Diophantine Equation

$$P_{n+1}^{l_{n+1}} = \frac{P_{n+2} + \cdots + P_{2n+1} + b}{P_1^{l_1} + \cdots + P_n^{l_n} + b}$$

Has Infinitely Many Prime Solutions

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Abstract

By using the arithmetic function $J_{2n+1}(w)$ we prove that Diophantine equation

$$P_{n+1}^{l_{n+1}} = \frac{P_{n+2} + \cdots + P_{2n+1} + b}{P_1^{l_1} + \cdots + P_n^{l_n} + b}$$

has infinitely many prime solutions.It is the Book proof. The $J_{2n+1}(w)$ ushers in a new era in the prime numbers theory.

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A new branch of number theory: Santilli's additive isoprime theory is introduced. By using the arithmetic function $J_n(w)$ the following prime theorems have been proved [1-8]. It is the Book proof.

1. There exist infinitely many twin primes.
2. The Goldbach theorem. Every even number greater than 4 is the sum of two odd primes.
3. There exist finitely many Mersenne primes, that is, primes of the form $2^P - 1$ where P is prime.
4. There exist finitely many Fermat primes, that is, primes of the form $2^{2^n} + 1$.
5. There exist finitely many repunit primes whose digits (in base 10) are all ones.
6. There exist infinitely many primes of the forms: $x^2 + 1$, $x^4 + 1$, $x^8 + 1$, $x^{16} + 1$.
7. There exist infinitely many primes of the form: $x^2 + b$.
8. There exist infinitely many prime m-chains, $P_{j+1} = mP_j \pm (m-1)$, $m = 2, 3, \cdots$, including the Cunningham chains.
9. There exist infinitely many triplets of consecutive integers, each being the product of k distinct primes, (Here is an example: 1727913=3×11×52361, 1727914=2×17×50821, 1727915=5×7×49369.)
10. Every integer m may be written in infinitely many ways in the form

$$m = \frac{P_2 + 1}{P_1^k - 1}$$

where $k = 1, 2, 3, \cdots$, P_1 and P_2 are primes.

11. There exist infinitely many Carmichael numbers, which are the product of three primes, four primes, and five primes.
12. There exist infinitely many prime chains in the arithmetic progressions.
13. In a table of prime numbers there exist infinitely many k-tuples of primes, where $k = 2, 3, 4, \cdots, 10^5$.

14. Proof of Schinzel's hypothesis.

15. Every large even number is representable in the form $P_1 + P_2 \cdots P_n$. It is the n primes theorem which has no almost-primes.

In this paper by using the arithmetic function $J_n(w)$ Diophantine equations are studied.

Theorem 1. Diophantine equation

$$P_{n+1}^{l_{n+1}} = \frac{P_{n+2} + \cdots + P_{2n+1} + b}{P_1^{l_1} + \cdots + P_n^{l_n} + b}, \quad (1)$$

has infinitely many prime solutions, where b is the integer.

We rewrite (1)

$$P_{2n+1} = (P_1^{l_1} + \cdots + P_n^{l_n} + b)P_{n+1}^{l_{n+1}} - P_{n+2} \cdots - P_{2n} - b. \quad (2)$$

The arithmetic function [1-8] is

$$J_{2n+1}(w) = \prod_{3 \le P \le P_i} \left((P-1)^{2n} - H(P)\right) \ne 0, \quad (3)$$

where $w = \prod_{2 \le P \le P_i} P$ is called the primorials.

Let $H(P)$ denote the number of solutions of the congruence

$$\left(q_1^{l_1} + \cdots + q_n^{l_n} + b\right)q_{n+1}^{l_{n+1}} - q_{n+2} \cdots - q_{2n} - b \equiv 0 \pmod{P}, \quad (4)$$

where $q_j = 1, 2, \cdots, P-1$, $j = 1, 2, \cdots, 2n$.

Since $J_{2n+1}(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes $P_1, \cdots, P_{2n}$ such that P_{2n+1} is also a prime. It is the Book proof. It is a generalization of the Euler proof for the existence of infinitely many primes.

The best asymptotic formula [1-8] is

$$p_2(N, 2n+1) = \left|\{P_1, \cdots, P_{2n} : P_1, \cdots, P_{2n} \le N; P_{2n+1} = \text{prime}\}\right|$$

$$= \frac{J_{2n+1}(w)w}{(2n)!(l_{n+1} + l_{\max})f^{2n+1}(w)} \frac{N^{2n}}{\log^{2n+1} N}\left(1 + O(1)\right). \quad (5)$$

where $l_{\max}$ is a maximal value among $(l_1, \cdots, l_n)$, $f(w) = \prod_{2 \le P \le P_i} (P-1)$ is called the Euler function of the primorials.

