Mon. Not. R. Astron. Soc. **000**, 1–5 (2009) Printed 15 February 2010 (MN LaTeX style file v2.2)

Are Flyby Anomalies and the Pioneer Effect an ASTG Phenomenon?

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Received: / Accepted:

ABSTRACT

This reading expounds with expediency on the recently proposed Azimuthally Symmetric Theory *of* Gravitation (ASTG) set-up earlier. There-in, the ASTG was set up and it was demonstrated that it is capable (amongst others solar anomalies) of explaining the precession of the perihelion of solar planets. In the second reading, it was shown that this theory is capable – *in principle*; of explaining outflows as a repulsive gravitational phenomenon. In the present, we show that the ASTG is capable of explaining the puzzling observations of flyby anomalies *vis*, anomalous asymptotic speed increases at the perigee and the speed changes occurring to the osculating hyperbolic speed excess. It is shown that these flyby anomalies are a natural occurrence in the ASTG. We derive a modified formula of the empirical formula proposed by Anderson *et al.*, which up to now has no foundational basis except that experience suggest it. It is seen that the ASTG can *in principle* explain the Pioneer Anomaly. To say for sure the ASTG is the reason for the Pioneer Anomaly, there is need to obtain the complete set of the Pioneer ephemerides.

Key words: gravitation – astrometry – celestial mechanics – Solar system: general

1 INTRODUCTION

An Earth flyby anomaly is not just an unexpected increase in the outgoing osculating hyperbolic excess speed but also an asymptotic speed increase at the perigee during Earth flybys of spacecraft. In general a flyby anomaly is an unexpected increase in the outgoing osculating hyperbolic excess speed and as-well an asymptotic speed increase at the perigee during a flyby of a spacecraft past a planet for the purposes of gravity assist maneuver. This anomaly has been observed for spacecrafts sent to probe the secrets of deep space as they fly past the Earth as a shift in the ranging and Doppler data. For these spacecrafts, along their hyperbolic trajectory on their incoming path as they approach the Earth with a speed v_i and when they exit at a speed of v_o; spherically symmetric Newtonian and Einsteinian gravitation dictates that $v_i = v_o$. Observations give a completely different and surprising result that has baffled European Space Agency (ESA) and the National Aeronautic Space Administration (NASA) scientists for quite sometime now, *i.e.* they [observations] reveal that $v_i < v_o$, hence the incoming kinetic energy of the spacecraft is less than the outgoing kinetic energy of that spacecraft.

Also, as the spacecraft reach their perigee, *i.e.*, their distance of closest approach to planet Earth, it has been observed that these spacecrafts experience a hitherto unknown, mysterious and unexplained asymptotic speed increase. All this has come from the telemetry received from the spacecrafts. When the shift in the Doppler and the ranging data is interpreted, flyby anomalies are a very small *albeit* very significant unaccounted speed increase of up to $13.46\,\mathrm{mm/s}$ at perigee. The first flyby anomaly was noticed during a very careful inspection of Doppler data shortly after the Earth flyby of the Galileo spacecraft on 8 December 1990. While the Doppler residuals (observed minus computed data) were expected to remain flat, the analysis revealed an unexpected 66 mHz shift, which corresponds to a speed increase of $3.92\,\mathrm{mm/s}$ at perigee. An investigation of this effect at the Jet Propulsion Laboratory (JPL), the Goddard Space Flight Center (GSFC) and the University *of* Texas has not yielded a satisfactory explanation. It should be noted that no anomaly was detected after the second Earth flyby of the Galileo spacecraft in December 1992, because any possible velocity increase is believed to have been masked by atmospheric drag of the lower altitude of $303\,\mathrm{km}$.

