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The Tsallis Entropy and the Boltzmann Entropy Applicable to the Same Classical Generalized System

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***Abstract*— It is demonstrated clearly that for the same classical generalized system the Tsallis power-laws with both the $q > 1$ and the $q < 1$ can be induced by the constraint of the constant harmonic mean for the so-called reciprocal energies E_r and at the same time the Boltzmann distribution or the negative exponential probability distribution can be generated with the constraint of the constant arithmetic mean for the generalized energies E. The author thus argues that there might be no definite "extensive system" or "classical system" and there are only "classical physical parameters" and "classical constraints". For any physical system or generalized system, it is the non-natural constraints which determine both the forms of the entropies and the non-uniform equilibrium distributions.**

***Keywords*— Tsallis entropy, Boltzmann-Gibbs-Shannon entropy, extensive system, nonextensive systems, Tsallis power-law, Boltzmann distribution, Tsallis q-parameter.**

I. INTRODUCTION

It is well known that under the constraint of the constant arithmetic mean of the physical energy, the physical system will follow the Boltzmann distribution or the negative exponential probability distribution. This fact was explained by Jaynes through maximizing the Boltzmman-Gibbs-Shannon entropy under the given constraints of both the natural constraint and the constant arithmetic mean [1]. In previous work by the author it was demonstrated in theory that a constant statistical harmonic mean of the effective energies of a generalized system may induce the standard form of Tsallis' power law when the Tsallis Entropy with the Tsallis q-parameter larger than one is maximized [2][3][4][5]. In this paper, the author likes to show an interesting fact of that for the same classical generalized system while the constraint of the constant arithmetic mean for the generalized energies E results in the classical Boltzmann distribution or the negative exponential probability distribution, the constraint of the constant harmonic mean for the so-called reciprocal energies E_r may lead to the Tsallis power-laws with both the $q > 1$ and the $q < 1$. Based these facts of the numerical experiments, some discussions are made.

II. THE SIMULATION FOR A CLASSICAL GENERALIZED SYSTEM WITH MATLAB

Imagine you cut a tangled skein of jute into many segments with a sharp knife and ask yourself the following scientific question: "What is the probability distribution of the lengths of all the segments ? ". Zhang and his co-worker [6] made a numerical simulation and a theoretical analysis with the maximal entropy principle developed by Jaynes [1]. Both the theory and the results of the numerical experiments demonstrated clearly that the lengths of the segments follow a negative exponential probability distribution or the Boltzmann distribution. Zhang's simulation system is very classical and the results are well known. In this paper the author uses the same classical system to make a MATLAB simulation and to show some interesting new results. It is assumed that the readers are familiar with MATLAB and some sentences of MATLAB language are used directly to express some mathematical equations. The detailed algorithm is shown as follows.

1) Generate 10,000,000 random numbers with MATLAB simulating cutting a tangled skein of jute into segments with a sharp knife

$$r = a * rand(10000000,1) + b \qquad \text{(Eq.1)},$$

where $a = 10000$, and $b = 5000$.

2) Sort these random numbers in ascending order.

$$r_1 = sort(r) \qquad \text{(Eq.2)},$$

3) Add the possible minimum and maximum of the random number r into the expanded, sorted and transposed array of r_2. In the following equation 3, the r_1' is the transpose of the r_1.

$$r_2 = [5000\ \ r_1'\ \ 10000] \qquad \text{(Eq.3)},$$

4) Calculate the differences E for the pairs of the adjacent elements in the r_2 which are the lengths of the segments, and define the differences E as the generalized energies of the generalized system.

$$E = r_2(2:end) - r_2(1:end-1) \qquad \text{(Eq.4)},$$

5) Define the reciprocal energies E_r of the generalized energies E as

$$E_r = \frac{1}{(1+\gamma E)} \qquad \text{(Eq.5)},$$

It is easy to show theoretically that

$$\sum_{i=1}^{S} E_i = const \qquad \text{(Eq.6)},$$

and for a given γ,

$$\sum_{i=1}^{S} \frac{1}{E_{ri}} = const \quad \text{(Eq.7)},$$

where E_i and E_{ri} are the ith elements of the generalized energies E and the reciprocal energies E_r respectively, The $const$ is meant by a constant independent of the random numbers generated by the equation 1, and the S is the sample size of the segments. In this paper, $S = 10000001$. The results of the numerical experiments indicated that although the generalized energies E and the reciprocal energies E_r change greatly from one experiment to another, the sums in the equations 6-7 do not change except for a small numerical errors. It is meant by the equation 7 that the harmonic mean of the reciprocal energies E_r, proportional to the reciprocal of the sum shown in the equation 7, is a constant.

