Table 5.3 shows several special cases.

Table 2: Neutrosophical values for special cases (adapted from [25]).

ϕ is ...	in ... poss. world(s)	Neutrosophical value
true false indet.	**all**	$T(\phi) = 1^+ \equiv F(\phi) =^- 0$ $F(\phi) = 1^+ \equiv T(\phi) =^- 0$ $I(\phi) = 1^+$
true false indet.	**at least one**	$T(\phi) = 1 \equiv F(\phi) = 0$ $F(\phi) = 1 \equiv T(\phi) = 1$ $I(\phi) = 1$
indet.	**no**	$I(\phi) =^- 0$
not indet.	**at least one**	$I(\phi) = 0$

Furthermore, one can consider the unary operators of neutrosophic logic (Non-ϕ, Anti-ϕ, Neut-ϕ, ϕ') to model new epistemic concepts but also as a means to represent situational objects, such as neutral situation, environment.

6 Conclusion

In this paper, we proposed a discussion on neutrosophy and its capacities to encompass the situation analysis challenges. In particular, we underlined three basic principles that should guide the modelization in Situation Analysis: (1) allowing statements about uncertainty to be made, (2) enrichment of the universe of discourse, (3) allowing autoreference. It is in this frame, that the advantages of DSmT and neutrosophic logic were studied. In particular, we showed that it is feasible to build a DSm structure upon the possible worlds semantics, and we illustrated it by an example. Extending the classical set structure of DSmT to an hyperset one, doubled with the possible worlds semantics could allow auto-referential statements on mental states. Considering neutrosophic logic, we showed that is could be possible to extend Kripke structures in order to take into account triplets of truth assignments. We also show how to represent the concepts of belief and knowledge with hyperreal truth (resp. falsity, indeterminacy) assignments on possible worlds. This allows one to clearly distinguish certain belief from knowledge.

An extended version of this paper will be published in [26].

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