The Gravitational Field of a Condensed Matter Model of the Sun: The Space Breaking Meets the Asteroid Strip

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Abstract: This seminal study deals with the exact solution of Einstein's field equations for a sphere of incompressible liquid without the additional limitation initially introduced in 1916 by Karl Schwarzschild, according to which the space-time metric must have no singularities. The obtained exact solution is then applied to the Universe, the Sun, and the planets, by the assumption that these objects can be approximated as spheres of incompressible liquid. It is shown that gravitational collapse of such a sphere is permitted for an object whose characteristics (mass, density, and size) are close to the Universe. Meanwhile, there is a spatial break associated with any of the mentioned stellar objects: the break is determined as the approaching to infinity of one of the spatial components of the metric tensor. In particular, the break of the Sun's space meets the Asteroid strip, while Jupiter's space break meets the Asteroid strip from the outer side. Also, the space breaks of Mercury, Venus, Earth, and Mars are located inside the Asteroid strip (inside the Sun's space break).

Contents:

§1. Problem statement. The main task of this paper is to study the possibilities of applying condensed matter models in astrophysics and cosmology. A cosmic object consisting of condensed matter has a constant volume and a constant density. A sphere of incompressible liquid, being in the weightless state (as any cosmic object), is a kind of condensed matter. Thus, assuming that a star is a sphere of incompressible liquid, we can study the gravitational field of the star inside and outside it.

The Sun orbiting the center of the Galaxy meets the weightless condition (see Chapter 2 of [1] for detail)

$$\frac{GM}{r} = \mathrm{v}^2,$$

where $G = 6.67\times10^{-8}\ \mathrm{cm}^3/\mathrm{g}\times\mathrm{sec}^2$ is the Newtonian gravitational constant, M is the mass of the Galaxy, r is the distance of the Sun from the center of the Galaxy, and v is the Sun's velocity in its orbit. The planets of the Solar System also satisfy the weightless condition. Assuming that the planets have a similar internal constitution as the Sun, we can consider these objects as spheres of incompressible liquid being in a weightless state.

In addition to it, we assume that the Universe also is a sphere of incompressible liquid. Concerning the Universe, this problem is not solved in detail in this study: only several conditions specific to the liquid Universe model are considered in §4.

I will consider the problems by means of the General Theory of Relativity. First, it is necessary to obtain the exact solution of the Einstein field equations for the space-time metric induced by the gravitational field of a sphere of incompressible liquid.

The regular field equations of Einstein, with the λ-field neglected, have the form

$$R_{\alpha\beta} - \frac{1}{2}\, g_{\alpha\beta} R = -\,\varkappa\, T_{\alpha\beta}\,, \tag{1.1}$$

where $R_{\alpha\beta}$ is the Ricci tensor, R is the Riemann curvature scalar, $\varkappa = \frac{8\pi G}{c^2} = 18.6\times10^{-28}$ cm/g is the Einstein gravitational constant, $T_{\alpha\beta}$ is the energy-momentum tensor, and $\alpha, \beta = 0, 1, 2, 3$ are the space-time indices. The gravitational field of spherical island of substance should possess spherical symmetry. Thus, it is described by the metric of spherical kind

$$ds^2 = e^{\nu} c^2 dt^2 - e^{\lambda} dr^2 - r^2 (d\theta^2 + \sin^2\theta\, d\varphi^2)\,, \tag{1.2}$$

where e^{ν} and e^{λ} are functions of r and t.

In the case under consideration the energy-momentum tensor is that of an ideal liquid (incompressible, with zero viscosity), by the condition that its density is constant: $\rho = \rho_0 = const$. As known, the energy-momentum tensor in this case is

$$T^{\alpha\beta} = \left(\rho_0 + \frac{p}{c^2}\right) b^{\alpha} b^{\beta} - \frac{p}{c^2}\, g^{\alpha\beta}, \tag{1.3}$$

where p is the pressure of the liquid, while

$$b^{\alpha} = \frac{dx^{\alpha}}{ds}\,, \qquad b_{\alpha}b^{\alpha} = 1 \tag{1.4}$$

is the four-dimensional velocity vector, which determines the reference frame of the given observer. The energy-momentum tensor should satisfy the conservation law

$$\nabla_{\sigma} T^{\alpha\sigma} = 0\,, \tag{1.5}$$

where ∇_{σ} is the four-dimensional symbol of covariant differentiation.

Formally, the problem we are considering is a generalization of the Schwarzschild solution produced for an analogous case (a sphere of incompressible liquid). Karl Schwarzschild [2] solved the Einstein field equations for this case, by the condition that the solution must be regular. He assumed that the components of the fundamental metric tensor $g_{\alpha\beta}$ must satisfy the signature conditions (the space-time metric must have no singularities). Thus, the Schwarzschild solution, according to his initial assumption, does not include space-time singularities.

This limitation of the space-time geometry, initially introduced in 1916 by Schwarzschild, will not be used by me in this study. Therefore, we will be able to study the singular properties of the space-time metric associated with a sphere of incompressible liquid. Then we will apply the obtained results to the cosmic objects such as the Sun, the planets, and, ultimately, the Universe as a whole.

It should be noted that the problem of space-time singularities plays a very important rôle in the General Theory of Relativity and astrophysics, because it is linked indirectly with the problem of black holes: such an object as a gravitational collapsar possesses a space-time singularity on its surface.

The term "black hole" was coined due to David Hilbert's study of 1927, which followed 11 years after Schwarzschild's original solution (and his tragic death in 1916). Hilbert analyzed the Schwarzschild solution for the gravitational field of a mass-point [3]. He wrote this solution in the form

$$ds^2 = \left(1 - \frac{r_g}{r}\right) c^2 dt^2 - \frac{dr^2}{1 - \frac{r_g}{r}} - r^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right), \tag{1.6}$$

where $r_g = \frac{2GM}{c^2}$ is known as the Hilbert radius, while M is the mass of the field source (the mass-point). At $r = r_g$ the space-time region of the surface around the mass-point collapses: gravitational collapse, a state by which the component g_{00} is zero, occurs on the surface $r = r_g$.

It is easy to see that by $r = r_g$ two conditions are fulfilled

$$g_{00} = 0 \quad \text{and} \quad g_{11} \to \infty \,. \tag{1.7}$$

The first condition, $g_{00} = 0$, is known as the *collapse condition*. It is said that "time is stopped by gravitational collapse". This situation will be studied in detail in §4: it will be shown that the observed time is truly stopped, while the coordinate time continues its flow, uniformly. The second condition, $g_{11} \to \infty$, was completely ignored in the past, although it can be considered as the *condition of the breaking of the space*. From a formal viewpoint these conditions violate the requirements which determine the the space-time region of a real observer (a real observer has a real mass and can move only with a sublight velocity).

The conditions are linked with violation of the *space-time signature prescription*. This violation means that the given space-time has singularities in the regions (surfaces or volumes) wherein the aforementioned conditions are true. The signature conditions for a diagonal metric $(+---)$ have the form

$$\left.\begin{aligned} g_{00} &> 0 \\ g_{00}\, g_{11} &< 0 \\ g_{00}\, g_{11}\, g_{22} &> 0 \\ g = g_{00}\, g_{11}\, g_{22}\, g_{33} &< 0 \end{aligned}\right\} . \tag{1.8}$$

The first three are known as the *weak signature conditions*. The fourth is known as the *strong signature condition*. If one or all weak signature conditions are violated, while the strong signature condition is true, this is a *removable singularity*. If the strong signature condition is violated, the space-time has *unremovable singularity*: in this case the field solution is regularly failed from consideration, because it "has not physical meaning". Actually, someone did not see the physical meaning therein. However it is very meaningful mathematically. Therefore, I will direct my focus onto unremovable singularities in the Schwarzschild field of a sphere of incompressible liquid. The most important results obtained due to this approach will be discussed in §7.

The most known kind of space (space-time) containing a removable singularity is Schwarzschild space. It is an empty space (no continuous matter presented in the space, that means $T_{\alpha\beta} = 0$), filled with the spherically symmetric gravitational field of a mass-point (1.6). Given such a space, two weak signature conditions are violated ($g_{00} = 0$ and $g_{11} \to \infty$ by $r = r_g$) and the strong signature condition $g = -r^4 \sin^2 < 0$ is true by $r = r_g$ in it.

Because the Schwarzschild solution describes the gravitational field of a mass-point in a space free of distributed matter (empty space-time), and it includes the possibility of gravitational collapse, it is very popular amongst the theoretical physicists and astrophysicists working on the black hole problem. As a matter of fact, several cosmic objects like stars can collapse (the problem of gravitational collapse is often linked with stars at a later stage of their evolution). On the other hand, the aforementioned Schwarzschild solution means gravitational collapse of a mass-point's field, although stars are continuous bodies, not mass-points. Therefore, the problem of the space-time singularities (for instance, gravitational collapsars — black holes) should be solved by models of continuous bodies, not mass-points.

Thus, the main task of my study is to study singularities in the space filled with the gravitational field of a sphere of incompressible liquid, which can approximate the models an actual cosmic body like a star, a planet, or the Universe as a whole. In the framework of this approach, the Universe will be considered in §4, while the Sun and the planets will be considered in §7.

I will consider this problem employing the mathematical methods of physically observable quantities, known as chronometric invariants. This versatile mathematical apparatus was developed in 1944 by Abraham Zelmanov [4–6], then applied by him, very successfully, to relativistic cosmology. This mathematical technique gives the advantage that it is connected to a specific (chosen) observer and the physical standards of his laboratory, so we obtain the theoretical results expressed through the real quantities measurable in practice. All physically observable characteristics of the reference space of the space-time described by the metric (1.2) will be calculated in §2.

In §3, the exact solution of the field equations (1.1) will be obtained for the spherically symmetric metric (1.2) inside a sphere of incompressible liquid, which is described by the energy-momentum tensor (1.3). Because we do not limit the solution by that the metric must be regular, the obtained metric has two singularities: 1) collapse by $g_{00}=0$, and 2) break of the space by $g_{11}\rightarrow\infty$. It will be shown then that these singularities are unremovable, because the strong signature condition is also violated in both cases.

The singularities of the Schwarzschild field produced by a sphere of incompressible liquid, are studied in detail in §4. It will be shown that the conditions of collapse and breaking of the space depend on the density of the liquid sphere, and also on its total mass and radius. The spherical surface of the space breaking can either be inside the liquid sphere or outside it, depending on the numerical value of the mass density. Besides, the surface of the space breaking can meet the Schwarzschild sphere of collapse under particular conditions. The last situation realizes itself for the liquid model of the Universe. In the liquid model of the Sun, the surface of the space breaking is located outside the Sun itself. Also, we will arrive at the next conclusion: a liquid sphere of the Sun's radius, cannot be in the state of gravitational collapse. In contrast, the liquid Universe as a whole is represented as a collapsed cosmic object.

