On the Scale factor of the Universe and Redshift.

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ABSTRACT
It is proposed that there has been a longstanding misunderstanding of the relationship between scale factor of the universe and redshift. It is shown how value of omega(matter) of one quarter of the true value, (hence the apparent dark energy phenomenon) can result from such a misconception. Predictions for the magnitudes of supernovae against redshift are made and found to be in good agreement with supernovae data, without dark energy. .

Key words: Cosmology: distance scale, cosmological parameters, dark energy

INTRODUCTION

Modern cosmology relies on the hypothetical entities of dark energy and dark matter. The conclusion that dark energy must exist is from two main lines of evidence.

Due to the observations of distant supernovae (Riess et al 2007), cosmologists have concluded that the expansion of the universe is speeding up. Thus ‘dark energy’ has been inferred, the nature of which is poorly understood. There is a lack of an understanding of a physical mechanism, by which dark energy causes an accelerating expansion of the universe.

Measurements from WMAP show a flat universe, with a value of omega(matter) of approximately 0.25. This has led to a value for omega(lambda) of about 0.75.
X-ray measurements of galaxy clusters measures the baryon fraction, from which omega(matter) is deduced. However the value obtained relies on the value of omega(baryons) from WMAP data, and so is not independent.

These two methods seem to support each other, so it is understandable that many cosmologists (some reluctantly!) support the dark energy conclusion.

However, a simple change to our notions of how scale factor relates to redshift can remove these two main arguments in favour of dark energy, as shown below.

1. A NEW RELATIONSHIP BETWEEN REDSHIFT AND SCALE FACTOR

Traditionally, in cosmology, the ratio of the wavelength of light is proportional to the ratio of the scale factor of the universe.

$$1 + z = \frac{\lambda_1}{\lambda_2} = \frac{a_1}{a_2} \qquad (1)$$

With the new proposal there is a different relation

$$1+z_n=\frac{\lambda_1}{\lambda_2}=\left(\frac{a_1}{a_2}\right)^2 \qquad (2)$$

For a given ratio of scale factor, the redshifts for the two cosmologies are related by

$$z=\sqrt{1+z_n}-1 \qquad (3)$$

Hubbles constant, H, is assumed constant.

$$\frac{da/dt}{a}=H \qquad (4)$$

The observed value of (what is traditionally assumed to be) Hubbles constant of approx 72km/s/Mpc is not the true value. The value as defined by (4) is half, i.e approx 36km/s/Mpc. From (2), for small changes in time, the redshift depends on twice the value of Hubbles constant, and still matches observations.

$$1+z_n=(1+Hdt)^2=1+2Hdt \qquad (5)$$

Why (2) should be true is not discussed (although the author has reasons for expecting it to be the case). The consequences of such a relation, is the subject of this paper, particularly how it can lead to the conclusion of the dark energy phenomenon.

2. A SOLUTION OF EINSTEINS EQUATIONS.

For constant H, $a=a_0\exp(Ht)$, where a is the scale factor of the universe, Einsteins equations of General Relativity reduce to

$$8\pi G\frac{\rho}{c^2}=-\Lambda+\frac{3H^2}{c^2}+\frac{3k}{a^2} \qquad (6)$$

$$8\pi G\frac{p}{c^4}=\Lambda-\frac{3H^2}{c^2}-\frac{k}{a^2} \qquad (7)$$

so for a flat universe with k = 0, and $\Lambda=0$

$$p=-c^2\rho \qquad \text{(i.e. } \omega=-1\text{)} \qquad (8)$$

and

$$\rho=\frac{3H^2}{8\pi G} \qquad (9)$$

therefore the traditionally inferred value of omega(matter) would be

$$\Omega_m = \frac{3H^2 / 8\pi G}{3(2H)^2 / 8\pi G} = 0.25 \qquad (10)$$

Measurements from WMAP5, (Komatsu et al, 2008), lead to an inferred value for omega(matter) of 0.258 (0.030). Their preferred model is a flat ΛCDM model with k = 0, and an equation of state parameter, ω, of -1. A value for the maximum likelihood for omega(matter) is given as 0.249.

In reality $\Omega_m = 1$, as the denominator of (10) should contain H not $2H$, and $\Omega_\Lambda = 0$. It is not necessary to assume dark energy, as the universe is naturally at critical density.

The 'coincidence problem' that the values of omega(lambda) and omega(matter) are similar only at the time we live in, is avoided with this approach. At all times omega(matter) is 1 and omega(lambda) is zero.

