PROOF WITHOUT WORDS: THE EXPANSION OF $(1+x+x^2+\cdots+x^n)^3$

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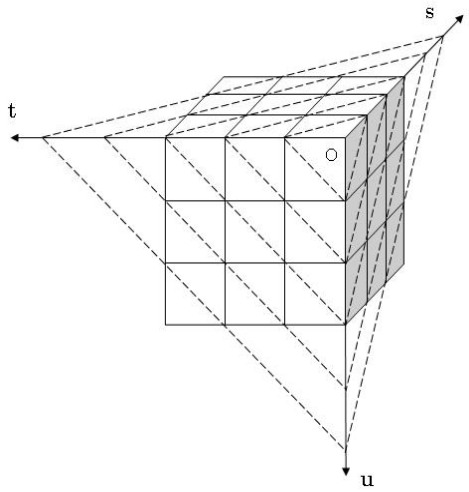
Abstract. In this paper, we show a geometry approach to the expansion of $(1+x+x^2+\cdots+x^n)^3$. This proof is a "Proof Without Words".

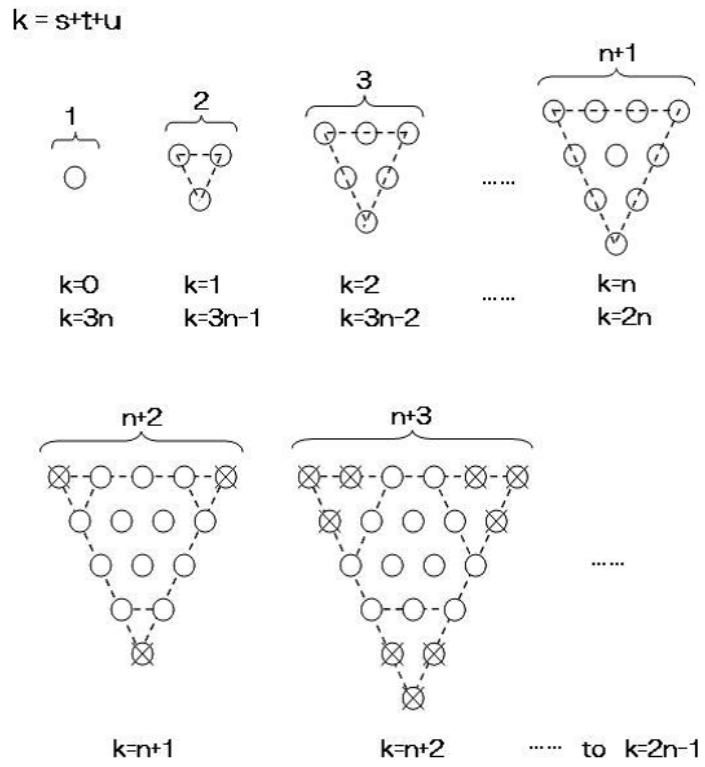
Theorem. Let n be a positive integer and let T_n denote the nth triangular number. Then the coefficient of x^k for the expansion of $(1+x+x^2+\cdots+x^n)^3$ is decided as follows.

$$\begin{cases} T_{k+1} & (0 \le k \le n) \\ T_{k+1} - 3T_{k-n} & (n+1 \le k \le 2n-1) \\ T_{3n-k+1} & (2n \le k \le 3n) \end{cases} \tag{0.1}$$

Proof.

$$\left(\sum_{i=0}^{n} x^i\right)^3 = \sum_{s=0}^{n}\sum_{t=0}^{n}\sum_{u=0}^{n} x^{s+t+u}$$





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