On 23 January 1998 the Near Earth Asteroid Rendezvous (NEAR) spacecraft experienced an anomalous speed increase of $13.46\,\mathrm{mm\,s^{-1}}$ after its Earth encounter. Cassini-Huygens gained about $0.11\,\mathrm{mm\,s^{-1}}$ in August 1999 and Rosetta $1.82\,\mathrm{mm\,s^{-1}}$ after its Earth flyby in March 2005. An analysis of the MESSENGER spacecraft (studying Mercury) did not reveal any significant unexpected velocity increase. The last Earth flyby was that by Rosetta in 2009. As she (Rosetta) bed farewell to humanity on her third and final Earth encounter at $08 : 45$ in the European morning of the 13^{th} of November 2009, on her trajectory to rendezvous with Comet 67P/Churyumov-Gerasimenko on $2014-May-22$, the ESA spacecraft approached the Earth before entering the depths of space in which event she left her highly expectant "onlookers" disappointed.

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While her "onlookers" watched her in the operation center, she approached and passed closest to Earth over the south of the island of Java, in Indonesia, at a speed of $13.34\,\mathrm{km/s}$ relative to the Earth, and at a height of 2481 km above its surface. In the operation center *i.e.*, the European Space Operation Center from ESA in Darmstadt (Germany), nothing special happened at that key moment. No applauses nor hugs from the pregnant engineers, *i.e.*, pregnant with expectations because everything had been planned to the minute and the millimeter weeks in advance and Rosseta did not yield any significant flyby anomaly as highly expected!

Researchers Anderson *et al.* (2008) had earlier deduced an empirical relationship from which they predicted a flyby anomaly of up to about $1\,\mathrm{mm/s}$ for the $13-Nov-2009$ Rosseta Earth encounter. This did not happen. What was measured is something to the tune of 0.004 ± 0.044 mm/s which for all practical purposes is a null result. The empirical relationship that Anderson *et al.* (2008) found is:

$$\frac{\Delta v}{v}=\kappa_A\left(\cos\delta_i-\cos\delta_o\right), \quad (1)$$

where $\kappa_A = 2\mathcal{R}_\oplus\omega_\oplus/c = 3.10\times 10^{-6}$ where $\omega_\oplus = 7.29\times 10^{-5}$ rad/s (see *e.g.* Stacey 1992, in Anderson *et al.* 2008) is the angular frequency of the Earth, $\mathcal{R}_\oplus = 6.40\times 10^6$ m (see *e.g.* Stacey 1992, in Anderson *et al.* 2008) is the radius of the Earth, and δ_i and δ_o are the incoming and outgoing osculating asymptotic velocity vectors. The Anderson formula (1) has up to now no substantial physical basics in that an acceptable/accepted physical theory is yet to furnish its very foundations.

The Anderson *et al.* (2008) relationship came about after realizing that the MESSENGER spacecraft had both approached and departed the Earth symmetrically about the equator (*i.e.* it approached at latitude 31 degrees north and; departed at latitude 32 degrees south). This was taken as a strong suggestion that the anomaly might be related to the Earth's rotation and this incoming and outgoing velocity vectors. As already shown above, this lead Anderson *et al.* (2008) to successfully seek an empirical relationship involving the incoming and outgoing declination angles of the orbit of the spacecrafts.

This empirical relationship of Anderson *et al.* (2008), as already said, suffers from the setback that it has no physical explanation. This reading seeks (and hopes) not only to give the Anderson *et al.* (2008) empirical relationship a foundational basics but to give a physical explanation of these seeming puzzling observations. It shall be demonstrated that flyby anomalies emerge naturally in the Azimuthally Symmetric Theory *of* Gravitation (ASTG) (Nyambuya 2010a).

It is known not whether this phenomenon of flyby anomalies may be related to the Pioneer Anomaly. Bona-fide; there is a significant number of researchers who (strongly) feel and suspect that these two phenomenon may very well be related. We shall deduce here-in that the component of the gravitational force responsible for the flyby anomalies produces both a radially repulsive and attractive component of the gravitational force. Whether this force is attractive or repulsive depends on the side of the spin equator. If this is to explain the Pioneer effect, we know that the Pioneer Anomaly is a radial attractive force. This means the Pioneers must be on side of the spin equator where this force is attractive. As shall be seen, that the force responsible for the flyby anomalies does produce a radially repulsive force does not entail one can explain the Pioneer Effect. One will need the complete set the Pioneer ephemerides to make this conclusion.

