Non-equilibrium Dynamics as Source of Asymmetries in High Energy Physics

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Abstract

Understanding the origin of certain symmetry breaking scenarios in high-energy physics remains an open challenge. Here we argue that, at least in some cases, symmetry violation is an effect of non-equilibrium dynamics that is likely to develop somewhere above the energy scale of electroweak interaction. We also find that, imposing Poincare symmetry in non-equilibrium field theory, leads to fractalization of the underlying space-time continuum.

"*Nature is simple in essence*"

Hideki Yukawa

1. Introduction and Motivation

Quantum Field Theory (QFT) is a well-tested body of ideas and methods with many successful applications in elementary particle interactions, astrophysics, cosmology and condensed matter phenomena. QFT supplies the foundation for the Standard Model of high-energy physics (SM), a framework that describes all forces observed in Nature with the exception of gravity.

A cornerstone of SM is the principle of *local gauge symmetry* which gives rise to the electromagnetic force, the weak interaction of radioactivity and the strong nuclear force that governs the structure of nuclei. These forces act on the primary constituents of matter which have been identified as point-like fermions (quarks and leptons). In SM two fundamental gauge models are brought together, the electroweak theory (EW) and quantum chromodynamics (QCD). Whereas EW deals with the electromagnetic and weak interaction of leptons and quarks, QCD applies to the strong interaction of quarks.

In QFT and classical field theory alike, *symmetry principles* play a key role. They express the invariance of physical phenomena under transformations of the way these phenomena are described. Symmetry principles underlie the existence of conserved currents and charges, the existence of antiparticles and the indistinguishable behavior of phenomena to arbitrary transformations of space-time coordinates [1, 2].

Despite being highly predictive, SM leaves out many open questions. For instance, the origin of *approximate* symmetries and *broken* symmetries is at best partially understood in SM. A typical example is that both EW and QCD break the symmetry between left-handed and right handed fermions, a phenomenon known as violation of chiral symmetry. Among other long-standing questions, we list the mechanism of mass generation through symmetry breaking in EW sector and the violation of parity (inversion of spatial coordinates) and time reversal symmetry in reactions involving K and B-mesons [3].

The basic premise of our work is that asymmetry in SM is a consequence of *non-equilibrium dynamics* that is presumed to develop beyond EW energy scale of about 200 GeV. High energy behavior is prone to prevent full thermalization of high order quantum corrections and to create conditions for an ever evolving dynamic regime in which the principles of QFT are likely to break down [4, 5]. In particular, chiral symmetry, reversibility, isotropy of space-time and locality may very well fail to hold in this high-energy environment.

The past two decades have convincingly shown that dynamical settings that are out of equilibrium are much more prevalent in Nature than equilibrium conditions. It is for this reason that non-equilibrium physics in QFT has recently attracted a great deal of attention. Interest involving non-equilibrium dynamics of quantum fields include inflationary stage of the early Universe, electroweak baryogenesis, chiral phase transition and quark-gluon plasma in heavy ion collisions, dynamics of phase transition in Bose-Einstein condensates, ultrafast spectroscopy of semiconductors, non-extensive statistics and fractional dynamics, models of the dark sector, non-equilibrium phase transitions in strongly correlated compounds, condensed matter phenomena with long range correlations, spin glasses and so on [6]. This impressive diversity of applications reveals the truly interdisciplinary character of non-equilibrium theory.

In the context of high-energy physics, non-equilibrium dynamics is attractive because it brings to the table at least two important insights:

A) It is a natural source for dissipative and anisotropic evolution [7].

B) It is also a natural source for multiplicity and the emergence of hierarchically organized structures [8].

By construction, QFT is a replica of equilibrium statistical mechanics built on Boltzmann-Gibbs distributions [1, 2]. QFT describes *local* quantum phenomena that are *fully reversible* in time and space. In contrast, non-equilibrium dynamics has the potential of violating time and space symmetries at the quantum level. It is apparent from these considerations that there is a fundamental tension between the *non-local* and *irreversible evolution* of non-equilibrium phenomena and the *local* and *conservative* description of dynamics postulated by QFT. Our view is that, to make progress, one need to show how non-equilibrium physics can gracefully coexist with QFT inside the narrow transition region from one regime to another. Investigating this transition is the main goal of this work.