The properties of particles located on a collapsar's surface are the subject of study in §5. It will be shown that these particles have imaginary rest-mass and imaginary three-dimensional momentum. The term "relativistic mass" is inapplicable to the particles, because they do not move in the usual sense of this word.

Physically observable properties of the space inside a sphere of incompressible liquid will be calculated in §6. It will be shown a tricky situation therein: the three-dimensional space inside the sphere is a constant negative curvature space, while the four-dimensional Riemann-Christoffel curvature tensor does not satisfy the constant curvature condition (its three-dimensional components satisfy the constant positive curvature, while the component R_{0101} does not satisfy the constant curvature condition in general). The component R_{0101} is zero on the surface of collapse, and is positive inside the collapsar. Therefore, a collapsar's surface is a bridge connecting two spaces of the negative and the positive curvature.

In §7, these results will be applied to the Sun and the planets, which will be considered as spheres of incompressible liquid. It will be shown that the collapse condition is not satisfied for these objects. The surface of the space breaking is located outside such an object. According to the detailed calculations, the surface of the breaking of the Sun's space meets the Asteroid strip. Analogous calculations manifest the intersections of the planets' space breaking.

§2. Physically observable characteristics of the gravitational field inside a sphere of incompressible liquid. In the framework of the mathematical apparatus of physically observable quantities (chronometric invariants), two three-dimensional quantities are necessary for further derivation of the physically observable properties of a space [4–6]: a three-dimensional scalar — the gravitational potential of the field produced by the reference body of the observer

$$\mathrm{w} = c^2 \left(1 - \sqrt{g_{00}}\right), \tag{2.1}$$

and a three-dimensional vector — the linear velocity of the reference space's rotation in the point of observation

$$v_i = -\,c\,\frac{g_{0i}}{\sqrt{g_{00}}}\,, \tag{2.2}$$

where Roman indices ($i=1,2,3$ in the present case) are signed for the three-dimensional spatial coordinates.

We can easily see, therefore, that the collapse condition $g_{00}=0$ is realized by $\mathrm{w}=c^2$. For the spherically symmetric metric (1.2), we have

$$\mathrm{w} = c^2\left(1 - e^{\frac{\nu}{2}}\right). \tag{2.3}$$

Because $g_{0i}=0$ in the metric (1.2), i.e. the space does not rotate, we have $v_i=0$. Hence the chr.inv.-tensor of the angular velocity of rotation of the reference space (the tensor of the space non-holonomity), determined in the theory of chronometric invariants, is zero

$$A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right) = 0\,, \tag{2.4}$$

while the chr.inv.-vector of gravitational inertial force is

$$F_i = \frac{c^2}{c^2-\mathrm{w}}\left(\frac{\partial \mathrm{w}}{\partial x^i} - \frac{\partial v_i}{\partial t}\right) = -\,\frac{c^2}{2}\,\nu'\,, \tag{2.5}$$

where the prime denotes the differentiation along the r-coordinate.

With these, the chr.inv.-metric tensor [4–6]

$$h_{ik} = -\,g_{ik} + \frac{1}{c^2}\,v_i v_k \tag{2.6}$$

(it determines the physically observable metric of the observer's three-dimensional space), in the space of the spherically symmetric metric (1.2) has the following components

$$h_{11} = e^{\lambda}, \quad h_{22} = r^2, \quad h_{33} = r^2\sin^2\theta\,, \tag{2.7}$$

$$h^{11} = e^{-\lambda}, \quad h^{22} = \frac{1}{r^2}\,, \quad h_{33} = \frac{1}{r^2\sin^2\theta}\,, \tag{2.8}$$

$$h = \det\|\,h_{ik}\,\| = e^{\lambda} r^4 \sin^2\theta\,. \tag{2.9}$$

Thus, the chr.inv.-tensor of the rate of deformation of the space [4–6]

$$D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t}\,, \qquad D^{ik} = -\,\frac{1}{2}\frac{{}^*\partial h^{ik}}{\partial t}\,, \tag{2.10}$$

where

$$\frac{^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t} \tag{2.11}$$

is the chr.inv.-operator of differentiation along the time coordinate, has the following non-zero components

$$D_{11} = \frac{\dot{\lambda}}{2}\, e^{\lambda - \frac{\nu}{2}}\,, \qquad D^{11} = -\,\frac{\dot{\lambda}}{2}\, e^{-\lambda - \frac{\nu}{2}}, \tag{2.12}$$

(the upper dot denotes differentiation along the time coordinate t).

We see, therefore, that a non-rotating spherical symmetric space contains gravitation, and can deform if the spatial metric h_{ik} does not depend on time. We will see later that the stationarity condition of the metric h_{ik} depends on the structure of the energy-momentum tensor of continuous matter filling the space.

Now, we calculate the characteristics of non-uniformity the space — the chr.inv.-Christoffel symbols of the first and second kinds

$$\Delta^k_{ij} = h^{km}\Delta_{ij,m} = \frac{1}{2}\, h^{km}\left(\frac{^*\partial h_{im}}{\partial x^j} + \frac{^*\partial h_{jm}}{\partial x^i} - \frac{^*\partial h_{ij}}{\partial x^m}\right), \tag{2.13}$$

where the chr.inv.-operator of differentiation along the spatial coordinates is

$$\frac{^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}\, v_i\, \frac{^*\partial}{\partial t}\,. \tag{2.14}$$

We obtain, after algebra, non-zero components of $\Delta_{ij,m}$

$$\Delta_{11,1} = \frac{\lambda'}{2}\, e^{\lambda}\,, \quad \Delta_{22,1} = -\,r\,, \quad \Delta_{33,1} = -\,r\,\sin^2\theta\,, \tag{2.15}$$

$$\Delta_{12,2} = r\,, \qquad \Delta_{33,2} = -\,r^2\sin\theta\cos\theta\,, \tag{2.16}$$

$$\Delta_{13,3} = r\,\sin^2\theta\,, \qquad \Delta_{23,3} = r^2\sin\theta\cos\theta\,, \tag{2.17}$$

thus non-zero components of Δ^k_{ij} are

$$\Delta^1_{11} = \frac{\lambda'}{2}\,, \quad \Delta^1_{22} = -\,r\,e^{-\lambda}, \quad \Delta^1_{33} = -\,r\,\sin^2\theta\,e^{-\lambda}, \tag{2.18}$$

$$\Delta^2_{12} = \frac{1}{r}\,, \quad \Delta^2_{33} = -\,\sin\theta\cos\theta\,, \tag{2.19}$$

$$\Delta^3_{13} = \frac{1}{r}\,, \qquad \Delta^3_{23} = \cos\theta\,. \tag{2.20}$$

The three-dimensional observable curvature of the space is characterized by the chr.inv.-curvature tensor C_{lkij}, which possesses all the algebraic properties of the Riemann-Christoffel tensor [4–6]

$$C_{lkij} = \frac{1}{4}\left(H_{lkij} - H_{jkil} + H_{kljij} - H_{iljk}\right), \tag{2.21}$$

where H_{lkij} is similar to the Schouten tensor of the theory of non-holonomic manifolds, and is derived from the non-commutativity of the second chr.inv.-derivatives of an arbitrary transferred three-dimensional vector Q_l along the spatial coordinates

$$^*\nabla_i{}^*\nabla_k Q_l - {}^*\nabla_k{}^*\nabla_i Q_l = \frac{2A_{ik}}{c^2}\frac{^*\partial Q_l}{\partial t} + H_{lki\cdot}^{\cdots j}Q_j, \tag{2.22}$$

where the chr.inv.-covariant differential from the vector is

$$^*\nabla_k Q^i dx^k = dQ^i + \Delta^i_{kl}Q^k dx^l. \tag{2.23}$$

The tensor $H_{lki\cdot}^{\cdots j}$ has the form [4–6]

$$H_{lki\cdot}^{\cdots j} = \frac{^*\partial \Delta^j_{il}}{\partial x^k} - \frac{^*\partial \Delta^j_{kl}}{\partial x^i} + \Delta^m_{il}\Delta^j_{km} - \Delta^m_{kl}\Delta^j_{im}\,, \tag{2.24}$$

it is connected with the curvature tensor C_{lkij} by

$$H_{lkij} = C_{lkij} + \frac{1}{c^2}\big(2A_{ki}D_{jl} + A_{ij}D_{kl} + A_{jk}D_{il} + {} + A_{kl}D_{ij} + A_{li}D_{jk}\big), \tag{2.25}$$

while the contracted tensors $H_{lk} = H_{lki\cdot}^{\cdots i}$ and $C_{lk} = C_{lki\cdot}^{\cdots i}$ are connected, according to the theory of chronometric invariants, as

$$H_{lk} = C_{lk} + \frac{1}{c^2}\left(A_{kj}D^j_l + A_{lj}D^j_k + A_{kl}D\right). \tag{2.26}$$

We see that H_{lkij} and C_{lkij} are the same if the reference space is free of rotation and deformation. It is obvious that this condition is true for H_{lk} and C_{lk} as well. The tensor $C_{lk} = h^{ij}C_{ilkj}$ has the form

$$C_{lk} = \frac{^*\partial}{\partial x^k}\left(\frac{^*\partial \ln\sqrt{h}}{\partial x^l}\right) - \frac{^*\partial \Delta^i_{kl}}{\partial x^i} + \Delta^m_{il}\Delta^i_{km} - \Delta^m_{kl}\frac{^*\partial \ln\sqrt{h}}{\partial x^m}\,. \tag{2.27}$$

Thus, we obtain non-zero components of C_{lk} for the spherically symmetric metric (1.2). They are

$$C_{11} = -\frac{\lambda'}{r}\,, \qquad C_{22} = \frac{C_{33}}{\sin^2\theta} = e^{-\lambda}\left(1 - \frac{r\lambda'}{2}\right) - 1. \tag{2.28}$$

So, we have calculated all the physically observable characteristics of the space, which are necessary for our further deduction of the exact solution of the Einstein field equations.

All that I have to add to these, are the physically observable components (chr.inv.-components) of the energy-momentum tensor of ideal liquid (1.3). Being calculated according to the theory of chronometric invariants, where $b^i = \frac{dx^i}{ds} = 0$ and $b^0 = \frac{1}{\sqrt{g_{00}}}$ [4–6], they are

$$\rho = \frac{T_{00}}{g_{00}} = \rho_0 \,, \quad J^i = \frac{c\,T_0^i}{\sqrt{g_{00}}} = 0 \,, \quad U^{ik} = c^2 T^{ik} = p h^{ik}, \qquad (2.29)$$

where ρ is the chr.inv.-density of the distributed matter, J^i is the chr.inv.-vector of the density of the momentum in the medium, U^{ik} is the chr.inv.-stress tensor. The condition $J^i = 0$ means that the observer's reference frame accompanies the mass, while $U^{ik} = ph^{ik}$ means that his reference frame accompanies the medium.

