The Deflection of Light in the Dynamic Theory of Gravity

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Abstract

In a new theory gravity called the dynamic theory, which is derived from thermodymical principles in a five dimensional space, the deflection of a light signal is calculated and compared to that of general relativity. This is achieved by using the dynamic gravity line element which is the usual four dimesional space-time element of Newtonian gravity modified by a negative inverse radial exponetial term. The dynamic theory of gravity predicts this modification of the original Newtonian potential by this exponential term.

Key words: Dynamic theory of gravity, thermodynamical, delay, general relativity.

1 Introduction

There is a new theory called the Dynamic Theory of Gravity [DTG]. It is based on classical thermodynamics and requires that Einstein's postulate of the constancy of the speed of light holds. [1]. Given the validity of the postulate Einstein's theory of special relativity follows right away [2]. The dynamic theory of gravity (DTG) through Weyl's quantum principle also leads to a non-singular electrostatic potential of the form:

$$V(r) = -\frac{K}{r} e^{-\frac{\lambda}{r}} . \tag{1}$$

where K and λ are constants defined by the theory. The DTG describes physical phenomena in terms of five dimensions: space, time and mass. [3] By conservation of the fifth dimension we obtain equations which are identical to Einstein's field equations and desribe the gravitational field in the theory of general relativity which are given below:

$$K_o T^{\alpha\beta} = G^{\alpha\beta} = R^{\alpha\beta} - \frac{g^{\alpha\beta}}{2} R . \tag{2}$$

Now $T^{\alpha\beta}$ is the surface energy-momentum tensor which may be found within the space tensor and is given by:

$$T^{\alpha\beta} = T_{sp}^{\alpha\beta} - \frac{1}{c^2}\left[F_4^{\alpha}F^{4\beta} - \frac{h^{\alpha\beta}}{2}F^{4\nu}F_{4\nu}\right] \tag{3}$$

and $T_{sp}^{\mu\nu}$ is the space energy-momentum tensor for matter under the influence of the gauge fields is also given by:[4]

$$T_{sp}^{ij} = \gamma u^i u^j + \frac{1}{c^2}\left[F_k^i F^{kj} + \frac{1}{4}a^{ij}F^{k\ell}F_{k\ell}\right] \tag{4}$$

which further can be written in terms of the surface metric as follows:[4]

$$T_{sp}^{\alpha\beta} = \gamma u^{\alpha}u^{\beta} + \frac{1}{c^2}\left[F_k^{\alpha}F^{k\beta} + F_4^{\alpha}F^{4\beta} + \frac{1}{4}\left(g^{\alpha\beta} - h^{\alpha\beta}\right)\left(F^{\mu\nu}F_{\mu\nu} + F^{4\nu}F_{4\nu}\right)\right] \tag{5}$$

since:

$$u^4 = \frac{dy^4}{dt} \Rightarrow \frac{\partial y^4}{\partial t} + \bar{\nabla}\bullet\left(y^4\,\bar{u}\right) = 0 \tag{6}$$

is the statement required by the conservation of the fifth dimension, and the surface indices ν, α,β=0,1,2,3 and space index i, j, k, l = 0,1,2,3,4, and $g_{\alpha\beta} = a_{ij}y_{\alpha}^{i}y_{\alpha}^{j} = a_{\alpha\beta} + h_{\alpha\beta} = a_{\alpha\beta} + 2a_{\alpha 4}y_{\beta}^{4} + a_{44}y_{\alpha}^{4}y_{\beta}^{4}$ and where the surface field tensor will be given by:

$$F_{\alpha\beta} = F_{ij}y_{\alpha}^{i}y_{\beta}^{j} \text{ and } y_{\alpha}^{i} = \frac{\partial y^i}{\partial x^{\alpha}} = \delta_{\alpha}^{i} \text{ for } i = 0,1,2,3 \text{ and } y_{\alpha}^{4} = \frac{\partial y^4}{\partial x^{\alpha}}. \tag{7}$$

$$F_{ij} = \begin{bmatrix} o & E_1 & E_2 & E_3 & V_o \\ -E_1 & o & B_3 & -B_2 & V_1 \\ -E_2 & -B_3 & o & B_1 & V_2 \\ -E_3 & B_2 & -B_1 & o & V_3 \\ -V_o & -V_1 & -V_2 & -V_3 & o \end{bmatrix} \tag{8}$$

