Consequences of squeezing initially coherent (semi classical) states with regards to finding if the cosmological constant is, or is not a 'vacuum' field

Andrew Beckwith,
beckwith@aibep.org,

Abstract

The following document is to prepare an analysis on if the cosmological constant is a vacuum field. Candidates for how to analyze this issue come up in terms of brane theory, modified treatments of WdM theory (pseudo time component added) and/ or how to look at squeezing of vacuum states, as initially coherent semi classical constructions, or string theory versions of coherent states. The article presents a thought experiment if there is a non zero graviton mass, and at the end, in the conclusion states experimental modeling criteria which may indicate if a non zero graviton mass is measurable, which would indicate the existence, de facto of a possible replacement for DE, based upon non zero mass gravitons, and indications for a replacement for a cosmological constant.

Introduction:

Dr. Karim[1] mailed the author with the following question which will be put in quotes:

The challenge of resolving the following question: At the Big Bang the only form of energy released is in the form of geometry – gravity. intense gravity field lifts vacuum fields to positive energies, So an electromagnetic vacuum of density 10^{122} kg/m^3 should collapse under its own gravity. But this does not happen - that is one reason why the cosmological constant cannot be the vacuum field. Why?

Answering this question delves into what the initial state of the universe should be, in terms of either vacuum energy, or something else. As was only recently seen, by works of 't Hooft, Susskind and others, a positive cosmological constant has surprising consequences, such as a finite (not enormous) maximum entropy of the observable universe (see the holographic principle). Lisa Dyson, Matthew Kleban, Leonard Susskind[2] in their 2002 article write the following

"The implication of such a description, as we have suggested in Section (1), is that Poincare recurrences are inevitable. Starting in a high entropy, "dead" configuration, if we wait long enough, a fluctuation will eventually occur in which the inflaton will wander up to the top of its potential, thus starting a cycle of inflation, re–heating, conventional cosmology and heat death. The frequency of such events is very low The typical time for (space–like boundary of de Sitter space) a fluctuation to occur is of order

$$T_r = \exp\left(S - S'\right) \tag{1.1}$$

where S is the equilibrium entropy and S' is the entropy of the fluctuation. The fluctuations we have in mind correspond to early inflationary eras during which the entropy is probably of order 10^{10}, while the equilibrium entropy is of order 10^{120} . Thus

$$T_r = \exp 10^{120}. \tag{1.2}$$

This seems like an absurdly big time between interesting events, which by comparison last for a very short time. Nevertheless dismissing such long times as "unphysical" may be a symptom of extreme temporal provincialism[2]."

The key point of **<u>their inquiry</u>** is their statement, as given by[2]

"Let us consider the entropy in observable matter in today's universe. It is of order 10^{100} This means that the number of microstates that are macroscopically indistinguishable from our world is exp (10^{100}). But only exp (10^{10}) of these states could have evolved from the low entropy initial state characterizing the usual inflationary starting point. The overwhelming majority of states which would have evolved into a world very similar to ours did not start in the usual low entropy ensemble."

I.e. how to make sense of a low entropy value of 10^{10} versus a present entropy of 10^{89} up to 10^{100}. Their question ends up with [2]

"To understand where they came from, imagine running these states backward in time until they thermalize in the eventual heat bath with entropy 10^{120}. Among the vast number exp (10^{120}) of possible initial starting points, a tiny fraction exp (10^{100}) will evolve into a world like ours. However, all but exp (10^{10}) of the corresponding trajectories (in phase space) are extremely unstable to tiny perturbations."

Having said that, the author wishes to come up with a model of how an initial cosmological constant could form which would be consistent with of 10^{10} counts of entropy

A way of doing so would be to require coherent states, initially very classical in nature, and to have deformation of these states in the initial states of the big bang. Consider the following, namely, the brane world treatment of initial generation of entropy. Note that *no where in the brane theory model is there a description of how the instantons formed in the first place:*

Mathur's [3] CQG article where he has a string winding interpretation of energy along the lines of putting as much energy E into string windings as possible via, $[n_1 + \bar{n}_1]LT = [2n_1]LT = E/2$, with T as tension, and where we are talking about n_1 wrappings of a string about a cycle of the torus , and $\bar{n}_1$ being 'wrappings the other way' , with the torus having a cycle of length L , which leads to an entropy defined in terms of an energy value of , if mass $m_i = T_P \prod L_j$ (with T_P being the tension of the i th brane, and L_j being spatial dimensions of a complex torus structure[3]

$$E_{Total} = 2\sum_i m_i n_i \tag{1.3}$$

This leads to entropy[3]

$$S_{Total} = A \cdot \prod_i^N \sqrt{n_i} \tag{1.4}$$

Our claim is that this very specific value of entropy for Eqn. (9) above will in Planck interval of time at about the onset of inflation lead to , after an adaptation of Seth Lloyd's formula [4]

$$\left| \left[S_{Total} = A \cdot \prod_i^N \sqrt{n_i} \right] \Big/ k_B \ln 2 \right| \approx [\#operations]^{3/4} \approx 10^8 - 10^{10} \tag{1.5}$$

In getting such values, the question to ask is how the particular value of n_i were formed in the beginning It is pretty obvious that Mathur [3] is using A , as in the above equation , to signify a counting of would be branes, , in some sort of structure, in the initial beginning. If or not the structure is emergent is not stated clearly in his article. Now how about using energy, instead of branes , in terms of forming initial entropy. Such an approach allows us to use the 'cosmological "constant" parameter' approach directly.

Difficulty in visualizing what g_* is in the initial phases of inflation.

Secondly, we look for a way to link initial energy states, which may be pertinent to entropy, in a way which permits an increase in entropy from about 10^{10} at the start of the big bang to about 10^{90} to 10^{100} today.

One such way to conflate entropy with an initial cosmological constant may be of some help, i.e. if $V_4\big|_{Threshold-volume-for-quantum-effects} \sim \left(10^{-4}cm\right)^3$ or smaller, i.e. in between the threshold value, and the cube of Planck length, one may be able to look at coming up with an initial value for a cosmological constant as given by Λ_{Max} as given by [5]

$$\frac{\Lambda_{Max}V_4}{8\cdot\pi\cdot G} \sim T^{00}V_4 \equiv \rho\cdot V_4 = E_{total} \tag{1.5}$$

Then making the following identification of total energy with entropy via looking at Λ_{Max} models, i.e. consider Park's model of a cosmological "constant" parameter scaled via background temperature[6]

$$\Lambda_{Max} \sim c_2 \cdot T^{\tilde{\beta}} \tag{1.6}$$

A linkage between energy and entropy, as seen in the construction, looking at what Kolb[7] put in, i.e.

$$\rho = \rho_{radiation} = (3/4)\cdot\left[\frac{45}{2\pi^2 g_*}\right]^{1/3} \cdot S^{4/3}\cdot r^{-4} \tag{1.7}$$

Here, the idea would be, possibly to make the following equivalence, namely look at,

$$\left[\left[\frac{\Lambda_{Max}r^4}{8\pi G}\right]\cdot(4/3)\cdot\left[\frac{2\pi^2 g_*}{45}\right]^{1/3}\right]^{3/4} \sim S_{initial} \tag{1.8}$$

Note that in the case that quantum effects become highly significant, that the contribution as given by $V_4\big|_{Threshold-volume-for-quantum-effects} \sim \left(10^{-4}cm\right)^3$ and potentially much smaller, as in the threshold of Plancks length, going down to possibly as low as 4.22419×10^{-105} m ^{3} = 4.22419×10^{-96} cm ^{3} leads us to conclude that even with very high temperatures, as an input into the initial entropy, that $S_{initial} \approx 10^{10}$ is very reasonable. Note though that Kolb and Turner[7] , however, have that g_* is at most about 120, whereas the author, in conversation with H. De La Vega[8], in 2009 indicated that even the exotic theories of g_* have an upper limit of about 1200, and that it is difficult to visualize what g_* is in the initial phases of inflation. De La Vega[8] stated in Como Italy, that he, as a conservative cosmologist, viewed defining g_* in the initial phases of inflation as impossible. One arguably needs a different venue as to how to produce entropy initially, and the way the author intends to present entropy, initially is through initial graviton production. The question of if gravitons, especially high frequency gravitons, can be detected will compose the last part of the manuscript. To start off with, consider what if entropy were in a near 1-1 relations with , in initially very strongly curved space time with information. To do this , look at what was stated by Sussind, et al[2], namely

"What then are the alternatives? We may reject the interpretation of de Sitter space based on complementarity. For example, an evolution of the causal patch based on standard Hamiltonian quantum mechanics may be wrong. What would replace it is a complete mystery. Another possibility is an unknown agent intervened in the evolution, and for reasons of its own restarted the universe in the state of low entropy characterizing inflation." We intend to put a structure in, which may influence the evolution, and to do it in terms of known squeezed state dynamics.

The basic problem. Formation of 'particle' states in curved space time is difficult

To see this, consider an argument by Wald [9] which outlines the problem. Wald references a quantum field theory construction for bi linear mappings $\mu : \varsigma \times \varsigma \to \Re$, where ς is for a space of solutions to the Klein Gordon field, as defined via $\nabla^a \nabla_a \phi - m^2 \phi = 0$, and $\varsigma \cong \left\{ set\, \phi \right\}$ satisfying $\nabla^a \nabla_a \phi - m^2 \phi = 0$, for curved space time, and Wald specifically states there exists no **unique** positive definite $\mu : \varsigma \times \varsigma \to \Re$ for defining high frequency solutions in curved space time to define a space H, in terms of a complete basis set. The best one can do in curved space time, is to use a unitary equivalence of equivalent norms, to deal with the problems of curved space time, in the very early universe, i.e. as defined by Wald [9], to have $C, C' > 0$ such that for all $\varsigma \cong \left\{ set\, \phi \right\}$ entries, that exist similar mappings μ_1, μ_2 for which

$$C\mu_1(\phi,\phi) \le \mu_2(\phi,\phi) \le C'\mu_1(\phi,\phi) \tag{1.9}$$

I.e. to have mappings μ_1, μ_2 which give locally almost the same basis set with μ_1, μ_2 defined on neighboring curved space time geometry. Even when space is highly curved. there can be locally speaking relatively close basis sets formed via $\varsigma \cong \left\{ set\, \phi \right\}$, for almost the same μ_1, μ_2. But in order to do this, one would need to have $\nabla^a \nabla_a \phi - m^2 \phi = 0$ obeyed, with $\varsigma \cong \left\{ set\, \phi \right\}$, and $\mu : \varsigma \times \varsigma \to \Re$, which places a premium upon having semi classical solutions, i.e. if one goes to purely quantum descriptions of entries for $\varsigma \cong \left\{ set\, \phi \right\}$ one will have difficulty forming the space H, in terms of a complete basis set in the sense specified by Wald. This leads us to the question of how to obtain solutions which are congruent to semi classical approximations, for the evolution of space time as we know it. I.e. our preferred procedure is to use coherent states, in terms of initial $\varsigma \cong \left\{ set\, \phi \right\}$, so as to get about the datum, as mentioned by Wald that the use of Eq. (1.9) in curved space time makes forming an initial particle in space time geometry very difficult to do. The best which could be done would be to make the functions μ_1, μ_2 as 'close to one another as possible' . So then what can be said is how to do that, and to make the process linked to structure formation later on.

Difficulties in obtaining a mostly classical treatment of $\varsigma \cong \left\{ set\, \phi \right\}$ initially speaking

Aside from the conundrum of squeezed states, which will be addressed in the next section, one of the issues to consider is what to do with the mix of DM states. As mentioned in the following . When DE becomes important, the density of DE contribution goes as[5]

$$\begin{aligned} \rho_{VAC} &\sim \frac{\Lambda_{observed}}{8\pi G} \sim \sqrt{\rho_{UV} \cdot \rho_{IR}} \\ &\sim \sqrt{l_{Planck}^{-4} \cdot l_H^{-4}} \sim l_{Planck}^{-2} \cdot H_{observed}^2 \end{aligned} \tag{1.10}$$

$$\Delta\rho \approx \text{a dark energy density} \sim H_{observed}^2 / G \tag{1.11}$$

This is, however, when DE is important, and that at best, we do not consider DE in the initial beginning of the universe. I.e. in the beginning the following mix of initial states is probably correct

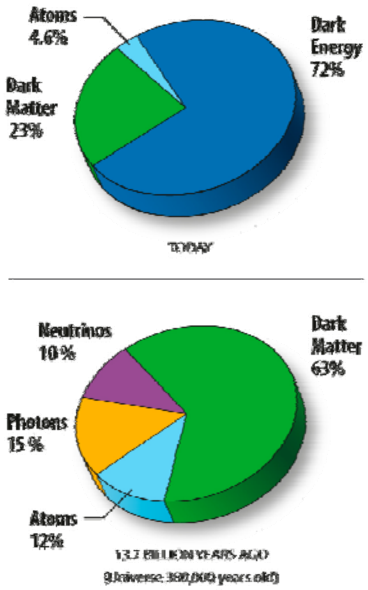

Figure 1: from G. Hingsaw presentation, in COMO, Italy, July 2009 at the ISAPP

How DM and other constituent parts of the early 380 thousand year old universe evolved to have connections with KK gravitons is connected closely with the following

