λ=h/p is universal?

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Abstract: de Broglie formula to photons in an unbounded space is $E=h\nu$ and $\lambda=h/p$. According to electrodynamics, nevertheless,we prove the ratio E/p in a waveguide is greater than the product $\nu\lambda$ which implies $E=h\nu$ and $p=h/\lambda$ cannot be tenable at the same time. Then the Casimir effect is applied to confirm $E=h\nu$ and $p<h/\lambda$. It is helpful to study quantum tunnelling and superluminality[1]~[2], Cavity-QED, origin of mass, etc. The microwave experiment to test is also presented.

Key words: de Broglie formula; waveguide; quantum tunnelling; Casimir effect; Higgs mechanism;

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1. Introduction: Photons in an Unbounded Space

To electromagnetic waves in an unbounded space, the phase velocity is

$$\frac{\omega}{\mathrm{k}} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}} = c \tag{1}$$

The relation between energy density $w = \frac{1}{2}(\varepsilon_0 \mathrm{E}^2 + \frac{\mathrm{B}^2}{\mu_0}) = \varepsilon_0 \mathrm{E}^2$ ($\mathrm{E}/\mathrm{B} = 1/\sqrt{\varepsilon_0 \mu_0}$) and momentum density $\mathbf{g} = \varepsilon_0 \mathbf{E} \times \mathbf{B}$ is

$$\frac{w}{\mathbf{g}} = \frac{\mathbf{S}/\mathbf{g}}{\mathbf{S}/w} \tag{2}$$

where $\mathbf{S} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}$ is the Poynting's vector. In electrodynamics,

$$\frac{\mathbf{S}}{\mathbf{g}} = \frac{\mathbf{E} \times \mathbf{B}/\mu_0}{\varepsilon_0 \mathbf{E} \times \mathbf{B}} = \frac{1}{\varepsilon_0 \mu_0} = c^2 \tag{3}$$

$$\frac{\mathbf{S}}{w} = \frac{\mathbf{E} \times \mathbf{B}/\mu_0}{\varepsilon_0 \mathrm{E}^2} = \frac{\mathrm{B}}{\varepsilon_0 \mu_0 \mathrm{E}}\mathbf{n^0} = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}\mathbf{n^0} = c\mathbf{n^0} \quad (\mathbf{n^0} \text{ is the unit vector}) \tag{4}$$

$$\frac{w}{g} = \frac{S/g}{S/w} = \frac{c^2}{c} = c \tag{5}$$

Suppose N is the number density of photons, the energy-momentum relation of a single photon is

$$\frac{E}{p} = \frac{w/N}{g/N} = \frac{w}{g} = c \tag{6}$$

Make a comparison with (6) and (1), $E = \hbar\omega$ and $p = \hbar\mathrm{k}$ is tenable indeed.

2. Photons in a Waveguide

In a rectangular waveguide, the basic relations are

$$\omega / \mathrm{k} = c \tag{1}$$

$$\frac{\omega}{\mathrm{k_z}} = \frac{c}{\sqrt{1 - \omega_c^2/\omega^2}} > c \tag{7}$$

$$\sqrt{1 - \omega_c^2/\omega^2} = \mathrm{k_z}/\mathrm{k} \tag{7'}$$

$$\frac{d\omega}{d\mathrm{k_z}} = c\sqrt{1 - \omega_c^2/\omega^2} < c \tag{8}$$

where $\omega_c = \pi c\sqrt{\frac{\mathrm{m}^2}{a^2} + \frac{\mathrm{n}^2}{b^2}}$ is the cut-off frequency (m and n is integer) .

Especially, the ratio between power P and linear energy density U of the electromagnetic field is no longer the phase velocity like that in an unbounded space. The result is well-known as [3]

$$\frac{P}{U}=\frac{\int_0^a\int_0^b \mathrm{S}\,dxdy}{\int_0^a\int_0^b w\,dxdy}=c\sqrt{1-\frac{\omega_c^2}{\omega^2}}<c \tag{4'}$$

Likewise, we define the linear momentum density to be $G=\int_0^a\int_0^b g\,dxdy$. Owing to (3),

$$\frac{P}{G}=\frac{\int_0^a\int_0^b \mathrm{S}\,dxdy}{\int_0^a\int_0^b g\,dxdy}=\frac{c^2\int_0^a\int_0^b g\,dxdy}{\int_0^a\int_0^b g\,dxdy}=c^2 \tag{3'}$$

The energy-momentum relation is

$$\frac{E}{p}=\frac{\int_0^a\int_0^b w\,dxdy}{\int_0^a\int_0^b g\,dxdy}=\frac{U}{G}=\frac{P/G}{P/U}=\frac{c^2}{c\sqrt{1-\omega_c^2/\omega^2}}=\frac{c}{\sqrt{1-\omega_c^2/\omega^2}}>c \tag{6'}$$

Clearly, $E=\hbar\omega$ and $p=\hbar\mathrm{k}$ is incompatible because $E/p>c$ and $\omega/\mathrm{k}=c$.

