# Electro – Mechanical contributions to low frequency dielectric responses of biological cells in Colloidal Suspension.

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## ABSTRACT

We investigate electro-mechanical contributions to the low frequency dielectric response of biological cells in colloidal suspension. Prior simulations of biological cells in colloidal suspension yield maximum dielectric constant values about $10^3$ in magnitude as the frequency of applied electric fields drops below the kHz range. Experimentally measured relative dielectric values in yeast cells , on the other hand, have maximal values up to $10^7$ -$10^8$. We consider both electrical and mechanical energy stored in cellular suspension and show that low frequency mechanical contributions can give rise to dielectric constant values of this magnitude.

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## I. Introduction

Biological cells in colloidal suspension are often modeled as having primarily electromagnetic interactions with an external ac electric field. Except for electro-rotation, there has been no discussion of mechanical effects in the $\alpha$ dispersion range. Prior numerical simulations [2] used formalism appropriate for $\beta$ dispersion effects, i.e. Maxwell – Wagner based dispersion models [1] Experimental values for the low frequency differ from what is predicted using Maxwell-Wagner [3] based calculations. We argue that this discrepancy is due to electro – mechanical effects which are not significant in higher frequencies because of inertial effects. We show that the mechanical contributions in the $\alpha$ dispersion range can result in effective dielectric constant values up to $10^7$ - $10^8$ , whereas $\beta$ dispersion effects only give maximum dielectric constant values of about $10^3$ in magnitude. In this paper we examine how electromechanical rotation of cells can contribute to a more realistic dielectric models of cells in colloidal suspension .

## II. Model

Experimentally , it is found that the complex dielectric constant $\varepsilon^*$ for N cells in a colloidal suspension of volume V has distinct dispersion regions denoted by $\alpha$, $\beta$, and $\gamma$. One of the most recent models for complex dielectric response [5] , is given by :

$$\varepsilon^* = \varepsilon_\infty + \frac{\Delta\varepsilon_{cell}}{1 - i\cdot\omega\cdot\tau_{cell}} + \frac{\Delta\varepsilon_{susp}}{1 - i\cdot\omega\cdot\tau_{susp}} - \frac{i\cdot\sigma}{\omega} \qquad (2.1)$$

$\varepsilon_\infty$ is a very high frequency contribution to the dielectric , and is about ten to twenty hertz in value. This is for $\gamma$ dispersion and simply is ignored in $\beta$ and $\alpha$ dispersion regimes when we consider lower frequency dispersion effects. The second and third terms are for $\beta$ dispersion and have a real valued magnitude of about $10^3$ which is in turn negated when we look at the real part of the fourth ( last ) term due to $\alpha$ dispersive effects with a real valued magnitude of about $10^8$ in upper value. The $\Delta\varepsilon_{cell}$ term represents the magnitude of $\beta$ dispersion effects due to cells , while $\Delta\varepsilon_{susp}$ is the magnitude of $\beta$ dispersion effects due to the fluid the cells are in suspension, in. Usually $\Delta\varepsilon_{cell} >> \Delta\varepsilon_{susp}$ and $\Delta\varepsilon_{cell}$ is about $10^3$ in upper value. In addition, we should look at the $\tau_{cell}$ as a relaxation time parameter for cell dispersion processes, and $\tau_{susp}$ is a relaxation parameter for fluid medium dispersion processes. Given that the $\beta$ dispersion effects occurred in frequencies between $10^5$ and $10^7$ , a top relaxation time of about $10^{-7}$ for $\tau_{cell}$ with $\tau_{susp} > \tau_{cell}$ .

Equation 2.1 is empirical. We should note that if we have no imaginary part in equation 2.1 that we no longer have dissipation of the applied electric field energy into

this suspension. The two $\beta$ dispersion terms are due to the Maxwell- Wagner relationship and represent a spatial mixing of dielectric regions of cells with the suspension material the cells are in. When we look at the first high frequency dielectric term, i.e. the $\gamma$ dispersion , 'Debye relaxation ' of molecular dipoles . We should note that in equation 2.1 that for our problem ( low frequency applied electric fields) ) that clearly the $\alpha$ term is the most important. However, we should note that our subsequent derivation will be to fill in details for this $\alpha$ term in terms of known physical processes which affects the cells in colloidal suspension.

We should begin by stating that our new model will be constructed by considering how the cells in colloidal suspension, plus the surrounding medium has a total energy expression which we may give as :

$$W_{Total} = W_r + W_{cell} + W_{med} \tag{2.2}$$

which we can write up as in a different form as looking like :

$$\varepsilon_{sus} \cdot \int E_{ap}^2 \cdot dV = \varepsilon_{cell} \cdot \int_{cell} E_{cell}^2 \cdot dV + \varepsilon_{mech} \cdot \int_{mech} E_{mech}^2 \cdot dV + N \cdot I \cdot \omega^2 \tag{2.3}$$

