The Interaction of Measurement

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Abstract: What is measurement and what can it tell us about the quantity measured? Can we know a quantity by measuring it? We mathematically demonstrate that the answer is no! We show how a continuous quantity *E(t)* that grows exponentially can in our measurements of it be seen as discrete and growing linearly. And if we further consider the practical limitations that render measurements as 'approximations' only, then the quantity *E(t)* that we measure can be *any* integrable function yet our measurements of it will *still* depict it as discrete and linear. Furthermore, and most surprising, the 'interaction of measurement' will be described by *Planck's Law*, whether *E(t)* is exponential or just integrable. Thus, we cannot know what the hidden quantity *E(t)* is as a function of time by the measurements of it.

Introduction: Measurement is the essence of Science. Observation and Understanding are forms of Measurement. We 'size up' what we 'observe' with what we 'know' – whether in relationships , Politics or Physics. Direct measurement involves an interaction between the 'observer' and the 'observed', the 'sensor' and the 'source'. The 'value' of the interaction is attained when an interaction equilibrium is reached between the two (when there is balance and stability) and a calibrated standard is used to quantify this value. What often is measured, however, are 'changes' ΔE and 'averages' $\overline{E}$ of a physical quantity E, but not the *instantaneous* value $E(t_0)$ of the physical quantity. Quantum Physics has raised many physical and philosophical questions and voluminous discussions. All stemming, in my view, from such aberrations of measurement and observation. Many of these counterintuitive phenomena and strained explanations hinge on QM's 'uncertainty principle' and the broader realization that the 'observation of an entity' molds the 'entity observed'.

We do not dispute or wish to debate any of the mathematical formalism or philosophical speculations of Quantum Physics. Rather, in this short note we consider a well defined simple question: *Are the measurements of a quantity* $E(t)$ *equate to the quantity itself?* If we were to plot the *measurements* of $E(t)$ over time, will this be the same as the graph of $E(t)$ *?* We mathematically demonstrate that the answer is no!

Notation:

$E(t)$ *is the value at time t of a quantity we measure*

$\eta = \int_0^\tau E(t)dt$ *is the 'accumulation' over a time interval* τ *of this quantity*

$E_{av} = \frac{\eta}{\tau}$ *is the average value over a time interval* τ *of this quantity*

$\Delta E = E(\tau) - E(0)$ *is the incremental change over a time interval* τ *of this quantity*

The Interaction of Measurement: For a (direct) measurement of a physical quantity to occur we must have the following,

1. A physical interaction between the 'source' of the physical quantity $E(t)$ measured and the 'sensor' that measures the quantity.
2. An interval of time $\tau = \Delta t = t - t_0$ for the interaction of measurement to occur.
3. An interaction equilibrium between the 'source' and the 'sensor', when the 'average energy at the source' equals the 'average energy from the sensor' during an interaction cycle.
4. An amount $\Delta E = E(t) - E(t_0)$ absorbed (sampled) by the 'sensor' when equilibrium is attained.

The *Interaction of Measurement* is a functional relationship between the value of $E(t_0)$, the actual instantaneous value of the physical quantity, the amount ΔE absorbed by the 'sensor' and the average E_{av} at the 'sensor' at each interaction cycle.

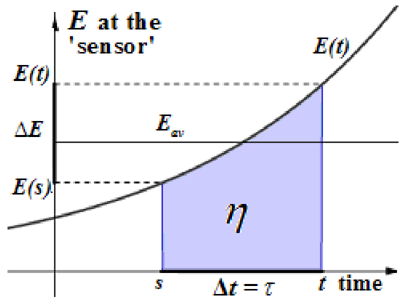

figure 1

Above, *figure 1* shows the quantity $E(t)$ at time *t* at the 'sensor', the time interval τ , the 'accumulation' η of the quantity E during that time interval, the incremental change ΔE over that time interval and the average quantity E_{av} at the 'sensor' over that time interval. The 'sensor' in physical interaction with the 'source' will absorb ('sample') an amount ΔE when the 'source' and the 'sensor' attain equilibrium. When this amount is absorbed by the 'sensor' the quantity at the sensor is reduced by that amount and so the function $E(t)$ 'collapses' to its initial value. The 'accumulation' of *E* will start anew until another interaction equilibrium is reached. This process stops once the 'sensor' reaches a saturation point and a calibrated reading is made. In *figure 2 and 3* we see graphically this process.