The paper is structured in a way that enables a progressive introduction of ideas. Section 3 explores how a minimal extension of action principle for systems near equilibrium can be consistently formulated. Following the general framework of non-equilibrium phenomena, in sections 4, 5 and 6 we expand on the idea that action functional emerges from an underlying network of generic, short scale degrees of freedom. Next sections show how non-equilibrium dynamics is able to qualitatively explain two symmetry breaking scenarios of particle physics (chiral symmetry breaking and symmetry violation due to mass terms). Emergence of fractal space-time as a result of enforcing Poincare symmetry in non-equilibrium dynamics is discussed in section 9. Last section includes a brief summary and concluding remarks. Three appendix sections are included to make the paper self-contained.

We caution that the intent of this contribution is limited to a tentative and informal introduction to the topic. Further developments are required to confirm, expand or discard these preliminary conclusions.

3. Minimal extension of the action principle

It is well known that evolution of physical systems in classical and quantum physics follows from the action principle [1, 2]. Since non-equilibrium dynamics may be inconsistent with the action principle [9], it makes sense to begin with a conservative approach that connects non-equilibrium dynamics and field theory for systems that are in *near e*quilibrium conditions.

Let $\psi^{\alpha}(x)$; $\alpha = 1, 2, 3....., N$ represent a set of classical fields that may be scalar, vector, spinor or tensor functions of the four-vector $x \equiv x^{\mu}$, $\mu = 0, 1, 2, 3$. Fields are assumed to belong to a generic statistical ensemble $q(x) = \{\psi^{\alpha}(x)\}$ whose evolution is determined by Liouville equation [10],

$$\frac{\partial \rho}{\partial t} = \{H, \rho\} \tag{1}$$

Here $\rho = \rho(p, q, t)$ is the probability density measured in phase space, H is the Hamilton function, $p = \{\partial_{\mu}\psi^{\alpha}(x)\} \equiv \left\{\frac{\partial \psi^{\alpha}(x)}{\partial x^{\mu}}\right\}$ and $\{\cdot\}$ denotes the Poisson bracket. Non-equilibrium evolution is described by a time-dependent probability density and a non-vanishing bracket

$$\frac{\partial \rho}{\partial t} = \{H, \rho\} \neq 0 \tag{2}$$

A concept closely related to the probability density in equilibrium statistical physics is the canonical partition function [1, 2, 11]

$$Z \propto \int \exp[-\beta H(p, q)]\, dp\, dq \tag{3}$$

in which $\beta = 1/kT$ is the inverse temperature. The probability density that the system settles in the stationary state $\rho_e(p, q)$ is defined by

$$\rho_e(p, q) = \frac{\exp[-\beta H(p, q)]}{Z} \tag{4}$$

The inverse temperature can be understood as a fictitious time variable $\tau = 1/kT$. This interpretation highlights the formal analogy between $\rho_e(p, q)$ and the action functional of classical field theory, that is,

$$\rho_e(p,q) \propto \exp[-S(p,q)] \quad (5)$$

The Lagrangian of the system,

$$L = L\,(\psi^{\alpha}(x), \partial_{\mu}\psi^{\alpha}(x)) \quad (6)$$

satisfies the action principle

$$\delta S = \delta\int_R L\; dx = 0\,,\; dx = d^4x = dx_0 dx_1 dx_2 dx_3 \quad (7)$$

in which R denotes the four-dimensional region of integration. (2) and (7) suggest that a minimal extension of (7) near equilibrium amounts to

$$\delta S = \delta S(t) \neq 0 \quad (8)$$

It is often convenient to specify R using two space-like surfaces σ_1 and σ_2 extending to infinity [12]. Let us adopt this choice and perform an arbitrary transformation on fields and coordinates in (7). Introducing the plausible assumption that all fields and their derivatives vanish at spatial infinity leads to

$$\delta S = \delta\int_R L\; dx = G(\sigma_2) - G(\sigma_1) \quad (9)$$

where $G(\sigma)$ is called the *generator* of variation δ. Furthermore, choosing $d\sigma = d\sigma^{\mu}$ along the time direction and carrying out the integration over the spatial region Ω yields

$$\delta S = \delta\int_{\Omega} L\; d^3x = G(t_2) - G(t_1) \quad (10)$$

It is apparent that $G(\sigma)$ represents an *invariant* if and only if (7) holds true. For time dependent dynamical systems, such as the ones described by (2), $G(\sigma)$ is no longer invariant and $\delta G(\sigma) \neq 0$. In this case condition (10) becomes

$$\delta G(t) \neq 0 \quad (11)$$

The weakest form of (11) is given by constraining the first order variation of the generator to a non-vanishing constant, or

$$\delta G(t) = \text{const} \neq 0 \tag{12}$$

4. Large scale physics as emergent behavior

To make progress from this point on, we assume the following:

1) As previously stated, the analysis is limited to classical fields. This ansatz is partly motivated by simplicity and partly the fact that large statistical ensembles of quantum particles behave like classical systems [13].

2) Action functional is an emergent property from an underlying large network of *short scale* degrees of freedom $\mathbf{X} = \{X_i\}$. Thus the action functional describes only the *large scale* behavior of fields (Appendix A).

3) Transition from short scale to the large scale dynamics is driven by a set of control parameters $\lambda = \{\lambda_i\}$, $i = 1, 2, 3...$. The precise nature of λ is irrelevant to our context[1].

Evolution from the large scale to the short scale dynamics may be understood as a continuous phase transition in which the two phases coexist only in narrow energy range near equilibrium $\Delta E \ll \Lambda$, that is, for $\Lambda - \Delta E \leq E \leq \Lambda + \Delta E$. Below this range ($E < \Lambda - \Delta E$) the action functional no longer depends *explicitly* on X_i.

We summarize these premises in the following table:

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[1] Specific examples include, but are not limited to, the mass scale Λ of effective field theories [14], the Wilson-Fisher parameter of the Renormalization Group program $\varepsilon = 4 - d$ [1, 2], the occupation probability p in percolation phenomena or self-organized criticality [15], the spatial correlation range in spin networks [16] and so on.

Large scale dynamics, $E < \Lambda - \Delta E$	**Short scale dynamics,** $E \geq \Lambda + \Delta E$
Equilibrium and unitary evolution $\frac{\partial \rho}{\partial t} = \{H, \rho\} = 0$	Out of equilibrium and non-unitary evolution $\frac{\partial \rho}{\partial t} = \{H, \rho\} \neq 0$
Principle of least action $\delta S = 0$	Evolution of short scale degrees of freedom $\frac{d\mathbf{X}}{dt} = f_i(\mathbf{X}, \{\lambda\})$
Control parameters reach critical values $\delta\lambda = \lambda - \lambda_c = 0$	Control parameters deviate from criticality $\delta\lambda = \lambda - \lambda_c \neq 0$

Tab. 1: Comparison of large and short scale dynamics

5. Compensating role of non-equilibrium dynamics

One can reasonably argue that conditions (11) and (12) violate the principle of action invariance of classical and quantum theory. According to this principle, physics laws are independent of any particular reference frame chosen to describe space-time coordinates and fields. With regard to systems that are in near equilibrium conditions, the object of this section is to reformulate the dynamics of (6) in a way that restores full symmetry of the action.

The generator of the change involving both space-time coordinates and fields is defined by [12]

$$G(\sigma) = \int_{\sigma} d\sigma [\frac{\partial L}{\partial(\partial^{\mu}\psi^{\alpha})} \delta_0 \psi^{\alpha} - \theta^{\mu\nu} \delta x_{\nu}] \tag{13}$$

Here, $\delta_0\psi^\alpha$ represents an *internal* field transformation (Appendix B), $\theta^{\mu\nu}$ the energy-momentum tensor,

$$\theta^{\mu\nu} = \frac{\partial L}{\partial(\partial^\mu \psi^\alpha)}\partial_\nu \psi^\alpha - \eta^{\mu\nu} L \tag{14}$$

and δx_ν is the four-vector measuring the change in coordinates ($\nu = 0,1,2,3$).