§3. The Einstein equations inside a sphere of incompressible liquid: the exact solution. In order to obtain the exact internal solution of the Einstein field equations with respect to a given distribution of matter, it is necessary to solve two systems of equations: the Einstein field equations (1.1), and the equations of the conservation law (1.5).

We will solve the equations in terms of physically observable quantities. The Einstein field equations expressed through the physically observable quantities (the chr.inv.-Einstein equations) are [4–6]

$$\frac{{}^*\partial D}{\partial t} + D_{jl}D^{lj} + A_{jl}A^{lj} + \left({}^*\nabla_j - \frac{1}{c^2}\,F_j\right)F^j = -\frac{\varkappa}{2}\left(\rho c^2 + U\right), \quad (3.1)$$

$${}^*\nabla_j\left(h^{ij}D - D^{ij} - A^{ij}\right) + \frac{2}{c^2}\,F_j A^{ij} = \varkappa J^i, \qquad (3.2)$$

$$\frac{{}^*\partial D_{ik}}{\partial t} - \left(D_{ij} + A_{ij}\right)\left(D_k^j + A_{k\cdot}^{\cdot j}\right) + DD_{ik} - D_{ij}D_k^j + $$
$$+ 3A_{ij}A_{k\cdot}^{\cdot j} + \frac{1}{2}\left({}^*\nabla_i F_k + {}^*\nabla_k F_i\right) - \frac{1}{c^2}\,F_iF_k - c^2 C_{ik} =$$
$$= \frac{\varkappa}{2}\left(\rho c^2 h_{ik} + 2U_{ik} - Uh_{ik}\right), \qquad (3.3)$$

where $U = h^{ik}U_{ik}$ is the trace of the stress-tensor U_{ik}. The chr.inv.-form of the conservation law is [4–6]

$$\frac{{}^*\partial\rho}{\partial t} + D\rho + \frac{1}{c^2}\,D_{ij}U^{ij} + {}^*\widetilde{\nabla}_i J^i - \frac{1}{c^2}\,F_i\,J^i = 0\,, \qquad (3.4)$$

$$\frac{{}^*\partial J^k}{\partial t} + DJ^k + 2\left(D_i^k + A_{i\cdot}^{\cdot k}\right)J^i + {}^*\widetilde{\nabla}_i\,U^{ik} - \rho F^k = 0\,, \qquad (3.5)$$

where the chr.inv.-differential operator ${}^*\widetilde{\nabla}_i = {}^*\nabla_i - \frac{1}{c^2}F_i$ is constructed on the basis of chr.inv.-divergence ${}^*\nabla_i$ which is according to (2.23).

To solve the system of the Einstein field equations we substitute, into (3.1–3.3), the chr.inv.-characteristics of the space obtained in §2. We substitute also the chr.inv.-components of the energy-momentum tensor (2.29), from which we conclude, additionally, that

$$U = 3p \tag{3.6}$$

in the spherically symmetric liquid model.

Then, after algebra, we obtain the chr.inv.-Einstein field equations in the spherically symmetric space (1.2) inside a sphere of incompressible liquid. The obtained equations, in component notation, are

$$e^{-\nu}\left(\ddot{\lambda} - \frac{\dot{\lambda}\dot{\nu}}{2} + \frac{\dot{\lambda}^2}{2}\right) - c^2 e^{-\lambda}\left[\nu'' - \frac{\lambda'\nu'}{2} + \frac{2\nu'}{r} + \frac{(\nu')^2}{2}\right] = \\ = -\varkappa\left(\rho_0 c^2 + 3p\right), \tag{3.7}$$

$$\frac{\dot{\lambda}}{r}\, e^{-\lambda-\frac{\nu}{2}} = \varkappa J^1 = 0\,, \tag{3.8}$$

$$e^{\lambda-\nu}\left(\ddot{\lambda} - \frac{\dot{\lambda}\dot{\nu}}{2} + \frac{\dot{\lambda}^2}{2}\right) - c^2\left[\nu'' - \frac{\lambda'\nu'}{2} + \frac{(\nu')^2}{2}\right] + \frac{2c^2\lambda'}{r} = \\ = \varkappa\left(\rho_0 c^2 - p\right)e^{\lambda}, \tag{3.9}$$

$$\frac{c^2\left(\lambda' - \nu'\right)}{r}\, e^{-\lambda} + \frac{2c^2}{r^2}\left(1 - e^{-\lambda}\right) = \varkappa\left(\rho_0 c^2 - p\right). \tag{3.10}$$

The second equation manifests that $\dot{\lambda} = 0$ in this case. Hence, the space inside the sphere of incompressible liquid does not deform. Taking this circumstance into account, and also that the stationarity of λ, we reduce the field equations (3.7–3.10) to the final form

$$c^2 e^{-\lambda}\left[\nu'' - \frac{\lambda'\nu'}{2} + \frac{2\nu'}{r} + \frac{(\nu')^2}{2}\right] = \varkappa\left(\rho_0 c^2 + 3p\right)e^{\lambda}, \tag{3.11}$$

$$-\,c^2\left[\nu'' - \frac{\lambda'\nu'}{2} + \frac{(\nu')^2}{2}\right] + \frac{2c^2\lambda'}{r} = \varkappa\left(\rho_0 c^2 - p\right)e^{\lambda}, \tag{3.12}$$

$$\frac{c^2\left(\lambda' - \nu'\right)}{r}\, e^{-\lambda} + \frac{2c^2}{r^2}\left(1 - e^{-\lambda}\right) = \varkappa\left(\rho_0 c^2 - p\right)e^{\lambda}. \tag{3.13}$$

To solve the equations (3.11–3.13), a formula for the pressure p is necessary. To find the formula, we now deal with the conservation equations (3.4–3.5). Because, as was found, $J^i = 0$ and $D_{ik} = 0$ in the case under consideration, the chr.inv.-scalar conservation equation (3.4) leads to the trivial result $\frac{{}^*\partial\rho}{\partial t} = 0$. Thus $\rho = \rho_0 = const$ inside the sphere, and the chr.inv.-vectorial conservation equations (3.5) take the form

$$ {}^*\nabla_i \left(ph^{ik}\right) - \left(\rho_0 + \frac{p}{c^2}\right) F^k = 0\,, \tag{3.14}$$

which, since ${}^*\nabla_i h^{ik} = 0$ in the case, reads

$$ h^{ik}\frac{{}^*\partial p}{\partial x^i} - \left(\rho_0 + \frac{p}{c^2}\right) F^k = 0\,. \tag{3.15}$$

Taking into account that $\frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i}$ in the case, we obtain, this formula reduces to only a single nontrivial equation

$$ p' e^{-\lambda} + \left(\rho_0 c^2 + p\right)\frac{\nu'}{2}\, e^{-\lambda} = 0\,, \tag{3.16}$$

where $p' = \frac{dp}{dr}$, $\nu' = \frac{d\nu}{dr}$, $e^{\lambda} \neq 0$. Dividing both parts of (3.16) by $e^{-\lambda}$, we arrive at

$$ \frac{dp}{\rho_0 c^2 + p} = -\frac{d\nu}{2}\,, \tag{3.17}$$

which is a plain differential equation with separable variables. It can be easily integrated as

$$ \rho_0 c^2 + p = Be^{-\frac{\nu}{2}}\,, \qquad B = const. \tag{3.18}$$

Thus we have to express the pressure p as the function of ν,

$$ p = Be^{-\frac{\nu}{2}} - \rho_0 c^2. \tag{3.19}$$

In look for an r-dependent function $p(r)$, we integrate the field equations (3.11–3.13). Summarizing (3.11) and (3.12), we find

$$ \frac{c^2\left(\lambda' + \nu'\right)}{r} = \varkappa B e^{\lambda - \frac{\nu}{2}}. \tag{3.20}$$

Then, expressing ν' from this equation, and substituting the result into (3.13), we obtain

$$ \frac{2c^2}{r}\,\lambda' + \frac{2c^2}{r^2}\left(e^{\lambda} - 1\right) - \varkappa B e^{-\lambda - \frac{\nu}{2}} = \varkappa\left(\rho_0 c^2 - p\right) e^{\lambda}. \tag{3.21}$$

Substituting p from (3.19) into (3.21), we obtain the following differential equation with respect to λ

$$ \lambda' + \frac{e^{\lambda} - 1}{r} - \varkappa\rho_0 r e^{\lambda} = 0\,. \tag{3.22}$$

We introduce a new variable $y = e^{\lambda}$. Thus $\lambda' = \frac{y'}{y}$. Substituting into this equation y and y', we obtain the Bernoulli equation (see Kamke [7], Part III, Chapter I, §1.34)

$$y' + f(r)\,y^2 + g(r)\,y = 0\,, \tag{3.23}$$

where

$$f(r) = \frac{1}{r} - \varkappa\rho_0 r\,, \qquad g(r) = -\frac{1}{r}\,. \tag{3.24}$$

It has the following solution

$$\frac{1}{y} = E(r)\int\frac{f(r)\,dr}{E(r)}\,, \tag{3.25}$$

$$E(r) = e^{\int g(r)dr}. \tag{3.26}$$

Integrating (3.26), we obtain $E(r)$ which is

$$E(r) = e^{-\int\frac{dr}{r}} = e^{\ln\frac{L}{r}} = \frac{L}{r}\,, \qquad L = const > 0\,, \tag{3.27}$$

thus we obtain $\frac{1}{y} = e^{-\lambda}$ which is

$$e^{-\lambda} = \frac{L}{r}\int\frac{r}{L}\left(\frac{1}{r} - \varkappa\rho_0 r\right)dr = 1 - \frac{\varkappa\rho_0 r^2}{3} + \frac{Q}{r}\,, \quad Q = const. \tag{3.28}$$

To find Q, we rewrite equation (3.21) as

$$e^{-\lambda}\left(\frac{\lambda'}{r} - \frac{1}{r^2}\right) + \frac{1}{r^2} = \varkappa\rho_0\,. \tag{3.29}$$

This equation has a singularity at the point $r = 0$, therefore the numerical value of the right side term of the equation (the density of the liquid) grows to infinity by $r \to 0$, i.e. in the center of the sphere. This is in contradiction to the initially assumed condition $\rho_0 = const$, which is specific to incompressible liquids. As a matter of fact, this contradiction should not be in the theory. We remove this contradiction by re-writting (3.29) in the form

$$e^{-\lambda}\left(1 - r\lambda'\right) = \frac{d}{dr}\left(re^{-\lambda}\right) = 1 - \varkappa\rho_0 r^2. \tag{3.30}$$

After integration, we obtain

$$re^{-\lambda} = r - \frac{\varkappa\rho_0 r^3}{3} + A\,, \qquad A = const. \tag{3.31}$$

Because $A = 0$ at the central point $r = 0$, it should be zero at any other point as well. Dividing this equation by $r \neq 0$, we obtain

$$e^{-\lambda} = 1 - \frac{\varkappa \rho_0 r^2}{3}\,. \tag{3.32}$$

Comparing this solution with the value $e^{-\lambda}$ obtained earlier (3.28), we see that they meet each other if $Q = 0$. Besides, we should suggest that $e^{\lambda_0} = 1$ at the central point $r = 0$, consequently $\lambda_0 = 0$.

