Algebraic Generalization[1] of Venn Diagram

Florentin Smarandache
University of New Mexico
Gallup, NM 87301, USA

Abstract.

It is easy to deal with a Venn Diagram for $1 \leq n \leq 3$ sets. When n gets larger, the picture becomes more complicated, that's why we thought at the following codification. That's why we propose an easy and systematic algebraic way of dealing with the representation of intersections and unions of many sets.

Introduction.

Let's first consider $1 \leq n \leq 9$, and the sets $S_1, S_2, \ldots, S_n$.
Then one gets 2^n-1 disjoint parts resulted from the intersections of these n sets. Each part is encoded with decimal positive integers specifying only the sets it belongs to. Thus:
part 1 means the part that belongs to S_1 (set 1) only, part 2 means the part that belongs to S_2 only, ..., part n means the part that belongs to set S_n only.
Similarly, part 12 means that part which belongs to S_1 and S_2 only, i.e. to $S_1 \cap S_2$ only.
Also, for example part 1237 means the part that belongs to the sets S_1, S_2, S_3, and S_7 only, i.e. to the intersection $S_1 \cap S_2 \cap S_3 \cap S_7$ only. And so on. This will help to the construction of a base formed by all these disjoint parts, and implementation in a computer program of each set from the power set $\mathcal{P}(S_1 \cup S_2 \cup \ldots \cup S_n)$ using a binary number.
The sets $S_1, S_2, \ldots, S_n$, are intersected in all possible ways in a Venn diagram. Let $1 \leq k \leq n$ be an integer. Let's denote by: $i_1 i_2 \ldots i_k$ the Venn diagram region/part that belongs to the sets S_{i1} and S_{i2} and … and S_{ik} only, for all k and all n. The part which is outside of all sets (i.e. the complement of the union of all sets) is noted by 0 (zero). Each Venn diagram will have 2^n disjoint parts, and each such disjoint part (except the above part 0) will be formed by combinations of k numbers from the numbers: 1, 2, 3, …, n.

Example.

Let see an example for n = 3, and the sets S_1, S_2, and S_3.

---

[1] It has been called the **Smarandache's Codification** (see [4] and [3]).

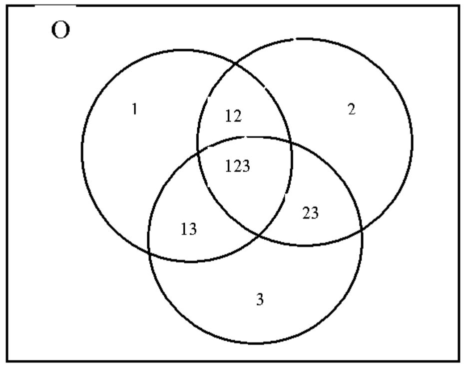

Fig. 1.

Unions and Intersections of Sets.

This codification is user friendly in algebraically doing unions and intersections in a simple way.
Union of sets S_a, S_b, …, S_v is formed by all disjoint parts that have in their index either the number a, or the number b, …, or the number v.
While intersection of S_a, S_b, …, S_v is formed by all disjoint parts that have in their index all numbers a, b, …, v.
For n = 3 and the above diagram:
$S_1 \cup S_{23} = \{1, 12, 13, 23, 123\}$, i.e. all disjoint parts that include in their indexes either the digit 1, or the digits 23;
and $S_1 \cap S_2 = \{12, 123\}$, i.e. all disjoint parts that have in their index the digits 12.

Remarks.

When $n \geq 10$, one uses one space in between numbers: for example, if we want to represent the disjoint part which is the intersection of S_3, S_{10}, and S_{27} only, we use the notation [3 10 27], with blanks in between the set indexes.
Depending on preferences, one can use other character different from the blank in between numbers, or one can use the numeration system in base n+1, so each number/index will be represented by a unique character.

References:

[1] J. Dezert, F. Smarandache, *An introduction to DSmT*, in <Advances and Applications of DSmT for Information Fusion>, ARPress, Vol. 3, pp. 3-73, 2009.

[2] J. Dezert & F. Smarandache *On the generation of hyper-powersets for the DSmT*, Proc. Fusion 2003 Conf., Cairns, Australia.

[3] A. Martin, *Implementing general belief function framework with a practical codification for low complexity*, in <Advances and Applications of DSmT for Information Fusion>, Vol. 3, pp. 217-273, 2009;
and in http://arxiv.org/PS_cache/arxiv/pdf/0807/0807.3483v1.pdf .

[4] N. J. A. Sloane, <Encyclopedia of Integer Sequences>, *Sequence A082185. This sequence is also called Smarandache's Codification and is used in computer programming*, http://www.research.att.com/~njas/sequences/A082185 .

[5] F. Smarandache and J. Dezert, *Advances and Applications of DSmT for Information Fusion*, Vol. 1, Am. Res. Press, 2004, pp. 42-46.