Theorem 2. Diophantine equation

$$P_2 = \frac{P_3 + b}{P_1 + b} \quad (6)$$

has infinitely many prime solutions.

The arithmetic function [1-8] is

$$J_3(w) = \prod_{3 \le P \le P_i} \left(P^2 + 3P + 3 - c(P)\right) \ne 0, \quad (7)$$

where $c(P) = -P + 2$ if $P|b$; $c(P) = 0$ otherwise.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula [1-8] is

$$p_2(N,3) = \left|\{P_1, P_2 : P_1, P_2 \le N; P_3 = \text{prime}\}\right|$$

$$= \frac{J_3(w)w}{4f^3(w)} \frac{N^2}{\log^3 N}\left(1 + O(1)\right). \quad (8)$$

Theorem 3. Diophantine equation

$$P_2 = \frac{P_3 + b}{P_1^2 + b} \quad (9)$$

has infinitely many prime solutions.

The arithmetic function is

$$J_3(w) = \prod_{3 \le P \le P_i} \left(P^2 - 3P + 3 + c(P)\right) \ne 0, \quad (10)$$

where $c(P) = P-2$ if $P|b$; $c(P) = \left(\frac{-b}{P}\right)$ otherwise.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3) = |\{P_1, P_2 : P_1, P_2 \le N; P_3 = \text{prime}\}|$$

$$= \frac{J_3(w)w}{6f^3(w)} \frac{N^2}{\log^3 N}(1+O(1)). \qquad (11)$$

Theorem 4. Diophantine equation

$$P_2^2 = \frac{P_3+1}{P_1+1} \qquad (12)$$

has infinitely many prime solutions.

The arithmetic function is

$$J_3(w) = \prod_{3 \le P \le P_i} (P^2 - 3P + 4) \ne 0, \qquad (13)$$

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3) = |\{P_1, P_2 : P_1, P_2 \le N; P_3 = \text{prime}\}|$$

$$= \frac{J_3(w)w}{6f^3(w)} \frac{N^2}{\log^3 N}(1+O(1)). \qquad (14)$$

Theorem 5. Diophantine equation

$$P_2^2 = \frac{P_3+1}{P_1^2+1}, \qquad (15)$$

has infinitely many prime solutions.

The arithmetic function is

$$J_3(w) = \prod_{3 \le P \le P_i} \left(P^2 - 3P + 3 + c(P)\right) \ne 0\,, \qquad (16)$$

where $c(P) = 3$ if $4 | (P-1)$; $c(P) = 1$ otherwise.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3) = \left|\{P_1,\, P_2 : P_1, P_2 \le N;\; P_3 = \text{prime}\}\right|$$

$$= \frac{J_3(w)w}{8 f^3(w)} \frac{N^2}{\log^3 N}\left(1 + O(1)\right). \qquad (17)$$

Theorem 6. Diophantine equation

$$P_2 = \frac{P_3 + b}{P_1^{P_0} + b} \qquad (18)$$

has infinitely many prime solutions, where P_0 is an odd prime.

The arithmetic function is

$$J_3(w) = \prod_{3 \le P \le P_i} \left(P^2 - 3P + 3 - c(P)\right) \ne 0\,, \qquad (19)$$

where $c(P) = -P + 2$ if $P | b$; $c(P) = -P_0 + 1$ if $b^{\frac{P-1}{P_0}} \equiv 1 \pmod{P}$;

$c(P) = 1$ if $b^{\frac{P-1}{P_0}} \not\equiv \pmod{P}$; $c(P) = 0$ otherwise.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3) = \left|\{P_1,\, P_2 : P_1, P_2 \le N;\; P_3 = \text{prime}\}\right|$$

$$= \frac{J_3(w)w}{2(P_0+1)f^3(w)} \frac{N^2}{\log^3 N}\left(1+O(1)\right). \qquad (20)$$

Theorem 7. Diophantine equation

$$P_2^{P_0} = \frac{P_3+b}{P_1+b}, \qquad (21)$$

has infinitely many prime solutions, where P_0 is an odd prime.

