The Hardy-Littlewood prime *k*-tuple conjecture is false

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Abstract

Using Jiang function we prove Jiang prime k-tuple theorem. We prove that the Hardy-Littlewood prime k-tuple conjecture is false. Jiang prime k-tuple theorem can replace the Hardy-Littlewood prime k-tuple conjecture.
Hardy-Littlewood 论文作为数论圣经，一百年来华罗庚、王元和一大批数论家必读，这是当代最高数论水平，数学天才陶哲轩拼命在学习他们论文，下一步仔细研究他们论文，到底有多少猜想是正确的，素数太复杂，蒋春暄已打开素数大门。

(A) Jiang prime k-tuple theorem [1, 2].

We define the prime k-tuple equation

$$p,\ p+n_i\,, \qquad (1)$$

where $2\mid n_i,\ i=1,\cdots k-1$.

we have Jiang function [1, 2]

$$J_2(\omega)=\prod_P (P-1-\chi(P)), \qquad (2)$$

where $\omega=\prod_P P$, $\chi(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q+n_i)\equiv 0 \pmod P,\ \ q=1,\cdots,p-1. \qquad (3)$$

If $\chi(P)<P-1$ then $J_2(\omega)\neq 0$. There exist infinitely many primes P such that each of $P+n_i$ is prime. If $\chi(P)=P-1$ then $J_2(\omega)=0$. There exist finitely many primes P such that each of $P+n_i$ is prime. $J_2(\omega)$ is a subset of Euler function $\phi(\omega)$ [2].

If $J_2(\omega)\neq 0$, then we hae the best asymptotic formula of the number of prime P [1, 2]

$$\pi_k(N,2)=\left|\{P\le N: P+n_i=prime\}\right|\sim\frac{J_2(\omega)\omega^{k-1}}{\phi^k(\omega)}\frac{N}{\log^k N}=C(k)\frac{N}{\log^k N} \qquad (4)$$

$$\phi(\omega)=\prod_P(P-1)\ ,$$

$$C(k)=\prod_P\left(1-\frac{1+\chi(P)}{P}\right)\left(1-\frac{1}{P}\right)^{-k} \qquad (5)$$

Example 1. Let $k=2, P, P+2$, twin primes theorem.

From (3) we have

$$\chi(2)=0,\quad \chi(P)=1 \text{ if } P>2\ , \qquad (6)$$

Substituting (6) into (2) we have

$$J_2(\omega)=\prod_{P\ge 3}(P-2)\ne 0 \qquad (7)$$

There exist infinitely many primes P such that $P+2$ is prime. Substituting (7) into (4) we have the best asymptotic pormula

$$\pi_k(N,2)=\left|\{P\le N: P+2=prime\}\right|\sim 2\prod_{P\ge 3}(1-\frac{1}{(P-1)^2})\frac{N}{\log^2 N}. \qquad (8)$$

Example 2. Let $k=3, P, P+2, P+4$.

From (3) we have

$$\chi(2)=0,\quad \chi(3)=2 \qquad (9)$$

From (2) we have

$$J_2(\omega)=0\ . \qquad (10)$$

It has only a solution $P=3$, $P+2=5$, $P+4=7$. One of $P, P+2, P+4$ is always divisible by 3.

Example 3. Let $k=4, P, P+n$, where $n=2,6,8$.

From (3) we have

$$\chi(2)=0, \chi(3)=1, \chi(P)=3 \text{ if } P>3. \qquad (11)$$

Substituting (11) into (2) we have

$$J_2(\omega)=\prod_{P\ge 5}(P-4)\ne 0\ , \qquad (12)$$

There exist infinitely many primes P such that each of $P+n$ is prime.

Substituting (12) into (4) we have the best asymptotic formula

$$\pi_4(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim \frac{27}{3}\prod_{P\ge 5}\frac{P^3(P-4)}{(P-1)^4}\frac{N}{\log^4 N} \qquad (13)$$

Example 4. Let $k=5$, P, $P+n$, where $n=2,6,8,12$.

From (3) we have

$$\chi(2)=0,\ \chi(3)=1,\ \chi(5)=3,\ \chi(P)=4 \ \text{ if } \ P>5 \qquad (14)$$

Substituting (14) into (2) we have

$$J_2(\omega)=\prod_{P\ge 7}(P-5)\ne 0 \qquad (15)$$

There exist infinitely many primes P such that each of $P+n$ is prime. Substituting (15) into (4) we have the best asymptotic formula

$$\pi_5(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim\frac{15^4}{2^{11}}\prod_{P\ge 7}\frac{(P-5)P^4}{(P-1)^5}\frac{N}{\log^5 N} \qquad (16)$$

Example 5. Let $k=6$, P, $P+n$, where $n=2,6,8,12,14$.
From (3) and (2) we have

$$\chi(2)=0,\ \ \chi(3)=1,\ \ \chi(5)=4,\quad J_2(5)=0 \qquad (17)$$

It has only a solution $P=5$, $P+2=7$, $P+6=11$, $P+8=13$, $P+12=17$, $P+14=19$. One of $P+n$ is always divisible by 5.