Thus we have obtained the components $h^{11} = e^{-\lambda}$ and $h_{11} = e^{\lambda}$ of the chr.inv.-metric tensor h_{ik}, expressed through the coordinate r, i.e.

$$h^{11} = e^{-\lambda} = 1 - \frac{\varkappa \rho_0 r^2}{3}\,, \qquad h_{11} = e^{\lambda} = \frac{1}{1 - \frac{\varkappa \rho_0 r^2}{3}}\,. \tag{3.33}$$

So forth, we should introduce a boundary condition on the surface of the sphere. We have on the surface: $r = a$, where a is the radius of the sphere. Thus

$$e^{-\lambda_a} = 1 - \frac{\varkappa \rho_0 a^2}{3}\,. \tag{3.34}$$

On the other hand, the solution of this function is also the Schwarzschild solution in emptiness. Hence,

$$e^{-\lambda_a} = 1 - \frac{2GM}{c^2 a}\,, \tag{3.35}$$

where M is the mass of the sphere. Comparing both expressions, and taking into account that the Einstein gravitational constant is $\varkappa = \frac{8\pi G}{c^2}$, we find

$$M = \frac{4\pi a^3 \rho_0}{3} = \rho_0 V, \tag{3.36}$$

where $V = \frac{4\pi a^3}{3}$ is the volume of the sphere. Thus, we have obtained the regular relation between the mass and the volume of a homogeneous sphere.

Our next step is the looking for the solution $e^{-\lambda}$ outside the sphere, i.e. for $r > a$. Since outside the sphere the density of the substance (liquid) is $\rho_0 = 0$, we obtain, after integration of (3.30),

$$r\,e^{-\lambda} = \int_0^r dr - \int_0^a \varkappa \rho_0\, r^2\, dr = r - \frac{\varkappa \rho_0\, a^3}{3}\,. \tag{3.37}$$

We obtain, from this formula, that

$$e^{-\lambda} = 1 - \frac{\varkappa \rho_0\, a^3}{3r}\,. \tag{3.38}$$

Taking (3.38) into account, we obtain the Schwarzschild solution in emptiness

$$e^{-\lambda} = 1 - \frac{2GM}{c^2 r}\,. \tag{3.39}$$

To obtain ν we again use equation (3.20). Substituting, into this equation,

$$\lambda' = \frac{\frac{2\varkappa\rho_0 r}{3}}{1 - \frac{\varkappa\rho_0 r^2}{3}} \tag{3.40}$$

and e^{λ}, we obtain, after transformations,

$$\nu' + \frac{\frac{2\varkappa\rho_0 r^2}{3}}{1 - \frac{\varkappa\rho_0 r^2}{3}} - \frac{\varkappa B}{c^2}\frac{r e^{-\frac{\nu}{2}}}{1 - \frac{\varkappa\rho_0 r^2}{3}} = 0\,. \tag{3.41}$$

We introduce a new variable $e^{-\frac{\nu}{2}} = y$. Thus, $\nu' = -\frac{2y'}{y}$. Substituting these into (3.41), we obtain the Bernoulli equation

$$y' + \frac{\varkappa B}{2c^2}\frac{r y^2}{1 - \frac{\varkappa\rho_0 r^2}{3}} - \frac{\frac{\varkappa\rho_0 r}{3}\, y}{1 - \frac{\varkappa\rho_0 r^2}{3}} = 0\,, \tag{3.42}$$

where

$$f(r) = \frac{\varkappa B}{2c^2}\frac{r}{1 - \frac{\varkappa\rho_0 r^2}{3}}\,, \qquad g(r) = -\frac{\frac{\varkappa\rho_0 r}{3}}{1 - \frac{\varkappa\rho_0 r^2}{3}}\,. \tag{3.43}$$

Thus, we have the integral

$$\int g(r)\, dr = -\int \frac{\frac{\varkappa\rho_0 r}{3}}{1 - \frac{\varkappa\rho_0 r^2}{3}} = \ln N\sqrt{\left|1 - \frac{\varkappa\rho_0 r^2}{3}\right|}\,, \quad N = const, \tag{3.44}$$

then

$$E(r) = N\sqrt{\left|1 - \frac{\varkappa\rho_0 r^2}{3}\right|}\,. \tag{3.45}$$

In the region where the signature condition $h_{11} = e^{\lambda} > 0$ is satisfied, we have

$$1 - \frac{\varkappa\rho_0 r^2}{3} > 0\,, \tag{3.46}$$

therefore we use the modulus of the function here.

Next, we look for $\frac{1}{y} = e^{\frac{\nu}{2}}$, which is

$$e^{\frac{\nu}{2}} = \frac{\varkappa B}{2c^2}\sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}\int \frac{r\, dr}{\sqrt{\left(1 - \frac{\varkappa\rho_0 r^2}{3}\right)^3}}\,. \tag{3.47}$$

We obtain, after integration,

$$e^{\frac{\nu}{2}} = \frac{\varkappa B}{2c^2}\left(\frac{3}{\varkappa\rho_0} + K\sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}\right), \qquad K = const. \tag{3.48}$$

Now, we look for the constants B and K. To find B, we rewrite the formula of p by the condition that $p=0$ on the surface of the sphere $(r=a)$. Thus, we obtain

$$B = \rho_0 c^2 e^{\frac{\nu_a}{2}}, \tag{3.49}$$

where $e^{\frac{\nu_a}{2}}$ is the value of the function $e^{\frac{\nu}{2}}$ on the surface. As a result, we have

$$e^{\frac{\nu}{2}} = \frac{\varkappa\rho_0}{2}\, e^{\frac{\nu_a}{2}}\left(\frac{3}{\varkappa\rho_0} + K\sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}\right). \tag{3.50}$$

To find K, we take the value of $e^{\frac{\nu}{2}}$ on the surface $(r=a)$

$$e^{\frac{\nu_a}{2}} = \frac{\varkappa\rho_0\, e^{\frac{\nu_a}{2}}}{2}\left(\frac{3}{\varkappa\rho_0} + K\sqrt{1-\frac{\varkappa\rho_0 a^2}{3}}\right). \tag{3.51}$$

We obtain, from this formula, that

$$K = -\frac{1}{\varkappa\rho_0}\frac{1}{\sqrt{1-\frac{\varkappa\rho_0 a^2}{3}}}\,. \tag{3.52}$$

The quantity $e^{\frac{\nu_a}{2}}$ means the numerical value of $e^{\frac{\nu}{2}}$ by $r=a$, therefore we can apply it to the Schwarzschild solution (a mass-point's field) in emptiness at $r=a$, i.e.

$$e^{\frac{\nu_a}{2}} = \sqrt{1-\frac{2GM}{c^2 a}}\,. \tag{3.53}$$

Taking the expressions for $e^{\frac{\nu_a}{2}}$, (3.34) and (3.35), into account, we obtain

$$e^{\frac{\nu}{2}} = \frac{1}{2}\, e^{\frac{\nu_a}{2}}\left(3 - \sqrt{\frac{1-\frac{\varkappa\rho_0 r^2}{3}}{1-\frac{\varkappa\rho_0 a^2}{3}}}\right) =$$

$$= \frac{1}{2}\left(3\sqrt{1-\frac{2GM}{c^2 a}} - \sqrt{1-\frac{2GMr^2}{c^2 a^3}}\right). \tag{3.54}$$

This formula on the surface $(r=a)$ meets the Schwarzschild solution in emptiness: $e^{\frac{\nu_a}{2}} = \sqrt{1-\frac{2GM}{c^2 a}} = \sqrt{1-\frac{\varkappa\rho_0 a^2}{3}}$.

Thus the space-time metric of the gravitational field inside a sphere of incompressible liquid is, since the formulae of ν and λ have already been obtained, as follows

$$ds^2 = \frac{1}{4}\left(3\,e^{\frac{\nu_a}{2}} - \sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}\right)^2 c^2 dt^2 - \\ - \frac{dr^2}{1 - \frac{\varkappa\rho_0 r^2}{3}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \quad (3.55)$$

Taking (3.34) and (3.35) into account, we rewrite (3.55) as

$$ds^2 = \frac{1}{4}\left(3\,e^{\frac{\nu_a}{2}} - \sqrt{1 - \frac{2GMr^2}{c^2a^3}}\right)^2 c^2 dt^2 - \\ - \frac{dr^2}{1 - \frac{2GMr^2}{c^2a^3}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \quad (3.56)$$

Since $\frac{2GM}{c^2} = r_g$ is the Hilbert gravitational radius, we rewrite (3.56) in the form

$$ds^2 = \frac{1}{4}\left(3\sqrt{1 - \frac{r_g}{a}} - \sqrt{1 - \frac{r^2 r_g}{a^3}}\right)^2 c^2 dt^2 - \\ - \frac{dr^2}{1 - \frac{r^2 r_g}{a^3}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \quad (3.57)$$

It is therefore obvious that this "internal" metric completely coincides with the Schwarzschild metric in emptiness on the surface of the sphere of incompressible liquid ($r = a$).

Our next step is to obtain the space-time metric outside the sphere ($r > a$). We already obtained the "external" solution for $e^{-\lambda}$, which completely coincides with the "external" Schwarzschild solution for this function (3.39). Outside the sphere, (3.20) takes the form

$$\lambda' + \nu' = 0\,, \quad (3.58)$$

consequently where according to (3.39)

$$\lambda' = \frac{2GM}{c^2r^2}\,\frac{1}{1 - \frac{2GM}{c^2r}}\,. \quad (3.59)$$

Substituting (3.59) into (3.58) and integrating, we find

$$\nu = \ln\left(1 - \frac{2GM}{c^2r}\right) + P, \qquad P = const, \quad (3.60)$$

thus

$$e^{\nu} = P\left(1 - \frac{2GM}{c^2 r}\right). \tag{3.61}$$

Since this function is

$$e^{\nu} = 1 - \frac{2GM}{c^2 a}\,,$$

on the surface ($r = a$), we obtain $P = 1$. Thus we have established that the space-time outside a sphere of incompressible liquid is described by the Schwarzschild metric in emptiness, which is (1.6).

