New prime K-tuple theorem(1)

$$P_1, P_2, P_1 + jP_2 + j (j = 1, \cdots, k)$$

and

$$P_1, P_2, P_1 + jP_2 - j (j = 1, \cdots, k)$$

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Abstract

Using Jinag funciton we prove that there exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 + j$ is prime and there exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 - j$ is prime.

Theorem 1.

$$P_1, P_2, P_1 + jP_2 + j (j = 1, \cdots, k) \qquad (1)$$

There exist infinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 + j$ is prime.

Proof. We have Jiang function [1, 2]

$$J_3(\omega) = \prod_P [(P-1)^2 - \chi(P)] \qquad (2)$$

where $\omega = \prod_P P$,

$\chi(P)$ is the number of solutions of congruence

$$\prod_{j=1}^{k} (q_1 + jq_2 + j) \equiv 0 \pmod P, \qquad q_i = 1, \cdots, P-1, i = 1, 2. \qquad (3)$$

From (3) we have

If $k < P$ then $\chi(P) = k(P-2)$. If $P \le k$ then $\chi(P) = (P-1)(P-2)$.

From (3) and (2) we have

$$J_3(\omega) = \prod_{P=3}^{P \le k} (P-1) \prod_{k<P} [(P-1)^2 - k(P-2)] \ne 0 \qquad (4)$$

For any positive integer k there exist inifinitely many primes P_1 and P_2 such that each of $P_1 + jP_2 + j$ is prime.

We have asymptotic formula [1, 2]

$$\pi_{k+1}(N, 3) = \left| \{ P_1, P_2 \le N : P_1 + jP_2 + j = prime \} \right| \sim \frac{J_3(\omega)\omega^k}{\phi^{k+2}(\omega)} \frac{N^2}{\log^{k+2} N} , \qquad (5)$$

where $\phi(\omega) = \prod_P (P-1)$.

Example 1.

$$P_1, P_2, P_1 + P_2 + 1. \qquad (6)$$

From (4) we have

$$J_3(\omega)=\prod_{3\le P}[(P-1)^2-(P-2)]\neq 0 \qquad (7)$$

From (5) we have

$$\pi_2(N,3)\sim 2\prod_{3\le P}\left(1+\frac{1}{(P-1)^3}\right)\frac{N^2}{\log^3 N} \quad . \qquad (8)$$

Example 2.

$$P_1,\ P_2,\ P_1+P_2+1,\ P_1+2P_2+2 \quad . \qquad (9)$$

From (4) we have

$$J_3(\omega)=\prod_{3\le P}[(P-1)^2-2(P-2)]\neq 0 \qquad (10)$$

From (5) we have

$$\pi_3(N,3)\sim\frac{J_3(\omega)\omega^2}{\phi^4(\omega)}\frac{N^2}{\log^4 N} \quad . \qquad (11)$$

Example 3.

$$P_1,\ P_2,\ P_1+jP_2+j(j=1,\cdots,8) \quad . \qquad (12)$$

From (4) we have

$$J_3(\omega)=48\prod_{11\le P}[(P-1)^2-8(P-2)]\neq 0 \quad . \qquad (13)$$

From (5) we have

$$\pi_9(N,3)\sim\frac{J_3(\omega)\omega^8}{\phi^{10}(\omega)}\frac{N^2}{\log^{10} N} \quad . \qquad (14)$$

Theorem 2.

$$P_1,\ P_2,\ P_1+jP_2-j(j=1,\cdots,k) \quad , \qquad (15)$$

we have Jiang function

$$J_3(\omega)=\prod_{P=3}^{P\le k}(P-1)\prod_{k<P}[(P-1)^2-k(P-2)]\neq 0 \quad . \qquad (16)$$

For any positive integer k there exist infinitely many primes P_1 and P_2 such that each of P_1+jP_2-j is prime.

we have asymptotic formula

$$\pi_{k+1}(N,3)=\left|\{P_1,P_2\le N: P_1+jP_2-j=prime\}\right|\sim\frac{J_3(\omega)\omega^k}{\phi^{k+2}(\omega)}\frac{N^2}{\log^{k+2} N} \quad . \qquad (17)$$

References

[1] Chun-Xuan Jiang, Jiang's function $J_{n+1}(\omega)$ in prime distribution. (http:// www. Wbabin. net/math/xuan2. pdf) (http://vixra.org/pdf/0812.0004v2.pdf)

[2] Chun-Xuan Jiang, The Hardy-Littlewood prime $k-$tuple conjecture is false. http://www.wbabin.net/math/xuan77.pdf