The emphasis is placed on DM, as opposed to DE, and this means, among other things, considering different mixtures of DM to consider as contributing to space time evolution. i.e. what can be said about a power spectra

$$P \sim \frac{1}{150\pi^2} \cdot \frac{V(\phi)}{\in} \tag{1.12}$$

This is assuming a slow roll parameter treatment with $\in << 1$, and for $t > t_P$. Eq. (1.12) would be growing fairly rapidly in line with what is said about Eq. (1.12) above. An increase in scalar power, is then proportional to an increase in entropy

$$\left|\frac{\Delta E}{l_P^3}\right| \sim \left|\frac{\Delta P \in 150\pi^2}{l_P^3}\right| \approx |\Delta S| \tag{1.13}$$

The issue of Eq(1.13) would be in its own way resolvable if one is consider a way to differentiate different values of DM, in a contribution to the term ΔE , which could reflect themselves in the delineation of a density contribution to ΔE along the lines of Alexey Boyarsky, Julien Lesgourgues, Oleg Ruchayskiy and Matteo Viel [10] (2009) have strict Baysian statistical limits as to what sort of warm to cold dark matter mixes are allowed. A mix of warm and cold DM mixes would allow for . Here, we can assume that $\Delta E / l_P^3 \sim \delta\rho$ A worthy exercise is to consider if $\Delta E / l_P^3 \sim \delta\rho$ can reflect DM warm and cold candidates, and to what degree this would reflect how the cosmological "constant" would come about. Begin by considering the possible identification, namely if H_{early} is the initial behavior of the Hubble parameter, and one starts with

$$\frac{[\Delta E / l_P^3]}{[H_{early}^2 / l_P^2]} \approx \frac{\Delta S}{[H_{early}^2 / l_P^2]} \sim \frac{l_P^2 \cdot \Delta S}{H_{Early}^2} \sim \frac{\delta\rho}{\rho} \tag{1.14}$$

where we will put in a candidate for the ΔS for initial conditions, and then use that as far as answering questions as far as formulating an answer as far as entropy fluctuations, and candidates for density fluctuations, as well as early values of the Hubble parameter. One of their basic result, which is put here,

$\rho_{Baryons}, \rho_{Cold-Dark-Matter}, \rho_{Warm-Dark-Matter}$ refer to density profiles, of the respective baryons, CDM, and WDM candidates, whereas, the density fluctuations $\delta_{Baryons}, \delta_{Cold-Dark-Matter}, \delta_{Warm-Dark-Matter}$ are with regards to the fluctuations of these density values. So[10]

$$\left(\frac{\delta\rho}{\rho}\right) \equiv \frac{\rho_{Baryons}\delta_{Baryons} + \rho_{Cold-Dark-Matter}\delta_{Cold-Dark-Matter} + \rho_{Warm-Dark-Matter}\delta_{Warm-Dark-Matter}}{\rho_{Baryons} + \rho_{Cold-Dark-Matter} + \rho_{Warm-Dark-Matter}} \qquad (1.15)$$

In rough details, if axions are CDM, and KK gravitons are for WDM, then up to a point, $\rho_{Warm-Dark-Matter}$ would dominate Eqn. (1.13) in earlier times, ie. Up to Z~1100. However, Boyarsky, et al (2009) also stress that as of the recent era, i.e. probably for Z~.55 to Z~0 today , they would expect to see the following limiting behavior[10]

$$\begin{aligned} &\delta_{Baryons} \equiv \delta_{CDM}, \\ &\delta_{WDM} << \delta_{CDM} \end{aligned} \qquad (1.16)$$

In earlier times, what is put in, with regards to eqn. (1.16) would be probably far different . However, up in the present era, the denominator of Eq (1.15) would be dominated by KK DM, whereas there would be rough equality in the contributions $\rho_{Cold-Dark-Matter}\delta_{Cold-Dark-Matter}, \rho_{Warm-Dark-Matter}\delta_{Warm-Dark-Matter}$, with the baryon contribution to the numerator being ignorable, due to how small baryon values would be for Z~.55 to Z~0 today. Somehow, contributions as to Eqn (1.14) should be compared with.

$$\left(\frac{\delta\rho}{\rho}\right)_{Horizon} \cong \frac{k^{3/2}\left|\delta_k\right|}{\sqrt{2}\pi} \propto \frac{k^{(3/2)+3\alpha-3/2}}{\sqrt{2}\pi} \approx \left(1/\sqrt{2}\pi\right)\cdot k^{3\alpha} \qquad (1.17)$$

, where $-.1 < \alpha < 0.2$, and $\alpha \equiv 0 \Leftrightarrow n_s \equiv 1$ and to first order, $k \cong Ha$. following limits as to what is picked as of Eq. (1.14) in early and later times should be reconciled with.

$$\left(\frac{\delta\rho}{\rho}\right)_{Horizon} \cong \left(1/\sqrt{2}\pi\right)\cdot k^{3\alpha} \sim \frac{H^2}{\dot{\phi}} \propto 10^{-4} - 10^{-5} \qquad (1.18)$$

Having such a relatively small value of $l_P^2 \propto \left[1.616\times10^{-35}\,meters\right]^2$ as placed with $\Delta S \sim 10^{10}$

$$10^{-4} - 10^{-5} \sim \frac{l_P^2 \cdot \Delta S}{H_{Early}^2} \qquad (1.19)$$

This will lead to comparatively low values for H_{Early}^2 which will be linked to the behavior of a cosmological 'constant' parameter value, which subsequently changes in value later. I.e, Eq. (1.19) will be for a configuration just before the onset of the big bang itself, which will have implications for Dr. Karims question. Assuming that one has a particle count, which will be how $\Delta S \sim 10^{10}$ is initially formulated, will lead to questions in how the cosmological constant evolves, and reaches its present value.Note that if H_{Early}^2 is picked as initially small, it becomes enormous when $\Delta S \sim 10^{10}$ pushes to value a bit later to approaching much later time values of 10^{89} up to 10^{100}.This shift in value for H_{Early}^2, from a small to a much later value later on, will go to the main issue of initially[9] , with $l_P^2 \propto \left[1.616\times10^{-35}\,meters\right]^2$

$$H_{Early}^2 \sim \left[\Lambda_{Cosmological} \cdot l_P^2 / 8\pi G\right] \qquad (1.20)$$

And, also,

$$\frac{l_P^2 \cdot \Delta S}{H_{Early}^2} \approx \frac{8\pi G \cdot \Delta S}{\Lambda_{Cosmological}} \sim 10^{-4} - 10^{-5} \tag{1.21}$$

An initially small value for $\Lambda_{Cosmological}$, in line with $\Delta S \sim 10^{10}$ just at the onset of inflation, plus increases of $\Lambda_{Cosmological}$ as $\left[\Delta S \sim 10^{10}\right]_{initial} \to 10^{58}$ at the end of inflation and about the start of the radiation era corresponds to initially un squeezed initial states which become squeezed coherent states . The discussion of coherent squeezed states comes next.

How squeezed state conditions at the onset of inflation affects usual attempts at measurement of coherent relic graviton states.

Now what could be said about forming states close to classical representations of gravitons? Venkatartnam, and Suresh,[11] 2007 built up a coherent state via use of a displacement operator $D(\alpha) \equiv \exp(\alpha \cdot a^+ - \alpha^* \cdot a)$, applied to a vacuum state, where α is a complex number, and a, a^+ as annihilation, and creation operations $\left[a, a^+\right] = 1$, where one has

$$|\alpha\rangle = D(\alpha) \cdot |0\rangle \tag{1.22}$$

It is appropriate to call the initial vacuum state as initially coherent, and not necessarily formed via purely quantum processes. The vacuum state, $|0\rangle$ may be explained as stated by Mauricio Bellini[12], with the initial vacuum state as described by using the initial time to be the Planckian time $G^{1/2}$. Later, the universe may undergo a second-order phase transition. A five dimensional vacuum state may or may not have a quantum genesis. For the sake of argument, consider if $|0\rangle$ arises as due to what G.E. Volovik[13]stated about . an initial state $|0\rangle$, consistent with stating that " *its vacuum energy density is exactly zero without fine tuning, if: there are no external forces acting on the liquid; there are no quasiparticles which serve as matter; no space-time curvature; and no boundaries which give rise to the Casimir effect. Each of these four factors perturbs the vacuum state and induces the nonzero value of the vacuum energy density of order of energy density of the perturbation. This is the reason, why one must expect that in each epoch the vacuum energy density is of order of matter density of the Universe, or/and of its curvature, or/and of the energy density of smooth component*"

However, what one sees in string theory, is a situation where a vacuum state as a template for graviton nucleation is built out of an initial vacuum state, $|0\rangle$. To do this though, as Venkatartnam, and Suresh[11] did, involved using a squeezing operator $Z[r, \vartheta]$ defining via use of a squeezing parameter r as a strength of squeezing interaction term , with $0 \le r \le \infty$, and also an angle of squeezing, $-\pi \le \vartheta \le \pi$ as used in $Z[r,\vartheta] = \exp\left[\frac{r}{2} \cdot \left(\left[\exp(-i\vartheta)\right] \cdot a^2 - \left[\exp(i\vartheta)\right] \cdot a^{+2}\right)\right]$, where combining the $Z[r,\vartheta]$ with (1.23) leads to a single mode squeezed coherent state, as they define it via

$$|\varsigma\rangle = Z[r,\vartheta]|\alpha\rangle = Z[r,\vartheta]D(\alpha) \cdot |0\rangle \xrightarrow[\alpha \to 0]{} Z[r,\vartheta] \cdot |0\rangle \tag{1.23}$$

The right hand side. of Eq. (1.23) given above becomes a highly non classical operator, i.e. in the limit that the super position of states $|\varsigma\rangle \xrightarrow[\alpha \to 0]{} Z[r,\vartheta] \cdot |0\rangle$ occurs, there is a many particle version of a 'vacuum state' which has highly non classical properties. Squeezed states, for what it is worth, are thought to occur at the onset of vacuum nucleation, but what is noted for $|\varsigma\rangle \xrightarrow[\alpha \to 0]{} Z[r,\vartheta] \cdot |0\rangle$ being a super position of vacuum states, means that classical analog is extremely difficult to recover in the case of squeezing, and general non classical behavior of squeezed states. Can one, in any case, faced with $\left|\alpha\right\rangle = D(\alpha)\cdot\left|0\right\rangle \neq Z[r,\vartheta]\cdot\left|0\right\rangle$ do a better job of constructing coherent graviton states, in relic conditions, which may not involve squeezing ?. Note L. Grishchuk [14] wrote in (1989) in "On the quantum state of relic gravitons", where he claimed in his abstract that 'It is shown that relic gravitons created from zero-point quantum fluctuations in the course of cosmological expansion should now exist in the squeezed quantum state. The authors have determined the parameters of the squeezed state generated in a simple cosmological model which includes a stage of inflationary expansion. It is pointed out that, in principle, these parameters can be measured experimentally'. Grishchuk, et al,[14] (1989) reference their version of a cosmological perturbation h_{nlm} via the following argument. How we work with the argument will affect what is said about the necessity, or lack of, of squeezed states in early universe cosmology. From[14] Class. Quantum Gravity: 6 (1989), L 161-L165, where h_{nlm} has a component $\mu_{nlm}(\eta)$ obeying a parametric oscillator equation, where K is a measure of curvature which is $= \pm 1,0$, $a(\eta)$ is a scale factor of a FRW metric, and $n = 2\pi \cdot [a(\eta)/\lambda]$ is a way to scale a wavelength, λ, with n, and with $a(\eta)$

$$h_{nlm} \equiv \frac{l_{Planck}}{a(\eta)} \cdot \mu_{nlm}(\eta) \cdot G_{nlm}(x) \tag{1.24}$$

$$\mu''_{nlm}(\eta) + \left(n^2 - K - \frac{a''}{a} \right) \cdot \mu_{nlm}(\eta) \equiv 0 \tag{1.25}$$

If $y(\eta) = \frac{\mu(\eta)}{a(\eta)}$ is picked, and a Schrodinger equation is made out of the Lagrangian used to formulate (1.25) above, with $\hat{P}_y = \frac{-i}{\partial y}$, and $M = a^3(\eta)$, $\Omega = \frac{\sqrt{n^2 - K^2}}{a(\eta)}$, $\breve{a} = [a(\eta)/l_{Planck}] \cdot \sigma$, and $F(\eta)$ an arbitrary function. $y' = \partial y / \partial \eta$. Also, we have a finite volume $V_{finite} = \int \sqrt{{}^{(3)}g}\, d^3x$

Then the Lagrangian for deriving (1.25) is (and leads to a Hamiltonian which can be **<u>also</u>** derived from the Wheeler De Witt equation), with $\varsigma = 1$ for zero point subtraction of energy

$$L = \frac{M \cdot y'^2}{2a(\eta)} - \frac{M^2 \cdot \Omega^2 a \cdot y^2}{2} + a \cdot F(\eta) \tag{1.26}$$

$$\frac{-1}{i} \cdot \frac{\partial \psi}{a \cdot \partial \eta} \equiv \hat{H}\psi \equiv \left[\frac{\hat{P}^2{}_y}{2M} + \frac{1}{2} \cdot M\Omega^2 \hat{y}^2 - \frac{1}{2} \cdot \varsigma \cdot \Omega \right] \cdot \psi \tag{1.27}$$

Then there are two possible solutions to the S.E.[14], one a non squeezed state, and another a squeezed state. So in general we work with

$$y(\eta) = \frac{\mu(\eta)}{a(\eta)} \equiv C(\eta) \cdot \exp(-B \cdot y) \tag{1.28}$$