The arithmetic function is

$$J_3(w) = \prod_{3\le P\le P_i} \left(P^2 - 3P + 3 - c(P)\right) \neq 0, \qquad (22)$$

where $c(P) = -P + 2$ if $P|b$; $c(P) = -P_0 + 1$ if $P_0|(P-1)$; $c(P) = 0$ otherwise.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3) = \left|\{P_1, P_2 : P_1, P_2 \le N; P_3 = \text{prime}\}\right|$$

$$= \frac{J_3(w)w}{2(P_0+1)f^3(w)} \frac{N^2}{\log^3 N}\left(1+O(1)\right). \qquad (23)$$

Theorem 8. Diophantine equation

$$P_2^2 = \frac{P_3+1}{P_1^3+1}, \qquad (24)$$

has infinitely many prime solutions

The arithmetic function is

$$J_3(w) = \prod_{3\le P\le P_i} \left(P^2 - 3P + 3 - c(P)\right) \neq 0, \qquad (25)$$

where $c(P) = -P + 8$ if $3|(P-1)$; $c(P) = -1$ otherwise.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2

such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3)=\left|\{P_1,\ P_2:P_1,P_2\le N;\ P_3=\text{prime}\}\right|$$

$$=\frac{J_3(w)w}{10f^3(w)}\frac{N^2}{\log^3 N}\left(1+O(1)\right). \tag{26}$$

Theorem 9. Diophantine equation

$$P_2^4=\frac{P_3+1}{P_1+1}, \tag{27}$$

has infinitely many prime solutions

The arithmetic function is

$$J_3(w)=\prod_{3\le P\le P_i}\left(P^2-3P+3-c(P)\right)\ne 0, \tag{28}$$

where $c(P)=-3$ if $4|(P-1)$; $c(P)=-1$ otherwise.

Since $J_3(w)\to\infty$ as $w\to\infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3)=\left|\{P_1,\ P_2:P_1,P_2\le N;\ P_3=\text{prime}\}\right|$$

$$=\frac{J_3(w)w}{10f^3(w)}\frac{N^2}{\log^3 N}\left(1+O(1)\right). \tag{29}$$

Theorem 10. Diophantine equation

$$P_2=\frac{P_3+1}{P_1^4+1}, \tag{30}$$

has infinitely many prime solutions.

The arithmetic function is

$$J_3(w) = \prod_{3 \le P \le P_i} \left(P^2 - 3P + 3 - c(P)\right) \ne 0, \qquad (31)$$

where $c(P) = 1$ if $4 | (P-1)$; $c(P) = -3$ if $8 | (P-1)$; $c(P) = 0$ otherwise.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3) = \left|\{P_1, P_2 : P_1, P_2 \le N; P_3 = \text{prime}\}\right|$$

$$= \frac{J_3(w)w}{10 f^3(w)} \frac{N^2}{\log^3 N}\left(1 + O(1)\right). \qquad (32)$$

Theorem 11. Diophantine equation

$$P_2 = \frac{P_3 + 1}{P_1^6 + 1}, \qquad (33)$$

has infinitely many prime solutions.

The arithmetic function is

$$J_3(w) = \prod_{3 \le P \le P_i} \left(P^2 - 3P + 3 - c(P)\right) \ne 0, \qquad (34)$$

where $c(P) = -5$ if $12 | (P-1)$; $c(P) = 1$ if $6 | (P-1)$; $c(P) = -(-1)^{\frac{P-1}{2}}$ otherwise.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3) = \left|\{P_1, P_2 : P_1, P_2 \le N; P_3 = \text{prime}\}\right|$$

$$= \frac{J_3(w)w}{14 f^3(w)} \frac{N^2}{\log^3 N}\left(1 + O(1)\right). \qquad (35)$$

Theorem 12. Diophantine equation

$$P_2 = \frac{P_3 + 1}{P_1^8 + 1}, \tag{36}$$

has infinitely many prime solutions.

The arithmetic function is

$$J_3(w) = \prod_{3 \le P \le P_i} \left(P^2 - 3P + 3 - c(P)\right) \ne 0, \tag{37}$$

where $c(P) = -7$ if $16 | (P-1)$; $c(P) = -3$ if $8 | (P-1)$; $c(P) = 1$ if $4 | (P-1)$; $c(P) = 1$ otherwise.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime.

The best asymptotic formula is

$$p_2(N,3) = \left|\{P_1, P_2 : P_1, P_2 \le N; P_3 = \text{prime}\}\right|$$

$$= \frac{J_3(w) w}{18 f^3(w)} \frac{N^2}{\log^3 N}\left(1 + O(1)\right). \tag{38}$$

Theorem 13. Diophantine equation

$$P_3 = \frac{P_4 + P_5 + 1}{P_1 + P_2 + 1}, \tag{39}$$

has infinitely many prime solutions.