3. Casimir Effect and Relativistic Mechanics

The ratio from classical electrodynamics reflects the contradiction but we do not know which one breaks down. In fact, theories and experiments of the Casimir effect affirm the energy expression of a photon is still $E=\hbar\omega=\hbar c\mathrm{k}$ between two planes that can be regarded as a one-dimensional rectangular waveguide ($b\to\infty$) [4]. Thus, the total momentum is

$$p=E/\frac{c}{\sqrt{1-\omega_c^2/\omega^2}}=\hbar\mathrm{k}\sqrt{1-\omega_c^2/\omega^2}<\hbar\mathrm{k} \tag{9}$$

rather than $p=\hbar\mathrm{k}$. Substituting (9) into the equation $E^2=p^2c^2+m_0^2c^4$ of special relativity,

$$\hbar^2\omega^2=(\hbar\mathrm{k}\sqrt{1-\omega_c^2/\omega^2})^2c^2+m_0^2c^4$$

The photon in a waveguide can be treated as a massive particle whose rest energy is $m_0c^2=\hbar\omega_c>0$. Here, we get a new interpretation to the high-pass function of the waveguide as a filter. It is because of the total energy $\hbar\omega$ of a photon cannot be less than its rest energy $\hbar\omega_c$.[2]

For the existence of (7') , (9) is also

$$p=\hbar\mathrm{k}_z<\hbar\mathrm{k} \tag{9'}$$

and the ratio between energy and momentum in quantum mechanics should be

$$\frac{E}{p}=\frac{\hbar\omega}{\hbar\mathrm{k}_z}=\frac{\omega}{\mathrm{k}_z}\xrightarrow{(7)}\frac{c}{\sqrt{1-\omega_c^2/\omega^2}} \tag{6''}$$

Moreover, the definition of the velocity of motion in mechanics is $V = \frac{dE}{dp}$. So,

$$V = \frac{d(\hbar\omega)}{d(\hbar \mathrm{k}_z)} = \frac{d\omega}{d\mathrm{k}_z} \xrightarrow{(8)} c\sqrt{1-\omega_c^2/\omega^2} \tag{4''}$$

which is just the velocity of energy transmission P/U in Equ.(4'). Actually, to use the velocity of energy transmission as the definition of the velocity of electromagnetic field is better than concepts from the wave theory like the phase velocity V_p and group velocity V_g. For instance, the phase velocity equals to V in the unbounded space but larger than it in a waveguide.

On the basis of $m_0 = \hbar\omega_c / c^2$ and $V = c\sqrt{1-\omega_c^2/\omega^2}$, the relativistic equations are

$$E = \frac{m_0 c^2}{\sqrt{1-V^2/c^2}} = \frac{\hbar\omega_c}{\omega_c/\omega} = \hbar\omega$$

$$p = \frac{m_0 V}{\sqrt{1-V^2/c^2}} = \frac{\frac{\hbar\omega_c}{c^2}\, c\sqrt{1-\omega_c^2/\omega^2}}{\omega_c/\omega} = \hbar \mathrm{k}\sqrt{1-\omega_c^2/\omega^2} = \hbar \mathrm{k}_z < \hbar \mathrm{k} \tag{9''}$$

$$\frac{E}{p} = \frac{m_0 c^2/\sqrt{1-V^2/c^2}}{m_0 V/\sqrt{1-V^2/c^2}} = \frac{c^2}{V} = \frac{c^2}{c\sqrt{1-\omega_c^2/\omega^2}} = \frac{c}{\sqrt{1-\omega_c^2/\omega^2}} > c \tag{6'''}$$

All these equations from electromagnetism, quantum mechanics and special relativity are compatible. Normal photons are subluminal particles in the waveguide. If the waveguide is full of dielectric media instead of vacuum, $\varepsilon_0 \to \varepsilon$, $\mu_0 \to \mu$ and $c \to \frac{1}{\sqrt{\varepsilon\mu}}$ [5].

4. Momentum Components and de Broglie Formula

There have two possible states corresponding to the total momentum $p = \hbar \mathrm{k}_z$.

