

# DECELERATION PARAMETER Q(Z) IN FOUR AND FIVE DIMENSIONAL GEOMETRIES, AND IMPLICATIONS OF GRAVITON MASS IN MIMICKING DE IN BOTH GEOMETRIES

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The case for a four dimensional graviton mass (non zero) influencing reacceleration of the universe in both four and five dimensions is stated, with particular emphasis upon if four and five dimensional geometries as given below give us new physical insight as to cosmological evolution. The author finds that both cases give equivalent reacceleration one billion years ago which leads to an inquiry if other criteria as to cosmology can determine the benefits of adding additional dimensions to cosmology models

## 1 Introduction

### 1.1 What can be said about gravitational wave density value detection?

We will start with a first-principle introduction to detection of gravitational wave density using the definition given by Maggiore <sup>1</sup>

$$\Omega_{gw} \equiv \frac{\rho_{gw}}{\rho_c} \equiv \int_{f=0}^{f=\infty} d(\log f) \cdot \Omega_{gw}(f) \Rightarrow h_0^2 \Omega_{gw}(f) \cong 3.6 \cdot \left[ \frac{n_f}{10^{37}} \right] \cdot \left( \frac{f}{1kHz} \right)^4 \quad (1)$$

where  $n_f$  is the frequency-based numerical count of gravitons per unit phase space. The author suggests that  $n_f$  may also depend upon the interaction of gravitons with neutrinos in plasma during early-universe nucleation, as modeled by M. Marklund *et al* <sup>2</sup>. Having said that, the question is, what sort of mechanism is appropriate for considering macro affects of gravitons, and the author thinks that he has one, i.e. reacceleration of the universe, as far as a function of graviton mass, i.e. what Beckwith<sup>3</sup> did was to make the following presentation. Assume Snyder geometry and look at use of the following inequality for a change in the HUP, <sup>4</sup>

$$\Delta x \geq \left[ (1/\Delta p) + l_s^2 \cdot \Delta p \right] \equiv (1/\Delta p) - \alpha \cdot \Delta p \quad (2)$$

and that the mass of the graviton is partly due to the stretching alluded to by Fuller and Kishimoto,<sup>5</sup> a supposition the author<sup>3</sup> is investigating for a modification of a joint KK tower of gravitons, as given by Maartens<sup>6</sup> for DM. Assume that the stretching of early relic neutrinos that would eventually lead to the KK tower of gravitons--for when  $\alpha < 0$ , is<sup>4</sup>, which can be understood as given by

$$m_n(\text{Graviton}) = \frac{n}{L} + 10^{-65} \text{ grams} \quad (3)$$

Note that Rubakov<sup>7</sup> writes KK graviton representation as, after using the following normalization  $\int \frac{dz}{a(z)} [h_m(z) \cdot h_{\tilde{m}}(z)] \equiv \delta(m - \tilde{m})$  where  $J_1, J_2, N_1, N_2$  are different forms of Bessel functions, to obtain the KK graviton/ DM candidate representation along RS dS brane world

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \quad (4)$$

This Eq. (4) is for KK gravitons having a TeV magnitude mass  $M_Z \sim k$  (i.e. for mass values at .5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK graviton, which is moving with a speed proportional to  $H^{-1}$  with regards to the negative tension brane with  $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$  as an initial starting value for the KK graviton mass,

before the KK graviton, as a ‘massive’ graviton moves with velocity  $H^{-1}$  along the RS dS brane. If so, and if  $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$  represents an initial state, then one

may relate the mass of the KK graviton, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass lower in value, i.e. of  $m_{\text{graviton}}(4\text{-Dim GR}) \sim 10^{-48} eV$ , as opposed to  $M_X \sim M_{\text{KK-Graviton}} \sim .5 \times 10^9 eV$ . Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky et.al.<sup>8</sup> who argue for graviton mass using CMBR measurements, of  $M_{\text{KK-Graviton}} \sim 10^{-20} eV$  Dubosky et. al.<sup>8</sup> results can be conflated with Alves et. al.<sup>9</sup> arguing that non zero graviton mass may lead to an acceleration of our present universe, in a manner usually conflated with DE, i.e. their graviton mass would be about  $m_{\text{graviton}}(4\text{-Dim GR}) \sim 10^{-48} \times 10^{-5} eV \sim 10^{65}$  grams.

