DECELERATION PARAMETER Q(Z) IN 4D AND 5D GEOMETRIES, AND IMPLICATIONS OF GRAVITON MASS IN MIMICKING DARK ENERGY IN BOTH GEOMETRIES

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The case for a four-dimensional graviton mass (non zero) influencing reacceleration of the universe in both four and five dimensions is stated, with particular emphasis on the question whether 4D and 5D geometries as given here yield new physical insight as to cosmological evolution. Both cases give equivalent reacceleration one billion years ago, which leads to the question whether other criteria can determine the relative benefits of adding additional dimensions to cosmology models.

Keywords: graviton mass; deceleration parameter; 4D and 5D geometries

1. Introduction

A first-principle introduction to the detection of gravitational wave density is the definition given by Maggiore:[1]

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \Rightarrow h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[\frac{n_f}{10^{37}}\right] \cdot \left(\frac{f}{1kHz}\right)^4 \quad (1)$$

Here, n_f is the frequency-based numerical count of gravitons per unit phase space, which may also depend on the interaction of gravitons with neutrinos in plasma during early-universe nucleation, as modeled by Marklund et al.[2] However, it is not clear what sort of mechanism is appropriate for considering macro effects of gravitons. Reacceleration of the universe, as a function of graviton mass, could be such a mechanism. We assume Snyder geometry and use the following inequality for a change in the Heisenberg Uncertainty Principle (HUP) (see Battisti[3]):

$$\Delta x \geqslant \left[(1/\Delta p) + l_s^2 \cdot \Delta p\right] \equiv (1/\Delta p) - \alpha \cdot \Delta p \quad (2)$$

For brane worlds, $\alpha < 0$, whereas $\alpha > 0$ for loop quantum gravity (LQG). Next, we assume that the mass of the graviton is partly due to the stretching alluded to by Fuller and Kishimoto,[4] and investigate this for a modification of a joint Kaluza-Klein (KK) tower of gravitons, as given by Maartens[5] for dark matter (DM). The assumption that the stretching of early relic neutrinos would eventually lead to the KK tower of gravitons - for when $\alpha < 0$ - can be understood as follows (Beckwith[6]):

$$m_n(graviton) = \frac{n}{L} + 10^{-65} grams \tag{3}$$

Eq. (3) is the starting point for a KK tower version of the Friedman equation (Maartens[5]):

$$\dot{a}^2 = \left[\left(\frac{\tilde{\kappa}^2}{3} \left[\rho + \frac{\rho^2}{2\lambda} \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \right] \tag{4}$$

Maartens[5] also gives a second Friedman equation:

$$\dot{H}^2 = \left[- \left(\frac{\tilde{\kappa}^2}{2} \cdot [p + \rho] \cdot \left[1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2\frac{m}{a^4} + \frac{K}{a^2} \right] \tag{5}$$

For $\rho \cong -P$, we will have red-shift values z between zero and 1.0-1.5 with exact inequality for z between zero and 0.5. We then obtain (Beckwith[6]):

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \equiv -1 - \frac{\dot{H}}{H^2} = -1 + \frac{2}{\tilde{\kappa}^2 \left[m/a^4 \right] \cdot \left(\rho + \rho^2/2\lambda \right) + 1} \approx -1 + \frac{2}{2 + \delta\left(z \right)} \tag{6}$$

Eq. (6) assumes $\Lambda = 0 = K$ and we also use $a \equiv [a_0 = 1]/(1+z)$. The net effect of presenting how gravitons with a small mass in four dimensions can account for reacceleration of the universe is a substitute for dark energy (DE). This is usually acquired with $\Lambda \neq 0$, even if curvature κ is set equal to zero. The deceleration parameter q in Eq. (6) has higher-dimension contributions in the brane theory case, but not in the LQG case.