The arithmetic function [6] is

$$J_5(w) = \prod_{3 \le P \le P_i} \left(\frac{(P-1)^5 + 1}{P}\right) \ne 0. \tag{40}$$

Since $J_5(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1, P_2, P_3 and P_4 such that P_5 is also a prime.

The best asymptotic formula is

$$p_2(N,5) = \left|\{P_1, \cdots, P_4 : P_1, \cdots, P_4 \le N; P_5 = \text{prime}\}\right|$$

$$= \frac{J_5(w)w}{48f^5(w)} \frac{N^4}{\log^5 N}\left(1+O(1)\right). \quad (41)$$

Theorem 14. Diophantine equation

$$P_{n+1} = \frac{P_{n+2} + \cdots + P_{2n+1} + 1}{P_1 + \cdots + P_n + 1}, \quad (42)$$

has infinitely many prime solutions.

The arithmetic function [6] is

$$J_{2n+1}(w) = \prod_{3 \le P \le P_i} \left(\frac{(P-1)^{2n+1} + 1}{P} \right) \ne 0, \quad (43)$$

Since $J_{2n+1}(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes $P_1 \cdots, P_{2n}$ such that P_{2n+1} is also a prime.

The best asymptotic formula is

$$p_2(N, 2n+1) = \left|\{P_1, \cdots, P_{2n} : P_1, \cdots, P_{2n} \le N; P_{2n+1} = \text{prime}\}\right|$$

$$= \frac{J_{2n+1}(w)w}{2 \times (2n)! f^{2n+1}(w)} \frac{N^{2n}}{\log^{2n+1} N}\left(1+O(1)\right). \quad (44)$$

Theorem 15. Diophantine equation

$$P_{n+1} = \frac{P_{n+2} + \cdots + P_{2n+1}}{P_1 + \cdots + P_n}, \quad n \ge 2, \quad (45)$$

has infinitely many prime solutions.

The arithmetic function [6] is

$$J_{2n+1}(w) = \prod_{3 \le P \le P_i} \left(\frac{(P-1)^{2n+1} + 1}{P} + 1 \right) \ne 0, \quad (46)$$

Since $J_{2n+1}(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes $P_1 \cdots, P_{2n}$ such that P_{2n+1} is also a prime.

The best asymptotic formula [6] is

$$p_2(N,2n+1)=\left|\{P_1,\cdots,P_{2n}:P_1,\cdots,P_{2n}\le N;\ P_{2n+1}=\text{prime}\}\right|$$

$$=\frac{J_{2n+1}(w)w}{2\times(2n)!f^{2n+1}(w)}\frac{N^{2n}}{\log^{2n+1}N}\left(1+O(1)\right). \qquad (47)$$

Theorem 16. Diophantine equation

$$P_{n+1}^m=\frac{P_{n+2}^l+\cdots+P_{2n}^l+P_{2n+1}+b}{P_1^l+\cdots+P_n^l+b}, \qquad (48)$$

has infinitely many prime solutions.

The arithmetic function [6] is

$$J_{2n+1}(w)=\prod_{3\le P\le P_i}\left((P-1)^{2n}-H(P)\right)\ne 0, \qquad (49)$$

Let $H(P)$ denote is the number of solutions of the congruence

$$(q_1^l+\cdots+q_n^l+b)q_{n+1}^m-q_{n+2}^l\cdots-q_{2n}^l-b\equiv 0 \pmod{P}, \qquad (50)$$

where $q_j=1,\cdots,P-1,\ j=1,\cdots,2n.$

Since $J_{2n+1}(w)\to\infty$ as $w\to\infty$, there exist infinitely many primes $P_1\cdots,P_{2n}$ such that P_{2n+1} is also a prime.

The best asymptotic formula [6] is

$$p_2(N,2n+1)=\left|\{P_1,\cdots,P_{2n}:P_1,\cdots,P_{2n}\le N;\ P_{2n+1}=\text{prime}\}\right|$$

$$=\frac{J_{2n+1}(w)w}{(2n)!(m+l)f^{2n+1}(w)}\frac{N^{2n}}{\log^{2n+1}N}\left(1+O(1)\right). \qquad (51)$$

Theorem 17. For every integer ***m*** Diophantine equation

$$m=\frac{P_{2n+1}+\cdots+P_{4n}}{P_1+\cdots+P_{2n}}, \qquad (52)$$

has infinitely many prime solutions.