Let $G_\lambda(\sigma, \delta\lambda)$ denote the *external* contribution to the action due to a small deviation from criticality $\delta\lambda = \lambda - \lambda_c$. Here $G_\lambda(\sigma, \delta\lambda)$ embodies the contribution of short scale physics which, by previous assumptions, is out of equilibrium. Invariance of the action is recovered by demanding that the change in $G(\lambda)$ be compensated by an equal and opposite change in $G_\lambda(\sigma, \delta\lambda)$ near the transition boundary between equilibrium and non-equilibrium, that is,

$$\delta G(\sigma) = -G_\lambda(\sigma, \delta\lambda) \quad \text{if} \quad E < \Lambda - \Delta E \tag{15}$$

As stated above, the two generators of (15) couple only within the coexisting range ΔE and decouple outside it. In this region we set

$$G_\lambda(\sigma, \delta\lambda) = f[G(\sigma), \delta\lambda] \tag{16}$$

such as, when the dynamics reaches full equilibrium,

$$\lim_{\delta\lambda \to 0} f[G(\sigma), \delta\lambda] = 0 \quad \text{if} \quad E < \Lambda - \Delta E \tag{17}$$

The challenge is to search for a function $G_\lambda(\sigma, \delta\lambda)$ that fulfills two requirements:

a) as shown in (17), it decouples from Lagrangian (6) outside ΔE and,

b) it arises as an emergent property from the short scale dynamics of $\mathbf{X} = \{X_i\}$.

Finding this function is the goal of next section.

6. Fixed point solution of the normal form equation

With reference to center manifold theory introduced in Appendix A, it is natural to identify δG with the order parameter z of (A3). In general, dynamics of (A3) is controlled by two parameters λ_1 and $\lambda_2 = u$ with critical values λ_{1c} and $\lambda_{2c} = 0$. It is often convenient to study the dynamics of a nonlinear system in discrete time [17]. The discrete analogue of (A3) is the iterated quadratic map

$$\frac{d(\delta G)}{dt} = (\lambda_1 - \lambda_{1c}) - u(\delta G)^2 \Rightarrow \delta G_{n+1} = \delta G_n + \tau(\lambda_1 - \lambda_{1c}) - \tau u(\delta G_n)^2 \tag{18}$$

where τ is the time step and $n \in \{N\}$ the iteration index. Assuming $u \neq 0$, the fixed point analysis of (18) yields a trivial result $(\delta G = 0)$ and a pair of non-trivial solutions

$$\delta G = \pm (\frac{\lambda_1 - \lambda_{1c}}{u})^{1/2} \tag{19}$$

When λ_1 is tuned towards λ_{1c}, the approach to chaos in (18) is driven by the by the geometric progression

$$\lambda_{1,N} - \lambda_{1,c} \approx \lambda_0 \delta^{-N} \tag{20}$$

where $N = 2^p \gg 1$ is the index counting the number of periodic orbits and $\delta \approx 4.669...$ represents the Feigenbaum constant for the quadratic map [17]. Replacing (20) in (19) yields an infinite series of fixed point solutions given by

$$\delta G_{2^p} \propto \delta^{-2^{p-1}} \quad \text{for } p \gg 1 \tag{21}$$

Series (21) is limited by the upper bound $N = 1$ for which

$$\delta G_0 = \pm (\frac{\lambda_0}{\delta u})^{1/2} \tag{22}$$

Refer again to (13) and consider the case where there is only a transformation of fields with no change of space-time location. The first term in (13) then corresponds to a conserved current

$$J^{\mu} = \frac{\partial L}{\partial(\partial^{\mu}\psi^{\alpha})}\delta_0\psi^{\alpha} \Rightarrow \partial_{\mu}J^{\mu} = 0 \tag{23}$$

It is apparent from (23) that any symmetry breaking transformation of fields can be associated with a dissipative current J^{μ} whose divergence is non-vanishing ($\partial_{\mu}J^{\mu} \neq 0$). Combining (21), (22) and (23) yields two possibilities. In symbolic form we write

$$\boxed{\partial_{\mu}J^{\mu} = \begin{cases} \delta G_0 \\ \text{or} \\ \delta G_{2^p} \end{cases}} \tag{24}$$

(24) is the main result of our work. It shows that the external source of non-conserving currents in QFT is either a fixed deviation from equilibrium (δG_0) or, more generally, a tower of deviations from equilibrium ordered according to the Feigenbaum series (δG_{2^p}).

7. Chiral symmetry breaking

A field theory is said to obey chiral symmetry if no distinction is made between left-handed (L) and right-handed components ® of the fermion field, that is, if they are treated on equal footing. It is known that free fermions are described in SM by the Dirac Lagrangian [1, 2]

$$L_D = i\overline{\Psi}\gamma^{\mu}\partial_{\mu}\Psi - m\overline{\Psi}\Psi \tag{25}$$

where

$$\Psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \tag{26}$$

and m is the rest-frame mass of the fermion. In (25) γ^{μ} stands for the set of Dirac matrices and

$$\overline{\Psi} = \Psi^{+}\gamma^{0} = \begin{pmatrix} \psi_{R}^{+} & \psi_{L}^{+} \end{pmatrix} \tag{27}$$

denotes the doublet of antiparticles corresponding to (26). If we consider massless fermions, the Lagrangian has a global symmetry for its both left-handed and right-handed components. It is represented by (Appendix B)

$$\psi_{L} \rightarrow \exp(i\theta_{L})\psi_{L} \ , \ \ \psi_{R} \rightarrow \exp(i\theta_{R})\psi_{R} \tag{28}$$

where ψ_{L} and ψ_{R} are rotated by two independent angles θ_{L} and θ_{R}. The transformation with $\theta_{L} = \theta_{R} \equiv \varphi$ can be written as

$$\Psi \rightarrow \exp(i\varphi)\Psi \tag{29}$$

The transformation having $\theta_{R} = -\theta_{L} \equiv \eta$ assumes a similar form, namely

$$\Psi \rightarrow \exp(i\eta\gamma^{5})\Psi \tag{30}$$

in which γ^{5} denotes the chiral Dirac matrix [1, 2]. Transformation (29) is called a *vector* symmetry whose conserved current is

$$j_{V}^{\mu} = \overline{\Psi}\gamma^{\mu}\Psi \tag{31}$$

Likewise, transformation (30) is called an *axial* symmetry and its conserved current is given by

$$j_{A}^{\mu} = \overline{\Psi}\gamma^{\mu}\gamma^{5}\Psi \tag{32}$$

It can be shown that, if fermions have non-vanishing masses ($m \neq 0$), the vector symmetry remains exact while axial symmetry is broken. In this case the divergence of axial current (32) is non-vanishing and we have

$$\partial_\mu j^\mu_A = 2im\overline{\Psi}\gamma^5\Psi \qquad (33)$$

This result indicates that massive fermions break chiral symmetry between L and R components of the fermion field. Following (24), we interpret the emergence of massive particles (and the consequent violation of chiral symmetry) as the effect produced by a deviation from equilibrium. This argument will be developed in the next section.

A particular signature for chiral symmetry breaking occurs in the EW model and it stems from the fact that right-handed fermions do not respond to the weak interaction. With reference to Appendix B, consider the infinitesimal unitary transformation

$$\delta_0\psi = \psi'(x) - \psi(x) = \exp(ig_2\frac{\bar{\tau}\cdot\overline{W}}{2})\psi(x) - \psi(x) \approx (1 + ig_2\frac{\bar{\tau}\cdot\overline{W}}{2} - 1)\psi(x) \qquad (34)$$

where g_2 measures the coupling strength of weak interaction. In the case of massless fermions, from (13) and (25) we obtain

$$\frac{\partial L_D}{\partial(\partial_\mu\psi)} = i\overline{\psi}\gamma^\mu \qquad (35)$$

and

$$\frac{\partial L_D}{\partial(\partial_\mu\psi)}\delta_0\psi = -\overline{\psi}_L\gamma^\mu g_2\frac{\bar{\tau}\cdot\overline{W}_\mu}{2}\psi_L \qquad (36)$$

(36) represents the term that does not have a counterpart built from right-handed fermions and, as a result, breaks chiral symmetry of the EW model even when no massive particles are present.

To summarize, this section points out that the intrinsic ability of non-equilibrium dynamics to break the symmetry between L and R objects provides a natural motivation for the violation of chiral symmetry in SM. This occurs through two distinct channels: a) by generation of massive fermions and b) by making right handed fermions insensitive to the weak interaction.

8. Symmetry breaking due to mass terms

Symmetry considerations forbid the SM Lagrangian to contain massive fermion terms such as [1-3]

$$L_{m,f} = -m\overline{\Psi}\Psi = -m(\overline{\psi}_L \psi_R + \overline{\psi}_R \psi_L) \tag{37}$$

To streamline the ensuing derivation, it is convenient to work in the approximation of homogeneous (space-independent) fields and assume that the factor quadratic in fermions is an arbitrary function of time. Thus,

$$L_{m,f} = -m\,\Phi_f(t) \tag{38}$$

On account of (18) – (20) and using the identification $\delta G = L_{m,f}$ leads to the continuous time representation of the normal form equation

$$\Phi_f(t)\frac{dm}{dt} + m\frac{d\Phi_f(t)}{dt} = (\lambda_{1c} - \lambda_1) + u\,m^2\Phi_f^2(t) \tag{39}$$

Furthermore, if for sufficiently small time intervals $\tau = O(\varepsilon)$ function Φ_f can be well approximated by the series expansions

$$\Phi_f(\tau) = \Phi_f^0 + \sum_n \tau^n \Phi_f^{(n)}(\tau) \tag{40}$$

the leading order formulation of (39) in discrete time assumes the quadratic form

$$m_{n+1} = m_n + a(\lambda_{1c} - \lambda) + b\,m_n^2 \tag{41}$$

in which $a = \tau(\Phi_f^0)^{-1}$ and $b = \tau u \Phi_f^0$. The hierarchical pattern of fermion masses computed from (21) and (41) is found to be in good agreement with experimental data for a “δ” whose numerical value matches the Feigenbaum constant for hydrodynamic flows, namely $\bar{\delta} = 3.9$ (Appendix C).

Symmetry under local gauge transformations also prohibits the Lagrangian to include terms containing massive gauge fields ($M \neq 0$) such as

$$L_{m,b} = \frac{1}{2} M^2 \overline{W} \cdot \overline{W} \tag{42}$$

There is, however, a fundamental difference between free fermions and free gauge bosons with regard to the mechanism of mass generation. Gauge bosons are self-interacting objects and the contribution of self-interacting energy needs to be factored in when computing their masses [18]. Following the arguments of [18, 19], the mass of the gauge boson is expected to scale as reciprocal of its coupling strength. For two consecutive flavors of gauge bosons we obtain

$$\frac{M_r}{M_{r+1}} = (\frac{g_{r+1}}{g_r})^2 \tag{43}$$

with $r = 1, 2, 3...$. The case of EW corresponds to $r = 1$ and the ratio of W and Z masses is given by (Appendix C)

$$(\frac{M_W}{M_Z})^2 = \frac{1}{1 + (\frac{e}{g_2})^2} \approx 1 - \frac{1}{\delta} \tag{44}$$

in which “e” denotes the electric charge.

9. Fractal space-time from Poincare symmetry

It is well known that space-time of both Relativity and QFT is considered a differentiable continuum. This property underlies the use of conventional calculus, vector analysis and ordinary symmetry operations. It seems natural to ask if this fundamental model of space-time continues to stand in an environment that favors the onset of non-equilibrium dynamics. This section explores the implications of demanding that four-momentum is exactly preserved in near-equilibrium conditions. To this end, let us return to (13) and consider the situation where there no internal field transformations take place ($\delta_0\psi^\alpha = 0$). The generator of space-time transformations becomes, in this case,

$$G(\sigma) = -\int_\sigma d\sigma_\mu \, \theta^{\mu\nu} \, \delta x_\nu \qquad (45)$$

where the infinitesimal changes of coordinates are described by

$$\delta x_\nu = \omega_{\nu\rho} x^\rho + a_\nu \qquad (46)$$

Here, a_ν is a constant vector and $\omega_{\nu\rho} = -\omega_{\rho\nu}$ a constant anti-symmetric tensor. The generator corresponding to translations is the four-momentum

$$P^\nu = \int_\sigma d\sigma_\mu \, \theta^{\mu\nu} \qquad (47)$$

Conveniently choosing a frame such that the "t = constant" is the space-like surface yields

$$P^\mu = \int_\Omega \theta^{0\mu} d^3x \qquad (48)$$

(48) denotes a set of invariants, that is

$$\frac{\partial}{\partial t}\left(\int_\Omega \theta^{\mu 0} d^3x\right) = 0 \qquad (49)$$

In particular, total energy corresponds to $\mu = 0$ and is a constant. From (45) we derive

$$G(t) = -\int_\Omega d^3x \, \theta^{0\nu}(\omega_{\nu\rho} x^\rho + a_\nu) \qquad (50)$$

whose differential can be presented as

$$\delta G(t) = -\int_\Omega d^3x \, \theta^{0\nu} \omega_{\nu\rho} \delta x^\rho \qquad (51)$$

On account of (49), the normal form equation (18) corresponding to (51) reads

$$-\int_{\Omega} d^3x\, \theta^{0\nu}\omega_{\nu\rho} \frac{\partial(\delta x^{\rho})}{\partial t} = (\lambda_1 - \lambda_{1c}) - u[\int_{\Omega} d^3x\, \theta^{0\nu}\omega_{\nu\rho}\delta x^{\rho}]^2 \tag{52}$$

This equation can be further streamlined with help from additional assumptions. For small enough volumes ($\Omega = O(\varepsilon)$) and under some mild requirements concerning time behavior of integrands, one ends up with a quadratic equation containing spatial averages of δx^{ρ}. Passing to a map representation and invoking the universal transition to chaos in unimodal maps leads to the conclusion that, near the Feigenbaum attractor $N = 2^{p} \gg 1$ of (20), underlying space-time is prone to acquire a *fractal structure*. Emergence of fractal space-time in high-energy physics is a speculative conjecture that has been widely explored during the last two decades [19][2].

10. Concluding remarks

The likely onset of non-equilibrium dynamics near or beyond the EW scale may provide a unified explanation for the origin of asymmetries in SM. In particular, chiral symmetry breaking and the mechanism of mass generation appear to arise via a minimal extension of the action principle. A surprising finding is that, enforcing the Poincare symmetry in near equilibrium conditions, leads to fractalization of the space-time background. A follow-up analysis will examine if the same approach is able to resolve the puzzle of the

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[2] It is important to emphasize that the onset of fractal space-time in the high-energy sector of field theory and its lack of differentiability makes the concept of "speed of light in vacuum" ill-defined. As a result, the notion of invariance under Poincare symmetry in *far-from equilibrium* settings requires a careful redefinition of concepts through use of fractal operators [20].

so-called strong CP problem in QCD [3, 19]. We plan on reporting these results elsewhere.

APPENDIX A: The center manifold theory

We assume below that short-scale degrees of freedom aggregate in a large ensemble of classical fields whose dynamics may be modeled as an autonomous many-body system. Often times, the evolution of autonomous dynamical systems can be cast in the form [20]

$$\frac{d\mathbf{X}}{dt} = f_i(\mathbf{X}, \{\lambda\}) \tag{A1}$$

where $\mathbf{X}(t) = \{X_i(t)\}$, $i = 1, 2, ..., n$ with $n \gg 1$ denotes the state vector of short-scale fields, f_i are the rate laws and $\{\lambda\} = \lambda_j$, $j = 1, 2, ..., m$ represents a vector of generic control parameters. Let $\mathbf{X}_s(t)$ stand for a stable reference state of (A1) and let $\mathbf{x}(t) = \mathbf{X}(t) - \mathbf{X}_s(t)$ be the vector of linear perturbations from the stable state. Linear stability analysis enables one to map (A1) onto the equivalent system of differential equations

$$\frac{dx_i}{dt} = \sum_j L_{ij}(\lambda) x_j + h_i(\{x_j\}, \lambda) \tag{A2}$$

Here, L_{ij} are the coefficients of the linear part in perturbations and h_i are nonlinear corrections. Depending on the rate of growth of perturbations, a multivariable system such as (A1) can display a rich spectrum of behaviors. It can be shown that, under some well-defined conditions, when λ reaches a set of critical values (λ_c), a bifurcation of solutions takes place. If perturbations are non-oscillatory at $\lambda = \lambda_c$, the bifurcating branches correspond to steady-state solutions. A remarkable outcome of this stability analysis is that an *order parameter* (z) emerges which obey a universal quadratic equation referred to as *normal form* equation. The original multivariable dynamics (A2) is effectively reduced to

$$\frac{dz}{dt} = (\lambda - \lambda_c) - uz^2 \tag{A3}$$

where " u " stands for a non-vanishing coefficient.

