The New Prime theorem（19）

$$P_n = (P_1 P_2 \cdots P_{n-1})^2 - 2$$

Chun-Xuan Jiang
P. O. Box 3924, Beijing 100854, P. R. China
jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that such that $P_n = (P_1 P_2 \cdots P_{n-1})^2 - 2$ has infinitely many prime solutions.

Theorem. The prime equation

$$P_n = (P_1 P_2 \cdots P_{n-1})^2 - 2 \quad (1)$$

has infinitely many prime soultions

Proof. We have Jiang function[1]

$$J_n(\omega) = \prod_P [(P-1)^{n-1} - \chi(P)], \quad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$(q_1 q_2 \cdots q_{n-1})^2 - 2 \equiv 0 \pmod P, \quad q_i = 1, \cdots, P-1, i = 1, \cdots, n-1, \quad (3)$$

From (3) we have

$$\left(\frac{2}{P}\right) = (-1)^{\frac{P^2-1}{8}}, \text{ if } \left(\frac{2}{P}\right) = 1 \text{ then } \chi(P) = 2(P-1)^{n-2}, \text{ if } \left(\frac{2}{P}\right) = -1 \text{ then } \chi(P) = 0.$$

Substituting it into (2) we have.

$$J_n(\omega) = \prod_{3 \le P} [(P-1)^{n-2}(P - 2 - (-1)^{\frac{P^2-1}{8}}] \ne 0. \quad (4)$$

We prove that (1) has infinitely many prime soultions. $J_n(\omega) \subset \phi^{n-1}(\omega)$

We have the best asymptotic formula

$$\pi_2(N, n) = \left|\{P_1, \cdots, P_{n-1} \le N : P_n = prime\}\right| \sim \frac{J_n(\omega)\omega}{2 \times (n-1)! \phi^n(\omega)} \frac{N^{n-1}}{\log^n N}. \quad (5)$$

Example 1. Let $n = 2$. From (1) we have

$$P_2 = P_1^2 - 2 \quad (6)$$

From (4) we have

$$J_2(\omega) = \prod_{3 \le P} [P - 2 - (-1)^{\frac{P^2-1}{8}}] \ne 0 \quad (7)$$

Example 2. Let $n = 3$. From (1) we have

$$P_3=(P_1P_2)^2-2\,. \qquad (8)$$

From (4) we have

$$J_3(\omega)=\prod_{3\le P}[(P-1)(P-2-(-1)^{\frac{P^2-1}{8}}]\ne 0\,. \qquad (9)$$

Note. The prime numbers theory is to count the Jiang function $J_{n+1}(\omega)$ and Jiang singular series $\sigma(J)=\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)(1-\frac{1}{P})^{-k}$ [1], which can count the number of prime number. The prime number is not random. But Hardy singular series $\sigma(H)=\prod_P\left(1-\frac{\nu(P)}{P}\right)(1-\frac{1}{P})^{-k}$ is false. [2], which can not count the number of prime numbers.

References

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf. http://wbabin.net/xuan.htm#chun-xuan.

[2] G. H. Hardy and J. E. Littlewood, Some problems of "Partitio Numerorum", III: On the expression of a number as a sum of primes. Acta Math., 44(1923)1-70.