2. Consequences of Small Graviton Mass for Reacceleration of the Universe

In a revision of the work of Alves et al.,[7] Beckwith[6] used a higher-dimensional model of the brane world and the KK graviton towers of Maartens.[5] The density of the brane world Alves et al.[7] used in the Friedman equation is then applied for a non-zero graviton (Beckwith[6]):

$$\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a} \right)^3 - \left[\frac{m_g c^6}{8\pi G\hbar^2} \right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2} \right) \tag{7}$$

Eq. (6) creates a joint DM and DE model, with all of Eq. (6) being for KK gravitons and DM, and 10^{-65} grams being a four-dimensional DE. Eq. (5) is part of a KK graviton presentation of DM/DE dynamics. Beckwith[6] found that one billion years ago, at z $\sim$ 0.423, the acceleration of the universe did not slow down but increased instead (shown in Fig. 1).

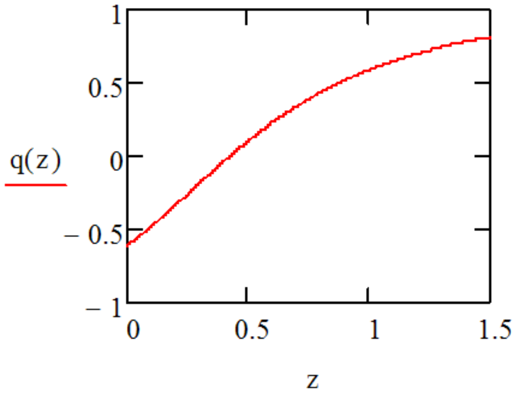

Fig. 1. Reacceleration of the universe based on Eq. (3); q < 0 if z < 0.4 and z ∼ 1.5 is 4.5 billion years after the Big Bang.

3. Connecting Neutrinos with Gravitons by Looking at Their Wavelengths

Assuming $m_0(graviton) \approx 10^{-65}$ grams for gravitons in four dimensions, Bashinsky[8] and Beckwith[6] suppose that density fluctuations are influenced by a modification of overall cosmological density ρ in the Friedmann equations by the proportionality factor given by Bashinsky:[8] $\left[1 - 5 \cdot (\rho_{neutrino}/\rho) + \vartheta\left([\rho_{neutrino}/\rho]^2\right)\right]$. This proportionality factor for ρ should be taken as an extension of the results of Marklund et al.,[2] where neutrinos interact with plasmons and plasmons interact with gravitons, thereby implying neutrino-graviton interactions. Also, graviton wavelengths have the same order of magnitude as those of neutrinos. Note (Valev[9]):

$$m_{graviton}|_{RELATIVISTIC} < 4.4 \times 10^{-22} h^{-1} eV/c^2 \quad (8)$$
$$\Leftrightarrow \lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 2.8 \times 10^{-8} meters$$

An extension on the work of Marklund et al.[2] and Valev[9] is the suggestion that some gravitons may become larger (Will[10]), i.e. $\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 10^4$ m.

4. Are Inflaton and Quintessence Manifestations of a Complex Field? Link Between Graviton Wave/Gravitons and Initial/Final Inflation?

Yurov[11] brought up that the following field could take on both inflaton and quintessence phenomenology:

$$\Phi(t) = \varphi(t) exp(i\theta(t))/\sqrt{2} \quad (9)$$

In the model of Yurov,[11] who assumes cyclic behavior where the value of $M = \Phi^2 \cdot \dot{\theta}$ is a constant, and supposes an overall chaotic inflationary-style potential, this dual-use, complex scalar field (Eq. 10) is part of a relatively simple chaotic potential:

$$V = \overset{\leftrightarrow}{m}^2 \Phi^* \Phi \tag{10}$$

Re-emergence of inflation allegedly occurs at the end of first inflation, and also at a second inflationary period, commencing at or before red shift Z ~ 0.423. The re-emergent second inflationary emergent field φ_+ allegedly is of the following form, with time t taken a billion years ago to the present (Yurov[11]):

$$\varphi_+ = \left[\varphi_{0,+}^3 - \sqrt{3/2} \cdot \frac{3M^2 t}{\overset{\leftrightarrow}{m}} \right]^{1/3} \tag{11}$$