The **<u>non squeezed state</u>** has a parameter $B\big|_{\eta} \xrightarrow[\eta \to \eta_b]{} B(\eta_b) \equiv \omega_b/2$ where η_b is an initial time, for which the Hamiltonian given in (1.27) in terms of raising/ lowering operators is 'diagonal', and then the rest of the time for $\eta \neq \eta_b$, the **<u>squeezed state</u>** for $y(\eta)$ is given via a parameter B for squeezing which when looking at a squeeze parameter r, for which $0 \leq r \leq \infty$, then (1.28) has, instead of $B(\eta_b) \equiv \omega_b/2$

$$B|_{\eta} \xrightarrow[\eta \neq \eta_b]{} B(\omega, \eta \neq \eta_b) \equiv \frac{i}{2} \cdot \frac{(\mu/a(\eta))'}{(\mu/a(\eta))} \equiv \frac{\omega}{2} \cdot \frac{\cosh r + [\exp(2i\vartheta)] \cdot \sinh r}{\cosh r - [\exp(2i\vartheta)] \cdot \sinh r} \tag{1.29}$$

Taking the formalism literally [14], a state for a graviton/ GW is not affected by squeezing when we are looking at an initial frequency, so that $\omega \equiv \omega_b$ initially corresponds to a non squeezed state which may have coherence, but then right afterwards, if $\omega \neq \omega_b$ which appears to occur whenever the time evolution, $\eta \neq \eta_b \Rightarrow \omega \neq \omega_b \Rightarrow B(\omega, \eta \neq \eta_b) \equiv \frac{i}{2} \cdot \frac{(\mu/a(\eta))'}{(\mu/a(\eta))} \neq \frac{\omega_b}{2}$ A reasonable research task would be to determine, whether or not $B(\omega, \eta \neq \eta_b) \neq \frac{\omega_b}{2}$ would correspond to a vacuum state being initially formed right after the point of nucleation, with $\omega \equiv \omega_b$ at time $\eta \equiv \eta_b$ with an initial cosmological time some order of magnitude of a Planck interval of time $t \approx t_{Planck} \propto 10^{-44}$ seconds The next section will be to answer whether or not there could be a point of no squeezing, as Grishchuck implied, for initial times, and initial frequencies, and an immediate transition to times, and frequencies afterwards, where squeezing was mandatory. Note that in 1993, Grischchuk[15] further extended his analysis, with respect to the same point of departure, i.e. what to do with when $|\alpha\rangle = D(\alpha) \cdot |0\rangle \neq Z[r, \vartheta] \cdot |0\rangle$. Having $|\alpha\rangle = D(\alpha) \cdot |0\rangle$ with $D(\alpha)$ a possible displacement operator, seems to be in common with $B(\eta_b) \equiv \omega_b/2$, whereas $|\alpha\rangle = Z[r, \vartheta] \cdot |0\rangle$ which is highly non classical seems to be in common with a solution for which $B(\omega_b) \neq (\omega_b/2)$ This leads us to the next section, i.e. does $B(\eta_b) \equiv \omega_b/2$ when of time $t \approx t_{Planck} \propto 10^{-44}$ seconds, and then what are the initial conditions for forming 'frequency' $\omega \equiv \omega_b$?

Necessary & sufficient conditions for String/Brane theory graviton coherent states?

A curved space-time is a coherent background of gravitons, and therefore in string theory is a coherent state Joseph Gerard Polchinski[16] starting with the typical small deviation from flat space times as can be written up by $G_{uv}(X) = \eta_{uv} + h_{uv}(X)$, with η_{uv} flat space time, and the Polyakov action, is generalized as follows, the S_{σ} Polyakov action is computed and compared with exponentiated values

$$S_{\sigma} = \frac{1}{4\pi\alpha'} \cdot \int_M d^2\sigma \cdot \sqrt{g} \cdot g^{ab} \cdot G_{uv}(X) \cdot \partial_a X^{\mu} \partial_b X^{\nu} \tag{1.30}$$

Becomes

$$\exp(-S_{\sigma}) = [\exp(-S_P)] \cdot \left[1 - \frac{1}{4\pi\alpha'} \cdot \int_M d^2\sigma \cdot \sqrt{g} \cdot g^{ab} \cdot h_{uv}(X) \cdot \partial_a X^{\mu} \partial_b X^{\nu} + \ldots\right] \tag{1.31}$$

Polochinski[16] writes that the term of order h in equation (1.31) is the vertex operator for the graviton state of the string, with $h_{uv}(X) \equiv -4\pi g_c \cdot \exp\left[-ikX_{S_{uv}}\right]$, and the action of S_{σ} a coherent state of a graviton. Now the important question to ask is if this coherent state of a graviton, as mentioned by Polochinski can hold up in relic, early universe conditions. Rainer Dick,[17] in 2001, argued as stating that the "graviton multiple as one particular dark matter source in heterotic string theory. In particular, it is pointed out that an appreciable fraction of dark matter from the graviton multiplet requires a mass generating phase transition around $T_c \simeq 10^8$ GeV, where the symmetry partners of the graviton would evolve from an ultra hard fluid to pressure less dark matter. Indicates m $\simeq$ 10 MeV for the massive components of the graviton multiplet". This has a counter part in a presentation made by Berkenstein[18] (2004) with regards to BPS states, and SHO models for $AdS_5 \times S^5$ geometry. The upshot is that string theory appears to construct coherent graviton states, but it has no answer to the problem that Ford [19](1995), and Grishchuck [20], wrote on if the existing graviton coherent states would be squeezed into non classical configurations in relic conditions.

A suggestion from the author as to initial graviton production, and its links to $|0\rangle$

Beckwith[21] has concluded that the only way to give an advantage to higher dimensions as far as cosmology would be to look at if a fifth dimension may present a way of actual information exchange to give the following parameter input from a prior to a present universe, i.e. the fine structure constant, as given by

$$\tilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\breve{\lambda}}{hc} \tag{1.32}$$

The wave length as may be chosen to do such an information exchange would be part of a graviton as being part of an information counting algorithm as can be put below, namely: if we wish to simplify entropy and graviton inter relationships,

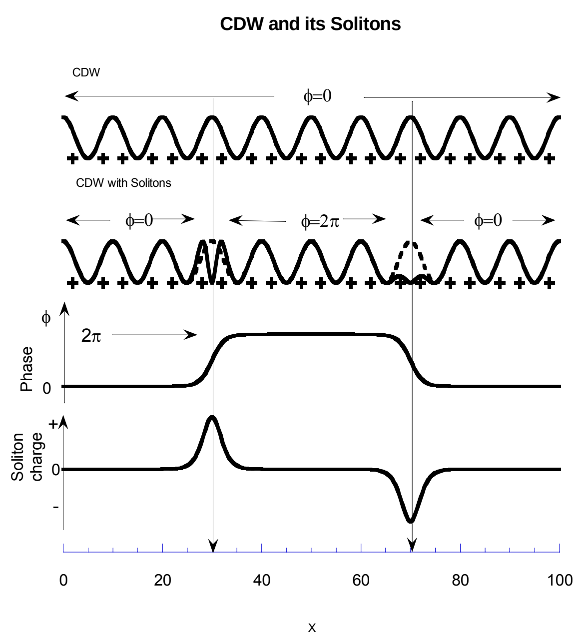

Fig 2a, **The pop up effects of an intanton-anti-instanton in Euclidian space[21,22]**

that the 1/N term drops out. As used by Ng[23]

$$Z_N \sim (1/N!) \cdot \left(V/\breve{\lambda}^3\right)^N \tag{1.33}$$

This, according to Ng,[23] leads to entropy of the limiting value of, if $S = (\log[Z_N])$ will be modified by having the following done, namely after his use of quantum infinite statistics, as commented upon by Beckwith[3]

$$S \approx N \cdot \left(\log\left[V/\lambda^3\right] + 5/2\right) \approx N \tag{1.34}$$

Eventually, the author hopes to put on a sound foundation what 'tHooft[24] is doing w.r.t. t'Hooft [24]deterministic quantum mechanics and equivalence classes embedding quantum particle structures. If one uses the wave functional

$$\Psi_{i,f}\ \left[\phi(\mathbf{x})\right]\Big|_{\phi\equiv\phi_{ci,cf}} = c_{i,f}\cdot\exp\left\{-\int d\mathbf{x}\ \alpha\left[\phi_{Ci,f}(\mathbf{x})-\phi_0(\mathbf{x})\right]^2\right\} \tag{1.35}$$

With $\phi_0(x)$ being equivalence classes to fit in a kink anti kink structure with t'Hooft's work[24] and tied it in with equivalence classes, and mixed it in with a kink anti kink structure given by the following figures from Beckwith's dissertation[25] . The first one is involving the use of instantons and what is known as domain wall approximations. Fig 2a. above represents how a Cooper pair charge can be used to ascertain an instanton- anti instanton structure would be organized as of CDW, for quasi one dimensions. The second, Fig 2b is how an equivalence class structure could be put in, and what the consequences would be. I.e. Doing so will answer the questions Kay[26] raised about particle creation, and the limitations of the particle concept in curved and flat space, i.e. the global hyperbolic space time which is flat everywhere expect in a localized "bump" of curvature. Furthermore, making a count of gravitons with $S \approx N \sim 10^{10}$ gravitons[3], with use of the formula from Lloyd [27] , of $I = S_{total}/k_B \ln 2 = [\#operations]^{3/4} \sim 10^{10}$ as implying at least one operation per unit graviton, with gravitons being one unit of information, per produced graviton[21]. What the author, Beckwith, sees is that since instanton- anti instanton pairs does not have to travel slowly, as has been proved by authors in the 1980s that gravitons if nucleated in a fashion as indicated by Fig. 2b will be in tandem and not be influenced as indicated by Isbanez and *Verdaguer* [28] . The instanton – anti instanton structure allows for rapid travel.

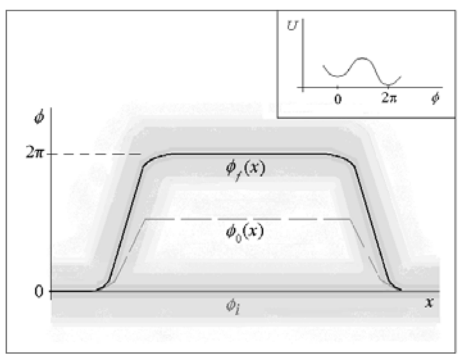

Fig. 2b: The pop up effects of an intanton-anti-instanton in Euclidian space[25]

Also, an instanton - anti instanton structure may allow us to be able to answer the following. The stretch-out of a graviton wave, greater than the size of the solar system, gives, an upper limit of a graviton mass due to wave length $\lambda_{graviton} > 300 \cdot h_0 kpc \Leftrightarrow m_{graviton} < 2\times 10^{-29} h_0^{-1} eV$ [3]. I. e. stretched graviton wave, at ultra-low frequency, may lead to a low mass limit. However, more careful limits due to experimental searches, as presented by Buonanno[29] have narrowed the upper limit to $10^{-20} h_0^{-1} eV$. An instanton – anti instanton structure to the graviton, if confirmed, plus experimental confirmation of mass, plus perhaps $n \sim 10^{10}$ gravitons $\approx 10^{10}$ entropy counts implies up to $\approx 10^{13}$ operations. If so, there is a one-to-one relationship between an operation and a bit of information, so a graviton has at least one bit of information. Starting with $S \approx N$ as a way to relate the graviton count with entropy may be a way to make inter connection between the inflaton picture of entropy generation and entropy connected/ generated with a numerical count of gravitons. The author contends that the above formalism for a graviton as an emergent particle, with a slight mass in four dimensions is consistent with what Sahni and Habib[30] worked with, in 1998. Experimental verification of this would be important for determining if or not theories purporting to show increasing or decreasing values of the gravitational constant were valid, e.g. of the sort given by Singh[31] are based upon firm experimental foundations.

Does LQG give us more direct arguments as to coherent states, squeezed states, and the break down of classical behavior at the onset of inflation?

Carlo Rovelli[32],[33], in 2006, in a PRL article states that a vertex amplitude that contributes to a coherent graviton state is the exponential of the Regge action: the other terms, that have raised doubts on the physical viability of the model, are suppressed by the phase of the vacuum state, and Rovelli[33] writes a coherent vacuum state as given by a Gaussian peaked on parts of the boundary Σ_d of a four dimensional sphere.

$$\Psi_q[s] = \Psi_q\left(\Gamma, j_{m,n}\right) \tag{1.36}$$

Rovelli [33] states that "bad" contributions to the behavior of eqn. (1.36) are cancelled out by an appropriate (Gaussian?) vacuum wave functional which has 'appropriately' chosen contributions from the boundary Σ_d of a four dimensional sphere. This is to avoid trouble with "bad terms" from what is known as the Barret – Crane vertex amplitude contributions, which are can be imitated by an appropriate choice of vacuum state amplitude being picked. Rovelli calculated some components of the graviton two-point function and found that the Barrett-Crane vertex yields a wrong long-distance limit. A problem, as stated by Lubos Motel[34] (2007), that there are infinitely many other components of the correlators in the LQG that are guaranteed not to work unless an infinite number of adjustments are made. The criticism is harsh, but until one really knows admissible early universe geometry one cannot rule out the Rovelli approach, or confirm it. In addition, Jakub Mielczarek et al.[35] (2009) considered tensor perturbations produced at a bounce phase in presence of the holonomy corrections. Here bounce phase and holonomy corrections originate from Loop Quantum Cosmology What comes to the fore are corrections due to what is called quantum holonomy, l.. A comment about the quantum bounce. i.e. what is given by Dah-Wei Chiou, Li-Fang Li[36],(2009) is that there is a branch match up between a prior to a present set of Wheeler De Witt equations for a prior to present universe, as far as modeling how the quantum bounce links the two Wheeler De Witt solution branches, i.e. one Wheeler De Witt wave function for a prior universe, and another wave function for a present universe. Furthermore, Abhay Ashtekar[37] (2006) wrote a simple treatment of the Bounce causing Wheeler De Witt equation along the lines of, for $\rho_* \approx const \cdot (1/8\pi G\Delta)$ as a critical density, and Δ the eignvalue of a minimum area operator. Small values of Δ imply that gravity is a repulsive force, leading to a bounce effect.