Also assume that to calculate the deceleration, the following modification of the HUP is used:  $[2] \Delta x \geq [(1/\Delta p) + l_s^2 \cdot \Delta p] \equiv (1/\Delta p) - \alpha \cdot \Delta p$ , where the LQG condition is  $\alpha > 0$ , and brane worlds have, instead,  $\alpha < 0$ <sup>4</sup>. The deceleration parameter in (1.4) will have either higher-dimensional contributions, in the brane theory case, or no higher-dimensional contributions, in the LQG case. Also Eq. (5) will be the starting point used for a KK tower version of Eq. (6) below. So from Maarten’s<sup>10</sup> 2005 paper,

$$\dot{a}^2 = \left[ \left( \frac{\tilde{\kappa}^2}{3} \left[ \rho + \frac{\rho^2}{2\lambda} \right] \right) a^2 + \frac{\Lambda \cdot a^2}{3} + \frac{m}{a^2} - K \right] \quad (5)$$

Maartens <sup>10</sup> also gives a 2<sup>nd</sup> Friedman equation, as

$$\dot{H}^2 = \left[ -\left( \frac{\tilde{\kappa}^2}{2} \cdot [p + \rho] \cdot \left[ 1 + \frac{\rho^2}{\lambda} \right] \right) + \frac{\Lambda \cdot a^2}{3} - 2 \frac{m}{a^4} + \frac{K}{a^2} \right] \quad (5)$$

Also, if we are in the regime for which  $\rho \cong -P$ , for red shift values  $z$  between zero to 1.0-1.5 with exact equality,  $\rho = -P$ , for  $z$  between zero to .5. The net effect will be to obtain, due to Eq. (6), and use  $a \equiv [a_0 = 1]/(1+z)$ . As given by Beckwith<sup>3</sup>

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \equiv -1 - \frac{\dot{H}}{H^2} = -1 + \frac{2}{1 + \tilde{\kappa}^2 [\rho/m] \cdot (1+z)^4 \cdot (1 + \rho/2\lambda)} \approx -1 + \frac{2}{2 + \delta(z)} \quad (6)$$

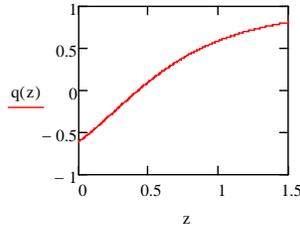
Eq. (6) assumes  $\Lambda = 0 = K$ , and the net effect is to obtain, a substitute for DE, by presenting how gravitons with a small mass in four dimensions can account for reacceleration of the universe, a datum usually done with  $\Lambda \neq 0$ , even if curvature  $\mathbf{K} = 0$

## 2 Consequences of small graviton mass for reacceleration of the universe

In a revision of Alves *et. al.*,<sup>9</sup> Beckwith<sup>3</sup> used a higher-dimensional model of the brane world and Marsden<sup>6</sup> KK graviton towers. The density  $\rho$  of the brane world in the Friedman equation as used by Alves *et. al.*<sup>9</sup> is use by Beckwith<sup>3</sup> for a non-zero graviton

$$\rho \equiv \rho_0 \cdot (1+z)^3 - \left[ \frac{m_g \cdot (c=1)^6}{8\pi G(\hbar=1)^2} \right] \cdot \left( \frac{1}{14 \cdot (1+z)^3} + \frac{2}{5 \cdot (1+z)^2} - \frac{1}{2} \right) \quad (7)$$

I.e. Eq. (6) above is making a joint DM and DE model, with all of Eq. (6) being for KK gravitons and DM, and  $10^{-65}$  grams being a 4 dimensional DE. Eq. (5) is part of a KK graviton presentation of DM/ DE dynamics. Beckwith<sup>11</sup> found at  $z \sim .4$ , a billion years ago, that acceleration of the universe increased, as shown in Fig. 1.