(B) The Hardy-Littlewood prime k-tuple conjecture[3-8].

We define the prime k-tuple equation

$$P,\ P+n_i \qquad (18)$$

where $2|n_i,\ i=1,\cdots,k-1$.

In 1923 Hardy and Littlewood conjectured the asymptotic formula

$$\pi_k(N,2)=\left|\{P\le N: P+n_i=prime\}\right|\sim H(k)\frac{N}{\log^k N}, \qquad (19)$$

where

$$H(k)=\prod_P\left(1-\frac{\nu(P)}{P}\right)\left(1-\frac{1}{P}\right)^{-k} \qquad (20)$$

$\nu(P)$ is the number of solutions of congruence

$$\prod_{i=1}^{k-1}(q+n_i)\equiv 0 \pmod P, \qquad q=1,\cdots,P. \qquad (21)$$

From (21) we have $\nu(P)<P$ and $H(k)\ne 0$. For any prime k-tuple equation there exist infinitely many primes P such that each of $P+n_i$ is prime, which is false.

Conjectore 1. Let $k=2, P, P+2$, twin primes theorem

Frome (21) we have

$$\nu(P)=1 \quad (22)$$

Substituting (22) into (20) we have

$$H(2)=\prod_{P}\frac{P}{P-1} \quad (23)$$

Substituting (23) into (19) we have the asymptotic formula

$$\pi_2(N,2)=\left|\{P\le N: P+2=prime\}\right|\sim\prod_{P}\frac{P}{P-1}\frac{N}{\log^2 N} \quad (24)$$

which is false see example 1.

Conjecture 2. Let $h=3, P, P+2, P+4$.

From (21) we have

$$\nu(2)=1,\ \nu(P)=2 \text{ if } P>2 \quad (25)$$

Substituting (25) into (20) we have

$$H(3)=4\prod_{P\ge 3}\frac{P^2(P-2)}{(P-1)^3} \quad (26)$$

Substituting (26) into (19) we have asymptotic formula

$$\pi_3(N,2)=\left|\{P\le N: P+2=prime, P+4=prim\}\right|\sim 4\prod_{P\ge 3}\frac{P^2(P-2)}{(P-1)^3}\frac{N}{\log^3 N} \quad (27)$$

which is false see example 2.

Conjecutre 3. Let $k=4$, $P, P+n$, where $n=2,6,8$.

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(P)=3 \text{ if } P>3 \quad (28)$$

Substituting (28) into (20) we have

$$H(4)=\frac{27}{2}\prod_{P>3}\frac{P^3(P-3)}{(P-1)^4} \quad (29)$$

Substituting (29) into (19) we have asymptotic formula

$$\pi_4(N,2)=\left|\{P\le N: P+n=prime\}\right|\sim\frac{27}{2}\prod_{P>3}\frac{P^3(P-3)}{(P-1)^4}\frac{N}{\log^4 N} \quad (30)$$

Which is false see example 3.

Conjecture 4. Let $k=5$, $P, P+n$, where $n=2,6,8,12$

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(5)=3,\ \nu(P)=4 \text{ if } P>5 \quad (31)$$

Substituting (31) into (20) we have

$$H(5)=\frac{15^4}{4^5}\prod_{P>5}\frac{P^4(P-4)}{(P-1)^5} \tag{32}$$

Substituting (32) into (19) we have asymptotic formula

$$\pi_5(N,2)=\left|\{P\le N:P+n=prime\}\right|\sim\frac{15^4}{4^5}\prod_{P>5}\frac{P^4(P-4)}{(P-1)^5}\frac{N}{\log^5 N} \tag{33}$$

Which is false see example 4.

Conjecutre 5. Let $k=6$, P, $P+n$, where $n=2,6,8,12,14$.

From (21) we have

$$\nu(2)=1,\ \nu(3)=2,\ \nu(5)=4,\ \nu(P)=5\ \text{ if }\ P>5 \tag{34}$$

Substituting (34) into (20) we have

$$H(6)=\frac{15^5}{2^{13}}\prod_{P>5}\frac{(P-5)P^5}{(P-1)^6} \tag{35}$$

Substituting (35) into (19) we have asymptotic formula

$$\pi_6(N,2)=\left|\{P\le N:P+n=prime\}\right|\sim\frac{15^5}{2^{13}}\prod_{P>5}\frac{(P-5)P^5}{(P-1)^6}\frac{N}{\log^6 N} \tag{36}$$

which is false see example 5.

Conclusion. The Hardy-Littlew [illegible] conjecture is false. Jiang prime k-tuple theorem can replace Har [illegible] k-tuple Conjecture.

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