<u>APPENDIX B: Unitary field transformations</u>

Unitary transformations of fields (UT) are fundamental symmetry operators in QFT. For example, chiral symmetry relates L and R components of fields and represents an UT. An infinitesimal UT of angle $\theta_a \ll 1$ can be presented as

$$\psi^\alpha \to \psi^\alpha - i\theta_a T^a_{\alpha\beta}\psi^\beta \tag{B1}$$

where the matrix $T^a_{\alpha\beta}$ is the generator of UT and the index "a" indicates that there might be several generators associated with the corresponding symmetry. Equation (B1) is the expansion for small angles of the general UT

$$\psi^\alpha \to \exp(-i\theta^a T^a_{\alpha\beta})\psi^\beta \tag{B2}$$

From (23) and (B1) we find the following expression for conserved currents

$$J^\mu = \frac{\partial L}{\partial(\partial^\mu \psi^\alpha)}\theta_a T^a_{\alpha\beta}\psi^\beta \tag{B3}$$

The exponential operator in (B2) may be understood as generating rotations in internal field space $\psi(x) \to \psi'(x)$. These are performed with no change of space-time location and preserve the modulus of the rotating field. Using for simplicity the label $\beta = \alpha'$, the field differential is given by

$$\delta_0\psi^\alpha = \psi'^\alpha - \psi^\alpha = \psi^\alpha[\exp(-i\theta_a T^a_{\alpha\beta}) - 1] \tag{B4}$$

Local gauge symmetry in EW model is described by a UT belonging to the SU(2) group. Field transformation of fermions in this model takes the form

$$\psi'(x) = \exp[ig_2 \bar{\tau} \cdot \overline{W}(x)]\psi(x) \tag{B5}$$

in which $\bar{\tau}$ denotes the triplet of 2 x 2 Pauli matrices, $[\cdot]$ stands for matrix multiplication and $\overline{W}(x)$ for the triplet of gauge fields carrying the SU(2) charge (known as *weak isospin*). Likewise, QCD exhibits local gauge invariance described by the SU(3) group and internal field transformation of fermions is given by

$$\psi'(x) = \exp[ig_s \bar{\lambda} \cdot \overline{G}(x)]\psi(x) \tag{B6}$$

Here, $\bar{\lambda}$ is the octet of 3 x 3 matrices, g_s the coupling describing strong interactions and $\overline{G}(x)$ the octet of gauge fields that carry the SU(3) charge (known as *color*).

Appendix 3: Feigenbaum attractor in particle physics

The table shown below is a summary of results published in [21]. It contains a side-by-side comparison of estimated versus actual mass ratios for charged leptons and quarks, massive gauge bosons and ratios of interaction strengths. All masses are reported in MeV and evaluated at the energy scale set by the top quark mass (m_t). Using recent results issued by the Particle Data Group [22], we take

$$m_u = 2.12, \quad m_d = 4.22, \quad m_s = 80.9$$

$$m_c = 630, \quad m_b = 2847, \quad m_t = 170{,}800$$

Coupling strengths are evaluated at the scale set by the mass of the "Z" boson, namely

$$\alpha_{EM} = 1/128 \,, \quad \alpha_W = 0.0338 \,, \quad \alpha_s = 0.123$$

Here, “u”, “d”, “s”, “c”, “b” and “t” stand for the six quark flavors, “e”, “μ” and "τ" represent the three flavors of charged leptons, "W" and “Z” the two flavors of massive gauge bosons and “α_{EM}”, “α_W”, “α_s” the coupling strengths associated with the electromagnetic, weak and strong interactions.

Parameter ratio	**Behavior**	**Actual**	**Predicted**
m_u / m_c	$\bar{\delta}^{-4}$	3.365×10^{-3}	4.323×10^{-3}
m_c / m_t	$\bar{\delta}^{-4}$	3.689×10^{-3}	4.323×10^{-3}
m_d / m_s	$\bar{\delta}^{-2}$	0.052	0.066
m_s / m_b	$\bar{\delta}^{-2}$	0.028	0.066
m_e / m_μ	$\bar{\delta}^{-4}$	4.745×10^{-3}	4.323×10^{-3}
m_μ / m_τ	$\bar{\delta}^{-2}$	0.061	0.066
M_W / M_Z	$(1-\frac{1}{\bar{\bar{\delta}}})^{1/2}$	0.8823	0.8623
$(\alpha_{EM} / \alpha_W)^2$	$\bar{\delta}^{-2}$	0.053	0.066
$(\alpha_{EM} / \alpha_s)^2$	$\bar{\delta}^{-4}$	4.034×10^{-3}	4.323×10^{-3}

Tab 2: Actual versus predicted ratios of SM parameters

References

To follow