6) Get the probability distributions for the 100 uniform intervals of both the generalized energies E and the reciprocal energies E_r. The centers of the intervals or the discrete samples denoted as X_1 and X_2 for both the generalized energies E and the reciprocal energies E_r are calculated with the following formula.

$$X_1 = \min(E) + (rank - \frac{1}{2}) * step_1 \quad \text{(Eq.8)},$$

$$X_2 = \min(E_r) + (rank - \frac{1}{2}) * step_2 \quad \text{(Eq.9)},$$

where $rank = [1:100]$, and

$$step_1 = \frac{\max(E) - \min(E)}{100} \quad \text{(Eq.10)}$$

$$step_2 = \frac{\max(E_r) - \min(E_r)}{100} \quad \text{(Eq.11)}$$

The uniformity of the 100 intervals makes the probabilities proportional approximately to the values of the pdf or the probability density functions about the both the generalized energies E and the reciprocal energies E_r.

III. THE SIMULATION RESULTS

The standard form of the Tsallis power-law for the reciprocal energies E_r as an effective energies shown in [2] can be expressed as [2][5]

$$P_k \propto (E_r)^{\frac{-1}{(q-1)}} \quad \text{(Eq.12)},$$

and the normalized probabilities can thus be described as

$$p_k = \frac{(E_r)^{\frac{-1}{(q-1)}}}{\sum (E_r)^{\frac{-1}{(q-1)}}} \quad \text{(Eq.13)},$$

For the generalized energies E the probability following the Boltzmann distribution or the negative exponential probability distribution can be described as

$$P_{Bk} \propto \exp(-\alpha E) \quad \text{(Eq.14)},$$

and the normalized probability distribution can be written as

$$p_{Bk} = \frac{\exp(-\alpha E)}{\sum \exp(-\alpha E)} \quad \text{(Eq.15)},$$

The simulation results are summarized in Fig.1-3.

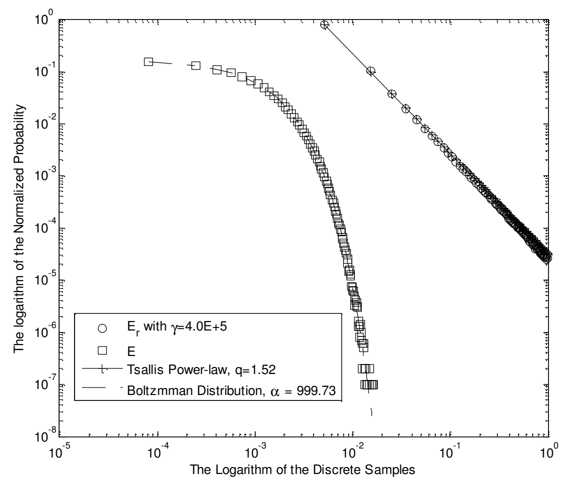

Figure 1 illustrates that the probability distribution of the reciprocal energies E_r fits the Tsallis power law with the $q = 1.52$ well while the probability distribution of the generalized energies E fits the Boltzmman distribution with $\alpha = 999.73$ well . The logarithm of the normalized probability for the reciprocal energies E_r looks like a straight line as a function of the logarithm of the discrete samples of E_r, which indicates a power-law.