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv \frac{8\pi G}{3} \cdot \rho \cdot \left(1 - \left(\rho/\rho_*\right)\right) + H.O.T. \tag{1.37}$$

Furthermore, Bojowald[38] (2008) specified criteria as to how to use an updated version of Δ and $\rho_* \approx const \cdot (1/8\pi G\Delta)$ in his GRG manuscript on what could constitute grounds for the existence of generalized squeezed initial (graviton?) states. Bojowald[38] (2008) was referring to the existence of squeezed states, as either being necessarily, or NOT necessarily a consequence of the quantum bounce. As Bojowald[38] (2008) wrote it up, in both his equation (26) which has a quantum Hamiltonian $\langle \hat{V} \rangle \approx H$, with

$$\left.\frac{d\langle \hat{V} \rangle}{d\phi}\right|_{\phi \approx 0} \xrightarrow[existence-of-un-squeezed-states \Leftrightarrow \phi \approx 0]{} 0 \tag{1.38}$$

, and $\hat{V}$ is a 'volume' operator where the ' volume' is set as V , Note also, that Bojowald has , in his initial Friedman equation, density values $\rho \equiv \frac{H_{matter}(a)}{a^3}$, so that when the Friedman equation is quantized, with an initial internal time given by ϕ , with ϕ becoming a more general evolution of state variable than 'internal time'. If so, Bojowald [38] (2008) writes, when there are squeezed states

$$\left.\frac{d\langle\hat{V}\rangle}{d\phi}\right|_{\phi\neq 0} \xrightarrow[existence-of-squeezed-states]{} N(value) \neq 0 \tag{1.39}$$

For his equation (1.39), which is incidentally when links to classical behavior breaks down, and when the bounce from a universe contracting goes to an expanding present universe. Bojowald also writes that if one is looking at an isotropic universe, that as the large matter 'H' increases, that in certain cases, one observes more classical behavior, and a reduction in the strength of a quantum bounce. Bojowald states that "Especially the role of squeezed states is highlighted. The presence of a bounce is proven for uncorrelated states, but as squeezing is a dynamical property and may change in time" The upshot is that although it is likely in a quantum bounce state that the states should be squeezed, it is not a pre requisite for the states to always start off as being squeezed states. .So a physics researcher can ,look at if an embedding of the present universe in a higher dimensional structure which could have lead to a worm hole from a prior universe to our present for re introduction of inflationary growth

. Other models. Do worm hole bridges between different universes allow for initial un squeezed states? Wheeler De Witt solution with pseudo time component put in.

This discussion is to present a not so well known but useful derivation of how instanton structure from a prior universe may be transferred from a prior to the present universe.

1. The solution as taken from L. Crowell's (2005)[39] book, and re produced here, as referenced by Beckwith (2008,2009) has many similarities with the WKB method. I.e. it is semi CLASSICAL.
2. left unsaid is what embedding structure is assumed
3. A final exercise for the reader. Would a WKB style solution as far as transfer of 'material' from a prior to a present universe constitute procedural injection of non compressed states from a prior to a present universe? Also if uncompressed, coherent states are possible, how long would they last in introduction to a new universe?

This is the Wheeler-De-Witt equation with pseudo time component added. From Crowell

$$-\frac{1}{\eta r}\frac{\partial^2\Psi}{\partial r^2}+\frac{1}{\eta r^2}\cdot\frac{\partial\Psi}{\partial r}+rR^{(3)}\Psi=\left(r\eta\phi-r\ddot{\phi}\right)\cdot\Psi \tag{1.40}$$

This has when we do it $\phi \approx \cos(\omega\cdot t)$, and frequently $R^{(3)} \approx$ constant, so then we can consider

$$\phi \cong \int_0^\infty d\omega\left[a(\omega)\cdot e^{ik_\varpi x^\mu}-a^+(\omega)\cdot e^{-ik_\varpi x^\mu}\right] \tag{1.41}$$

In order to do this, we can write out the following for the solutions to Eq (1.41) above.

$$C_1=\eta^2\cdot\left(4\cdot\sqrt{\pi}\cdot\frac{t}{2\omega^5}\cdot J_1(\omega\cdot r)+\frac{4}{\omega^5}\cdot\sin(\omega\cdot r)+(\omega\cdot r)\cdot\cos(\omega\cdot r)\right)$$
$$+\frac{15}{\omega^5}\cos(\omega\cdot r)-\frac{6}{\omega^5}Si(\omega\cdot r) \tag{1.42}$$

And

$$C_2=\frac{3}{2\cdot\omega^4}\cdot\left(1-\cos(\omega\cdot r)\right)-4e^{-\omega\cdot r}+\frac{6}{\omega^4}\cdot Ci(\omega\cdot r) \tag{1.43}$$

This is where $Si(\omega\cdot r)$ and $Ci(\omega\cdot r)$ refer to integrals of the form $\int_{-\infty}^{x}\frac{\sin(x')}{x'}dx'$ and $\int_{-\infty}^{x}\frac{\cos(x')}{x'}dx'$. .

Next, we should consider whether or not the instanton so formed is stable under evolution of space-time leading up to inflation. To model this, we use results from Crowell[39] (2005) on quantum fluctuations in space-time, which gives a model from a pseudo time component version of the Wheeler-De-Witt equation, with use of the Reinssner-Nordstrom metric to help us obtain a solution that passes through a thin shell separating two space-times. The radius of the shell $r_0(t)$ separating the two space-times is of length l_P in approximate magnitude, leading to a domination of the time component for the Reissner – Nordstrom metric

$$dS^2 = -F(r)\cdot dt^2 + \frac{dr^2}{F(r)} + d\Omega^2 \tag{1.44}$$

This has:

$$F(r) = 1 - \frac{2M}{r} + \frac{Q^2}{r^2} - \frac{\Lambda}{3}\cdot r^2 \xrightarrow[T\to 10^{32}\,Kelvin \sim \infty]{} -\frac{\Lambda}{3}\cdot(r = l_P)^2 \tag{1.45}$$

This assumes that the cosmological vacuum energy parameter has a temperature dependence as outlined by Park [6] (2003), leading to

$$\frac{\partial F}{\partial r} \sim -2\cdot\frac{\Lambda}{3}\cdot(r \approx l_P) \equiv \eta(T)\cdot(r \approx l_P) \tag{1.46}$$

As a wave functional solution to a Wheeler-De-Witt equation bridging two space-times, similar to two space-times with "instantaneous" transfer of thermal heat, as given by Crowell [39](2005)

$$\Psi(T) \propto -A\cdot\{\eta^2\cdot C_1\} + A\cdot\eta\cdot\omega^2\cdot C_2 \tag{1.47}$$

This has $C_1 = C_1(\omega, t, r)$ as a pseudo cyclic and evolving function in terms of frequency, time, and spatial function. This also applies to the second cyclical wave function $C_2 = C_2(\omega, t, r)$, where the values $C_1 =$ Eq (1.42) and $C_2 =$ Eq. (1.43) Here, Eq. (1.47) is a solution to the pseudo time WDM equation. The question which will be investigated is if eqn (1.47) is a way to present either a squeezed or un squeezed state. A way forward is to note that Prado Martin-Moruno, Pedro F. Gonzalez-Diaz [40]in July (2009) wrote up about thermal phantom-like radiation process coming from the wormhole throat. Note that the Crowell construction of a worm hole bridge is in some ways similar to Cavaglià's [41](1994,1998) treatment of use of conjugate momentum π^{ij} of h_{ij} generalized momentum variables, also known as conjugate momentum $\hat{\pi}^{ij} \equiv \frac{\hbar}{i}\cdot\frac{\partial}{\partial\cdot h_{ij}}$, leading to the sort of formalism as attributed to Luis J. Garay [42] (1991) article , of

$$\Psi(h_{ij}) \approx \left\lfloor \exp\int d^3x\cdot\pi^{ij}\cdot h_{ij} \right\rfloor_T \tag{1.48}$$

Now in the case of what can be done with the worm hole used by Crowell[39], with, if $\hbar \equiv 1$, $\hat{\pi}^{ij} \equiv -i\frac{\delta}{\delta\cdot g_{ij}}$,$\hat{\pi}^{\theta\theta} \equiv -\frac{i}{2r}\frac{\partial}{\partial\cdot r}$, $\hat{\pi}^{tt} \equiv -i\cdot\left(\frac{\partial F(r)}{\partial r}\right)^{-1}\cdot\frac{\partial}{\partial\cdot r}$, and a kinetic energy value as given of the form $\hat{\pi}^{\theta\theta}\hat{\pi}^{tt} + \hat{\pi}^{tt}\hat{\pi}^{\theta\theta}$. The supposition which we have the worm hole wave functional may be like, so, use the wave functional looking like $\Psi(g_{ij}) \approx \left\lfloor \exp\int d^3x\cdot[\pi^{ij}]\cdot g_{ij} \right\rfloor_T$ where the g_{ij} for the Weiner- Nordstrom metric will be

$$dS^2 = -F(r)\cdot dt^2 + \frac{dr^2}{F(r)} + d\Omega^2 \equiv g^{ij}dx^i dx^j \equiv \frac{\Lambda}{3}\cdot(r = l_P)^2\cdot dt^2 - \frac{dr^2}{\frac{\Lambda}{3}\cdot(r = l_P)^2} + d\Omega^2 \tag{1.49}$$

The main problem in these assumptions about how likely one can measure GW at all is in the assumed impossibility of measuring a 'strain factor' $h << 10^{-21}$ According to Li, et al (2009), $h \sim 10^{-30}$ is the sensitivity factor needed to measure GW. Weiss, in personal communications (2009) states flatly in personal communications with the author that measurements of $h \sim 10^{-30}$ are impossible with currently achievable GW technology. To answer this, the author states that there does exist an argument by Dr. Fangyu Li's personal notes and personal communications (2009)[43], which implies that relic GW, and by implicit assumption, gravitons, are not to be ruled out as Weiss stated was the case in personal communications with the author The assumptions the author is making is that with with careful calibration, there is a way to obtain measurable relic GW, and also, possibly, graviton measurements. The author wishes to thank Professor Rainer Weiss[44], of MIT, in ADM 50, in November 7th (2009) for explaining the implications of a formula for HFGW of at least 1000 Hertz for GW which is a start in the right direction i.e., a strain value of, if L is the Interferometer length, and N is the number of quanta / second at a beam splitter, and τ is the integration time.

$$h \sim \frac{\lambda}{Lb\sqrt{N\tau}} \qquad (1.50)$$

For LIGO systems, and their derivatives, the usual statistics and technologies of present lasers as bench marked by available steady laser in puts given by Eq (1.50) appear to limit $h \sim 10^{-23}$. The problem is that as Weiss explained to the author, one of the most active, and perhaps guaranteed to obtain GW sources involves the interaction of super massive black holes in the center of colliding galaxies, which would need $h \sim 10^{-25}$ to obtain verifiable data. Going significantly below $h \sim 10^{-23}$ involves an argument as given as follows: The following question was posed by a reviewer of a document given to Dr. Fangyu Li, and the author has reproduced Dr. Li's response below[43]

Quote:

"*The most serious is that a background strain* $h \sim 10^{-30}$ *at 10GHz corresponds to a* Ω_g *(total)* $\sim 10^{-3}$ *which violates the baryon nuclei-synthesis epoch limit for either GWs or EMWs.* Ω_g *(Total) needs to be smaller than* 10^{-5} *otherwise the cosmological Helium/hydrogen abundance in the universe would be strongly affected......"*The answer, which the author copied from Dr. Li, i.e. if $v = 10GHz, \quad h = 10^{-31}$, then $\Omega_g = 8.3 \times 10^{-7} < \Omega_{g\max}$, is an answer to this supposition