2 REVISITING THE ASTG

The ASTG as formulated in Nyambuya (2010a) is unable to explain the asymptotic change in the speed of the spacecraft that occurs at the perigee and also it will fail to explain the resultant change in the speed of the spacecraft as it move away to infinity. However, with a closer look, we realize that for a different set of boundary conditions determining the A_ℓ's an B_ℓ's in Nyambuya (2010a), one will be able to add an extra term in to the gravitational potential and this new addition does the explanation of the flyby anomalies.

In the formulation of the ASTG in Nyambuya (2010a), the Poisson equation for empty space: $\nabla^2\Phi=0$, was solved by means of separation of variables, *i.e.* by setting: $\Phi(r,\theta)=\Phi(r)\Phi(\theta)$ and then inserting this into the Poisson equation where-after some basic algebraic operations one would naturally arrive at:

$$\frac{1}{\Phi(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi(r)}{\partial r}\right)+\frac{1}{\Phi(\theta)}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi(\theta)}{\partial\theta}\right)=0. \quad (2)$$

The radial and the angular portions of this equation must equal some constant since they are independent of each other and following tradition, one must set:

$$\frac{1}{\Phi(r)}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Phi(r)}{\partial r}\right)=\ell(\ell+1), \quad (3)$$

and the solution to this is: $\Phi_\ell(r) = A_\ell r^\ell + B_\ell r^{-(\ell+1)}$, where A_ℓ and B_ℓ are constants and $\ell = 0,1,2,3,...,\infty$. At his point in the reading Nyambuya (2010a), the boundary conditions; ($\Phi_\ell(r=\infty)=0$, then $A_\ell=0$ for all ℓ) where set and their justification is found therein. The revision that we make herein is to shift the boundary conditions to: $d\Phi_\ell(r)/dr$ must be measurable at $r=\infty$. By measurable, we mean the value of $d\Phi_\ell(r)/dr$ at $r=\infty$ must be finite. The quantity $d\Phi_\ell(r)/dr$ is actually a measure of the force and requiring that it be finite at infinity means the gravitational force at infinity must be equal to zero or some finite number at least.

The new boundary conditions mean that $A_\ell=0$ for all $\ell>2$ and for $\ell=0,1$, we will not have $A_{0,1}=0$. However, whether A_0 zero or a constant it does not matter at all as this quantity will no appear in the equations of motion. For this reason we shall forget it by setting $A_0=0$ and leaving $A_1\neq 0$. Lets us set $A_1=-a_*$ where this is a space independent parameter/constant. This parameter/constant may well vary with time. Certainly we have no business in the present hour to investigate possible time variation of this. Also, this parameter/constant may very well be specific to the body in question, in which case it is not universal constant.

Now, at this point, if one where to go through the same steps as in the reading Nyambuya (2010a), then, it should be difficult to deduce that the resultant gravitational potential will be:

$$\Phi(r,\theta)=-\sum_{\ell=0}^{\infty}\left[\lambda_\ell c^2\left(\frac{G\mathcal{M}}{rc^2}\right)^{\ell+1}P_l(\cos\theta)\right]-a_*r\cos\theta. \quad (4)$$

From this potential, we shall show that flyby anomalies can be explained.

3 SOLUTION FROM THE ASTG

As argued in Nyambuya (2010*b*), the ASTG must be taken only up to its second order approximation because third and higher order terms are practically equal to zero. With the new addition of $a_* r\cos\theta$, the second order approximation of the gravitational potential will be given by:

$$\Phi(r,\theta) = -\frac{G\mathcal{M}}{r}\left[1+\frac{\lambda_1 G\mathcal{M}\cos\theta}{c^2 r}+\lambda_2\left(\frac{G\mathcal{M}}{rc^2}\right)^2\frac{3\cos^2\theta-1}{2}\right] - a_* r\cos\theta, \tag{5}$$

