The New Prime theorem（13）

$n\times a^n \pm 1$ and $n\times 2^n \pm 1$

Chun-Xuan Jiang
P. O. Box 3924, Beijing 100854, P. R. China
jiangchunxuan@vip.sohu.com

Abstract

Using Jiang function we prove that $n\times a^n \pm 1$ have infinitely many prime solutions and $n\times 2^n \pm 1$ have finite prime solutions.

Theorem. We define the irreducible prime equation

$$P_1 = n\times(P-1)^n + 1 \quad (1)$$

For every positive integer n there exist infinitely many primes P such that P_1 is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \quad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$n\times(q-1)^n+1\equiv 0 \pmod P, \quad q=1,\cdots,P-1. \quad (3)$$

From (3) we have that if $n=3b+2$ then $\chi(3)=1$, $\chi(3)=0$ otherwise, $\chi(P)<P-1$. We have

$$J_2(\omega)\neq 0. \quad (4)$$

We prove that there exist inifinitely many primes P such that P_2 is a prime.

We have asymptotic formula [1]

$$\pi_2(N,2)=\left|\left\{P\le N: n\times(P-1)^n+1=prime\right\}\right| \sim \frac{J_2(\omega)\omega}{n\phi^2(\omega)}\frac{N}{\log^2 N} \quad (5)$$

where $\phi(\omega)=\prod_P(P-1)$.

Let $P=3$. From (1) we have Cullen equation

$$P_1 = n\times 2^n + 1 \quad (6)$$

From (5) we have

$$\pi_2(3,2) \sim \frac{J_2(\omega)}{n\phi^2(\omega)} \frac{3}{\log^2 3} \to 0 \quad \text{as} \quad n \to \infty \qquad (7)$$

We prove the finite Cullen primes.

In the same way we are able to prove that $n \times a^n - 1$ has infinitely many prime solutions, $n \times 2^n - 1$ has definite prime solutions and $h \times 2^n \pm 1$ have finite prime solutions.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf.