New prime K-tuple theorem (3)

$P, jP+j+1 (j=1,\cdots,k)$

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Abstract

Using Jiang function we prove that for every positive integer k there exist infinitely many primes P such that each of $jP+j+1$ is prime.

Theorem

$$P, jP+j+1 (j=1,\cdots,k). \quad (1)$$

For every psitive integer k there exist infinitely many primes P such that each of $jP+j+1$ is prime.

Proof. We have Jiang function [1, 2]

$$J_2(\omega)=\prod_P (P-1-\chi(P)), \quad (2)$$

where $\omega=\prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k}(jq+j+1)\equiv 0 \pmod{P}, \quad (3)$$

where $q=1,\cdots,P-1$.

From (3) we have

If $P\le k+1$ then $\chi(P)=P-2$, if $k+1<P$ then $\chi(P)=k$.

From (3) and (2) we have

$$J_2(\omega)=\prod_{k+1<P}(P-k-1)\neq 0. \quad (4)$$

We prove that for every positive integer k there exist infinitely many primes P such that each of $jP+j+1$ is primie.

We have the best asymptotic formula [1, 2]

$$\pi_{k+1}(N,2)=\left|\{P\le N: jP+j+1=prime\}\right|\sim \frac{J_2(\omega)\omega^k}{\phi^{k+1}(\omega)}\frac{N}{\log^{k+1}N}. \quad (5)$$

The auther takes a day to write this paper.

References

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (http://www. wbabin. net/math/xuan2.pdf) (http://vixra.org/pdf/0812.0004v2.pdf)

[2] Chun-xuan Jiang, The Hardy-Littlewood prime k-tuple conjecture is false. http:// www. wbabin.net/math/xuan77.pdf