where the symbols have the same meaning as in Nyambuya (2010*a*). We shall take $\lambda_1^{\oplus}=10$ and justification for this shall come in latter during this reading. With this, it is seen that the terms involving the λ's are for the Earth so small we can neglect them. One can check that the term, $\lambda_1 G\mathcal{M}\cos\theta/c^2 r$: $(0\leqslant \lambda_1 G\mathcal{M}\cos\theta/c^2 r\leqslant 9.33\times10^{-12})$, which for practical purposes is small enough to be neglected. If this term is this small, clearly that involving λ_2 is even much smaller. Clearly. we can neglect these terms without harm. Now, having dropped these terms, the emergent azimuthally symmetric gravitational field intensity is: $\vec{\mathbf{a}}(r,\theta) = \mathbf{a}_r(r,\theta)\hat{\mathbf{r}} + \mathbf{a}_\theta(r,\theta)\hat{\boldsymbol{\theta}}$, where:

$$\mathbf{a}_r(r,\theta) = -\frac{G\mathcal{M}}{r^2}+a_*\cos\theta, \tag{6}$$

$$\mathbf{a}_\theta(r,\theta) = -a_*\sin\theta. \tag{7}$$

Now, applying the above formulae to the trajectory of the spacecrafts, we know that for the incoming orbit ($r=r_i$, $\theta=\theta_i=180^\circ+\delta_i$), thus we will have: $\mathbf{a}_r(r_i,\theta_i) = -G\mathcal{M}/r_i^2 - a_*\cos\delta_i$ and $\mathbf{a}_\theta(r_i,\theta_i) = a_*\sin\delta_i$, and for the outgoing orbit ($r=r_o$, $\theta=\theta_o=270^\circ+\delta_o$) and this leads to: $\mathbf{a}_r(r_o,\theta_o) = -G\mathcal{M}/r_o^2 + a_*\sin\delta_o$ and $\mathbf{a}_\theta(r_o,\theta_o) = -a_*\cos\delta_o$. From these equations, one sees that there will be a change in the asymptotic gravitational acceleration at infinity (a_∞) both in the radial and azimuthal directions and these are given by:

$$\left(\frac{\Delta a_\infty}{a_\infty^i}\right)_r = -\left(1+\frac{\cos\delta_o}{\sin\delta_i}\right) \text{ and } \left(\frac{\Delta a_\infty}{a_\infty^i}\right)_\theta = 1+\frac{\cos\delta_o}{\sin\delta_i}. \tag{8}$$

From this we see that the forces acting on the spacecrafts at infinity pre-and-post perigee are not the same. This points to the fact that the kinetic energies will exhibit the same behavior. In order to deduce this, we shall have to look at the equation for the orbit.

3.1 Anomalous Speed Changes of Spacecraft at Infinity

In the circumstances as in the present where an additional radial force per unit mass $a_*\cos\theta$ is present, the Newtonian equation of motion describing orbits around a massive body as the Earth (whose mass is $\mathcal{M}_\oplus$) is given by:

$$\frac{d^2u}{d\varphi^2}+u+\frac{G\mathcal{M}_\oplus}{J^2} = \left(\frac{a_*\cos\theta}{J^2}\right)u^{-2}, \tag{9}$$

where $J=\sqrt{G\mathcal{M}_\odot l}$ is the specific angular momentum, *i.e.*, it is the angular momentum per unit mass of a test body orbiting the massive body, and $u=1/r$, where r is the radial distance from the centre of the massive body and φ is the azimuthal angle. For a nearly Newtonian orbit $u=(1+\epsilon_N\cos\varphi)/l$ where ϵ_N is the Newtonian eccentricity of the orbit. To first order approximation: $u^{-2}\simeq(1-2\epsilon\cos\varphi+\ldots+\ldots)/l^2$. As argued in Nyambuya (2010*a*), this kind of approximation holds good for nearly Newtonian orbits as those of the spacecrafts making their Earth swing-bys. Effective this into (9) reduces this equation to:

$$\frac{d^2u}{d\varphi^2}+u+\frac{1}{l} = \left(\frac{a_*\cos\theta}{G\mathcal{M}_\oplus/l}\right) - \left(\frac{2a_*\epsilon_N\cos\theta}{G\mathcal{M}_\oplus/l}\right)\cos\varphi, \tag{10}$$