**Fig. 1: Reacceleration of the universe based on Beckwith<sup>3</sup> (note that  $q < 0$  if  $z < .423$ )**

### 3. Suggesting a non standard way to accommodate small graviton mass in 4 D

If one is adding the small mass of  $m_n(Graviton) = \frac{n}{L} + 10^{-65}$  grams<sup>3</sup>, with  $m_0(Graviton) \approx 10^{-65}$  grams, then the problem being worked with is a source term problem of the form given by Peskins<sup>11</sup> as of the type

$$\psi_n(x) \equiv \int d^3 p \cdot \frac{1}{(2\pi)^3} \cdot \frac{1}{\sqrt{2E_p}} \cdot \left\{ \left( a_p + \frac{i}{\sqrt{2E_p}} \cdot FT(m_0(graviton)) \right) \exp(-ipx) + H.C. \right\} \quad (8)$$

This is, using the language V.A. Rubakov<sup>7</sup> put up equivalent to<sup>3, 9</sup>,

$$\psi_m(x) \approx h_m(x) + \int d^3 p \cdot \frac{1}{(2\pi)^3} \cdot \left( \frac{1}{\sqrt{2E_p}} \right)^2 \cdot \{ (i \cdot FT(m_0(graviton))) \exp(-ipx) + H.C. \} \quad (9)$$

If  $m_0(graviton)$  is a constant, then the expression (9) has delta functions. This is the field theoretic identification. Another way is to consider an instanton-anti instanton treatment of individual gravitons, and to first start with the supposed stretch out of gravitons to enormous lengths. Assuming  $m_0(Graviton) \approx 10^{-65}$  grams for gravitons in 4 dimensions, the supposition by Bashinsky<sup>12</sup> and Beckwith<sup>3</sup> is that density fluctuations are influenced by a modification of cosmological density  $\rho$  in the Friedmann equations by the proportionality factor given by Bashinsky,<sup>12</sup>  $\left[ 1 - 5 \cdot (\rho_{neutrino} / \rho) + \mathcal{G} \left( (\rho_{neutrino} / \rho)^2 \right) \right]$  This proportionality factor for  $\rho$  as showing up in the Friedmann equations should be taken as an extension of results from Marklund *et al.*<sup>2</sup>, due to graviton-neutrino interactions as proposed by Marklund *et al.*<sup>2</sup>, where neutrinos interact with plasmons and plasmons interact with gravitons. Thereby implying neutrino- graviton interactions Also, graviton wavelengths have the same order of magnitude of neutrinos. Note, from Valev,<sup>13</sup>

$$\begin{aligned} m_{graviton} \Big|_{RELATIVISTIC} &< 4.4 \times 10^{-22} h^{-1} eV / c^2 \\ \Leftrightarrow \lambda_{graviton} &\equiv \frac{\hbar}{m_{graviton} \cdot c} < 2.8 \times 10^{-8} \text{ meters} \end{aligned} \quad (10)$$

Extending M. Marklund *et al.*<sup>2</sup> and Valev<sup>13</sup>, some gravitons may become larger<sup>14</sup>, i.e.

$$\lambda_{graviton} \equiv \frac{\hbar}{m_{graviton} \cdot c} < 10^4 \text{ meters or larger} \quad (11)$$

A way to accommodate this wave length has been suggested by Beckwith,<sup>3</sup> as to an instanton-anti instanton packaging of gravitons, was to start with an analogy between Giovannini,<sup>15</sup> from a least action version of the Einstein – Hilbert action for ‘quadratic’ theories of gravity involving Euler- Gauss-Bonnet. Then Giovannini’s<sup>15</sup> equation 6 corresponds to

$$\phi = \tilde{v} + \arctan((bw)^v) \quad (12)$$

Givannini<sup>15</sup> represents of Eq. (12) as a kink, and makes references to an anti-kink solution, in Fig. 1 in Givannini<sup>15</sup>. Furthermore the similarity between Eq. (12) and

$$\phi_+(z, \tau) = 4 \cdot \arctan \left( \exp \left\{ \frac{z + \beta \cdot \tau}{\sqrt{1 - \beta^2}} \right\} \right)$$

in Beckwith’s<sup>3,16</sup> treatment with regards to density wave physics instantons is obvious. If  $\arctan((bw)^v)$  is part of representing a graviton as a kink-anti-kink combination, arising from a 5 dimensional line element,<sup>15</sup>

$$dS^2 = a(w) \cdot [\eta_{uv} dx^u dx^v - dw^2] \quad (13)$$

Then, noting as Beckwith<sup>3</sup> mentioned, there is the possibility of using t’Hoofts<sup>17</sup> classical embedding of “deterministic quantum mechanics” as a way to embed a nearly four dimensional graviton as having almost zero mass, in a larger non linear theory.