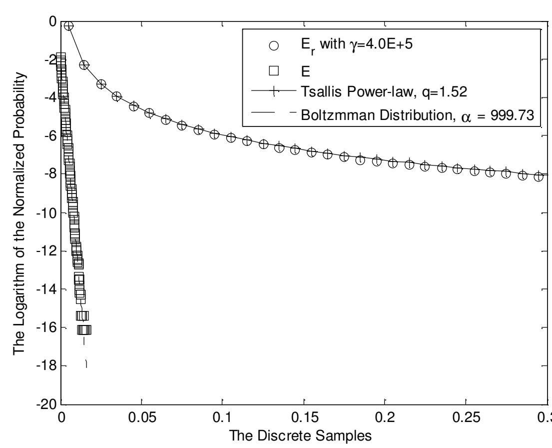

Figure 2 illustrates that the probability distribution of the generalized energies E follows the Boltzmann distribution.

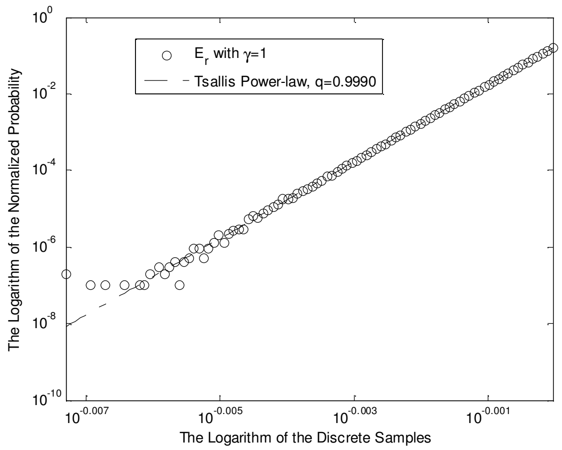

Figure 3 illustrates that the probability distribution of the reciprocal energies E_r fits the Tsallis power law with the $q = 0.9990$ well when $\gamma = 1$. A clear comparison is formed between the positive slope with $\gamma = 1$ and the negative slope with $\gamma = 400000$ as shown in Fig.1.

From the basically repeatable numerical results as shown in Fig 1-3, one may see clearly that for the same generalized system the Tsallis power laws with both the $q > 1$ and the $q < 1$ and the Boltzmann distribution can be formed under different constraints and with the different values of the parameter γ for two closely related system parameters of the generalized energies E and the reciprocal energies E_r. The MATLAB's optimization function called fmincon is used to optimize the fitting.

IV. THE IMPLICATIONS OF THE SIMULATION

Tsallis and his co-workers use the q-parameter to distinguish a nonextensive system from an extensive system [4]. When the $q -> 1$, the Tsallis entropy becomes the Boltzmann-Gibbs-Shannon entropy and the system is called an extensive system or classical system. When the $q > 1$, the system is called a nonextensive system or a non-classical system. The simulation results shown in this paper, however, have clearly and repeatedly demonstrated that the Tsallis entropies with essentially different q-parameters and the Boltzmann-Gibbs-Shannon entropy as the special case of the Tsallis entropy with $q \to 1$ are all applicable to the same generalized system for the generalized energy E and its reciprocal energy E_r under different constraints. Is the generalized system an extensive system or a nonextensive system ? There might be no definite "extensive system " or "classical system" and there are only "classical parameters" and "classical constraints". For any physical system or generalized system, it is the non-natural constraints which determine both the forms of the entropies and the non-uniform equilibrium distributions. The reasons are that we can only get a uniform distribution if there is no non-natural constraint and the uniform distribution can be obtained with the maximizing of both the Tsallis entropy with $q \neq 1$ and the Boltzmann-Gibbs-Shannon entropy as a special case of the Tsallis entropy with $q \to 1$ Therefore the form of the entropy will not be exclusive if there is no non-natural constraint [3].

V. CONCLUSION AND DISCUSSIONS

It has been demonstrated clearly that for the same generalized system the Boltzmann distribution can be induced for the generalized energies E under the constraint of the constant arithmetic mean and at the same time the Tsallis power-law may be generated under the constraint of the constant harmonic mean for the reciprocal energies E_r of the energies E. The author thus once again argues that [3] for any physical system or generalized system, it is the non-natural constraints which determine both the forms of the entropies and the non-uniform equilibrium distributions.

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