The arithmetic function [6] is

$$J_{4n}(w)=\prod_{3\le P\le P_i}\left(\frac{(P-1)^{4n}-1}{P}-c(P)\right)\ne 0\,, \tag{53}$$

where $c(P)=-\frac{(P-1)^{4n-1}+1}{P}$ if $P|m$; $c(P)=-1$ otherwise.

Since $J_{4n}(w)\to\infty$ as $w\to\infty$, there exist infinitely many primes $P_1\cdots,P_{4n-1}$ such that P_{4n} is also a prime.

The best asymptotic formula [6] is

$$p_2(N,4n)=\left|\{P_1,\cdots,P_{4n-1}:P_1,\cdots,P_{4n-1}\le N;\ P_{4n}=\text{prime}\}\right|$$

$$=\frac{J_{4n}(w)w}{(4n-1)!f^{4n}(w)}\frac{N^{4n-1}}{\log^{4n}N}\left(1+O(1)\right). \tag{54}$$

Theorem 18. Diophantine equation

$$P_3=mP_1^3+nP_2^3,\quad (m,n)=1,\quad 2|mn,\quad n\ne\pm b^3\ , \tag{55}$$

has infinitely many prime solutions.

The arithmetic function [1-8] is

$$J_3(w)=\prod_{3\le P\le P_i}\left(P^2-3P+3-c(P)\right)\ne 0\,, \tag{56}$$

where $c(P)=2P-1$ if $m^{\frac{P-1}{3}}\equiv n^{\frac{P-1}{3}}$ (mod P); $c(P)=-P+2$ if $m^{\frac{P-1}{3}}\not\equiv 2^{\frac{P-1}{3}}$ (mod P); $c(P)=-P+2$ if $P|mn$; $c(P)=1$ otherwise.

Since $J_3(w)\to\infty$ as $w\to\infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime. It is the Book proof.

The best asymptotic formula [1-8] is

$$p_2(N,3)=\left|\{P_1,P_2:P_1,P_2\le N;\ P_3=\text{prime}\}\right|$$

$$= \frac{J_3(w)w}{6f^3(w)} \frac{N^2}{\log^3 N} (1+O(1)). \qquad (57)$$

Let $m=1$ and $n=2$. From (55) we have $P_3 = P_1^3 + 2P_2^3$ [9].

Theorem 19. Diophantine equation

$$P_3 = aP_1^2 + cP_2^4, (a,c)=1, 2|ac, a+c \neq 3d, \qquad (58)$$

has infinitely many prime solutions.

The arithmetic function [1-8] is

$$J_3(w) = \prod_{3 \le P \le P_i} (P^2 - 3P + 3 - c(P)) \neq 0, \qquad (59)$$

where $c(P) = P$ if $\left(\frac{-ac}{P}\right) = 1$; $c(P) = -P+2$ if $\left(\frac{-ac}{P}\right) = -1$ and $P|ac$.

Since $J_3(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P_1 and P_2 such that P_3 is also a prime. It is the Book proof.

The best asymptotic formula [1-8] is

$$p_2(N,3) = |\{P_1, P_2 : P_1, P_2 \le N; P_3 = \text{prime}\}|$$

$$= \frac{J_3(w)w}{8f^3(w)} \frac{N^2}{\log^3 N} (1+O(1)). \qquad (60)$$

Let $a=4$ and $c=1$. From (58) we have $P_3 = (2P_1)^2 + P_2^4$ [10].

Remark. $aP_1^2 + bP_1P_2 + cP_2^2$ and $aP_1^2 + bP_1P_2^n + cP_2^{2n}, n \ge 2$ have the same arithmetic function and the same property. If a, b and c have no prime factor in common, they represent infinitely many primes as P_1 and P_2 run through the positive primes. Gauss proved that there are infinitely many primes of the form $x^2 + y^2$. It is shown that there are infinitely many primes of the form $x^2 + y^n, n \ge 2$ [8].

Theorem 20. Two primes represented by $P^2 - 6^2$.

Suppose that

$$P^2-6^2=(P+6)(P-6)\,. \tag{61}$$

Form (61) we have two equations

$$P_1=P+6 \text{ and } P_2=P-6. \tag{62}$$

The arithmetic function [1-8] is

$$J_2(w)=2\prod_{5\le P\le P_i}(P-3)\ne 0\,, \tag{63}$$

Since $J_2(w)\to\infty$ as $w\to\infty$, there exist infinitely many primes P such that P_1 and P_2 are primes. It is the Book proof.

