On the Precession of Mercury's Orbit

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Abstract.

The Sun's orbital motion around the Solar System barycentre contributes a small quadrupole component to the gravitational energy of Mercury. The effect of this component has until now gone unnoticed, but it actually induces a significant part of the observed precession of Mercury's orbit.

Keywords: precession, Mercury, planets.

1. Introduction

The orbit of planet Mercury has been calculated by several investigators; see Clemence (1947), Brouwer & Clemence (1961), a review in Pireaux & Rozelot (2003). In these calculations, Newton's inverse square law has been applied to set up the differential equations of motion using the instantaneous measured distances and velocities between Mercury, the Sun and planets. The individual contributions to the motion of the perihelion of Mercury have been listed as due to the general precession in longitude, the planets, solar oblateness, and relativity. However, an omission from this list has been identified because the slow motion of the Sun around the barycentre produces a small quadrupole component in the potential energy of Mercury, which induces some precession of Mercury's orbit.

Thus, the calculated orbit of Mercury incorporates the moving Sun, and the total precession is known very accurately, but it is the attribution of 43arcsec/cy to a general relativity effect which is wrong. Contrary to received wisdom, such precession due to the wobbling Sun is not zero, nor automatically nulled, nor is it included in the calculation of Jupiter's effect on Mercury. This obscure precession now destroys any credible application of the Schwarzschild solution to the Solar System.

2. Derivation of precession due to the moving Sun

Imagine Mercury *alone* orbiting the Sun in absolute free space, in an ideal closed elliptical orbit with no perturbations nor precession of the orbit. Now, imagine the Sun moving rapidly in a circle of radius equal to 1.068 solar radii, due to a rapidly moving Jupiter. Mercury can still orbit this "effectively-toroidal" Sun, but its orbit will certainly not be exactly elliptical as before, and its binding energy will become stronger. The *average* gravitational field directed towards the *centre of mass* could no longer be an inverse square law, but it would be like the field of an oblate Sun containing a small additional toroidal component. Even if subsequently, the Sun's period were made greater than the period of Mercury, the toroidal field would still exist in essence but at a reduced level.

Thus, some precession of Mercury's orbit will be caused specifically by the remaining toroidal field component. It cannot be reduced to zero by calculating planetary trajectories in heliocentric coordinates. The precession of Mercury due to Jupiter itself has always been calculated using heliocentric coordinates, but this is totally independent of the above effect of a moving Sun on Mercury's orbit.

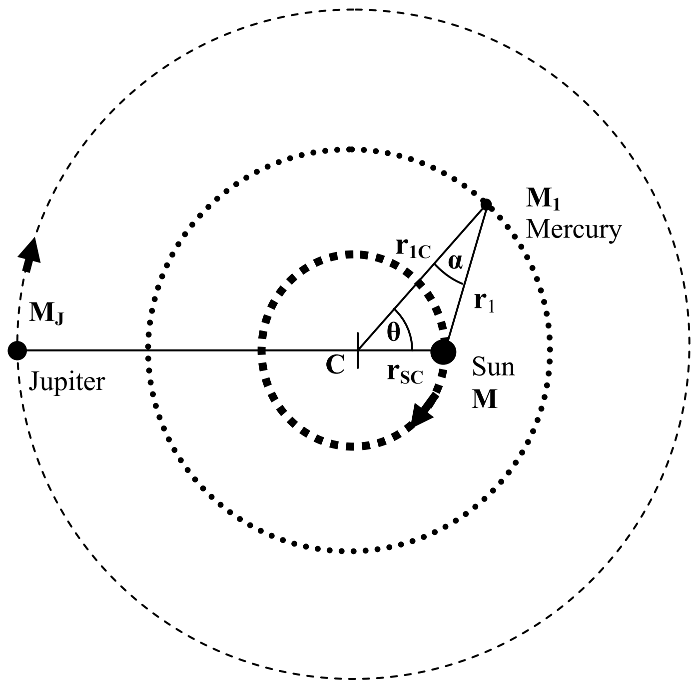

Figure 1. Schematic diagram showing Jupiter and the Sun moving around their centre of mass C. Mercury is considered to be stationary during one orbit of the Sun.

The binding energy of Mercury, in the field of the Sun orbiting around the barycentre, may be calculated by using Newton's law. First, from Figure 1, let Mercury (mass M_1) be regarded as stationary while the Sun (mass M) travels rapidly around the centre C at radius r_{SC} . Then, for the Sun at distance r_1 from Mercury we can write:

$$r_1^2 = r_{1C}^2 + r_{SC}^2 - 2r_{1C}r_{SC}\cos\theta \quad . \qquad (1a)$$

The *instantaneous* gravitational force exerted by the Sun on Mercury is given by the inverse square law, ($F_1 = -GMM_1 / r_1^2$); therefore, the force directed towards C is ($F = F_1\cos\alpha$), where α is the angle between the Sun and centre C given by:

$$r_{SC}^2 = r_{1C}^2 + r_1^2 - 2r_{1C}r_1\cos\alpha \ . \qquad (1b)$$

Therefore we get:

$$F = F_1\cos\alpha = -\left(\frac{GMM_1}{r_1^3}\right)(r_{1C} - r_{SC}\cos\theta) \ . \qquad (2a)$$

After simplifying, so that all the $\cos\theta$ terms are in the numerator before averaging over a complete orbit of the Sun, then the average force towards C derived from Eq.(2a) becomes:

$$\tilde{F} \approx -\left(\frac{GMM_1}{r_{1C}^2}\right)\left[1 + \frac{3}{4}\left(\frac{r_{SC}^2}{r_{1C}^2}\right)\right] \ . \qquad (2b)$$

Thus, a small additional term distinguishes this from the inverse square law of force. The absolute potential energy of Mercury can be calculated by integrating this force from infinity to r_{1C} :

$$PE_{av} \approx -\left(\frac{GMM_1}{r_{1C}}\right)\left[1 + \frac{1}{4}\left(\frac{r_{SC}^2}{r_{1C}^2}\right)\right] \ . \qquad (3)$$

This also equals the kinetic energy that Mercury would gain if it fell straight from infinity to its position r_{1C} from C. It follows that the angular momentum, kinetic energy, and trajectory of orbiting Mercury would be different from that around a stationary Sun.