#### 4. Other than five dimensions for cosmology ? Problems which need resolutions

If a way to obtain a graviton mass in four dimensions is done which fits in with the as

given higher 5 dimensions specified by a slight modification of brane theory, or Maarten’s cosmological evolution<sup>3,10</sup> equations, what benefits could this approach accrue for other outstanding problems in cosmology ? Beckwith<sup>3</sup> claims that a re do of the Friedmann equations would result in deceleration parameter  $q(z)$  similar to Fig. 1 above. Snyder geometry for the four dimensional case with would specify Friedmann equations along the lines of  $\alpha > 0$  in Eq. (2) above. If one follows  $\alpha < 0$ , then the Friedmann equations appear as giving details to the following equation<sup>3,19</sup>

$$\mathfrak{S} = -\frac{1}{2} \int d^4 x h_{uv}^0 \cdot T^{uv} \sim L^2 \approx \delta^+ \geq 0 \quad (14)$$

The construction done from sections 1 to 3 are for  $\alpha < 0$ . When  $\alpha > 0$ , the claim is that almost all the complexity is removed  $\alpha > 0$ , and what is left is a Taveras<sup>20</sup> treatment of the Friedmann equations, where he obtains, to first order, if  $\rho$  is a scalar field density,

$$\left( \frac{\dot{a}}{a} \right)^2 = [\kappa/3] \cdot \rho \quad (15)$$

and

$$\left(\frac{\ddot{a}}{a}\right) = -[2 \cdot k/3] \cdot \rho \quad (16)$$

The interpretation of  $\rho$  as a scalar field density<sup>20</sup>, and if one does as Alves et al<sup>9</sup> uses Eq. (7) above. We need to interpret the role of  $\rho$ . In the LQG version by<sup>21</sup>, Eq. (15) may be rewritten as follows: If conjugate momentum is in many cases, "almost" or actually a constant, using  $\dot{\phi} = -[\hbar/i] \cdot [\partial/\partial p_\phi]$

$$\left(\frac{\dot{a}}{a}\right)^2 \equiv [\kappa/6] \cdot [p_\phi^2/a^6] \quad (17)$$

Beckwith<sup>11</sup> claims that the deceleration parameter  $q(z)$  incorporating Eq. (15), Eq. (16) and Eq. (17) should give much the same behavior as Fig. 1 above. If so, then if one is differentiating between four and five dimensions by what is gained, in cosmology, one needs having it done via other criteria. The following is a real problem. As given by Maggiore<sup>1</sup>, the massless equation of the graviton evolution equation takes the form

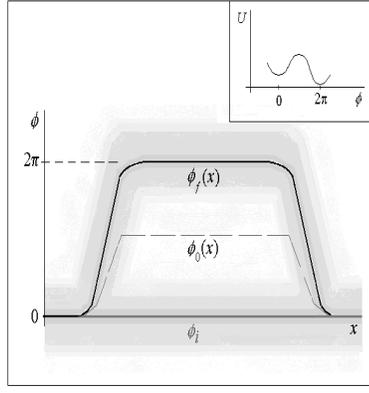
$$\partial_\mu \partial^\sigma h_{\mu\nu} = \sqrt{32\pi G} \cdot \left( T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\mu_\mu \right) \quad (18)$$

When  $m_{\text{graviton}} \neq 0$ , the above becomes

$$\left( \partial_\mu \partial^\sigma - m_{\text{graviton}} \right) \cdot h_{\mu\nu} = \left[ \sqrt{32\pi G} + \delta^+ \right] \cdot \left( T_{\mu\nu} - \frac{1}{3} \eta_{\mu\nu} T^\mu_\mu + \frac{\partial_\mu \partial_\nu T^\mu_\mu}{3m_{\text{graviton}}} \right) \quad (19)$$