3. THE SUPERNOVAE DATA

The flux F due to distant supernovae is given by

$$F = \frac{L}{4\pi {d_L}^2} = \frac{L}{4\pi(1+z)^2 d_p^2} \qquad (11)$$

d_L is the luminosity distance, L is luminosity and d_p is the 'proper distance'. Traditionally, for a flat universe

$$d_p = \frac{1}{H}\int_0^z \frac{dz'}{h(z')} \qquad (12)$$

With the new approach H is half of the traditional value and

$$d_p = \frac{c}{H}(\sqrt{1+z}-1) \qquad (13)$$

(there is a derivation in Appendix A), so

$$d_L = \frac{c}{H}(1+z)(\sqrt{1+z}-1) \qquad (14)$$

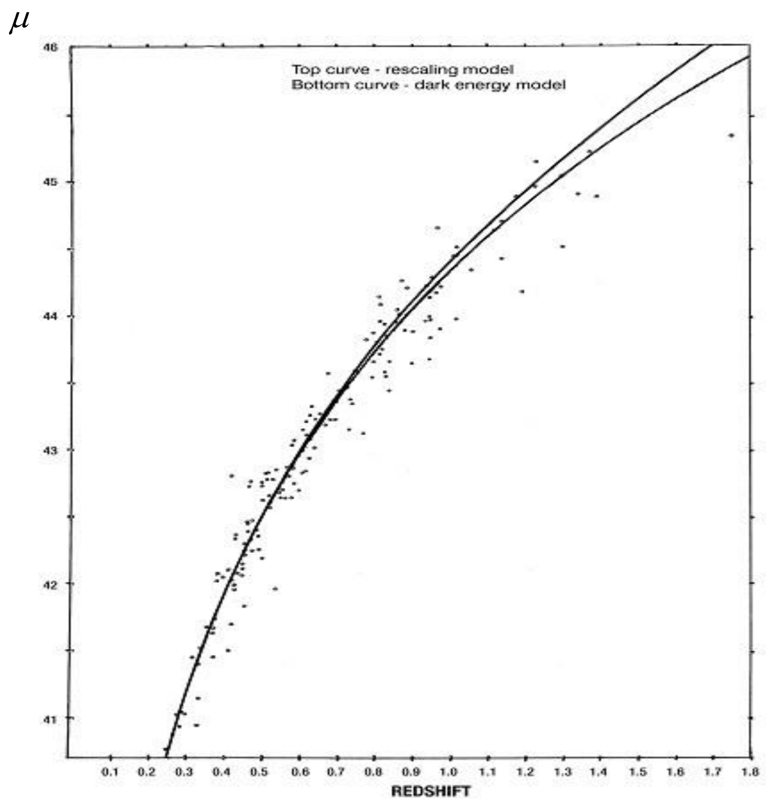
the distance modulus is

$$\mu = 25 + 5\log d_L \qquad (15)$$

Using (14) in (15), there is a good match to the supernovae data (Riess 2007), gold set. The chi-squared fit is 183.8 for 182 degrees of freedom. This close match is with H, constant, with no requirement for a dark energy component of the universe.

Figure 3 shows a comparison between the new approach and the dark energy model, The curves are very similar. The top curve is for $2H$ = 65.1kms^{-1}Mpc^{-1}. The bottom curve is for the best flat dark energy model with $2H$ = 63.8kms^{-1}Mpc^{-1} (Wright 2007). The dark energy model has a variable parameter, the matter density, for the curve shown omega(matter) = 0.27. The new approach uses no extra variable parameter. With the new approach, the deceleration parameter q(z) = -1 (constant), for dark energy q(z) varies, in a way that is not understood (Shapiro & Turner, 2006).

Figure 3 Supernova moduli with redshift, for rescaling and dark energy models.



4. CONCLUSIONS AND PREDICTIONS.

There has been a serious and long-standing misinterpretation of the relationship between the ratio of scale factor and redshift. The relation should be as in (2). If this is indeed the case, then we would expect the following.

i) The inferred value of omega(matter) will be exactly 0.25.
ii) The distance moduli (15) for supernovae, will be according (14)

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APPENDIX A DERIVATION OF (13) AND (14)

Starting from the Robertson-Walker metric

$$ds^2 = -c^2 dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right] \qquad \text{(A1)}$$

in terms of the co-moving co-ordinates, χ has the role of the radial co-ordinate

$r = \sin\chi$ if k = +1
$r = \chi$ if k = 0
$r = \sinh\chi$ if k = -1

For ray of light moving along a radial path with θ and φ constant , for a flat universe,

$$ds^2 = -c^2 dt^2 + a(t)^2 d\chi^2 = 0 \qquad \text{(A2)}$$

so

$$ad\chi = -cdt = -c\frac{dt}{da}da = -c\frac{da}{\dot{a}} = -c\frac{da}{aH} \qquad \text{(A3)}$$

for the new relation (2)

$$1 + z = \left[\frac{a_0}{a_t}\right]^2 \qquad \text{(A4)}$$

$$dz = -2\frac{a_0^2}{a^3}da \qquad \text{(A5)}$$

so from (A3)

$$ad\chi = \frac{c}{2H}\frac{a^2}{a_0^2}dz \qquad \text{(A6)}$$

$$a_0 d\chi = \frac{c}{2H}\frac{a}{a_0}dz \qquad \text{(A7)}$$

from (2)

$$a_0 d\chi = \frac{c}{2H}\frac{1}{\sqrt{1+z}}dz \qquad \text{(A8)}$$

$$d_p = \int_0^z \frac{c}{2H}(1+z)^{-\frac{1}{2}}dz \qquad \text{(A9)}$$

$$d_p = \frac{c}{H}(\sqrt{1+z}-1) \qquad \text{(A10)}$$

which is (13), and H is about 36km/s/Mpc, and from (11)

$$d_L = \frac{c}{H}(1+z)(\sqrt{1+z}-1) \qquad \text{(A11)}$$

which is (14).