If the computation of Mercury's trajectory around this orbiting Sun were done with heliocentric coordinates for the equations of motion employing the inverse square law, then it would be necessary to introduce this modified potential during the integration process. Clearly, the choice of heliocentric coordinates or barycentric coordinates (centre C) should not matter. We are interested in the particular form of the force, which adds some precession to the perihelion of Mercury.

In practice, the Sun's period around C due to Jupiter is 49 times longer than Mercury's period, so the toroidal component of potential energy must be decreased. The aim now is to determine how the constant angular momentum of Mercury around this moving Sun stabilises the near-elliptical orbit against the perturbation by the Sun.

Let us consider this as a perturbation problem such that the long-term mean distance of Mercury from the Sun is (r_{1C} = 57.91x10^{6}km), but the Sun can shift distance r_x in any direction from its position at the origin of coordinates. When the Sun moves towards Mercury, the potential of Mercury might increase to:

$$P_1 = -\frac{GM}{(r_{1C} - r_x)} \quad , \tag{4a}$$

but the Sun moving away by r_x might reduce the potential:

$$P_2 = -\frac{GM}{(r_{1C} + r_x)} \quad . \tag{4b}$$

The average of these is derived by using the binomial expansion:

$$\widetilde{P} = \frac{(P_1 + P_2)}{2} \approx -\frac{GM}{r_{1C}}\left[1 + \left(\frac{r_x}{r_{1C}}\right)^2\right] \quad , \tag{4c}$$

then the average gravitational acceleration on Mercury is:

$$a_r = -\frac{d\widetilde{P}}{dr_{1C}} \approx -\frac{GM}{r_{1C}{}^2}\left[1 + 3\left(\frac{r_x}{r_{1C}}\right)^2\right] \quad . \tag{4d}$$

Upon substituting ($u = 1/r_{1C}$), plus specific angular momentum h, then orbit theory leads to a differential equation for Mercury

$$\frac{d^2u}{d\phi^2} + u = \frac{-a_r}{h^2u^2} = \frac{GM}{h^2} + \left(\frac{GM}{h^2}3r_x{}^2\right)u^2 \quad . \tag{5}$$

General relativity theory produces a similar expression for the trajectory of Mercury, (see Rindler, 2001):

$$\frac{d^2u}{d\phi^2} + u = \frac{GM}{h^2} + \left(\frac{3GM}{c^2}\right)u^2 \quad . \tag{6}$$

The final term in this expression is responsible for 43arcsec/cy precession of Mercury's orbit, so we can calculate the precession to be expected from using Eq.(5):

$$\delta\omega = \left(\frac{c}{h} r_x\right)^2 \times 43 \text{arc sec/cy} \quad . \tag{7}$$

In the analysis leading to Eq.(2b), Mercury was held stationary, but now it is free to move in harmony with the Sun's motion, in an effort to maintain its angular momentum and average distance r_{1C} . This means that r_x in Eq.(4) is not equal to the actual motion of the Sun around C at radius r_{SC} , but is greatly reduced and given by:

$$r_x = \left(\frac{2}{\pi} r_{SC}\right) \times \left[\frac{(2/\pi)\, r_{SC}}{r_{1C}}\right] \approx 3860\text{km} \quad , \tag{8}$$

where $(2/\pi)r_{SC}$ is the mean perturbation amplitude of the Sun relative to Mercury, for arbitrary positions of Mercury and the Sun; and ($r_{SC} = 7.43\text{x}10^5$km) is due to Jupiter. The term in the square bracket is the corresponding perturbation parameter. Upon substituting this value of r_x in Eq.(7), we get precession:

$$\delta\omega = 0.174 \times 43 = 7.48 \text{arc sec/cy} \quad . \tag{9}$$

There is an adjustment to be made due to the effect of $\cos\alpha$ in Figure 1, which has not been included yet. The factor (3/4) in Eq.(2b) is due to $\cos\alpha$, so the final precession should be:

$$\delta\omega = 7.48 \times (3/4) = 5.61 \text{ arc sec/cy} \quad . \tag{10}$$

Inclusion of an eccentricity factor $(1\text{-}e^2)$ would make this become 5.86arcsec/cy.

Further precession due to the effect of the other planets on the Sun's motion will be smaller than this but has not been derived, yet. Precessions attributable to general relativity effects in the orbits of Venus, Earth and Icarus, (Shapiro et al (1968), Lieske & Null (1969), Sitarski (1992)) will be likewise decreased.

3. Conclusion

Motion of the Sun, around the Solar System barycentre, adds a small quadrupole term to the gravitational binding energy of Mercury. This term has been overlooked previously, but it is responsible for significant precession in the orbit of Mercury. The residual part of the total measured precession is therefore less than the acclaimed 43arcsec/cy. Fortunately, Einstein's general relativity theory is capable of describing the world without singularities, and with a real prospect of unification

(Wayte, 1983). This is very welcome news for physicists because singularities have invariably been due to a breakdown of theory.

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