As it was shown by Weyl the gauge fields may be derived from these gauge potentials and the componets of the 5-dimensional field tensor F_{ij} given by the 5×5 matrix given by (8). [4]
Now the determination of the fifth dimension may be seen, for the only physically real property that could give Einstein's equations is the gravitating mass or it's equivalent, mass [5]. Finally the dynamic theory of gravity further argues that the gravitational field is a gauge field linked to to the electromagnetic field in a 5-dimensional manifold of space-time and mass, but, when conservation of mass is imposed, it may be described by the geometry of the 4-

dimensional hypersyrface of space-time, embedded into the 5-dimensional manifold by the conservation of mass. The 5 dimensional field tensor can have only one nonzero component V_0 which must be related to the graviational field and the fifth gauge potential must be related to the gravitational potential. In the dynamic theory, one obtains the non singular gravitational potential by differentiating the electrostatic potential with respect to the mass. This is required by the inductive coupling introduced by the unity scale factor. Therefore, the gravitaional potential retains the same non-singular form as the electrostatic potential and is given below:

$$V(r) = -\frac{GM}{r} e^{-\frac{\lambda}{r}} \tag{9}$$

The theory makes its predictions for redshifts by working in the five dimensional geometry of space, time, and mass, and determines the unit of action in the atomic states in a way that can be calculated with the help of quantum Poisson brackets when covariant differentiation is used: [6]

$$\left[x^{\mu}, p^{\nu}\right]\Phi = i\hbar g^{\nu q}\left\{\delta_{\mu q} + \left|\Gamma^{\mu}_{s,q}\right| x^{s}\right\}\Phi \tag{10}$$

In (8) the vector curvature is contained in the Christoffel symbols of the second kind and the gauge function Φ is a multiplicative factor in the metric tensor $g^{\nu q}$, where the indices take the values $\nu, q = 0,1,2,3,4$. In the commutator, x^{μ} and p^{ν} are the space and momentum variables respectively, and finally $\delta_{\mu q}$ is the Cronecker delta. In DTG the momentum ascribed, as a variable canonically conjugated to the mass is the rate at which mass may be converted into energy. The canonical momentum is defined as follows:

$$p_4 = mv_4 \tag{11}$$

where the velocity in the fifth dimension is given by:

$$v_4 = \frac{\dot{\gamma}}{\alpha_o} \tag{12}$$

and gamma dot is a time derivative and gamma has units of mass density (kg/m^3) and α_o is a density gradient with units of kg/m^4. In the absence of curvature (8) becomes:

$$\left[x^{\mu}, p^{\nu}\right]\Phi = i\hbar\delta^{\nu q}\Phi \quad . \tag{13}$$

2 The line element of the dynamic theory of gravity

In the DTG the metric is not different than that of general relativity except an exponential term with an 1/r dependence, and λ is a constant. Therefore we can write for the line element:

$$ds^2 = c^2\left(1-\frac{2GM}{c^2 r}e^{-\lambda/r}\right)dt^2 - \left(1-\frac{2GM}{c^2 r}e^{-\lambda/r}\right)^{-1}dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right).$$

(14)

3. The diagram of the deflection of light

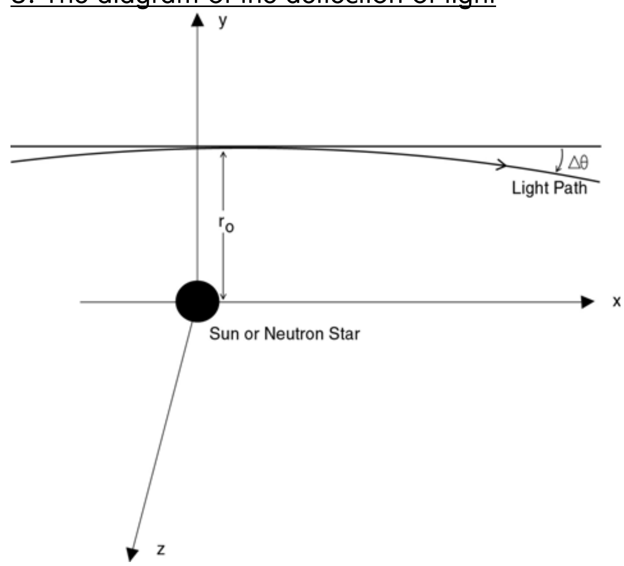

Fig 1: The trajectory of the light ray in the vicinity of a massive body. The light ray path is nearly a straight line $y = y_o$, $z = 0$ and r_o = is the distance of the closest approach to the center of the star.