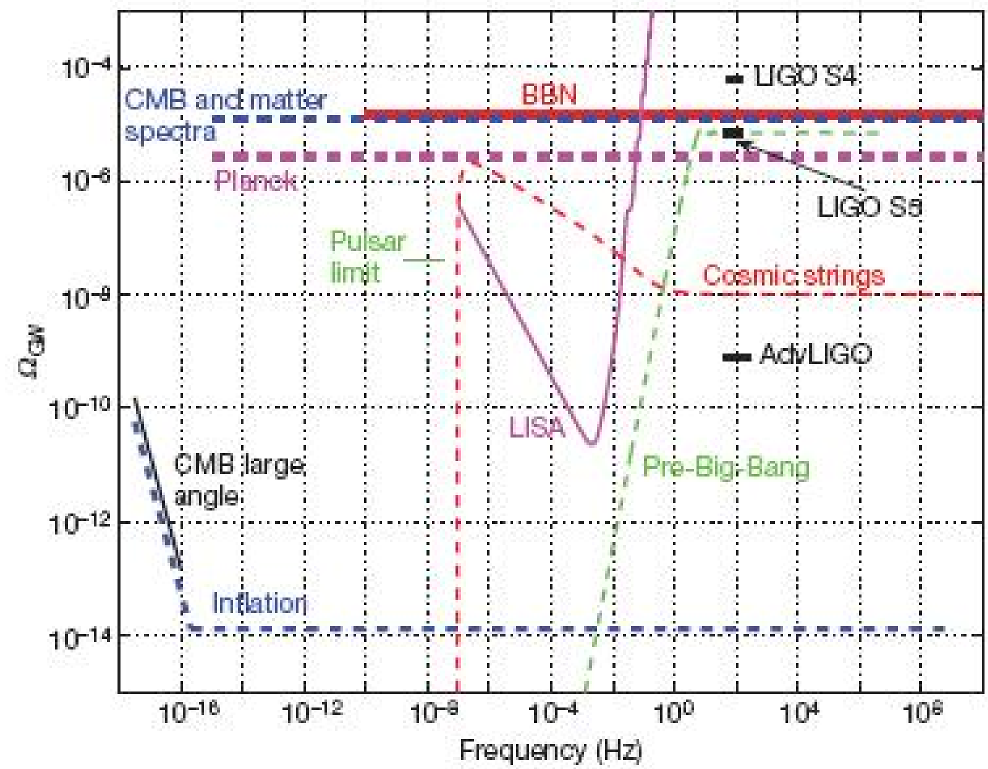

Figure3. This figure from B.P. Abbott[45], et.al., shows the relation between Ω_g and frequency. The curve of the pre-big-bang models shows that Ω_g of the relic GWs is almost constant $\sim 6.9\times10^{-6}$ from 10^{-1}Hz to 10^{10}Hz. Ω_g of the cosmic string models is about 10^{-8} in the region 1Hz to 10^{10}Hz; its peak value region is about 10^{-7}-10^{-6}Hz. According to more accepted by the general astro physics community values, the estimate, the upper limit of Ω_g on relic GWs should be smaller than 10^{-5}, while recent data analysis (B.P. Abbott et al[45], (2009)) shows the upper limit of Ω_g, as in figure FIX should be 6.9×10^{-6} FIX. By using such parameters, Dr. Li [43]estimates the spectrum $h(v_g,\tau)$ FIX and the RMS amplitude h_{rms} FIX. The relation between Ω_g and the spectrum $h(v_g,\tau)$ is often expressed as L. P. Grishchuk [46], as

$$\Omega_g \approx \frac{\pi^2}{3}\left(\frac{v}{v_H}\right)^2 h^2(v,\tau), \tag{1.51}$$

so

$$h(v,\tau) \approx \frac{\sqrt{3\Omega_g}}{\pi}\frac{v_H}{v}, \tag{1.52}$$

Where $v_H = H_0 \square 2\times10^{-18}\,Hz$, the present value of the Hubble frequency. From Esq. (1.51) and (1.52), we have

(a) If $\nu = 10GHz$, $h = 10^{-30}$, then $\Omega_g = 8.3 \times 10^{-5}$, (1.53)

If $\nu = 10GHz$, $h = 10^{-31}$, then $\Omega_g = 8.3 \times 10^{-7} < \Omega_{g\max}$, (1.54)

If $\nu = 10GHz$, $\Omega_g = \Omega_{g\max} = 6.9 \times 10^{-6}$, then $h = 2.9 \times 10^{-31}$ (1.55)

(b) If $\nu = 5GHz$, $h = 10^{-30}$

Then $\Omega_g = 2.1 \times 10^{-5}$ (1.56)

If $\nu = 5GHz$, h=10^{-31} then $\Omega_g = 2.1 \times 10^{-7} < \Omega_{g\max}$ (1.57)

If $\nu = 5GHz$, $\Omega_g = \Omega_{g\max} = 6.9 \times 10^{-5}$, then $h = 5.7 \times 10^{-31}$ (1.58)

Such values of $\nu = 5GHz$, $\Omega_g = \Omega_{g\max} = 6.9 \times 10^{-5}$, would be essential to ascertain the possibility of detection of GW from relic conditions, whereas Ω_g, or in integral form $\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c}$, as given by $\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f)$ [47]. Furthermore, one could also write $h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f[graviton] + n_f[neutrino]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1kHz} \right)^4$ [48]for a very narrow range of frequencies, that to first approximation, make a comparison between an integral representation of Ω_g and $h_0^2 \Omega_{gw}(f)$. Note also that Dr. Li suggests, as an optimal upper frequency to investigate, $\nu_g = 2.9GHz, \Delta\nu = 3Khz$ then

And
$$h \approx \frac{\sqrt{3\Omega_g}}{\pi} \frac{\nu_H}{\nu_g} \approx 1.0 \times 10^{-30}, \tag{1.59}$$

$$h_{rms} = \sqrt{\langle h^2 \rangle} \approx h \left[\frac{\Delta\nu}{\nu_g} \right]^{\frac{1}{2}} \approx 1.02 \times 10^{-33} \tag{1.60}$$

These are upper values of the spectrum, and should be considered as preliminary. Needed in this mix of calculations would be a way to ascertain a set of input values for $n_f[graviton], n_f[neutrino]$ into $h_0^2 \Omega_{gw}(f)$. The objective is to get a set of measurements to confirm if possible the utility of using , experimentally the numerical count of

$$h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f[graviton] + n_f[neutrino]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1kHz} \right)^4 \tag{1.61}$$

. If there is roughly a 1-1 correspondence between gravitons and neutrios (highly unlikely) , then

$$h_0^2 \Omega_{gw}(f) \sim 3.6 \cdot \left[\frac{n_f[graviton]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1kHz} \right)^4. \tag{1.62}$$

counting the number of gravitons per cell space should also consider what Buoanno[29] wrote, for Les Houches if one looks at BBN, the following upper bound should be considered:

$$h_0^2 \Omega_{gw}(f) \leq 4.8 \times 10^{-9} \cdot (f / f_*)^2 \tag{1.63}$$

Here, Buoanno[29] is using $f > f_* = 4.4 \times 10^{-9} Hz$, and a reference from Kosowoky, Mack, and Kahniashhvili [49] (2002) as well as Jenet et al [50](2006). Using this upper bound, if one insist upon assuming, as Buoanno[29] (2006) does, that the frequency today depends upon the relation

$$f \equiv f_* \cdot [a_*/a_0] \tag{1.64}$$

The problem in this is that the ratio $[a_*/a_0] << 1$, assumes that a_0 is "today's" scale factor. In fact, using this estimate, Buoanno[29] comes up with a peak frequency value for relic/\early universe values of the electroweak era-generated GW graviton production of

$$f_{Peak} \cong 10^{-8} \cdot [\beta/H_*] \cdot [T_*/16GeV] \cdot [g_*/100]^{1/6} Hz \tag{1.65}$$

By conventional cosmological theory, limits of g_* are at the upper limit of 100-120, at most, according to Kolb and Turner[7] (1991). $T_* \sim 10^2 GeV$ is specified for nucleation of a bubble, as a generator of GW. Early universe models with $g_* \sim 1000$ or so are not in the realm of observational science, yet, according to Hector De La Vega [8] (2009) in personal communications with the author,) at the Colmo, Italy astroparticle physics school , ISAPP, Furthermore, the range of accessible frequencies as given by Eq (1.65) is in sync with

$$h_0^2 \Omega_{gw}(f) \sim 10^{-10} \tag{1.66}$$

for peak frequencies with values of 10 MHz. The net affect of such thinking is to proclaim that all relic GW are inaccessible. If one looks at Figure , $\Omega_{GW} > 10^{-6}$ for frequencies as high as up to 10^6 Hertz, this counterswhat was declared by Turner and Wilzenk[51] (1990): that inflation will terminate with observable frequencies in the range of 100 or so Hertz. The problem is though, that after several years of LIGO, no one has observed such a GW signal from the early universe, from black holes, or any other source, yet. About the only way one may be able to observe a signal for GW and/or gravitons may be to consider how to obtain a numerical count of gravitons and/or neutrinos for

$$h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f [graviton] + n_f [neutrino]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1kHz} \right)^4 \tag{1.67}$$

. And this leads to the question of how to account for a possible mass/ information content to the graviton.

Consequences of small graviton mass for reacceleration of the universe

Where n_f is the frequency-based numerical count of gravitons per unit phase space. The author suggests that n_f may depend upon the interaction of gravitons with neutrinos in plasma during early-universe nucleation, as modeled by M. Marklund *et al*[52] , which is a supposition the author is investigating for a modification of a joint KK tower of gravitons, as given by Maartens[53] for DM. Assume the stretching of early relic neutrinos that would lead to the KK tower of gravitons--for when $\alpha < 0$, is, as explained in **Appendix A** [48]

$$m_n(Graviton) = \frac{n}{L} + 10^{-65} \text{ grams} \tag{1.68}$$

. Also Eq. (1.68) will be the starting point used for a KK tower version of Eq. (1.69) below. So from Maarten's 2005 paper [53],

$$\dot{a}^2 = \left[\left(\frac{\tilde{\kappa}^2}{3}\left[\rho + \frac{\rho^2}{2\lambda}\right]\right)a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K\right] \quad (1.69)$$

Maartens [53]also writes

$$\dot{H}^2 = \left[-\left(\frac{\tilde{\kappa}^2}{2}\cdot[p+\rho]\cdot\left[1+\frac{\rho^2}{\lambda}\right]\right) + \frac{\Lambda \cdot a^2}{3} - 2\frac{m}{a^4} + \frac{K}{a^2}\right]. \quad (1.70)$$

Also, if $\rho \cong -P$, for red shift values z between zero to 1.0-1.5 with equality, $\rho = -P$, for z between zero to **.5.** $a \equiv [a_0 = 1]/(1+z)$. As given by Beckwith[48]

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \equiv -1 - \frac{\dot{H}}{H^2} = -1 + \frac{2}{1+\tilde{\kappa}^2[\rho/m]\cdot(1+z)^4\cdot(1+\rho\ \ /2\lambda)} \approx -1 + \frac{2}{2+\delta(z)} \quad (1.71)$$

Eq. (1.71) assumes using in Eq. (1.69) and Eq. (1.70) $\Lambda = 0 = K$, and the net effect is to obtain, a substitute for DE, by presenting how gravitons with a small mass done with $\Lambda \neq 0$, even if curvature **K** =0 In a revision of Alves *et. al,* [54] Beckwith[48] used a higher-dimensional model of the brane world and Marsden[6] KK graviton towers. The density ρ of the brane world in the Friedman equation as used by Alves *et. al*[7] is use by Beckwith[48] for a non-zero graviton

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[\frac{m_g \cdot (c=1)^6}{8\pi G(\hbar = 1)^2}\right]\cdot\left(\frac{1}{14\cdot(1+z)^3} + \frac{2}{5\cdot(1+z)^2} - \frac{1}{2}\right) \quad (1.72)$$

I.e. Eq. (1.64) above is making a joint DM and DE model, with all of Eq. (1.67) being for KK gravitons and DM, and 10^{-65} grams being a 4 dimensional DE. Eq. (1.71) and Eq. (1.68) are part of a KK graviton presentation of DM/ DE dynamics. Beckwith[48] found at $z \sim$ **. 4**, a billion years ago, that acceleration of the universe increased, as shown in Fig. 4.

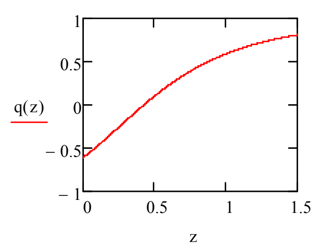

Fig. 4: Reacceleration of the universe based on Beckwith [48] **(note that q < 0 if z <.423)**

Answering Dr. Karim's questions about the Cosmological "constant" and how it may differ from Vacuum energy.

Assume that the cosmological "constant" is defined via the following equation, i.e. working with the values $\frac{l_P^2 \cdot \Delta S}{H_{Early}^2} \approx \frac{8\pi G \cdot \Delta S}{\Lambda_{Cosmological}} \sim 10^{-4} - 10^{-5}$ for varying entropy values, affecting the $\Lambda_{Cosmological}$ values, gives us a way to analyze what Dr. Karim asked [1], namely :

At the Big Bang the only form of energy released is in the form of geometry – gravity. intense gravity field lifts vacuum fields to positive energies, So an electromagnetic vacuum of density 10^{122} kg/m3 should collapse under its own gravity. But this does not happen - that is one reason why the cosmological constant cannot be the vacuum field. Why?

How could an electromagnetic vacuum of density of 10^{122} kg/m^3 conceivably form initially in the first place ? And what are the procedures which could determine if there **is, or is not**, a connection between $\Lambda_{Cosmological}$ and vacuum energy ?