§4. Singular properties of the external and internal Schwarzschild solutions. The space-time of a sphere of incompressible liquid is described by the metric (3.55) or, in the equivalent form, by (3.57). The singular properties of the space-time will be studied here. This study is a generalization of the originally Schwarzschild solution for such a sphere [2], and means that Schwarzschild's requirement to the metric to be free of singularities will not be used here. Naturally, the metric (3.57) allows singularities; they will be studied here in detail. This problem will be solved by analogy with the singular properties of the Schwarzschild solution in emptiness (a mass-point's field), which already gave black holes. As will be shown, there is a big difference between the Schwarzschild solutions. The mass-point solution in emptiness [3] will be considered at first, because it plays a key rôle in physics of black holes (gravitational collapsars).

As is known, the Schwarzschild metric of a mass-point's field (1.6) has singularities by the condition that the radial coordinate r equals the Hilbert radius

$$r = \frac{2GM}{c^2} = r_g\,. \tag{4.1}$$

One considers (4.1) as the condition of collapse of real cosmic objects like stars. It is supposed that stars can collapse in the last stage of their evolution. Of course, no doubts that stars can collapse in this way. However they are not mass-points; they are continuous objects consisting of substance. Therefore, the Schwarzschild metric of a mass-point's field (1.6) does not characterize a collapsing continuous object, but states that the field of a continuous object collapses at the distance $r = r_g$ from its centre of gravity. This distance is known as the *radius of the Schwarzschild sphere*. It is easy to see that r_g depends on the object's mass M only, and not on its characteristics such as the density of substance or the radius of the object itself.

Two singular conditions are realized in the metric (1.6) by the condition (4.1)

$$g_{00} = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 = 1 - \frac{r_g}{r} = 0\,, \tag{4.2}$$

$$g_{11} = -\,h_{11} = \frac{1}{1 - \frac{r_g}{r}} \to \infty\,. \tag{4.3}$$

The first singularity (4.2) is known as *gravitational collapse*. In this case, the gravitational potential is $\mathrm{w} = c^2$. The state of collapse is connected indirectly with the physically observable time τ, which is determined by the theory of chronometric invariants [4–6] as

$$d\tau = \sqrt{g_{00}}\,dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\,dx^i, \tag{4.4}$$

where t is the coordinate (ideal) time, which is according to $x^0 = ct$, and flows uniformly. As seen, τ depends on the gravitational potential and the rotation of the space. Since a space with the Schwarzschild metric does not rotate ($g_{0i} = 0$), we have

$$d\tau = \sqrt{g_{00}}\,dt = \sqrt{1 - \frac{r_g}{r}}\,dt\,, \tag{4.5}$$

consequently

$$\tau = 0 \tag{4.6}$$

on the surface of collapse in the field, which is located at the distance $r = r_g$ from the centre of gravity of the body. In other words, the observable time stops on the surface of collapse, being registered by a regular observer. (However the coordinate time t still be flowing uniformly.)

Consider the spatial part of the metric (1.6) by the condition (4.1). At first, we note that any four-dimensional metric ds^2 can be expressed through the interval of the physically observable time $d\tau$ and the physically observable space interval $d\sigma$ as [4–6]

$$ds^2 = c^2 d\tau^2 - d\sigma^2, \qquad d\sigma^2 = h_{ik}\,dx^i dx^k. \tag{4.7}$$

Since the three-dimensional observed space (the observer's spatial section) is curved, only distances σ are observable, while r are coordinate (photometric) distances. A physically observable distance between two points with radial coordinates r_1 and r_2 along the radial direction, for the metric (1.6) has the form

$$\sigma_r = \sqrt{h_{11}}\,dr = \int_{r_1}^{r_2} \frac{dr}{\sqrt{1 - \frac{r_g}{r}}}\,. \tag{4.8}$$

In the integration of this equation we should keep in mind that $r > r_g$ always for any regular observer, because the metric (1.6) does not describe the region inside the Schwarzschild sphere. The space-time inside the collapsar, created by a mass-point in emptiness, is described by another, non-stationary metric which is [8]

$$ds^2 = \frac{c^2 d\tilde{t}^2}{\frac{r_g}{c\tilde{t}} - 1} - \left(\frac{r_g}{c\tilde{t}} - 1\right) d\tilde{r}^2 - c^2\tilde{t}^2 \left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{4.9}$$

This metric is obtained from (1.6) by means of substitution among $r = c\tilde{t}$ and $ct = \tilde{r}$. We realize that, during a finite interval of time, $\tilde{t} = \frac{r_g}{c}$.

Let us find the observable distance between a point with the radial coordinate r_1 and a point on the Schwarzschild sphere $r = r_g$. Integrating (4.8) from r_g to r_1, we obtain

$$\sigma = \sqrt{r_1}\,\sqrt{r_1 - r_g} + r_g \ln\left|\frac{\sqrt{r_1} + \sqrt{r_1 - r_g}}{\sqrt{r_1}}\right|. \tag{4.10}$$

We have just integrated $d\sigma$ from $r_g = r_{sp}$ (a radial distance, where a breaking of the space takes place) to another radial coordinate which is $r = r_1 > r_g$, since we are presently considering only the space-time outside the collapsar. What is the collapsar according to the metric (1.6)? This is a region of the empty space-time inside the sphere of the radius $r = r_g$. We see, therefore, that the observed distance between the points with radial coordinates r_g and r_1 has a finite value, which becomes zero if $r_1 = r_g$. If $r_g \ll r_1$, we expand $\sqrt{1 - \frac{r_g}{r}}$ and $\ln\left|1 + \sqrt{1 - \frac{r_g}{r}}\right|$ into the series, then save only the terms of the first order with respect to $\frac{r_g}{r}$. We obtain, after algebra, for the metric (4.9), the approximate formula

$$\sigma_r = r_1 - \frac{r_g}{2} + r_g \ln\left|2 - \frac{r_g}{2r_1}\right| \simeq r_1 + 0.19\, r_g\,. \tag{4.11}$$

This formula is true for small r_g. Thus the observable distance σ_r between the points with the radial coordinates r_g and r_1 is larger than the coordinate (photometric) distance $r_1 - r_g$ between the points. It is important to remark that the elementary interval $d\sigma_r$ has a singularity by $r = r_g$, while the integral of it is continuous and has a finite value. The spatial interval of the metric (1.6) has the form

$$d\sigma^2 = \frac{dr^2}{1 - \frac{r_g}{r}} - r^2\left(d\theta^2 + \sin^2\theta\, d\varphi^2\right). \tag{4.12}$$

We see, therefore, that the three-dimensional metric form has a singularity at $r = r_g$. In this case, $h_{11} \to \infty$ (i.e. $d\sigma \to \infty$).

Thus the Schwarzschild metric of a mass-point's field (1.6) has two singularities: collapse and breaking of the space. Both singularities take place by the same condition $r = r_g$. We will refer to the state of the space-time by which the elementary observable spatial interval $d\sigma \rightarrow \infty$ as the *space breaking*, and denote the corresponding value of the radial coordinate as r_{br}. We see that $r_{br} = r_g$ in the space-time filled with the gravitational field of a mass-point. In other words, the space-time described by the Schwarzschild metric (1.6) has a singular surface, spherically covering the gravitating body at the distance $r = r_g$ from its centre of gravity (mass-point).

Now, we are going to study singularities in the space-time filled with the gravitational field of a sphere of incompressible (ideal) liquid. Inside such a sphere (its radius is $r = a$), space-time is described by the metric (3.55) or its equivalent form (3.57). As is seen, the metric has a spatial singularity (space breaking) by the condition

$$r_{br} = \sqrt{\frac{3}{\varkappa\rho_0}} = a\sqrt{\frac{a}{r_g}}\,, \tag{4.13}$$

thus we conclude something about the surface of the space breaking:

1) It meets the surface of the liquid sphere, if $a = r_g$;
2) It is located outside the liquid sphere, if $r_g < a$;
3) It is located inside the liquid sphere, if $r_g > a$.

Calculating the physically observable distance between the center of the liquid sphere and the spherical surface of the space breaking, in the r-direction, we obtain

$$\sigma_r = \int_0^{r_{br}} \sqrt{h_{11}}\, dr = \int_0^{r_{br}} \frac{dr}{\sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}} = \\ = \sqrt{\frac{3}{\varkappa\rho_0}}\arcsin\left(\sqrt{\frac{\varkappa\rho_0}{3}}\, r_{br}\right) = \frac{\pi}{2}\, r_{br}\,. \tag{4.14}$$

Thus, σ_r takes finite numerical values in the field of a liquid sphere. It is obvious that the physically observable distance $\frac{\pi}{2}\, r_{br}$ is larger than the coordinate (photometric) distance r_{br}.

Since r_g is determined only by the mass of the liquid sphere, r_{br} depends on ρ_0 as it does on the sphere's radius a. For example, considering the Sun as a sphere of incompressible liquid, whose density is $\rho_0 = 1.4$ g/cm^3, we obtain

$$r_{br} = 3.4\times10^{13} \text{ cm}, \tag{4.15}$$

while the radius of the Sun is $a = 7\times10^{10}$ cm and its Hilbert radius is $r_g = 3\times10^{5}$ cm. Therefore, the surface of the Sun's space breaking is located outside the surface of the Sun, far distant from it in the near cosmos.

Consider another example. Assume our Universe to be a sphere of incompressible liquid, whose density is $\rho_0 = 10^{-31}$ g/cm^3. The radius of its space breaking, according to (4.13), is

$$r_{br} = 1.3\times10^{29} \text{ cm}. \tag{4.16}$$

Observational astronomy provides the following numerical value of the Hubble constant

$$H = \frac{c}{a} = (2.3 \pm 0.3)\times10^{-18} \text{ sec}^{-1}, \tag{4.17}$$

where a is the observed radius of the Universe. It is easy obtain from here that

$$a = 2.3\times10^{28} \text{ cm}. \tag{4.18}$$

This value is comparable with (4.16), so the Universe's radius may meet the surface of its space breaking by some conditions. We calculate the mass of the Universe by (3.36) and (4.18). We have $M = 5\times10^{54}$ g. Thus, for the liquid model of the Universe, we obtain $r_g = 7.4\times10^{26}$ cm: the Hilbert radius (the radius of the surface of gravitational collapse) is located inside the liquid spherical body of the Universe.

Now, we are going to study the collapse condition of a sphere of incompressible liquid. On the first view, this problem statement makes nonsense, because the body of incompressible liquid cannot be compressed. Yes, it is true, if one would consider collapse as the process of compression of a liquid cosmic body. We do not do it. In contrast, we will consider a collapsar as a singular region of the space-time. In a particular case, a cosmic body consisting of incompressible liquid can be a collapsar, if the parameters of its field on its surface will correspond to the collapse condition $g_{00} = 0$ or the equivalent condition $\mathrm{w} = c^2$. But this rises to the occurrence of the physical conditions, not the evolutionary compression of a liquid cosmic body.