(i)

$$p_x = 0 \qquad\qquad \mathrm{k}_x$$

$$p_y = 0 \qquad\qquad \mathrm{k}_y$$

$$p_z = \hbar \mathrm{k}_z \qquad\qquad \mathrm{k}_z$$

$$p = \sqrt{p_x^2 + p_y^2 + p_z^2} = \hbar \mathrm{k}_z \qquad\qquad \mathrm{k} = \sqrt{\mathrm{k}_x^2 + \mathrm{k}_y^2 + \mathrm{k}_z^2}$$

In this case, $p_x = 0 \neq \hbar \mathrm{k}_x$, $p_y = 0 \neq \hbar \mathrm{k}_y$ and $p = \hbar \mathrm{k}_z \neq \hbar \mathrm{k}$.

(ii)

$$p_x = \hbar \mathrm{k}_x \sqrt{1-\omega_c^2/\omega^2} = \hbar \mathrm{k}_x \frac{\mathrm{k}_z}{\mathrm{k}} \qquad\qquad \mathrm{k}_x$$

$$p_y = \hbar k_y \sqrt{1-\omega_c^2/\omega^2} = \hbar k_y \frac{k_z}{k} \qquad\qquad k_y$$

$$p_z = \hbar k_z \sqrt{1-\omega_c^2/\omega^2} = \hbar k_z \frac{k_z}{k} \qquad\qquad k_z$$

$$p = \sqrt{p_x^2 + p_y^2 + p_z^2} = \hbar k \frac{k_z}{k} = \hbar k_z \qquad\qquad k = \sqrt{k_x^2 + k_y^2 + k_z^2}$$

In this case, $p_x \neq \hbar k_x$, $p_y \neq \hbar k_y$, $p_z \neq \hbar k_z$ and $p \neq \hbar k$.

Obviously,both states are inconsistent with de Broglie's wavelength formula $p = \hbar k$ ($p_x = \hbar k_x$, $p_y = \hbar k_y$, $p_z = \hbar k_z$).

5. Experiment to Test

We can measure energy, momentum and spin L_s of microwaves in a waveguide through the torsion balance, etc. That relations should be

$$\frac{E}{L_s} = \frac{\hbar\omega}{\hbar} = \omega \qquad [6]$$

$$\frac{p}{L_s} = \frac{\hbar k_z}{\hbar} = k_z < k$$

$$\frac{E}{p} = \frac{c}{\sqrt{1-\omega_c^2/\omega^2}} > c$$

The momentum p_z in the z direction of (i) is greater than that of (ii) . The difference can be applied to judge which state is real.

6. Massive Photons in Plasma

The dispersion formula of radio waves in ionosphere is

$$\omega^2 = k^2 c^2 + \omega_0^2 \qquad (\omega_0 = \sqrt{\frac{Ne^2}{\varepsilon M}})$$

Make a comparison with $E^2 = p^2c^2 + m_0^2 c^4$, we have

$$E = \hbar\omega$$

$$p = \hbar k$$

$$m_0 = \hbar\omega_0 / c^2 > 0$$

$$V = c\sqrt{1-\omega_0^2/\omega^2} = V_g < c$$

$$\frac{E}{p}=\frac{c^2}{V}=\frac{c^2}{V_g}=\frac{c}{\sqrt{1-\omega_0^2/\omega^2}}=V_p>c$$

The formula $p=\hbar\mathrm{k}$ is valid because the case is also unbounded. This is an ideal model to de Broglie's theory of matter wave.

Conclusion

The photon has a nonzero rest mass sometimes. In the waveguide,

$$E=\hbar\omega=\hbar\mathrm{k}c=\sqrt{\underbrace{\hbar^2\mathrm{k}_{//}^2c^2}_{\substack{p^2c^2=(p_{//}^2+p_{\perp}^2)c^2\\ \text{(i) } p_{\perp}=0,p_{//}=p=\hbar\mathrm{k}_{//}\\ \text{(ii) } p_{\perp}\neq 0,p_{//}<p=\hbar\mathrm{k}_{//}}}+\underbrace{\hbar^2\mathrm{k}_{\perp}^2c^2}_{m_0^2c^4}}$$

The rest mass depends on geometric parameters. It comes from boundary conditions breaking the symmetry of space and the solution of Maxwell's equations. As to photons in plasma,

$$E=\hbar\omega=\sqrt{\underbrace{\hbar^2\mathrm{k}^2c^2}_{p^2c^2=(p_x^2+p_y^2+p_z^2)c^2}+\underbrace{\hbar^2\omega_0^2}_{m_0^2c^4}}$$

Such a rest mass is induced by the interaction between the (electromagnetic) field and matter(plasma).Other ways to acquire mass except Higgs mechanism are necessary.

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