The mismatch between these two equations, when  $m_{\text{graviton}} \rightarrow 0$ , is due to  $m_{\text{graviton}} h^\mu_\mu \neq 0$  as  $m_{\text{graviton}} \rightarrow 0$ , which is due to setting a value of  $m_{\text{graviton}} \cdot h^\mu_\mu = -[\sqrt{32\pi G} + \delta^+] \cdot T^\mu_\mu$ . The semi classical method by t'Hooft, using Eq. (12) is the solution. We generalize to higher dimensions the following diagram as given by Beckwith<sup>3</sup>. Use an instanton- anti instanton structure, and t'Hooft<sup>18</sup> equivalence classes along the lines of Eq. (20) below with equivalence class structure in the below wave functional to be set by a family of admissible values<sup>3</sup>  $\phi_0(x)$

$$\Psi_{i,f} [\phi(\mathbf{x})]_{\phi=\phi_{ci,f}} = c_{i,f} \cdot \exp \left\{ - \int d\mathbf{x} \alpha \left[ \phi_{Ci,f}(\mathbf{x}) - \phi_0(\mathbf{x}) \right]^2 \right\} \quad (20)$$



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**Fig. 2: The pop up effects of an instanton-anti-instanton in Euclidian space<sup>11, 17</sup>**

### 5. Conclusion. Examining information exchange between different universes?

Beckwith<sup>3</sup> has concluded that the only way to give an advantage to higher dimensions as far as cosmology would be to look at if a fifth dimension may present a way of actual information exchange to give the following parameter input from a prior to a present universe, i.e. the fine structure constant, as given by<sup>3</sup>

$$\tilde{\alpha} \equiv e^2 / \hbar \cdot c \equiv \frac{e^2}{d} \times \frac{\lambda}{hc} \quad (21)$$

The wave length as may be chosen to do such an information exchange would be part of a graviton as being part of an information counting algorithm as can be put below, namely: Argue that when taking the log, that the 1/N term drops out. As used by Ng<sup>17</sup>

$$Z_N \sim (1/N!) \cdot (V/\lambda^3)^N \quad (22)$$

This, according to Ng,<sup>17</sup> leads to entropy of the limiting value of, if  $S = (\log[Z_N])$  will be modified by having the following done, namely after his use of quantum infinite statistics, as commented upon by Beckwith<sup>3</sup>

$$S \approx N \cdot (\log[V/\lambda^3] + 5/2) \approx N \quad (23)$$

Eventually, the author hopes to put on a sound foundation what 'tHooft<sup>18</sup> is doing with respect to 'tHooft<sup>18</sup> deterministic quantum mechanics and equivalence classes embedding quantum particle structures.. Doing so will answer the questions Kay<sup>21</sup> raised about particle creation, and the limitations of the particle concept in curved and flat space, i.e. the global hyperbolic space time which is flat everywhere except in a localized "bump" of curvature. Furthermore, making a count of gravitons with  $S \approx N \sim 10^{20}$  gravitons<sup>3,17</sup>, with  $I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} \sim 10^{20}$  as implying at least one operation per unit graviton, with gravitons being one unit of information, per produced graviton<sup>3</sup>. What the author, Beckwith, sees is that since instanton- anti instanton pairs do not have to travel slowly, as has been proved by authors in the 1980s, that gravitons if nucleated in a fashion as indicated by Fig. 2, may be able to answer the following. The stretch-out of a graviton wave, greater than the size of the solar system, gives, an upper limit of a graviton mass due to wave length

$\lambda_{\text{graviton}} > 300 \cdot h_0 \text{kpc} \Leftrightarrow m_{\text{graviton}} < 2 \times 10^{-29} h_0^{-1} \text{eV}$ . I. e. stretched graviton wave, at ultra-low frequency, may lead to a low mass limit. However, more careful limits due to experimental searches, as presented by Buonanno<sup>22</sup> have narrowed the upper limit to  $10^{-20} h_0^{-1} \text{eV}$ . An instanton – anti instanton structure to the graviton, if confirmed, plus experimental confirmation of mass, plus perhaps  $n \sim 10^{20}$  gravitons  $\approx 10^{20}$  entropy counts, Eq. (23) implies up to  $\approx 10^{27}$  operations. If so, there is a one-to-one relationship between an operation and a bit of information, so a graviton has at least one bit of information. And that may be enough to determine the conditions needed to determine if Eq. (21) gives information and structure from a prior universe to our present cosmos.

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