The general solution to this equation is:

$$\frac{l}{r} = 1+\left(\frac{a_*\cos\theta}{G\mathcal{M}_\oplus/l^2}\right)+\epsilon_N\left(1-\frac{a_*\cos\theta}{G\mathcal{M}_\oplus/l^2}\right)\cos\varphi. \tag{11}$$

Dividing by the term $[1-a_*\cos\theta/(G\mathcal{M}_\odot/l^2)]$, the above equation is to first order approximation given by:

$$\left(\frac{l}{r}\right)\left(1+\frac{a_*\cos\theta}{G\mathcal{M}_\oplus/l^2}\right) = 1+\left(\frac{2a_*\cos\theta}{G\mathcal{M}_\oplus/l^2}\right)+\epsilon_N\cos\varphi. \tag{12}$$

The eccentricity of an Earth Newtonian hyperbolic orbit is given by:

$$\epsilon_N = \left(\frac{v_\infty^2}{G\mathcal{M}_\oplus/\mathcal{R}_{min}}\right), \tag{13}$$

where $\mathcal{R}_{min}$ is the distance of closest approach. Now for the pre-perigee encounter, when $r=\infty$, $\varphi=180^\circ-\Psi/2$ and $\theta=90-\delta_i$, this means:

$$0 = 1-\left(\frac{2a_*\sin\delta_i}{G\mathcal{M}_\oplus/l^2}\right)-\left(\frac{v_{i,\infty}^2}{G\mathcal{M}_\oplus/\mathcal{R}_{min}}\right)\cos\left(\frac{\Psi}{2}\right). \tag{14}$$

Likewise, for the post-perigee encounter, when $r=\infty$, $\varphi=-(180^\circ-\Psi/2)$ and $\theta=90-\delta_i$, this means:

$$0 = 1-\left(\frac{2a_*\sin\delta_o}{G\mathcal{M}_\oplus/l^2}\right)-\left(\frac{v_{o,\infty}^2}{G\mathcal{M}_\oplus/\mathcal{R}_{min}}\right)\cos\left(\frac{\Psi}{2}\right). \tag{15}$$

Now subtracting (15) from (14) and thereafter performing some basic algebraic operations, one arrives at:

$$\left(\frac{\Delta v_\infty}{v_\infty}\right) = \left(\frac{a_*\mathcal{R}_{min}}{v_\infty^2\cos(\Psi/2)}\right)(\sin\delta_i-\sin\delta_o), \tag{16}$$

where clearly, $\Psi:(0\ll\Psi<180^\circ)$. In the above we have made use of $v_{i,\infty}^2-v_{o,\infty}^2=\Delta K_\infty/m=2v_\infty\Delta v_\infty$ where K and m are the kinetic energy at infinity and mass of the spacecraft respectively.

There is one unknown a_* in equation (16) thus we can calculated this given $\delta_i,\delta_o,\mathcal{R}_{min}$, and Ψ. These values are given in table (I). Less the value for Galileo I, the rest of the values for other spacecrafts of a_* are positive and lay in a reasonable narrow range $(0.14-3.23)\times10^{-5}\,\mathrm{ms^{-2}}$. The error margins in the values of a_* come from the error values of Δv_∞^{obs}. The obtained average of all the values of a_* is $a_*=(1.02\pm0.02)\times10^{-5}\,\mathrm{ms^{-2}}$. We are of the view that this value is acceptable and that this result strongly points to the ASTG as containing in it, a grain of truth to do with the flyby anomalies.

Table (I). Earth flyby parameters at the asymptotes of their orbits for Galileo, NEAR, Cassini, Rosetta, and MESSENGER spacecraft. Columns $1, 2\,\&\,3$ gives the name of the spacecraft, the date it made its gravity assist maneuver and the Agency responsible for this spacecraft respectively. Columns $4, 5, 6, 7\,\&\,8$ gives the inclination of the spacecraft's orbit relative to the Earth's equator, the incoming and outgoing Right Ascension, the incoming and outgoing Declination angle respectively. Columns $9, 10\,\&\,11$ is the osculating hyperbolic excess velocity, the altitude is referenced to an Earth geoid plus the radius of the Earth, and the change in osculating hyperbolic excess velocity. The data in this table except for that in column 13, is adapted from Anderson *et al.* (2008).