Tying the complex scalar field to evolving Friedman equations can be accomplished by using the linkage suggested by Beckwith.[12] It follows from initial inflationary conditions, assuming that m is a typical inflaton mass. Equivalence is given to the representation of inflaton physics by Yurov[11] (left-hand side) and the expression of a brane world in a Friedman equation by Beckwith[12] (right-hand side):

$$H^2 = \frac{1}{6} \cdot \left[\dot{\varphi}^2 + \overset{\leftrightarrow}{m}^2 \varphi^2 + \frac{M^2}{\varphi^2} \right] \leftrightarrow H^2 = \left(\frac{\tilde{\kappa}^2}{3} \left[\rho + \frac{\rho^2}{2\lambda} \right] \right) + \frac{m}{a^4} \tag{12}$$

Next, the representation of a Friedman equation given by Yurov[11] (left) is paired with the suggestion Beckwith[12] made for the second Friedman equation (right):

$$\dot{H} = V - 3H^2 \leftrightarrow \dot{H}^2 \cong \left[-2\frac{m}{a^4} \right] \tag{13}$$

Equivalence is also assigned to the typical bound between $\left| \overset{\leftrightarrow}{m} \right| \leq \left[\frac{l^2}{4} \right]$ as given by Yurov,[11] and the initial brane world line elements from the work of Beckwith[6,12] $dS^2\big|_{5-\mathrm{dim}} = \frac{l^2}{z^2} \cdot \left[\eta_{uv} dx^\mu dx^v + dz^2 \right]$. A linkage in early inflation and inflaton $\varphi_{0,-}$ to first inflationary dynamics for $\varphi(t)$ is then given by:

$$\varphi(t) = \varphi_{0,-} - \sqrt{2/3} \cdot \overset{\leftrightarrow}{m} \cdot t \tag{14}$$

5. Conclusions

If a joint DM and DE model as given by Eq. 6 is consistent with known astrophysical observations, connections between Eq. (8) and Eq. (3) should be proven, and further work is needed on Eq. (6), to get better results than provided by Chaplygin-gas style joint DM-DE models (Debnath and Chakarborty[13]). We want other measurements than dependence on baryon acoustic oscillation data and supernovas as proof for DE, when the DE leads to $\omega_{0de} = -1.08 < -1$. Answering these questions requires new developments to improve the sensitivity of graviton wave detectors (Maggiore[1]). Note that DE does not appear in the beginning of inflation, and Eq. (6) may link the DE with the emergence and nucleation of gravitons, i.e., if DE $\sim m_0(graviton) \propto 10^{-65}$ grams in four dimensions. We need improvements over $h \sim 10^{-23}$ GW sensitivity to investigate DE-DM, as discussed with Weiss.[14] Further connections could arise from determining whether we have tension $\lambda = 3M_P^2 / 4\pi l^2$ (Maartens;[5] Beckwith[12]). A small value of $l \sim \breve{\lambda}$ would be consistent with the used approximation $\rho/2\lambda \approx 0.01$, and would lead to the following (as derived by Ng[15]):

$$S \approx N \cdot \left(\log \left[V \Big/ \breve{\lambda}^3 \right] + 5/2 \right) \approx N = number\ of\ gravitons \tag{15}$$

Confirming whether Eq. (15) holds true for initial graviton production N would be priceless. Even better would be to determine whether Fig. 1 applies for both geometries present in the Snyder uncertainty bound in Eq. (2). In addition, there would be opportunities to confirm whether Beckwith[6,12] is correct about semi-classical treatments of graviton mass, in confirmation of the deterministic quantum mechanics of 't Hooft.[16]

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