The New Prime theorem（12）

$3\times a^3 \pm 1$

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Abstract

Using Jiang function we prove that $3\times a^3 \pm 1$ has infinitely many prime solutions

Theorem. We define the prime equation

$$P_1 = 3\times (P-1)^3 + 1 \qquad (1)$$

There exist infinitely many primes P such that P is a prime.

Proof. We have Jiang function[1]

$$J_2(\omega) = \prod_P [P-1-\chi(P)], \qquad (2)$$

where $\omega = \prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$3\times (q-1)^3 + 1 \equiv 0 \pmod P, \quad q = 1, \cdots, P-1 \qquad (3)$$

we have

$$3^{\frac{P-1}{3}} \equiv 1 \pmod P \qquad (4)$$

If (4) has a solution then $\chi(P) = 3$. If (4) has no solution then $\chi(P) = 0$, $\chi(P) = 1$ otherwise.

We prove $J_2(\omega) \neq 0$, there exist infinitely many primes P such that P_2 is a prime.

We have asymptotic formula [1]

$$\pi_2(N,2) = \left|\left\{P \le N : 3\times (P-1)^3 + 1 = prime\right\}\right| \sim \frac{J_2(\omega)\omega}{3\phi^2(\omega)} \frac{N}{\log^2 N} \qquad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

In the same way we are able to prove that $3\times a^3 - 1$ has infinitely many prime solutions.

Reference

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. http://www. wbabin.net/math /xuan2. pdf.