Spacecraft	**Date**	**Agency**	Ψ (1°)	α_i (1°)	α_o (1°)	δ_i (1°)	δ_o (1°)	v_∞ (km/s)	$\mathcal{R}_{min}$ (km)	Δv_∞^{obs} (mm/s)	a_* (10^{-5}m/s^2)
Galileo I	08/12/1990	NASA	47.7	266.76	219.97	12.52	34.15	8.949	7356	3.92 ± 0.08	-1.27 ± 0.03
Galileo II	12/12/1992	NASA	51.1	219.35	174.35	-34.26	-4.87	8.877	6703	-4.60 ± 1.00	3.23 ± 0.06
NEAR	23/01/1998	NASA	66.9	261.17	183.49	-20.76	-71.96	6.851	6939	13.46 ± 0.13	1.77 ± 0.04
Cassini	18/08/1999	NASA	19.7	334.31	352.54	-12.92	-4.99	1.601	7571	-2.00 ± 1.00	1.94 ± 0.04
Rosseta I	04/03/2005	ESA	99.3	346.12	246.51	-2.81	-34.29	3.863	8354	1.80 ± 0.05	0.31 ± 0.01
M"NGER	02/08/2005	Private	94.7	292.61	227.17	31.44	-31.92	4.056	8736	0.02 ± 0.01	0.14 ± 0.00
Rosseta II	13/11/2007	ESA	–	–	–	–	–	3.863	11722	~ 0	–
Rosseta III	13/11/2009	ESA	–	–	–	–	–	3.863	8883	~ 0	–
Mean											1.02 ± 0.02

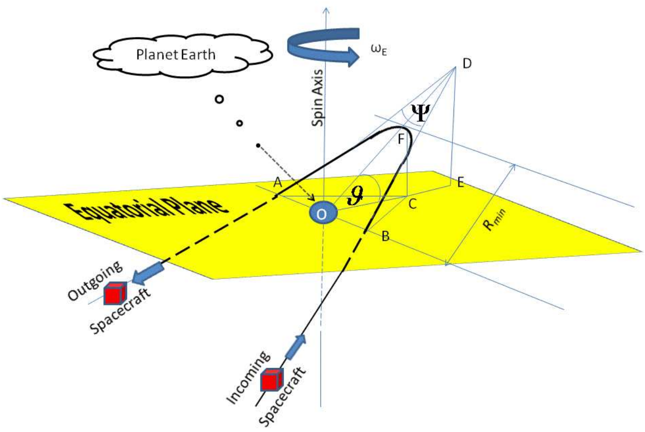

Figure (1). Schematic diagram showing the geometry of the orbit of a spacecraft making a planetary flyby.

3.2 Asymptotic Speed Changes at Perigee

How are we to explain the asymptotic speed changes that occur at the perigee? If this asymptotic change is a phenomenon explainable from the confines of the ASTG, then, this surely points to a component in the equations of motion that must change asymptotically. To answer this question, let us go to figure (3.2). When the spacecraft reaches the perigee, it encounters two different values for φ, *i.e.*: $\varphi = (0^\circ, 360^\circ)$. The functions $(\sin\varphi, \cos\varphi)$ do not have a problem with this apparent asymptotic change in the φ-value, thats is from $0^\circ \longmapsto 360^\circ$ (or $360^\circ \longmapsto 0^\circ$, this depends on the direction from which the spacecraft approaches the perigee).

A function like $e^{k\varphi}$ will have a problem, because it would have to jump from $1 \longmapsto e^{2\pi k}$. At this point we are taken aback, to Nyambuya (2010a) where the ASTG was first laid down. We did show there-in that the eccentricity of a orbit has an additional term $e^{k\varphi}$ such that $\epsilon = \epsilon_N e^{k\varphi}$ where for the Earth:

Table (II). Earth flyby parameters at closest approach for Galileo, NEAR, Cassini, Rosetta, and MESSENGER spacecraft. Columns 1 gives the name of the spacecraft. Columns 2 gives the inclination of the spacecraft's orbit relative to the Earth's equator. Columns $3\,\&\,4$ the velocity the perigee and the altitude is referenced to an Earth geoid plus the radius of the Earth, and column 5 gives asymptotic change in velocity at the perigee while column 6 gives the value of change in velocity at the perigee from the ASTG. The data in this table except for that in column 6, is adapted from Anderson *et al.* (2008).