The earliest treated version of a changing cosmological constant the author referenced was in having $\Lambda_{Cosmological} \sim c_1 T^{\beta}$, with T in this case a measure of background temperature. If one is trying to relate $\Lambda_{Cosmological}$ strictly to a vacuum energy state, initially, the following integral equations of motion become relevant to the inquiry. Namely look at a change of variables, with $\tau = H_0 t$, with H_0 the present Hubble parameter value, and with $a = a_0 q(\tau)$ so that "particles" moving in a particular cosmology operate under, as given by T. Padmanabhan, for 'particles' in a trajectory semi classically defined by[5]

$$\frac{1}{2}\cdot\left[\frac{dq}{d\tau}\right]^2 + V(q) = 0 \tag{1.73}$$

This assuming that the potential as given, namely $V(q)$ have the following contents in initial universe conditions

$$V(q) \cong -(1/2)\cdot\frac{\Omega_{radiation}}{q^2} \tag{1.74}$$

The expression for $\Omega_{radiation}$ could be related to an initially huge cosmological constant, which would die out in time, as stating succinctly that up to a degree one can write when $\rho_{critial} \equiv 3M_P^2 H_{initial}^2$, ie signifying a roughly spatially flat cosmological constant regime in the vicinity of the inflationary era of cosmology.

$$\Omega \sim \Omega_{radiation} \equiv \frac{\rho_{radiation}}{\rho_{critial}} \approx \frac{\rho_{radiation}}{3M_P^2 H_{initial}^2} \sim \frac{\left[\frac{\pi^2}{30}\cdot g_* T^4\right]}{3M_P^2 H_{initial}^2} \tag{1.75}$$

Recall that Park[6], in 2003, wrote an effective early cosmological "constant" parameter as $\Lambda_{Max} \sim c_2 \cdot T^{\tilde{\beta}}$ So that up to a point, one could write, for the early temperature dependence, having

$$\Omega \sim \frac{\left[\frac{\pi^2}{30}\cdot g_* \cdot \left[\Lambda_{Max}/c_2\right]^{4/\tilde{\beta}}\right]}{3M_P^2 H_{initial}^2} \sim \frac{\rho_{early-universe}}{3M_P^2 H_{initial}^2} \le 1 \tag{1.76}$$

The denominator in place only makes sense if there is 'flat space' and the term for $\rho_{critial} \equiv 3M_P^2 H_{initial}^2$ cannot be defined for when curved space conditions take hold.

Case I, $\Lambda_{Max} \sim c_2 \cdot T^{\tilde{\beta}}$ is a vacuum energy term. The problem is, what to do in the case that the temperature reaches $T \le 1.416 \times 0^{32} K \sim 10^{19} GeV$ as opposed to the Planck Mass term of setting the value of $M_P \sim 1.2 \times 10^{19} GeV/c^2$. Then the following ratio as can be rounded off leaves

$$\Omega \sim \frac{\left[\frac{\pi^2}{30}\cdot g_* \cdot \left[1.416\times 10^{19} GeV\right]^4\right]\cdot c^4}{3\cdot\left[1.2\times 10^{19} GeV\right]^2 H_{initial}^2} \sim \frac{\frac{\pi^2}{30}\cdot g_* \cdot c^4}{3}\cdot\frac{\left[10^{19} GeV\right]^2}{H_{initial}^2} \ge 1 \tag{1.77}$$

I.e. especially if the degrees of freedom rises above $g_* \geq 120$. Note that $g_* \approx 120$ at T ~ 100 KeV Unless the term for $H_{initial}$ were absolutely enormous, and if $g_* \approx 1000$, then $\Omega \geq 1$ could happen, which would be physically meaningless. The other situation is that there could be situations for which g_* would be undefined, especially if $T \leq 1.416 \times 0^{32} K \sim 10^{19} GeV$ were close to an equality.

$\Omega \sim \frac{\left[\frac{\pi^2}{30} \cdot g_* \cdot [\Lambda_{Max} / c_2]^{4/\tilde{\beta}}\right]}{3M_P^2 H_{initial}^2} \sim \frac{\rho_{early-universe}}{3M_P^2 H_{initial}^2} \leq 1$ would be altered, if there is a situation for which $\frac{l_P^2 \cdot \Delta S}{H_{Early}^2} \approx \frac{8\pi G \cdot \Delta S}{\Lambda_{Cosmo\log ical}} \sim 10^{-4} - 10^{-5}$ has entropy change from $\Delta S \sim 10^{10}$ to $\Delta S \sim 10^{58}$, corresponding to a major increase so as to observe $\frac{\left[\frac{\pi^2}{30} \cdot g_* \cdot [\Lambda_{Max} / c_2]^{4/\tilde{\beta}}\right]}{3M_P^2 H_{initial}^2} \geq 1$, instead of $\frac{\left[\frac{\pi^2}{30} \cdot g_* \cdot [\Lambda_{Max} / c_2]^{4/\tilde{\beta}}\right]}{3M_P^2 H_{initial}^2} \leq 1$, even if there is a decrease of $g_* \geq 1000$ to $g_* \leq 100$. I.e. a change in g_* by a factor of two, ie. Shrinking 10^{-2} in magnitude is a very small effect, as opposed to a change of $\Delta S \sim 10^{10}$ at $t \approx t_P \sim 10^{-44}$ seconds to $\Delta S \sim 10^{58}$ past the time $t \sim 10^{-35}$ seconds i.e. at the start of the big bang, up to past the grand unification era, where a significant amount of entropy would be released. So can there be quark gluon plasma models which take into account entropy generation in a sensible manner to avoid such problems?

Break down of Quark – Gluon models for generation of entropy

It gets worse if one is asserting that there is, in any case, a quark gluon route to determine the role of entropy. To begin this analysis, let us look at what goes wrong in models of the early universe. The assertion made is that this is due to the quark – Gluon model of plasmas having major 'counting algorithm' breaks with non counting algorithm conditions, i.e. when plasma physics conditions BEFORE the advent of the Quark gluon plasma existed. Here are some questions which need to be asked.

1. Is QGP strongly coupled or not? Note : Strong coupling is a natural explanation for the small (viscosity) Analogy to the RHIC: J/y survives deconfinement phase transition
2. What is the nature of viscosity in the early universe? What is the standard story? (Hint: AdS-CFT correspondence models). Question 2 comes up since

$$\frac{\eta}{s} = \frac{1}{4\pi} \tag{1.78}$$

typically holds for liquid helium and most bosonic matter. However, this relation breaks down. At the beginning of the big bang. As follows i.e. if Gauss- Bonet gravity is assumed, in order to still keep casuality , one needs $\lambda_{BG} \leq \frac{9}{100}$

This even if one writes for a viscosity over entropy ratio the following

$$\frac{\eta}{s} \equiv \frac{1}{4\pi} \cdot [1 - 4\lambda_{GB}] \leq \frac{1}{4\pi} \tag{1.79}$$

A careful researcher may ask why this is so important. If a causal discontinuity as indicated means the $\frac{\eta}{s}$ ratio is $\approx \frac{1}{4\pi}\cdot\frac{33}{50}$, or less in value, it puts major restrictions upon viscosity, as well as entropy. A drop in viscosity, which can lead to major deviations from $\frac{1}{4\pi}$ in typical models may be due to more collisions.

Then, more collisions due to WHAT physical process? Recall the argument put up earlier. I.e. the reference to causal discontinuity in four dimensions, and a restriction of information flow to a fifth dimension at the onset of the big bang/ transition from a prior universe? That process of a collision increase may be inherent in the restriction to a fifth dimension, just before the big bang singularity, in four dimensions, of information flow. In fact, it very well be true, that initially, during the process of restriction to a 5^{th} dimension, right before the big bang, that $\left|\frac{\eta}{s} \approx \varepsilon^{+}\right| << \frac{1}{4\pi}$. Either the viscosity drops nearly to zero, or else the entropy density may, partly due to restriction in geometric 'sizing' may become effectively nearly infinite. It is due to the following qualifications put in about Quark – Gluon plasmas which will be put up, here. **Namely,** more collisions imply less viscosity. More Deflections ALSO implies less viscosity. Finally, the more momentum transport is prevented, the less the viscosity value becomes. Say that a physics researcher is looking at viscosity due to turbulent fields. Also, perturbatively calculated viscosities: due to collisions . This has been known as *Anomalous Viscosity* in plasma physics ,(this is going nowhere, from pre-big bang to big bang cosmology). Appendix B gives some more details as far as the

So happens that **RHIC models for viscosity assume**

$$\frac{1}{\eta} \approx \frac{1}{\eta_A} + \frac{1}{\eta_C} \tag{1.80}$$

As Akazawa[55] noted in an RHIC study, equation 1.80 above makes sense if one has stable temperature T, so that

$$\frac{\eta_A}{s} = \bar{c}_0 \cdot \left(\frac{T}{g^2\left|\nabla u\right|}\right)^{\frac{2n-1}{2n+1}} \Leftrightarrow \frac{\eta_C}{s} = \text{constant} \tag{1.81}$$

If the temperature T wildly varies, as it does at the onset of the big bang, this breaks down completely. This development is **Mission impossible: why we need a different argument for entropy. I.e. Even for the RHIC, and in computational models of the viscosity for closed geometries—what goes wrong in computational models**

- **Viscous Stress is *NOT* $\propto$ *shear***
- **Nonlinear response: impossible to obtain on lattice (computationally speaking)**
- **Bottom line: we DO NOT have a way to even define SHEAR in the vicinity of big bang!!!!**

I.e. the quark gluon stage of production of entropy, and its connections to early universe conditions may lead to undefined conditions which , i.e. like shear in the beginning of the universe, cannot be explained. I.e. what does viscosity mean in the neighborhood of time where $10^{-44}s < time < 10^{-35}s$?

Gravitons as a source of entropy generation?

The author has come up with criteria as far as having $S \approx N\cdot\left(\log\left[V/\lambda^3\right]+5/2\right) \approx N$ [48] for a generation of entropy, and most likely this means having $S \approx N \sim 10^{10}$ in the vicinity of time at or right at the boundary of a volume of space where $10^{-44}s < time < 10^{-35}s$. If V ~ wavelength λ of nucleated particles, this is a way of indicating that the particles may have a very high frequency , i.e. if the particles are in any case, HFGW, then the early universe may create a number of them such that initial values for them lead to $S \approx N \sim 10^{10}$ What if the gravitons have, initially a tiny mass ? If $\Delta S \sim 10^{58}$ = # of gravitons , and the mass of the graviton initially was 10^{-65} grams per graviton, then since 1eV/c^2 = 1.78×10^{-33} grams, then if one sets $\Delta S \sim 10^{58}$, it is 10^{-7} grams, and having $S \approx N \sim 10^{10}$ = # of gravitons at about , which is just about after $10^{-44} s$ into the beginning of the universe, weighing at most about 10^{-55} grams, or about in total about 10^{-18} eV/c^2, or about 10^{-27} GeV/c^2. Getting up to the level of an electron volt would require $S \approx N \sim 10^{28}$ = # of gravitons, which would be significantly after time $t \sim t_P \sim 10^{-44} s$. Still though one should look at realistically defensible conditions for entropy generation, and avoid mistakes like the following example

Entropy is not exclusively generated by gravitons, i.e. one of the amusing aspects of identification of entropy with gravitons is a considering what todays conditions would imply. If we set V as the space-time volume, then look at $\nu_0 \sim 10^{-18}$ Hz, and $\nu_1 \sim 10^{11} (H_1/M_P)^{3/2} \sim 10^{11}$ Hz as an upper bound, assuming no relationship like the GW wavelength cubed, as proportional to early universe volume, which leads to $r(\nu) \equiv \ln \bar{n}_{gravitons}$, where $\bar{n}_{gravitons}$ refers to the number of produced gravitons over a very wide spectral range of frequencies. This assumes that we are working with $H_1 \propto M_P$,. The upshot would be a non physical result, as outlined by Giovannini[56]

$$S_{gw} = V \cdot \int_{\nu_0}^{\nu_1} r(\nu) \cdot \nu^2 d\nu \cong \left(10^{29}\right)^3 \cdot \left(H_1/M_P\right)^{3/2} \approx 10^{87} - 10^{88} \qquad (1.82)$$

This should be compared with the result that Sean Carroll[57] (2004) came up with: that for the universe as a whole

$$S_{Total} \sim 10^{88} - 10^{100} \qquad (1.83)$$

This Eq. (1.83) should be compared with the even odder result that the author discussed in a question and answer period in the Bad Honnef perspectives in quantum gravity (2008) meeting, April 2008 to reconcile Eq. (1.83) with the odd prediction given in Eqn. (1.84), namely , as presented by Carroll[57], (2004)

$$S_{Black-Hole} \sim 10^{90} \cdot \left[\frac{M}{10^6 \cdot M_{Solar-Mass}} \right]^2 \qquad (1.84)$$

I.e. the black hole in the center of our galaxy may have purportedly more entropy than the entropy of the entire KNOWN universe?

Conclusion. The author thinks what is called the cosmological constant may be relic graviton release, and graviton interaction with structure formation, and re acceleration (How the CMBR permits , via maximum frequency, and maximum wave amplitude values, an upper bound value for massive graviton mass m_g)

The conclusion of this document will outline how one may confirm if gravitons have a small mass and will conclude with analysis of how to ascertain what is needed for graviton detection, in terms of detectors to proceed with a confirmable theory about the cosmological parameter, for the “Cosmological constant”. We will start off with the general problem of black hole GW detection and from there move on to the cosmological initial conditions relevant to GW / graviton production in relic conditions. If relic gravitons are discovered, the author thinks that their production, and influence will be pertinent to a possible substation of what we call the cosmological “constant” for DE, and the re acceleration of the universe problem.

To begin, note that Camp and Cornish (2004)[58], as does Fangyu Li [59] (2002, 2008, 2009) use the typical transverse gravitational gauge h_{ij} with a typically traceless value summed as $0 + h_+ - h_+ + 0$ and off diagonal elements of h_x on each side of the diagnonal to mix with a value of

$$h_{ij} \equiv \frac{G_N}{c^4} \cdot \frac{2}{r} \cdot \left[\frac{d^2}{dt^2} Q_{ij} \right]^{TT}_{retarded} \tag{1.85}$$

This assumes r is the distance to the source of gravitational radiation, with the *retarded* designation on Eqn. (34) denoting $\frac{d}{dt}$ replaced by a retarded time derivative $\frac{d}{d[t-(r/c)]}$, while TT means take the transverse projections and subtract the trace. Here, we call the quadrupole moment, with $\rho(t,x)$ a density measurement. Now, the following value of the Q_{ij} as given gives a luminosity function L, where R is the 'characteristic size' of a gravitational wave source. Note that if M is the mass of the gravitating system

$$Q_{ij} = \int d^3x \left[x_i x_j - \frac{1}{3} \cdot \delta_{ij} \cdot x^2 \right] \cdot \rho(t,x) \tag{1.86}$$

$$L \approx \frac{1}{5} \cdot \frac{G_N}{c^5} \cdot \frac{d^3 Q_{ij}}{dt^3} \cdot \frac{d^3 Q^{ij}}{dt^3} \cong \frac{\pi \cdot c^5}{G_N} \cdot \left(\frac{G_N M}{R \cdot c^2} \right)^2 \tag{1.87}$$

After certain considerations reported by Camp and Cornish[58] (2004) , one can recover a net GW amplitude

$$h \sim 2 \cdot \left[\frac{G_N \cdot M}{R \cdot c^2} \right] \cdot \left[\frac{G_N \cdot M}{r \cdot c^2} \right] \tag{1.88}$$

This last equation requires that $R > R_G = \frac{G_N M}{c^2} \equiv$ gravitational radius of a system, with a black hole resulting if one sets $R < R_G = \frac{G_N M}{c^2}$. Note that when $R \sim R_G = \frac{G_N M}{c^2}$ we are at an indeterminate boundary where one may pick our system as having black hole properties.

Now for stars, Camp and Cornish[58] (2004) give us that

$$h \approx 10^{-21} \cdot \left[\frac{15 \cdot Mpc}{r} \right] \cdot \left[\frac{M}{2.8 M_{solar-mass}} \right]^2 \cdot \left(\frac{90km}{R} \right) \tag{1.89}$$

$$f \equiv \text{frequency} \approx \sqrt{\frac{M}{2.8 M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}} \cdot 100 Hz \tag{1.90}$$

As well as a mean time τ_{GW} for half of gravitational wave potential energy to be radiated away as

$$\tau_{GW} \approx \frac{R}{2\pi \cdot c} \cdot \left[\frac{G_N M}{R \cdot c^2} \right]^{-3} \sim \left(\frac{R}{90km} \right)^4 \cdot \left[\frac{2.8 M_{solar-mass}}{M} \right]^3 \cdot \left(\frac{1}{2} \cdot \sec \right) \tag{1.91}$$

The assumption we make is that if we model $R \sim R_G = \frac{G_N M}{c^2}$, for a sufficiently well posed net mass M that the star formulas roughly hold for early universe conditions, provided that we can have a temperature T for which we can use the approximation $\approx \sqrt{\frac{M}{2.8 M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}} \cdot 100 Hz$ that we also have $\left[\frac{T}{TeV}\right] \sim 10^{13}$ or higher, so, that at a minimum we recover Grishchuck's [60](2007) value of

$$
\begin{aligned}
f_{Peak} &\approx \left(10^{-3} Hz\right) \cdot \left[\frac{T}{TeV}\right] \sim 10^{10} Hz \\
&\approx \sqrt{\frac{M}{M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}}
\end{aligned}
\tag{1.92}
$$

Eq (1.91) places, for a specified value of R, which can be done experimentally, an upper bound as far as far as what a mass M would be . Can this be exploited to answer the question of if or not there is a minimum value for the Graviton mass?

The key to the following discussion will be that $\sqrt{\frac{M}{2.8 M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}} \approx 10^8 - 10^{10}$, or larger .for relic entropy/ graviton production in relic conditions We will end this final conclusion up by making the following as a specified future works project . Our supposition is that if GW from relic conditions are confirmed, that the choice for a possible substitute for both entropy production and the DE value as to re acceleration of the universe becomes testable, as an experiment in the making.

Inter relationship between graviton mass m_g and the problem of a sufficient number of bits of $\hbar$ from a prior universe, to preserve continuity between fundamental constants from a prior to the present universe?

V.A. Rubakov and , P.G. Tinyakov [61]gives that there is, with regards to the halo of sub structures in the local Milky Way galaxy an amplitude factor for gravitational waves of

$$
< h_{ij} > \sim 10^{-10} \cdot \left[\frac{2 \cdot 10^{-4} Hz}{m_{graviton}}\right] \tag{1.93}
$$

If we use LISA values for the Pulsar Gravitational wave frequencies , this may mean that the massive graviton is ruled out. On the other hand $\sqrt{\frac{M}{2.8 M_{solar-mass}}} \cdot \sqrt{\frac{90km}{R}} \approx 10^8 - 10^{10}$ leads to looking at , if

$$
< h_{ij} > \sim h \sim 10^{-5} \cdot \left[\frac{15Mpc}{r}\right]^{1/2} \cdot \left[\frac{M}{2.8 \cdot M_{solar-mass}}\right]^{1/2} \approx 10^{-30} \tag{1.94}
$$

If the radius is of the order of $r \geq 10$ billion light-years ~ 4300 *Mpc or much greater, so then we have , as an example* $< h_{ij} > \sim 10^{-10} \cdot \left[\frac{2 \cdot 10^{-4} Hz}{m_{graviton}}\right] \approx 5.9 \cdot 10^{-7} \cdot \left[\frac{M}{2.8 \cdot M_{solar-mass}}\right]^{1/2}$, so then one is

getting

$$\left[\frac{10^{-7}\,Hz}{m_{graviton}}\right] \approx \left[\frac{5.9}{\sqrt{5.6}}\right] \cdot \sqrt{\frac{M}{M_{solar-mass}}} \tag{1.97}$$

This Eqn. (1.97) is in units where $\hbar = c = 1$.

If $10^{-60} - 10^{-65}$ **grams per graviton, and 1 electron volt is in rest mass , so** $1.6 \times 10^{-33}\, grams \Rightarrow gram = 6.25 \times 10^{32}\, eV$ **. Then**

$$\left[\frac{10^{-7}\,Hz}{m_{graviton}}\right] \equiv \left[\frac{10^{-7}\,Hz \cdot \left[6.582 \times 10^{-15}\, eV \cdot s\right]}{\left[10^{-60}\, grams \equiv 6.25 \times 10^{-28}\, eV\right] \cdot \left[2.99 \times 10^{9}\, meter / \sec\right]^{2}}\right] \sim \frac{10^{-22}}{10^{-9}} \sim 10^{-13}$$

Then, exist

$$M \sim 10^{-26} M_{solar-mass} \approx 1.99 \times 10^{33-26} \equiv 1.99 \times 10^{7}\, grams. \tag{1.98}$$

If each photon, as stated above is 3.68×10^{-48} ***grams per photon,*** then

$$M \sim 5.44 \times 10^{54} \text{ initially transmitted photons.} \tag{1.99}$$

Futhermore, if there are , today for a back ground CMBR temperature of 2.7 degrees Kelvin $5 \times 10^{8}\, photons / cubic - meter$, with a wave length specified as $\lambda_{\max} \approx 1 \cdot cm$. This is for a numerical density of photons per cubic meter given by

$$n_{photon} := \frac{\sigma \cdot (T)^{4} \cdot (\lambda_{max})}{h \cdot c^{2}} \tag{1.200}$$

As a rough rule of thumb, if , as given by Weinberg [62](1972) that early quantum effects , for quantum gravity take place at a temperature $T \approx 10^{33}$ Kelvin, then, if there was that temperature for a cubic meter of space, the numerical density would be , roughly 10^{132} times greater than what it is today. Forget it. So what we have to do is to consider a much smaller volume area. If the radii of the volume area is $r \cong 4 \times 10^{-35}\, meters \equiv l_{P} = Planck - length$,then we have to work with a de facto initial volume $\approx 64 \times 10^{-105} \sim 10^{-103}\, (meters)^{3}$. I.e. the numerical value for the number of photons at $T \approx 10^{33}$, if we have a per unit volume area based upon planck length, in stead of meters, cubed is $10^{29} \times \left(5 \times 10^{8}\right) \approx 5 \times 10^{37}$ photons for a cubic area with sides $r \cong 4 \times 10^{-35}\, meters \equiv l_{P}$ at $T|_{quantum-effects} \approx 10^{33}$ Kelvin However, $M \sim 5.44 \times 10^{54}$ initially transmitted photons! Either the minimum distance ,i.e. the grid is larger, or $T|_{quantum-effects} >> 10^{33}$ Kelvin

We assert here, though, that if non zero mass gravitons are found, perhaps via the procedure outlined above may be a way to ascertain if a replacement/ modification of the usual cosmological constant is in order.

Acknowledgements

The author thanks **Dr. Raymond Weiss**, of MIT as of his interaction in explaining Advanced LIGO technology for the detection of GW for frequencies beyond 1000 Hertz and technology issues with the author in ADM 50, November 7th 2009. **Dr. Fangyu Li,** of Chonquing University is thanked for lending his personal notes to give substance to the content of page 10 of this document, plus his week long hospitality to the author in China. Also, **Dr. Munawar Karim** of Saint John Fisher's college is thanked for his questions which initiated the article write up.

Appendix A : Explanation of additional term put in Maartens Graviton eqn.

Note that Rubakov[63] writes KK graviton representation as, after using the following normalization $\int \frac{dz}{a(z)} \cdot [h_m(z) \cdot h_{\tilde{m}}(z)] \equiv \delta(m - \tilde{m})$ where J_1, J_2, N_1, N_2 are different forms of Bessel functions, to obtain the KK graviton/ DM candidate representation along RS dS brane world

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \tag{A1}$$

This Eq. (A1) is for KK gravitons having a TeV magnitude mass $M_Z \sim k$ (i.e. for mass values at .5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK graviton, which is moving with a speed proportional to H^{-1} with regards to the negative tension brane with $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ as an initial starting value for the KK graviton mass, before the KK graviton, as a 'massive' graviton moves with velocity H^{-1} along the RS dS brane. If so, and if $h \equiv h_m(z \to 0) = const \cdot \sqrt{\frac{m}{k}}$ represents an initial state, then one may relate the mass of the KK graviton, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass lower in value, i.e. of $m_{graviton}(4 - Dim \quad GR) \sim 10^{-48} eV$, as opposed to $M_X \sim \ M_{KK-Graviton} \sim .5 \times 10^9 eV$. Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky et.al.[64] who argue for graviton mass using CMBR measurements, of $M_{KK-Graviton} \sim 10^{-20} eV$ Dubosky et. al. [64] results can be conflated with Alves et. al. [54] arguing that non zero graviton mass may lead to an acceleration of our present universe, in a manner usually conflated with DE, i.e. their graviton mass would be about $m_{graviton}(4 - Dim \quad GR) \sim 10^{-48} \times 10^{-5} eV \sim 10^{65}$ grams.