4 Deflection of light analysis

First of all let us write the line element in (14) in the following form:

$$ds^2 = c^2\left(1-\frac{2GM}{c^2 r}e^{-\lambda/r}\right)dt^2 - \left(1+\frac{2GM}{c^2 r}e^{-r/\lambda}\right)dr^2 - r^2\left(d\theta^2 + \sin^2\theta d\phi^2\right)$$

(15)

and taking into account the transformations between rectangular and polar coordinates namely $r = (x^2+y^2)^{1/2}$ and $\cos\theta = y/(x^2+y^2)^{1/2}$ from which it follows that $(dr/dx)^2 = x^2/r^2$ and $(d\theta/dx)^2 = y^2/r^4$. Next rewrite (15) as follows:

$$ds^2 = c^2\left(1-\frac{2GM}{c^2 r}\right)dt^2 - \left(1+\frac{2GM}{c^2 r}\right)\left(\frac{dr}{dx}\right)^2 dx^2 - r^2\left(\frac{d\theta}{dx}\right)^2 dx^2 \qquad (16)$$

substituting their equal in (16) we finally obtain:

$$ds^2 = c^2\left(1-\frac{2GM}{c^2 r}e^{\lambda/r}\right)dt^2 - \left(1+\frac{2GMx^2}{c^2 r^3}e^{-r/\lambda}\right)dx^2 . \qquad (17)$$

Next making use that the coordinate velocity of light in the x direction which is given by:

$$c' = \left(\frac{dx}{dt}\right)_{ds^2=dy=dz=0} \tag{18}$$

to first order in M we obtain that:

$$c' = \left(\frac{dx}{dt}\right)_{ds^2=dy=dz=0} = c\left(1-\frac{GM}{c^2 r}e^{-\lambda/r}\right)\left(1-\frac{GMx^2}{r^3c^2}e^{-\lambda/r}\right)$$

(19)

$$c' = c\left[1-\frac{GM}{c^2 r}e^{-\lambda/r}\left(1+\frac{x^2}{r^2}\right)\right].$$

Sinc $\lambda < r$ we can expanding the exponential term to first order. Then the angular deflection rate is given by: [7]

$$\frac{d\theta}{dx}=\frac{1}{c'}\left(\frac{dc'}{dy}\right)=-\frac{4GMx^2y}{c^2\left(x^2+y^2\right)^3}\left[1-\frac{GM\left(1+\frac{x^2}{x^2+y^2}\right)\left(1-\frac{\lambda}{\sqrt{x^2+y^2}}\right)}{c^2}\right]^{-1}$$

$$+\frac{3GMx^2y}{c^2\left(x^2+y^2\right)^{5/2}}\left[1-\frac{GM\left(1+\frac{x^2}{x^2+y^2}\right)\left(1-\frac{\lambda}{\sqrt{x^2+y^2}}\right)}{c^2}\right]^{-1}$$

$$+\frac{2GM\lambda y}{c^2\left(x^2+y^2\right)^2}\left[1-\frac{GM\left(1+\frac{x^2}{x^2+y^2}\right)\left(1-\frac{\lambda}{\sqrt{x^2+y^2}}\right)}{c^2}\right]^{-1}$$

$$+\frac{GMy}{c^2\left(x^2+y^2\right)^{3/2}}\left[1-\frac{GM\left(1+\frac{x^2}{x^2+y^2}\right)\left(1-\frac{\lambda}{\sqrt{x^2+y^2}}\right)}{c^2}\right]^{-1} \qquad (20)$$

and therefore the total deflection can be obtained integrating from -∞ to +∞ and taking into account that y does not differ very much from r_o in the region where significant deflection occurs. Finally have omitting the second terms in square brackets from the denominators for being much smaller than one the expression can be simplified to:

$$\theta \approx \frac{2GMr_o}{c^2}\int_0^{\infty}\left[\frac{1}{r^3}+\frac{2\lambda}{r^4}+\frac{3x^2}{r^5}-\frac{4\lambda x^2}{r^6}\right]dx\,. \qquad (21)$$