Spacecraft	ϑ (1°)	v_{prg} (km/s)	$\mathcal{R}_{min}$ (km)	Δv_{prg}^{obs} (mm/s)	$\lambda_1^\oplus$
Galileo I	142.9	13.740	7356	2.560 ± 0.050	7.20 ± 0.10
Galileo II	138.9	–	6703	–	–
NEAR	108.0	12.739	6939	7.210 ± 0.070	20.0 ± 2.00
Cassini	25.4	19.026	7571	-1.700 ± 0.900	7.00 ± 4.00
Rosseta I	144.9	10.517	8354	0.670 ± 0.020	7.0 ± 4.00
M"NGER	133.1	10.389	8736	0.008 ± 0.004	8.00 ± 2.00
Rosseta II	–	12.49	11722	~ 0	–
Rosseta III	–	13.34	8883	-0.004 ± 0.044	–
Mean					10.00 ± 2.00

$$k_\oplus = \frac{\lambda_1^\oplus}{2}\left(\frac{G\mathcal{M}_\oplus}{c^2 l}\right)\sin\theta \qquad (17)$$

If $\epsilon = \epsilon_N e^{k\varphi}$ is the eccentricity predicted from the ASTG, the reader may ask why then did we not include this in our earlier calculation (11)? The reason is that to first order approximation, its inclusion would have resulted in the important term $k\varphi$ emerging as a second order term. To see this; to first order approximation $\epsilon \simeq \epsilon_N(1 + k\varphi + ... + ...)$ and clearly the term $\epsilon_N k\varphi$ would have dropped out, the meaning of which is it would have not been necessary to include this in (11). In the present, the term involving $k\varphi$ emerges as a first order approximation.

Now if the spacecraft approaches the perigee as shown in figure (3.2), it starts of from infinity with a ϑ-value of $\vartheta = 90^\circ + \delta_i$ and this decreases as it approaches the pre-perigee where upon is arrives at the perigee with a φ-value of $0^\circ = 0$rad, and the correspond-

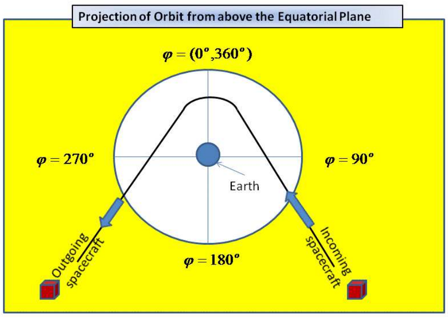

Figure (2). An illustration showing the equatorial view of spacecraft flyby orbits

ing eccentricity would be $\epsilon_1 = \epsilon_N$; this same spacecraft enters its post perigee journey with a φ-value of $360^\circ = 2\pi$rad and the corresponding eccentricity would be $\epsilon_2 = \epsilon_N(1 + 2\pi k)$ this will decrease as the spacecraft approaches infinity unit it reaches $90^\circ + \delta_o$. Now $(\Delta v_{prg}/v_{prg}) = (\epsilon_2 - \epsilon_1)/2\epsilon_1$ where Δv_{prg} is the asymptotic change in the speed at the perigee and v_{prg} is the speed at the perigee. From this, it follows that:

$$\left(\frac{\Delta v_{prg}}{v_{prg}}\right) = \frac{\lambda_1^{\oplus}}{4}\left(\frac{G\mathcal{M}_{\oplus}}{c^2\mathcal{R}_{min}}\right)\cos\vartheta. \quad (18)$$

All the values in the above are known less for $\lambda_1^{\oplus}$. As before, we have in table (II) calculated this from the known and available data. If these ideas are something worthwhile, the values of $\lambda_1^{\oplus}$ from each of the five data points available, must not vary widely from the mean value. The error values calculated in $\lambda_1^{\oplus}$ come from the error values in Δv_{prg}^{obs}. The final average value of $\lambda_1^{\oplus}$ obtained is $\lambda_1^{\oplus} = 10.00 \pm 2.00$. While the rest of the values are in the range $(7 - 8)$, that of NEAR is far off at 20. This means $4/5$ of these seem to strongly point to a correlation.

