The New Prime theorem（3）

$$P,\ jP+5-j\ (j=1,2,3,4)$$

Chun-Xuan Jiang
P. O. Box 3924, Beijing 100854, P. R. China
jiangchunxuan@vip.sohu.com

Abstrat

Using Jiang function we prove that there exist infinitely many primes P such that each $jP+5-j$ is a prime.

Theorem.

$$P,\ jP+5-j\ (j=1,2,3,4) \qquad (1)$$

There exist infinitely many primes P such that each of $jP+5-j$ is a prime.

Proof. We have Jiang function [1]

$$J_2(\omega)=\prod_P[P-1-\chi(P)], \qquad (2)$$

where

$$\omega=\prod_P P,$$

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{4}(jq+5-j)\equiv 0 \pmod P, \qquad (3)$$

$q=1,\cdots,P-1$.

From (3) we have $\chi(2)=0$, $\chi(3)=1$, $\chi(5)=1$, $\chi(P)=4$ otherwise.

From (3) and (2) we have

$$J_2(\omega)=3\prod_{7\le P}(P-5)\neq 0 \qquad (4)$$

We prove that there exist infinitely many primes P such that each of $jP+5-j$ is a prime.

We have the best asymptotic formula [1]

$$\pi_5(N,2)=\left|\{P\le P: jP+5-j=prime\}\right|\sim\frac{J_2(\omega)\omega^4}{\phi^5(\omega)}\frac{N}{\log^5 N} \qquad (5)$$

Reference

[1] Chun-Xuan Jiang, Jiang’s function in $J_{n+1}(\omega)$ prime distributio. http://www.wbabin. net/math/xuan2. pdf.