As is known, the collapse condition of a common case has the form

$$g_{00} = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 = 0\,, \tag{4.19}$$

thus a cosmic object is a collapsar, if the three-dimensional gravitational potential on its surface is

$$\mathrm{w} = c^2\,. \tag{4.20}$$

Consider the collapse condition for the space-time metric of the gravitational field inside a sphere of incompressible liquid (3.56). As is seen from the metric (3.56), the collapse condition (4.19) in this case is

$$3\,e^{\frac{\nu_a}{2}} = \sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}\;, \tag{4.21}$$

or, in terms of the Hilbert radius, when the metric takes the form (3.57), the collapse condition is

$$3\sqrt{1-\frac{r_g}{a}} = \sqrt{1-\frac{r_g r^2}{a^3}}\;. \tag{4.22}$$

We obtain that the numerical value of the radial coordinate r_c, by which the sphere's surface meets the surface of collapse, is

$$r_c = \sqrt{9a^2-8r_{br}^2} = a\sqrt{9-\frac{8a}{r_g}}\;. \tag{4.23}$$

Because we keep in mind really cosmic objects, the numerical value of r_c should be real. This requirement is obviously satisfied by

$$a < 1.125\, r_g\,. \tag{4.24}$$

If this condition holds not ($a \geqslant r_g$), the sphere, which is a spherical liquid body, has not the state of collapse.

It is obvious that the condition $a = r_g$ satisfies (4.24). Consider this interesting particular case in detail. We have, in this case, that

$$r_c = r_{br} = r_g = a\,. \tag{4.25}$$

This means that, in this case, given a sphere of incompressible liquid in the state of collapse, it has the radius of its surface a, the Hilbert radius r_g, and the radius of the space breaking r_{br} coinciding with the radius r_c characterizing its surface in the state of collapse.

Comparing (4.25) with (4.2–4.3), which characterize the Schwarzschild solution for a mass-point in emptiness, we see that a mass-point's field in emptiness satisfies the condition

$$r_g = r_{br}\,, \tag{4.26}$$

which is a particular case of (4.25): despite such characteristics as the proper radius a and the collapsed sirface's radius r_c are inapplicable to a mass-point, the common condition (4.25) still be working in the case, being represented in its particular form (4.26).

I repeat that the condition $a = r_g$ is only a partial case of (4.24). The common condition (4.24) includes three particular cases, concerning the location of the surface of a collapsed liquid sphere:

1) The radius of a collapsed liquid sphere is larger than the Schwarzschild sphere's radius ($a > r_g$);
2) The radius of a collapsed liquid sphere is lesser than the Schwarzschild sphere's radius ($a < r_g$);
3) The surface of a collapsed liquid sphere meets the Schwarzschild sphere ($a = r_g$).

It is obvious that r_c is imaginary for $r_g \ll a$, so collapse of such a sphere of incompressible liquid is impossible. For example, considering the Sun ($a = 7\times10^7$ cm, $M = 2\times10^{33}$ g, $r_g = 3\times10^5$ cm), we obtain from (4.24) that r_c has an imaginary value. This means that:

> A homogeneous sphere of incompressible liquid, whose parameters are the same as those of the Sun, cannot collapse.

One may ask: what does the condition $r_g \neq 0$ imply for the Sun? This means that the term r_g comes from another model of the Sun where it is approximated by a mass-point: the gravitational field of a mass-point includes a collapsed region inside the spherical surface of the radius r_g around the mass-point.

Another example. Consider the Universe as a sphere of incompressible liquid (the liquid model of the Universe). Assuming, according to the numerical value of the Hubble constant (4.17), that the Universe's radius is $a = 2.3\times10^{28}$ cm, we obtain the collapse condition, from (4.24),

$$r_g > 2.6\times10^{28} \text{ cm}, \tag{4.27}$$

and immediately arrive at the following conclusion:

> The observable Universe as a whole, being represented in the framework of the liquid model, is completely located inside its gravitational radius. In other words, the observable Universe is a collapsar — a huge black hole.

In another representation, this result means that a sphere of incompressible liquid can be in the state of collapse only if its radius approaches the radius of the observable Universe.

Compare the singularities of the liquid sphere's internal metric (3.57) and the mass-point's field metric (1.6). The weak signature conditions $g_{00} > 0$ and $g_{11} < 0$ are violated by $r = r_g$. The determinant of the fundamental metric tensor of the mass-point's field metric (1.6) equals

$$g = -\,r^4 \sin^2\theta < 0\,, \tag{4.28}$$

so the strong signature condition $g < 0$ is fulfilled, hence the singularity of the mass-point's field metric is removable. This means that the space-time collapses and has the same breaking the condition $r_{br} = r_g$. Thus, the collapse surface coincides with the space breaking surface in a mass-point's field in emptiness: in this case, both collapse and the space breaking are realized by the same condition (4.26).

A few words more on the singularities of the liquid sphere's internal metric (3.57). In this case, the determinant of the fundamental metric tensor equals

$$g = -\frac{1}{4}\left(3e^{\frac{\nu_a}{2}} - \sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}\right)^2 \frac{r^4 \sin^2\theta}{\sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}}\,, \tag{4.29}$$

so the strong signature condition $g < 0$ is always true for a sphere of incompressible liquid, except in two following cases: 1) in the state of collapse ($g_{00} = 0$), and 2) by the breaking of space ($g_{11} \to \infty$). These particular cases violate the weak signature conditions $g_{00} > 0$ and $g_{11} < 0$ correspondingly. If both weak signature conditions are violated, g has a singularity of the kind $\frac{0}{0}$. If collapse occurs in the absence of the space breaking, we have $g = 0$. If no collapse, while the space breaking is present, we have $g \to \infty$. In all the cases, the singularity is non-removable, because the strong singular condition $g < 0$ is violated.

So, as was shown above, a spherical object consisting of incompressible liquid can be in the state of gravitational collapse only if it is as large and massive as the Universe. Meanwhile, the space breaking realizes itself in the fields of all cosmic objects, which can be approximated by spheres of incompressible liquid. Besides, since $r_{br} \sim \frac{1}{\sqrt{\rho_0}}$, the r_{br} is then greater while smaller is the ρ_0. Assuming all these, we arrive at the following conclusion:

> A regular sphere of incompressible liquid, which can be observed in the cosmos or an Earth-bound laboratory, cannot collapse but has the space breaking — a singular surface, distantly located around the liquid sphere.

This problem will be considered in detail in §7.

§5. Collapsar as a special state of substance. Let us now consider the properties of substance inside a collapsar and on its surface. As was already shown above in the study, this consideration is applicable, on the one hand, to the internal gravitational field of a homogeneous liquid sphere (3.57), and, on the other hand, to the Schwarzschild gravitational field of a mass-point in emptiness (1.6).

So, we need to understand what sorts of particles inhabit these singular regions (the regions inside a collapsar and on its surface). This problem will be solved here by analogy with the study in [1].

First, we study substance on a collapsar's surface. This spherical surface is characterized by the condition $r = r_g$ for the metric of a mass-point's field (1.6), and by the condition $r = r_c$ for the internal metric of a liquid sphere (3.57). Because the space-times metrics are free of rotation, and taking the collapse condition $g_{00} = 0$ into account, they can be written in the common chr.inv.-form

$$ds^2 = -\,d\sigma^2 = -\,h_{ik}\,dx^i dx^k, \tag{5.1}$$

based on the general formula $ds^2 = c^2 d\tau^2 - d\sigma^2$ (4.7), which comes from the theory of chronometric invariants [4–6] and can be applied to any space-time metric.

Space-time trajectories are characterized by the four-dimensional velocity vector, which on the surface of a collapsar takes the form

$$b^\alpha = \frac{dx^\alpha}{|ds|} = \frac{dx^\alpha}{d\sigma}\,, \qquad b_\alpha b^\alpha = -1, \tag{5.2}$$

so it is a space-like vector on a collapsar's surface. Multiplying it by rest-mass m_0, we obtain a momentum world-vector

$$P^\alpha = m_0\,\frac{dx^\alpha}{d\sigma}\,, \qquad P_\alpha P^\alpha = -\,m_0^2\,, \tag{5.3}$$

which is a space-like vector therein as well, while the rest-masses take imaginary numerical values in the case (on a collapsar's surface).

According to the theory of chronometric invariants, the physically observable components of the momentum vector P^α (5.3) should be

$$\frac{P_0}{\sqrt{g_{00}}} \longrightarrow \frac{0}{0}\,, \qquad P^i = i\,|m_0|\,\frac{dx^i}{d\sigma}\,, \tag{5.4}$$

which have analogous physical meaning as the respective components

$$\frac{P_0}{\sqrt{g_{00}}} = \pm m\,, \qquad P^i = \frac{m_0}{\sqrt{1-\mathrm{v}^2/c^2}}\,\frac{dx^i}{c\,d\tau} = \frac{1}{c}\,m\mathrm{v}^i \tag{5.5}$$

of the momentum world-vector of a regular, real rest-mass particle

$$P^\alpha = m_0\,\frac{dx^\alpha}{ds}\,, \qquad P_\alpha P^\alpha = m_0^2\,. \tag{5.6}$$

We conclude therefore that the observable quantity analogous to relativistic mass is not determined for the particles which inhabit the surface of a collapsar, while the observable quantity analogous to three-dimensional momentum is imaginary for them. Thus, this sort of particles has special physical properties: such a particle has imaginary rest-masses and three-dimensional momentum, while the characteristic known as relativistic mass is not applicable to them.

Thus the surface of a collapsar cannot be considered as that of a regular physical body: this is a space-time region where the signature conditions are violated, and is inhabited with a singular sort of substance. Therefore, a regular observer whose rest-mass takes real numerical values cannot be there.

Another question: what sort of substance exists inside a collapsar, under its collapsed surface?

A space-time filled with the Schwarzschild field of a mass-point in emptiness has the metric (1.6), which is stationary. This metric is written for a regular observer, who is located outside the surface of collapse, and is watching the collapsar from outside. It is known [8] that the same Schwarzschild metric written for an internal observer, located inside the collapsar, is obtained from (1.6) by means of substitution among $r = c\tilde{t}$ and $ct = \tilde{r}$. The resulting metric (4.9) is non-stationary [8]. Thus, despite the invariance of the space-time metric as a whole, its stationarity under the collapsed surface (inside the collapsar) depends on the observer's reference frame: from views of an external observer the space inside the collapsar is stationary, while it is non-stationary being observed by an internal observer inside the collapsar.