The best asymptotic formula [1-8] is

$$p_3(N,2)=\left|\{P:P\le N;\ P_1,P_2=\text{primes}\}\right|$$

$$=\frac{J_2(w)w^2}{f^3(w)}\frac{N}{\log^3 N}(1+O(1))\,. \tag{64}$$

Theorem 21. Two primes represented by P^3-2^3.

Suppose that

$$P^3-2^3=(P-2)(P^2+2P+4)\,. \tag{65}$$

Form (65) we have two equations

$$P_1=P-2 \text{ and } P_2=P^2+2P+4\,. \tag{66}$$

The arithmetic function [1-8] is

$$J_2(w)=\prod_{5\le P\le P_i}\left(P-3-\left(\frac{-3}{P}\right)\right)\ne 0\,, \tag{67}$$

Since $J_2(w)\to\infty$ as $w\to\infty$, there exist infinitely many primes P such that P_1 and P_2 are primes.

The best asymptotic formula [1-8] is

$$p_3(N,2) = |\{P : P \le N; P_1, P_2 = \text{prime}\}|$$

$$= \frac{J_2(w)w^2}{2f^3(w)} \frac{N}{\log^3 N}(1+O(1)). \quad (68)$$

Theorem 22. Two primes represented by $P^5 + 2^5$.

Suppose that

$$P^5 + 2^5 = (P+2)(P^4 - 2P^3 + 4P^2 - 8P + 16). \quad (69)$$

Form (69) we have two equations

$$P_1 = P + 2 \text{ and } P_2 = P^4 - 2P^3 + 4P^2 - 8P + 16. \quad (70)$$

The arithmetic function [1-8] is

$$J_2(w) = \prod_{3 \le P \le P_i} (P - 3 - c(P)) \ne 0, \quad (71)$$

where $c(P) = 4$ if $5|(P-1)$; $c(P) = 0$ otherwise.

Since $J_2(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P such that P_1 and P_2 are primes.

The best asymptotic formula [1-8] is

$$p_3(N,2) = |\{P : P \le N; P_1, P_2 = \text{primes}\}|$$

$$= \frac{J_2(w)w^2}{4f^3(w)} \frac{N}{\log^3 N}(1+O(1)). \quad (72)$$

Theorem 23. Three primes represented by $P^4 - 30^4$.

Suppose that

$$P^4 - 30^4 = (P+30)(P-30)(P^2 + 900). \quad (73)$$

Form (73) we have three equations

$$P_1 = P + 30, \ P_2 = P - 30 \text{ and } P_3 = P^2 + 900. \quad (74)$$

The arithmetic function [1-8] is

$$J_2(w) = 8 \prod_{7 \le P \le P_i} \left(P - 3 - c(P)\right) \neq 0, \quad (75)$$

where $c(P) = 2$ if $4 | (P-1)$; $c(P) = 0$ otherwise.

Since $J_2(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P such that P_1, P_2 and P_3 are primes.

The best asymptotic formula [1-8] is

$$p_4(N,2) = \left| \{P : P \le N; P_1, P_2, P_3 = \text{primes}\} \right|$$

$$= \frac{J_2(w) w^3}{2 f^4(w)} \frac{N}{\log^4 N} \left(1 + O(1)\right). \quad (76)$$

Theorem 24. Four primes represented by $P^6 - 42^6$.

Suppose that

$$P^6 - 42^6 = (P+42)(P-42)(P^2 + 42P + 1764)(P^2 - 42P + 1764) \quad .$$

(77)

Form (77) we have four equations

$$P_1 = P + 42, P_2 = P - 42, P_3 = P^2 + 42P + 1764, P_4 = P^2 - 42P + 1764. \quad (78)$$

The arithmetic function [1-8] is

$$J_2(w) = 24 \prod_{11 \le P \le P_i} \left(P - 3 - c(P)\right) \neq 0, \quad (79)$$

where $c(P) = 4$ if $3 | (P-1)$; $c(P) = 0$ otherwise.

Since $J_2(w) \to \infty$ as $w \to \infty$, there exist infinitely many primes P such that P_1, P_2, P_3 and P_4 are primes.

The best asymptotic formula [1-8] is

$$p_5(N,2) = \left| \{P : P \le N; P_1, P_2, P_3, P_4 = \text{primes}\} \right|$$

$$= \frac{J_2(w) w^4}{4 f^5(w)} \frac{N}{\log^5 N} \left(1 + O(1)\right). \quad (80)$$

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