Appendix B **Formulation of criteria for a second-order phase transition at the onset of nucleation of a new universe ?**

Let us first review Torrieri's and Mushuntin's [65] contribution to stability analysis of a wave functional treatment of a QCD bulk viscosity-over-entropy constant-ratio state equation. The idea is that we have initially a super hot plasma reaching a peak value of viscosity for a given temperature T, which is less than or equal to a critical temperature, T_C reflecting the QCD plasma having a peak value for viscosity. For those who wish to understand how this may work out, we can refer to a paper by M. Asakawa et al [55] of (2006), which specified a sheer bulk viscosity approximated by a viscosity value with $d_f \approx O(100)$, which weakly depends upon the number of quark flavors n_f in the quark-gluon plasma

$$\eta_C = \left[d_f \cdot T^3 \big/ g^4 \ln g^{-1} \right] \tag{B1}$$

Here, g is fixed by the number of degrees of freedom of the system. M. Asakawa et al.[55](2006) also specify that in a quark-gluon plasma, frequently there is an additional anomalous contribution to viscosity , η_A

caused by turbulent fields within the quark-gluon plasma. M Asakawa et al.[55] (2006) concluded in their document that frequently we have

$$\eta_{Total}^{-1} = \eta_C^{=1} + \eta_A^{-1} \qquad \text{(B2)}$$

Frequently we also have for extremely high temperatures to a good first approximation, as given by Kolb and Turner[7]

$$s_{Density} = \frac{2 \cdot \pi^2}{45} \cdot g_* \cdot T^3 \qquad \text{(B3)}$$

Where $g_* \cdot$ is the net degrees of freedom of the plasma gas that we can model as an ultra-relativistic fluid. For high temperatures, if $g_* \cdot$ is on the order of 100-120 and possibly up to over 1000 [8], in the first few seconds of the big bang , i.e., reflecting many initial degrees of freedom [7],

$$\eta_{Total} / s_{Density} \approx const \sim [1/4\pi] \qquad \text{(B4)}$$

With classical fluid models, even for quark-gluon plasmas, this assumes we are working with η_A^{-1} as not a very strong contributing factor to Eq (B2) , leading to almost infinite viscosity if we have viscosity almost entirely dependent upon temperature, as the temperature climbs.With the model of entropy so offered above, we have if the temperature is not elevated and the two terms in Eq. (B2) contribute , trouble in obtaining a stable value for Eqn. (B4) above as a constant. It so happens that Torrieri's and Mushuntin's [60] (2008) idea is to incorporate a modification of the Bjorken equation for cosmology applications,

$$\tau^{-3} \frac{d[\tau^3 s]}{d\tau} = \frac{3s}{R\tau} \qquad \text{(B5)}$$

where τ is conformal time, and R is the Reynolds number, and s is entropy density. This Eqn. (B5) is well above the complexity level of what one expects from the simple linearized models, where we look at, say, if y represents space time "length," etc., with

$$s(\tau) = s_0(\tau) + \delta \cdot s(\tau, y) \exp[iky] \qquad \text{(B6)}$$

And a velocity $v \propto x/t$ so that eventually we look at $x_1 = \delta \cdot s/s$ and $x_2 \equiv y - y_{space-time}$. So the stability analysis we have is

$$\tau \frac{\partial}{\partial \tau} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \equiv \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} \\ \mathrm{A}_{21} & \mathrm{A}_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \qquad \text{(B7)}$$

This is when we have at high temperatures a major simplification of the A_{ij} terms in the matrix in the right hand side of Eq. (B7) .This simplification of the right hand side of Eq. (B7) happens when we write $\eta \approx T^3$ and $s \propto T^3$. We obtain with this simplification of entropy and viscosity a relatively constant Reynolds number R_0, and a relatively constant speed of "sound" in the viscous media c_s^0. The resulting simplification and drop out of terms in the evolution equation allows us to write

$$A_{11} = c_s^{02} R_0^{-1} \qquad \text{(B8)}$$

and

$$A_{12} = -k \cdot (1 - 2R_0^{-1}) \qquad \text{(B9)}$$

and

$$A_{21} = kc_s^{02} \cdot (1 - 3R_0^{-1}) / (1 - R_0^{-1}) \qquad \text{(B10)}$$

and

$$A_{22} = -[(1 - c_s^{02}) + c_s^{02} R_0^{-1} + 3c_s^{02} R_0^{-1} (1 - R_0^{-1}) + k^2 \cdot R_0^{-1}] / (1 - R_0^{-1}) \qquad \text{(B11)}$$

In this limit we have a stability analysis performed for the eigenvalues of

$$A + A^T \qquad \text{(B12)}$$

Where we are using $A \equiv \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$, and with the summarized results that for $\{\lambda_{\min}, \lambda_{\max}\}$ of Eqn (B12) are such that , if

$\lambda_{\min} > 0$ we always have instability (B13)

$\lambda_{\max} < 0$ we always have stability (B14)

$\lambda_{\min} < 0, \lambda_{\max} > 0$, we some times have stability, (B15)
and sometimes we do not have stability.

The forms of Eqn (B12) to Eqn (B15) remain the same, but we assert that if we deviate from strict adherence to $\eta \approx T^3$ and $s \propto T^3$ due to marked initial conditions, i.e., unusual contributions due to the anharmonic contribution to viscosity η_A we will have increasingly involved criteria for forming the matrix for Eqn. (B11) and Eqn. (B7) to Eqn. (B10). We are looking into what these criteria should be for very unstable initial GUT criteria, with the proviso that we are not able to use simple linearization in GUT initial conditions, but that the ratio of $\eta_{Total} / s_{Density} \sim [1/4\pi]$ holds.

Bibliography

[1] Private notes, Dr. Karim Munawar , sent to the author in correspondence

[2] L. Dyson, M. Kleban, L. Susskind, JHEP 0210:011,2002, http://arxiv.org/abs/hep-th/0208013

[3] S. Mathur, , and B. Chowdhury,, Classical and Quantum Gravity, 24 (2007), pp. 2689-2720;

[4] S. Lloyd , Phys. Rev. Lett. 88, 237901 (2002)

[5] T. **Padmanabhan,** *Int.J.Mod.Phys.,* **D 17,** 367-398 (2008) [gr-qc/0409089] ; Padmanabhan, T. "Invitation to Astro physics", World Scientific series in Astronomy and Astro physics, Vol 8

[6] D. Park, H.. Kim,, and S. Tamarayan, *Phys.Lett*. **B**535 (2002) pp. 5-10

[7] E. Kolb., and S. Turner,. "The Early Universe", Westview Press, Chicago, USA, 1994

[8] Dicussion with H. De La Vega, at Villa Olmo, Como , Italy July 11 2009; ISAPP

[9] R. Wald, Quantum Fields in Curved Spacetime and Black Hole Thermodynamics, University of Chicago press, 1995

[10] A. Boyarsky, J. Lesgourgues, O. Ruchayskiy and M.Viel (2009) Phys.Rev.Lett.102:201304,2009

[11] K. Venkatartnam, and P. Suresh, Int.J.Mod.Phys.D17:1991-2005,2008, http://arxiv.org/abs/0712.3129

[12] M. Bellini, Gen.Rel.Grav. 35 (2003) 35-41, http://arxiv.org/abs/gr-qc/0210009

[13] G.E. Volovik J. Low Temp. Phys. 124 (2001) 25-39, http://arxiv.org/abs/gr-qc/0101111

[14] L P Grishchuk and Yu V Sidorov 1989 Class. Quantum Grav. 6 L155

[15] L. P. Grishchuk, Class. Quantum Grav. 10, 2449 (1993)

16 J. G. Polchinski; http://www.kitp.ucsb.edu/~joep/JLBS.pdf; String Theory, Vol. 1 (Cambridge Monographs on Mathematical Physics) (Paperback)

[17] R. Dick, Acta Phys.Polon. B32 (2001) 3669-3682; http://arxiv.org/abs/hep-th/0110162

[18] J. Berkenstein ; Phys.Rev. D70 (2004) 083509; Erratum-ibid. D71 (2005) 069901; http://arxiv.org/abs/astro-ph/0403694

[19] L. Ford, Phys.Rev.D51:1692-1700,1995; http://arxiv.org/abs/gr-qc/9410047

[20] L. P. Grishchuk Phys. Rev. D 53, 6784–6795 (1996)

[21] A.W. Beckwith, http://vixra.org/abs/1003.0193, version 3

[22] A.W. Beckwith, Mod.Phys.Lett. B20 (2006) 849-861, http://arxiv.org/abs/math-ph/0411045

[23] Y. J. Ng, *Phys. Lett. B*, **657**, (2007), pp. 10-14 , Y.J. Ng, *Entropy* **2008**, *10*(4), 441-461; DOI: 10.3390/e10040441

[24] . G. 't Hooft, http://arxiv.org/PS_cache/quant-ph/pdf/0212/0212095v1.pdf (2002); G. 't Hooft.,in *Beyond the Quantum*, edited by Th. M. Nieuwenhuizen et al. (World Press Scientific)

[25] A.W. Beckwith, "Classical & Quantum Models of Density Wave Transport, a comparative study", PhD Dec/ 2001

[26] B. Kay, Quantum Field theory in Curved Spacetime, pp 180-190, in the Encyclopedia of Mathematical physics, Vol 5, General Relativity; Quantum Gravity; String theory and M. Theory, With J-P Franciose, G. Naber, and T. Tsun as editors

[27] S. Lloyd, Phys. Rev. Lett., 88(23):237901, 2002.

[28] Ibanez, J., and Verdaguer, E. (1985). Physical Review D, 31, 251

[29] A. Buonanno, "Gravitational waves", pp 10-52, from the Les Houches Section LXXXVI, 'Particle physics and Cosmology, the fabric of spacetime' which has F. Bernardeau, C. Grojean, and J. Dalibard as editors
[30] V. Sahni, & S. Habib, S. 1998, Phys. Rev. Lett. 81, 1766 [hep-ph/9808204]
[31] C. Singh, IJTP, Vol. 45, number 3 / March, 2006
[32] C. Rovelli, Quantum Gravity (Cambridge Monographs on Mathematical Physics) (Hardcover), 2004
[33] C. Rovelli, Phys. Rev. Lett. 97, 151301 (2006)
[34] L. Montel, http://motls.blogspot.com/2007/08/thiemann-dittrich-discreteness-of-lqg.html
[35] J. Mielczarek , O. Hrycyna and M. Szydłowski; JCAP11(2009)014
[36] Dah-Wei Chiou* and Li-Fang Li, Phys. Rev. D 79, 063510 (2009)
[37] A. Ashtekar, The issue of the beginning in quantum gravity, In Proceedings of the 7th International Conference on the history of general relativity, edited by C. Lehner et al, (Springer, Berlin, 2006).
[38] M. Bojowald, General Relativity and Gravitation, Volume 40, Issue 12, pp.2659-2683
[39] L. Crowell,"Quantum fluctuations of spacetime", World Press Scientific, 2005
[40] Prado Martin-Moruno, Pedro F. Gonzalez-Diaz Phys. Rev. D {80}, 024007 (2009); arXiv:0907.4055
[41] M. Cavaglia; http://arxiv.org/PS_cache/hep-th/pdf/9808/9808136v1.pdf, http://arxiv.org/PS_cache/gr-qc/pdf/9411/9411070v2.pdf
[42] L. GarayPhys. Rev. D 44, 1059–1066
[43] Fangyu Li, personal communication, quoted with his permission (2009)
[44] Rainer Weiss, private discussions with the author, at ADM 50
[45] B.P. Abbott, et.al., Nature 460, 991 (2009).
[46] L. P. Grishchuk Lect. Notes Phys. 562, 167 (2001)
[47] M. Maggiore, *Gravitational Waves , Volume 1 : Theory and Experiment,* Oxford Univ. Press(2008)
[48] A. Beckwith, http://vixra.org/abs/0912.0012, v 6 (newest version).
[49] A. Kosowsky, A. Mack, and T. Kahniashvili, Phys. Rev. D 66, 024030 (2002)
[50] F. A. Jenet, G. B. Hobbs, W. van Straten, R. N. Manchester, M. Bailes, J. P. W. Verbiest, R. T. Edwards, A. W. Hotan, J. M. Sarkissian, and S. M. Ord, ApJ 653, 1571–1576 (2006) ; F. A. **Jenet**, G. B. Hobbs, K. J. Lee, and R. N. Manchester, Astrophys. J. 625, L123 (2005)
[51] M.S. Turner and F. Wilzenk Phys. Rev. Lett. 65, 3080 (1990).
[52] M. Marklund, G. Brodin, and P. Shukla, *Phys. Scr*. **T82** 130-132 (1999).
[53] R. Maartens, *Brane-World Gravity*, http://www.livingreviews.org/lrr-2004-7 (2004); R, Maartens *Brane world cosmology*, pp **213-247** from the conference *The physics of the Early Universe*, editor Papantronopoulos, (Lect. notes in phys., Vol **653**, Springer Verlag, **2005**).
[54] M. Alves, O. Miranda. and J. de Araujo, J. ," Can Massive Gravitons be an Alternative to Dark Energy?", arXiv:0907.5190 (July 2009)
[55] M. *Asakawa* , S. Bass, and B. Müller B.," Anomalous viscosity of an expanding quark-gluon plasma", Physical review letters 96(25):252301 *2006* Jun 30 ; M. *Asakawa* , T. Hatsuda, and. Y. Nakahara,, Prog. Part. Nucl. Phys. 46, 459(2001); M.*Asakawa* , T. Hatsuda, T. and. Y. Nakahara, Prog. Part. Nucl. Phys. 46, 459(2001)
[56] M. Giovannini, " A Primer on the Physics of the Cosmic Microwave Background", World Press Scientific, 2008, Singapore, Republic of Singapore, (2008)
[57] *Spacetime and Geometry:: An Introduction to General Relativity*, Addison Wesley Co., San Francisco, 2005
[58] A. Hajian , T. Souradeep, and N. Cornish, Astrophys.J.618:L63-L66,2004, http://arxiv.org/abs/astro-ph/0406354
59 F. Li , M. Tang , International Journal of Modern Physics D Vol.11 No.7 1049-1059 (2002) ; F. Li, , R. Baker, Z Fang, G. Stephenson., and Z. Chen, Zhenya, European Physical Journal C., 56, 407- 423 (2008) ; F. Li, N. Yang, Z. Fang, R. Baker , G. Stephenson, and H. Wen; Phys. Rev. D 80, 064013 (2009)
[60] L. P. Grishchuk, http://en.scientificcommons.org/42957051 ; D. Baskaran , L. Grishchuk ,W. Zhao, http://arxiv.org/abs/1004.0804
[61] V.A. Rubakov and , P.G. Tinyakov , Phys. Usp. 51 759 (2008) http://ufn.ru/en/articles/2008/8/a/references.html ;
[62] S. Weinberg, Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity (1972)
[63] V. Rubakov,*Classical Theory of Gauge Fields* , Princeton University press, 2002.
[64] S. Dubovsky, R. Flauger, A. Starobinsky, I. Tkachev,,report UTTG-06-09, TCC-23-09, http://arxiv.org/abs/0907.1658
[65] G.Torrieri and I. Mushuntin, Phys.Rev.C78:021901,2008, http://arxiv.org/abs/0805.0442