A sphere of incompressible liquid cannot expand or compress. Meanwhile, if its characteristics satisfy the collapse conditions, it can be in the state of collapse, i.e. be a gravitational collapsar (black hole). As was shown above in §4, this is possible if the liquid sphere is as large and massive as the Universe. In this case, the internal metric of a liquid sphere (3.57) will be the metric inside the collapsar. This metric is stationary. It is written for a regular observer who is watching the collapsar from outside. It is easy to re-write the metric for an internal observer by the same substitution of the coordinates as for the Schwarzschild metric of a mass-point field. The resulting metric will be non-stationary. Thus, the space inside a collapsar consisting of incompressible liquid can expand or compress, being observed from inside it.

This tricky situation is due to the fact that we consider a very specific case: two different space-time regions, which are located outside and inside a collapser respectively, and are separated by a singular surface. If considering a regular sphere of incompressible liquid, whose surface's radius differs from the radius of collapse for this mass, this situation would be impossible.

Since $g_{00} < 0$ under the surface of collapse, relativistic masses $\frac{P_0}{\sqrt{g_{00}}}$ take imaginary numerical values. This result can be easily obtained with the deduction analogous to as formula (5.4) was obtained, but with replacement of $g_{00} = 0$ by $g_{00} < 0$. This is from the viewpoint of an external observer, located outside the collapsar. In the observer is located inside a collapsar, under the surface of collapse, in his reference frame all objects inside the collapsar will be observed as bearing real relativistic masses, while all objects outside the surface of collapse will be imaginary.

§6. Physical and geometric factors acting inside a sphere of incompressible liquid. Let us find the physical and geometric factors acting inside a sphere of incompressible liquid. The metric inside the sphere, expressed through the density of substance, is given by formula (3.55). According to the metric, the chr.inv.-vector of the gravitational inertial force (2.5) has only a single component which is non-zero

$$F^1 = -\frac{\varkappa\rho_0 c^2 r}{6}\frac{1}{3e^{\frac{\nu_a}{2}} - \sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}} = -\frac{GMr}{a^3}\frac{1}{3e^{\frac{\nu_a}{2}} - \sqrt{1-\frac{r_g r^2}{a^3}}}\,. \qquad (6.1)$$

We see here that this is a force of attraction, which is proportional to distance r. Its numerical value is zero in the center of the sphere. In the state of collapse, $F^1 \rightarrow \infty$. Since the numerical value of the Einstein constant $\varkappa$ is very small ($\varkappa = 18.6 \times 10^{-28}$ cm/g), it is obvious that this force is significant only by large distances r, for instance, in the case of "cosmological" objects such as the Universe.

Now, we are going to consider the singularities of pressure p concerning a sphere of incompressible liquid. Substituting B (3.49) and $e^{\frac{\nu}{2}}$ (3.54) into p (3.19), we obtain the formula

$$p = \rho_0 c^2\,\frac{\sqrt{1-\frac{\varkappa\rho_0 r^2}{3}} - e^{\frac{\nu_a}{2}}}{3e^{\frac{\nu_a}{2}} - \sqrt{1-\frac{\varkappa\rho_0 r^2}{3}}}\,. \qquad (6.2)$$

From here we see that, for the liquid model, $p \rightarrow \infty$ for the state of collapse. Also, we see that the space breaking occurs by the pressure $p = -\frac{\rho_0 c^2}{3}$. This is a *negative pressure of radiation*, because $p = \frac{\rho_0 c^2}{3}$ is the equation of state of radiation. It is obvious that this situation is possible only if the spherical surface case of the space breaking is located inside the liquid sphere (by $r<a$). Therefore, this particular case is important for our further understanding of the internal constitution of the cosmic objects which could be approximated by spheres of incompressible liquid.

Consider the space-time regions outside the singularities. Because $r \leqslant a$ means the space inside the liquid sphere, the numerator is positive outside the region of collapse always, except on the surface of the liquid sphere ($r=a$) where it equals zero.

So forth we consider the sign of this function in the region outside the collapse. The denominator is always positive in this region. Since

$$e^{\frac{\nu_a}{2}} = \sqrt{1-\frac{\varkappa\rho_0 a^2}{3}} = \sqrt{1-\frac{2GM}{c^2 a}}\ ,$$

the numerator is positive by $r \geqslant 0$, that is inside the sphere except the region inside the sphere of breaking ($r=r_{br}$, the numerator is strongly negative in this case). It follows from (4.13), if the sphere of incompressible liquid is not a collapsar, the sphere of the space breaking is located outside it ($r_{br}>a$). Consequently, $\rho=p=0$ in the layer.

Consider the pressure near the surface of the liquid sphere. The constant $\varkappa = 18.6\times10^{-28}$ cm/g is a very small value. Therefore, if ρ_0 is not very large, $\varkappa\rho_0$ is also very small. Supposing that

$$\sqrt{1-\frac{\varkappa\rho_0 r^2}{3}} \approx 1-\frac{\varkappa\rho_0 r^2}{6}\ ,$$

we obtain, after algebra, the approximate formula for p, which is

$$p \approx \frac{\varkappa\rho_0\left(a^2-r^2\right)}{2} = \frac{\rho_0 GM}{2a^2}\left(\frac{a^2-r^2}{a}\right), \tag{6.3}$$

where $\frac{GM}{a^2}=g$ is the free-fall acceleration.

Now we calculate the pressure of the liquid, with taking into account that the liquid has the density ρ_0, while the parameter $h=a-r$ is the distance from the surface of the sphere to the point of the measurement. Assuming that $h \ll r$, i.e. the measurement is processed in the upper layer of the sphere, near its surface, we obtain

$$a^2-r^2 = (a-r)\,(a+r) = h\,(2a+h) \approx 2ah\,.$$

Thus, we arrive at the regular formula for the pressure

$$p = \rho_0 g h\,. \tag{6.4}$$

Let us study the geometric properties of the three-dimensional space of a sphere of incompressible liquid. Calculating the components of the tensor H_{lkij} by the formula (2.25) for the metric (3.55), we obtain its non-zero components

$$H_{1212} = C_{1212} = -\,\frac{\varkappa\rho_0}{3}\,\frac{r^2}{1-\frac{\varkappa\rho_0 r^2}{3}}\,, \tag{6.5}$$

$$H_{1313} = C_{1313} = -\,\frac{\varkappa\rho_0}{3}\,\frac{r^2\sin^2\theta}{1-\frac{\varkappa\rho_0 r^2}{3}}\,, \tag{6.6}$$

$$H_{2323} = C_{2323} = -\,\frac{\varkappa\rho_0}{3}\,r^4\sin^2\theta\,. \tag{6.7}$$

We see, therefore, that the non-zero components of the observable space curvature tensor C_{iklj} satisfy the condition

$$C_{iklj} = -\frac{\varkappa\rho_0}{3}\left(h_{kl}h_{ij} - h_{il}h_{kj}\right), \tag{6.8}$$

where the constant $-\frac{\varkappa\rho_0}{3}$ is the observable three-dimensional curvature in the two-dimensional direction. This means that this is a constant negative curvature three-dimensional space. Calculating the observable scalar curvature $C = h^{ik}C_{ik}$, where non-zero components of C_{ik} are

$$C_{11} = -\frac{2\varkappa\rho_0}{3}\frac{1}{1-\frac{\varkappa\rho_0 r^2}{3}}\,, \tag{6.9}$$

$$C_{22} = \frac{C_{33}}{\sin^2\theta} = -\frac{2\varkappa\rho_0 r^2}{3}\,, \tag{6.10}$$

we obtain

$$C = -\,2\varkappa\rho_0 = const < 0\,. \tag{6.11}$$

Consequently, the components of the three-dimensional observable tensor of curvature C_{iklj} have the form

$$C_{iklj} = \frac{C}{6}\left(h_{kl}h_{ij} - h_{il}h_{kj}\right). \tag{6.12}$$

Thus the physically observable three-dimensional space has a constant negative curvature. The radius of the curvature $\Re$ in this case is imaginary. It is linked with C by the relation

$$C = -\,2\varkappa\rho_0 = \frac{1}{\Re^2}\,, \tag{6.13}$$

thus

$$\Re = \frac{i}{2\varkappa\rho_0}\,. \tag{6.14}$$

Let us estimate the numerical value of $|\Re|$ for this liquid model of the Universe. Assuming the density of the Universe to be $\rho_0 = 10^{-31}$ g/cm^3, we obtain $|\Re| = 2\times 10^{27}$ cm. Thus, the numerical value of $\Re$ is comparable with the Hubble radius of the Universe $a = 2.3\times 10^{28}$ cm (4.18).

We see from (6.5) and (6.6) that the three-dimensional space has breakings in the direction of two surfaces (x^1, x^2) and (x^1, x^3). Both breakings are realized by the condition $r = r_{br} = \sqrt{\frac{3}{\varkappa\rho_0}} = a\sqrt{\frac{a}{r_g}}$. It was shown above that $g_{11} = -h_{11} \to \infty$ by this condition.

Now let us study the geometrical properties of the space-time described by the metric (3.55). First, we calculate the components of the Riemann-Christoffel tensor

$$R_{\alpha\beta\gamma\delta} = \frac{1}{2}\left(\partial_{\beta\gamma}\, g_{\alpha\delta} + \partial_{\alpha\delta}\, g_{\beta\gamma} - \partial_{\alpha\gamma}\, g_{\beta\delta} - \partial_{\beta\delta}\, g_{\alpha\gamma}\right) + \\ + g^{\sigma\tau}\left(\Gamma_{\alpha\delta,\sigma}\Gamma_{\beta\gamma,\tau} - \Gamma_{\beta\delta,\sigma}\Gamma_{\alpha\gamma,\tau}\right), \qquad (6.15)$$

where $\Gamma_{\alpha\beta,\delta}$ are the four-dimensional Christoffel symbols of the 1st kind. We have, for the metric (3.55), $g_{ik} = -h_{ik}$ and $\Gamma_{ik,j} = -\Delta_{ik,j}$. Thus, calculating the other components of $\Gamma_{\alpha\beta,\delta}$, which are non-zero,

$$\Gamma_{01,0} = -\Gamma_{00,1} = \frac{\varkappa\rho_0 r}{12}\,\frac{3e^{\frac{\nu_a}{2}} - \sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}}{\sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}}\,, \qquad (6.16)$$

$$\Gamma_{11,1} = -\frac{\varkappa\rho_0 r}{3}\,\frac{1}{\left(1 - \frac{\varkappa\rho_0 r^2}{3}\right)^2}\,, \qquad (6.17)$$

and substituting these into (6.5), we obtain

$$R_{0101} = -\frac{\varkappa\rho_0}{12}\,\frac{3e^{\frac{\nu_a}{2}} - \sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}}{\sqrt{1 - \frac{\varkappa\rho_0 r^2}{3}}}\,, \qquad (6.18)$$

$$R_{1212} = -\,C_{1212}\,, \quad R_{1313} = -\,C_{1313}\,, \quad R_{2323} = -\,C_{2323}\,. \qquad (6.19)$$

We see from here that the four-dimensional space inside a sphere of incompressible liquid, described by the metric (3.55), is not a constant curvature space. This is because the component R_{0101}, determining the four-curvature in the radial-time (x^0, x^1)-direction, does not satisfy the condition

$$R_{\alpha\beta\gamma\delta} = Q\left(g_{\beta\gamma}g_{\alpha\delta} - g_{\beta\delta}g_{\alpha\gamma}\right), \qquad Q = const, \qquad (6.20)$$

which determines a four-dimensional constant curvature space.