Using the substitution $x = r_o \tan\theta$ and $dx = r_o \sec^2\theta\ d\theta$, $r = r/\cos\theta$ (21) becomes and integrates to:

$$\theta_{Dyn} \approx \frac{2GMr_o}{c^2}\int_0^{\pi/2}\left[\frac{\cos\theta}{r^2{}_o} + \frac{2\lambda\cos^2\theta}{r^3{}_o} + \frac{3\cos\theta\sin^2\theta}{r^2{}_o} - \frac{4\lambda\cos^2\theta\sin^2\theta}{r^3{}_o}\right]d\theta \quad (22)$$

and therefore the final expression for the light deflection in the dynamic theory of gravity become:

$$\theta_{Dyn} \approx \frac{4GM}{c^2 r_o} + \frac{\pi GM\lambda}{2c^2 r^2{}_o} = \frac{4GM}{c^2 r_o}\left[1 + \frac{\pi\lambda}{8r_o}\right] = \theta_{Rel}\left[1 + \frac{\pi\lambda}{8r_o}\right] \quad (23)$$

Since $\lambda = G\, M / c^2$ is a constat of the dynamic theory of gravity equation (23) can be written:

$$\theta_{Dyn} \approx \frac{4GM}{c^2 r_o} + \frac{\pi G^2 M^2}{2c^4 r_o{}^2} = \frac{4GM}{c^2 r_o}\left[1 + \frac{\pi GM}{8c^2 r_o}\right] = \theta_{Rel}\left[1 + \frac{\pi GM}{8c^2 r_o}\right] \quad (24)$$

5 Numerical calculations

To numerically calculate the deflection of the light in the dynamical theory of gravity let us first assume an 1 M_{solar}. Using (23) we have:

$$\theta_{Dyn} = \theta_{Rel} + \theta_{correction} = 1.750'' + 1.451\times 10^{-6''} = 4.861\times 10^{-4^{\circ}} + 4.030\times 10^{-10^{\circ}} \quad (25)$$

For a neutron star of mass $M_{min} = 0.0925\ M_{solar}$ and $R_{star} = r_o$ 164 Km [8] we have that:

$$\theta_{Dyn} = \theta_{Rel} + \theta_{correction} = 689.76'' + 0.226'' = 0.192^{\circ} + 6.286\times 10^{-5^{\circ}} \quad (26)$$

Finally for a neutron star with $M = 1.46\ M_{solar}$ and $R_{star} = r_o = 3.1\times 10^8$ cm $= 0.004\ R_{solar}$ [9] we obtain:

$$\theta_{Dyn} = \theta_{Rel} + \theta_{correction} = 637.66'' + 0.193'' = 0.177^{\circ} + 5.373\times 10^{-5^{\circ}} \quad (27)$$

6. Conclusions

In the dynamic theory of gravity an analytical expression for the deflection of light has been derived and evaluated numerically. First as the main body a star having the mass and the radius of the sun was used in calculating the light deflection and next two neutron stars of different masses and radii were also used Since neutron stars are stars of great gravity it should be only logical to expect that light would be more deflected in their vicinity by analogy of general relativity to dynamic gravity as well.

This is a standard calculation that somebody can performed as a test of a new gravitational theory, namely to investigate the behaviour of light close to a massive spherical body where gravity can be described by the appropriate potential, in this case the non-singular dynamic potential, and compare this with the established theory of general relativity.

We can now see that the final expression obtained for the deflection of light to a first order approximation for the exponential term which characterizes the dynamic potential is composed by the original relativistic term plus two more terms, which can be thought as corrections to the original general relativistic expression. For the sun and the two neutron stars it seems that the possible correction due to the dynamic gravity seems to be within our instrumental reach, in case that somebody wants to really investigate for this kind of gravity.

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