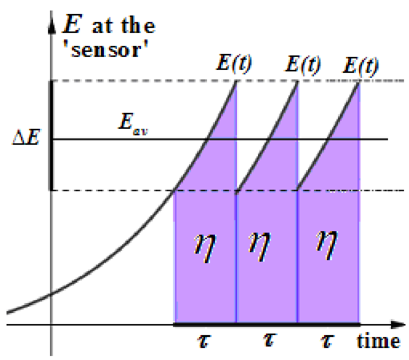

figure 2

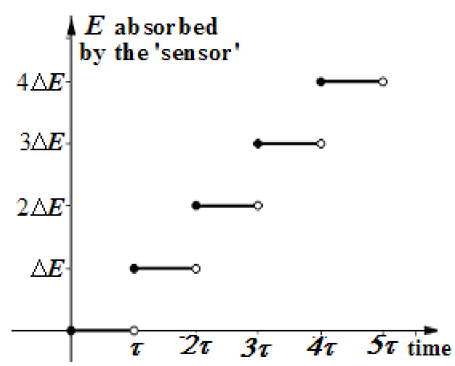

figure 3

Clearly in this formulation as shown in *figure 2*, $E(t)$ can be any integrable function of time. And according to the above discussion, when the interaction of measurement reaches equilibrium, a pulse of time τ will have lapsed while an discrete amount of energy ΔE will be absorbed by the 'sensor'. The function $E(t)$ will 'collapse' and a new interaction cycle will begin. The accumulation η during each such cycle will be the same. In *figure 3* we see the 'sensor' absorbs discrete fixed amounts ΔE over fixed time intervals τ . The experimental graph *E vs t* produced will be a *linear step function*. This is what we *see* as our measurements of $E(t)$. But this will be the *same* (linear step function) for *any* function $E(t)$. Clearly from the graph of *figure 3* we cannot possibly *know* the function that describes the quantity $E(t)$ we are measuring.

We have **proven elsewhere** the following mathematical results,

Theorem 1: $$E(t) = E_0 e^{\nu t} \textit{ if and only if } E_0 = \frac{\eta\nu}{e^{\eta\nu/\kappa\mathcal{T}} - 1} \quad \textbf{(1)}$$

where $E(t)$ is integrable, $\eta = \int_0^\tau E(u)du$, $\mathcal{T} = \left(\frac{1}{\kappa}\right)\frac{\eta}{\tau}$ and κ is a scalar constant.

Corollary: $$E(t) = E_0 e^{\nu t} \textit{ if and only if } E_0 = \frac{\Delta E}{e^{\Delta E/E_{av}} - 1} \quad \textbf{(2)}$$

where $E(t)$ is integrable, $\Delta E = E(\tau) - E(0)$ and $E_{av} = \frac{\eta}{\tau}$

Theorem 2: $$E(t) = E_0 e^{\nu t} \textit{ if and only if } \Delta E = \eta\nu \quad \textbf{(3)}$$

Theorem 3: For any integrable function *E(t)*, $\lim_{\tau \to 0} \frac{\eta\nu}{e^{\eta\nu/\kappa\mathcal{T}} - 1} = E_0$, i.e. $E_0 \approx \frac{\eta\nu}{e^{\eta\nu/\kappa\mathcal{T}} - 1}$ **(4)**

Corollary: For any integrable function *E(t)*, $\lim_{\tau \to 0} \frac{\Delta E}{e^{\Delta E/E_{av}} - 1} = E_0$, i.e. $E_0 \approx \frac{\Delta E}{e^{\Delta E/E_{av}} - 1}$

Thus, from the above, whether we assume an exponential representation of the quantity $E(t)$ we are measuring or simply assume that this quantity can be represented by some integrable function, we can describe the *Interaction of Measurement* by

$$E_0 = \frac{\Delta E}{e^{\Delta E/E_{av}} - 1} = \frac{\eta\nu}{e^{\eta\nu/\kappa\mathcal{T}} - 1} \qquad \text{(if } E(t) \text{ is exponential)} \quad \textbf{(5)}$$

or,

$$E_0 \approx \frac{\Delta E}{e^{\Delta E/E_{av}} - 1} = \frac{\eta\nu}{e^{\eta\nu/\kappa\mathcal{T}} - 1} \qquad \text{(if } E(t) \text{ is just integrable)} \quad \textbf{(6)}$$

Planck's Law can be written as $$E_0 = \frac{h\nu}{e^{h\nu/kT} - 1} \quad \textbf{(7)}$$

The mathematical comparison between *Planck's Law* **(7)** and the *Interaction of Measurement* **(5)** above is striking. Planck's constant *h* and the 'accumulation' η have the same units and play the same role in these equations. And so does temperature *T* and the quantity $\mathcal{T}$ we defined above. Furthermore, the Quantization of Energy $\Delta E = h\nu$ is likewise reflected in *Theorem 2* above. The following conclusion is inescapable:

Planck's Law is a Mathematical Identity that describes the interaction of measurement.

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