We see that depending on the direction from which the spacecraft approaches the perigee, there will either be a positive or negative jump in speed at the perigee. With respect to hyperbolic speed changes, whether the change in the speed at infinity is either positive or negative depends on the declination angles. From what we have presented, it appears that, if one is supplied with the data of the pre-perigee orbit, one will be able to make verifiable predictions of the anomalies.

4 DISCUSSION AND CONCLUSION

The fact that the unknown values of a_* and $\lambda_1^{\oplus}$ coming from the theory when-after it is weighed against experience, seem to lie in a narrow range strongly suggests that the ASTG has in it some element of truth to do with flyby anomalies. Clearly there is need for researchers to look into the ASTG as this theory flows from a natural solution of the well known Poisson equation. That we understand the Poisson equation is something almost taken for granted. Surely and clearly, we have made not any modification(s) to the Poisson equation but merely took its natural azimuthal solution and applied it to the scenario of a gravitational field of a spinning body.

Given the radial component of the gravitational field in equation (6), there is a region $r = \mathcal{R}_{crit}$ around a spinning body where the net gravitational acceleration cancels off, *i.e.* $a_r = 0$. This occurs at: $\mathcal{R}_{crit} = (G\mathcal{M}/a_* \cos\theta)^{1/2}$. Curiously enough – for the calculated value of a_* – one finds for the Sun that at the spin equatoral plane where $\theta = 90^\circ$, $\mathcal{R}_{crit} = 19.00 \pm 2.00$AU which is the distance where the Pioneer Anomaly begins to profusely manifest (Anderson *et al.* 1998). From the value of $a_* = (1.72 \pm 0.30) \times 10^{-5}\mathrm{ms}^{-2}$ obtained, assuming it where universal, the meaning of which is that it applies to the Sun and all the gravitating bodies in the Universe, it would mean that the Pioneer's, given their anomalous acceleration of $a_{\odot}^{p} = (8.74 \pm 1.34) \times 10^{-10}\mathrm{ms}^{-2}$ (Anderson *et al.* 1998); are moving on the spin equator of the Sun, because their inclination should be $0.0050 \pm 0.0006^\circ$. This data can be interpreted as saying when the Pioneers reached about 5AU, they made an ascent off the solar spin equator until at 20AU where the spacecrafts where about 0.0018 ± 0.0002AU above this plane and they traveled at a constant inclination of $0.005 \pm 0.0006^\circ$ above the solar spin equator until at 70AU where they had ascended a distance 0.0061 ± 0.0008AU. For this reason, there is a need to obtain the complete set (if at all possible) of the Pioneer ephemerides.

In closing, allow us to say the following, that; the formula we obtained for predicting the anomalous increase in hyperbolic excess speed is similar and not congruent to that of Anderson *et al.* (2008). Additionally, prior to the present reading, *i.e.* from Anderson *et al.* (2008), only two parameters appeared to matter in as far as predicting the observed anomalous speed increase of the spacecraft at infinity and these are the incoming hyperbolic excess speed and the declination angle (incoming and outgoing). In the present, we have added two more and these are the deflection angle (ϑ) and the perigee distance ($\mathcal{R}_{min}$) from the center. We have submitted all these data to the theory and from it we obtained what strongly appears to be a well behaved and related set of physical parameters. It appears to us highly unlikely that these parameters behave so well by chance, against this probability, we strongly believe we herein have a theory that strongly appears to contain in it, an element of truth. Given also that it has been suggested that the ASTG may have something to do with outflows (see Nyambuya 2010*b*). Perhaps, researchers should excogitate on the possibility that the gravitational field of a spinning body is not Newtonian, but azimuthally symmetric as laid down in Nyambuya (2010*a*) and in the present.

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