Also, we see that $R_{0101} \to \infty$ by the space breaking, while $R_{0101} = 0$ in the state of collapse. It is seen from (6.19) that all the spatial (three-dimensional) components of the Riemann-Christoffel tensor are positive. The mixed (space-time) component R_{0101} (6.18) is negative, except in the case of collapse where it equals zero. Because the numerator of (6.18) is proportional to $\sqrt{g_{00}} = 1 - \frac{\mathrm{w}}{c^2}$, the component R_{0101} will be positive inside the collapsar (since $\sqrt{g_{00}} < 0$ therein). Thus the four-curvature in the space-time direction (x^0, x^1) changes its sign by the state of collapse. Therefore we arrive at the conclusion that the surface of collapse is a bridge connecting two spaces of the negative and the positive curvature.

§7. The internal constitution of the Solar System: the Sun and the planets as spheres of incompressible liquid. First, we are going to consider the Sun as a sphere of incompressible liquid. Schwarzschild [2] was the first person who considered the gravitational field of a sphere of incompressible liquid. He however limited this consideration by an additional condition that the space-time metric should not have singularities. In this study the metric (3.55) will be used. It allows singularities, in contrast to the limited case of Schwarzschild: 1) collapse of the space, and 2) the space breaking.

We calculate the radius of the space breaking by formula (4.13), where we substitute the Sun's density $\rho_0 = 1.41$ g/cm^3. We obtain

$$r_{br} = 3.4 \times 10^{13} \text{ cm} = 2.3 \text{ AU}, \tag{7.1}$$

where 1 AU $= 1.49 \times 10^{13}$ cm (Astronomical Unit) is the average distance between the Sun and the Earth. So, we have obtained that the spherical surface of the Sun's space breaking is located inside the Asteroid strip, very close to the orbit of the maximal concentration of substance in it (as is known, the Asteroid strip is hold from 2.1 to 4.3 AU from the Sun). Thus we conclude that:

> The space of the Sun (actually — its gravitational field), as that of a sphere of incompressible liquid, has a breaking. The space breaking is distantly located from the Sun's body, in the space of the Solar System, and meets the Asteroid strip near the maximal concentration of the asteroids.

In addition to it, we conclude:

> The Sun, approximated by a mass-point according to the Schwarzschild solution for a mass-point's field in emptiness, has a space breaking located inside the Sun's body. This space breaking coincides with the Schwarzschild sphere — the sphere of collapse.

Object	Mass, gram	Proper radius, cm	Hilbert radius, cm
Sun	1.98×10^{33}	6.95×10^{10}	2.9×10^{5}
Mercury	2.21×10^{26}	2.36×10^{8}	0.03
Venus	4.93×10^{27}	6.19×10^{8}	0.73
Earth	5.97×10^{27}	6.38×10^{8}	0.88
Mars	6.45×10^{26}	3.44×10^{8}	0.10
Jupiter	1.90×10^{30}	7.11×10^{9}	2.8×10^{2}
Saturn	5.68×10^{29}	6.00×10^{9}	84
Uranus	8.72×10^{28}	2.55×10^{9}	13
Neptune	1.03×10^{29}	2.74×10^{9}	15
Pluto	1.31×10^{25}	1.20×10^{8}	0.002

Table 1: The proper radius and the Hilbert radius of the Sun and the planets, calculated in the framework of the model where they are approximated by spheres of incompressible liquid.

What is the Schwarzschild sphere? It is an imaginary spherical surface of the Hilbert radius $r_g = \frac{2GM}{c^2}$, which is not a radius of a physical body in a general case (despite it can be such one in the case of a black hole — a physical body whose radius meets the Hilbert radius calculated for its mass). The numerical value of r_g is determined only by the mass of the body, and does not depend on its other properties. The physical meaning of the Hilbert radius in a general case is as follows: this is the boundary of the region in the gravitational field of a mass-point M, where real particles exist; particles in the boundary (the Hilbert radius) bear the singular properties as shown in §5. In the region wherein $r \leqslant r_g$, real particles cannot exist. The Hilbert radius r_g calculated for the Sun and the planets is given in Table 1.

Let us turn back to the Sun approximated by a sphere of incompressible liquid. The space-time metric is (3.55) in this case. Substituting into (4.23) the Sun's mass $M = 2\times10^{33}$ g, radius $a = 7\times10^{7}$ cm, and the Hilbert radius $r_g = 3\times10^{5}$ cm calculated for its mass, we obtain that the numerical value of the radial coordinate r_c by which the Sun's surface meets the surface of collapse of its mass is imaginary. Thus, we arrive at the conclusion that a sphere of incompressible liquid, whose parameters are the same as those of the Sun, cannot collapse. This conclusion is as that before, see Page 24.

One can ask: then what does the Hilbert radius r_g mean for the Sun, in this context? Here is the answer: r_g is the photometric distance in the radial direction, separating the "external" region inhabited with real particles and the "internal" region under the radius wherein all particles bear imaginary masses. Particles which inhabit the boundary surface (its radius is r_g) bear singular physical properties. Note that no one real (external) observer can register events inside the singularity.

Now we apply this research method to the planets of the Solar System. Thus, we approximate the planets by spheres of incompressible liquid. All results of the calculation are given in Table 2.

The numerical values of r_c, calculated for the planets according to the same formula (4.23) as that for the liquid model of the Sun, are imaginary. Therefore, the planets being approximated by spheres of incompressible liquid cannot collapse as well as the Sun.

According to Table 1, the Hilbert radius r_g calculated for the planets is much smaller than the sizes of their physical bodies, and is in the order of 1 cm. This means that, given any of the planets of the Solar System, the singulary surface separating our world and the imaginary mass particles world in its gravitational field draws the sphere of the radius about one centimetre around its centre of gravity.

Table 2 gives the numerical values of the radius of the space breaking, calculated for each of the planets through the average density of substance inside the planet according to the formula (4.13).

The results of the summarizing and substraction associated with the planets, according to Table 2, lead to the next conclusions:

1. The spheres of the singularity breaking of the spaces of Mercury, Venus, and the Earth are completely located inside the sphere of the singularity breaking of the Sun's space;
2. The spheres of the singularity breaking of the internal spaces of all planets intersect among themselves, when being in the state of a "parade of planets";
3. The spheres of the singularity breaking of the Earth's space and Mars' space reach the Asteroid strip;
4. The sphere of the singularity breaking of Mars' space intersects with the Asteroid strip near the orbit of Phaeton (the hypothetical planet which was orbiting the Sun, according to the Titius–Bode law, at $r = 2.8$ AU, and whose distraction in the ancient time gave birth to the Asteroid strip).
5. Jupiter's singularity breaking surface intersects the Asteroid strip near Phaeton's orbit, $r = 2.8$ AU, and meets Saturn's singularity breaking surface from the outer side;
6. The singularity breaking surface of Saturn's space is located between those of Jupiter and Uranus;

Object	Density, gram/cm^3	Orbit, AU	Radius of the space breaking*, AU	Location of the space breaking sphere
Sun	1.41	—	2.3	Asteroid strip
Mercury	4.10	0.39	1.3	Completely inside the Sun's space breaking
Venus	5.10	0.72	1.2	Completely inside the Sun's space breaking
Earth	5.52	1.00	1.1	Completely inside the Sun's space breaking
Mars	3.80	1.52	1.4	Meets the Sun's space breaking at the outer side
Asteroid strip	—	2.5†	—	—
Jupiter	1.38	5.20	2.3	Meets the Sun's space breaking at one side and Saturn's space breaking at the opposite side
Saturn	0.720	9.54	3.2	Between Jupiter's space breaking and Uranus' space breaking
Uranus	1.30	19.2	2.4	Between Saturn's space breaking and Neptune's space breaking
Neptune	1.20	30.1	2.4	On the lower boundary of the Kuiper belt
Pluto	2.0	39.5	1.9	Completely on the lower strip of the Kuiper belt
Kuiper belt	—	30–100	—	—

*The distance (radius) of the singulary breaking of the respective cosmic body's space, measured from the body (in Astronomical Units).
†The density of the Asteroid strip's substance has a maximum registered at 2.5 AU, while the strip itself continues from 2.1 to 4.3 AU.

Table 2: Singularity breakings of the local spaces of the Sun and the planets.

7. The singularity breaking surface of Uranus's space is located between those of Saturn and Neptune;
8. The singularity breaking surface of Neptune's space meets, from the outer side, the lower boundary of the Kuiper belt (the strip of the aphelia of the Solar System's comets);
9. The singularity breaking surface of Pluto is completely located inside the lower strip of the Kuiper belt.

Just two small notes in addition to these. The intersections of the space breakings of the planets, discussed here, take place for only that case where the planets thenselves are in the state of a "parade of planets". However the conclusions concerning the location of the space breaking spheres, for instance — that the space breaking spheres of the internal planets are located inside the sphere of the Sun's space breaking, while the space breaking spheres of the external planets are located outside it, — are true for any position of the planets.

What does the "space breaking" mean from the physical viewpoint? Has this breaking a real action on a physical body appeared in it, or is it only a mathematical fiction? As was obtained in §6, the space (space-time) of a sphere of incompressible liquid has a breaking of its four-curvature $R_{\alpha\beta\gamma\delta}$ by the condition $r = r_{br}$: the quantity R_{0101} (6.18), which is the four-curvature of the space in the radial-time direction 0101, has a breaking $R_{0101} \rightarrow \infty$ (the curvature becomes infinite) at the distance $r = r_{br}$ from the centre of gravity of the liquid sphere. (See top of Page 32.) Because the curvature determines the gravitational field filling the space, the aforementioned breaking means the breaking in the gravitational field of the liquid sphere at $r = r_{br}$. This is the physical meaning of the space breaking we studied here.

The fact that the space breaking of the Sun meets the Asteroid strip, near Phaeton's orbit, allows us to say: yes, the space breaking considered in this study has a really physical meaning. As probable the Sun's space breaking did not permit the Asteroids to be joined into a common physical body, Phaeton. Alternatively, if Phaeton was an already existing planet of the Solar System, the common action of the space breaking of the Sun and that of another massive cosmic body, appeared near the Solar System in the ancient ages (for example, another star passing near it), has led to the distraction of Phaeton's body.

Thus the internal constitution of the Solar System was formed by the structure of the Sun's space (space-time) filled with its gravitational field, and according to the laws of the General Theory of Relativity.

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