We have that the total energy of the biological system modeled in equation 2.2 has its dominant energy contribution given by

$$W_r = \frac{1}{2} \cdot N \cdot I \cdot \omega^2 \tag{2.4}$$

Also,

$$W_{cell} = \frac{1}{2} \cdot \varepsilon_{cell} \cdot \int_{cell} E_{cell}^2 \cdot dV \tag{2.5}$$

and

$$W_{med} = \frac{1}{2} \cdot \varepsilon_{med} \cdot \int_{med} E_{med}^2 \cdot dV \tag{2.6}$$

we can change equation 2.3 to the expression:

$$\varepsilon^*_{susp} = \varepsilon^*_{cell} \cdot p + \varepsilon^*_{med} \cdot (1-p) + \frac{N \cdot I \cdot \omega^2}{\left|E_{ap}\right|^2 \cdot V_{Tot}} \tag{2.7}$$

where $\frac{N \cdot I \cdot \omega^2}{\left|E_{ap}\right|^2 \cdot V_{Tot}}$ is for $\alpha$ dispersion effects and is of the order of $10^8$ whereas we have $\varepsilon^*_{cell} \cdot p + \varepsilon^*_{med} \cdot (1-p)$ for $\beta$ and $\gamma$ dispersion effects and has $\varepsilon_{med} \equiv \varepsilon_{water} \approx 80$ with an upper bound value of the order of $10^3$ . We shall now explicitly show how equation 2.7 actually is from equation 2.2. To do this , note that

$$\varepsilon_{sus} \propto \frac{N \cdot I \cdot \omega^2}{\left|E_{ap}\right|^2 \cdot V_{Tot}} \tag{2.9}$$

whereas we can make the following approximations :

$$\frac{\varepsilon_{cell} \cdot \int_{cell} E^2_{cell} \cdot dV}{\int E^2_{ap} \cdot dV} \approx \frac{\varepsilon_{cell} \cdot \left\langle E^2_{cell} \right\rangle \cdot V_{cell}}{\left\langle E^2_{ap} \right\rangle \cdot V_{Tot}} \approx \frac{\varepsilon^*_{cell} \cdot \cdot V_{cell}}{\cdot V_{Tot}} \approx \varepsilon^*_{cell} \cdot p \tag{2.10}$$

and

$$\frac{V_{cell}}{V_{Tot}} = \cdot p \tag{2.11}$$

similarly we have that

$$\frac{\varepsilon_{med} \cdot \int_{cell} E^2_{med} \cdot dV}{\int E^2_{ap} \cdot dV} \approx \frac{\varepsilon_{med} \cdot \left\langle E^2_{med} \right\rangle \cdot V_{med}}{\left\langle E^2_{ap} \right\rangle \cdot V_{Tot}} \approx \frac{\varepsilon^*_{med} \cdot \cdot V_{med}}{\cdot V_{Tot}} \approx \varepsilon^*_{med} \cdot (1-p) \tag{2.12}$$

We should take into consideration that

$$\varepsilon^*_{cell} = \frac{\left\langle E^2_{cell} \right\rangle}{\left\langle E^2_{app} \right\rangle} \cdot \varepsilon_{cell} \tag{2.13}$$

and

$$\varepsilon^{*}_{med} = \frac{\left\langle E^{2}_{med} \right\rangle}{\left\langle E^{2}_{app} \right\rangle} \cdot \varepsilon_{med} \qquad (2.14)$$

refer to mean effective dielectric values of individual cells and the medium the cells are in suspension. . $V_{total}$ = total volume of space between the two capacitor plates . Frequently, we have that $.01 \leq p \leq .1$.. Here, $I$ is the moment of inertia of an individual cell. For the sake of convenience, we shall assume that the cells are nearly spherical. If so then we will write:

$$I \cong \frac{2}{5} M_{cell} \cdot a^{2}_{cell} \equiv \frac{2}{5} \cdot \rho_{cell} \cdot V_{cell} \cdot a^{2}_{cell} \qquad (2.15)$$

$\rho_{cell}$ refers to the net density of biological cells assumed, and $a_{cell}$ refers to the average radius of a biological cell. We can then obtain a general expression for cell values, i.e.

$$\varepsilon_{sus} \cong \frac{2}{5} \cdot \frac{N \cdot \rho_{cell} \cdot a^{2}}{\varepsilon_{0}} \cdot \left( \frac{\omega_{cell}}{E_{0}} \right)^{2} \qquad (2.16)$$

We shall now attempt to make a general derivation of $\omega_{cell}$ so as to give a detailed experimentally accessible formulation of how angular velocity of a cell influences formation of actual dielectric values, using equation 2.7 above.

## III. Rotational Spectra of Biological Cells in Electric Field

We are , here , setting up a time independent average value of the frequency of rotation ( actually the angular velocity ), which we will call $\langle \omega_{cell}^2(t,\theta) \rangle$ which is a spatial and time averaged quantity. In order to do this, we will set up a relationship between a polarization vector with regards to net charge in the cell, and the external electric field impinging upon the cell, to get a net torque, and then from there to set up a differential equation relating the net torque with angular velocity, cell moment of inertia, and an added damping coefficient we will call $D$ in order to set up a general expression for $\omega_{cell}(t,\theta)$ . This is then time wise and spatially averaged so as to obtain $\langle \omega_{cell}^2(t,\theta) \rangle$ which is then placed into equation 2.7 in place of the simple expression $\omega_{cell}^2$ in equation 2.7. We shall then compare this expression for $\langle \omega_{cell}^2(t,\theta) \rangle$ with rotational spectra from the research literature more appropriate for $\beta$ dispersion effects , and then use this new rotational spectra we derived to obtain $\varepsilon_{cell}$ values more in sync with known experimental values , so we can obtain Re $\varepsilon^*$ $\cong$ $10^7$ to even $10^8$ in magni
tude as the frequency of the applied AC electric field goes down to one hertz in value ( for $\alpha$ dispersion effects ).

Let us examine an external field torque upon a cell, with an equation of :

$$\tau_e(t) = -E_0 \times p \qquad (3.1)$$

Here, we have that we have an external electric field $E_0$ which is at a given angle $\theta$ with respect to a dipole moment $p$ of the cell in colloidal suspension. We shall be comparing this torque with moment of inertia of the cell times the time derivative of rotational frequency plus an additional term which is composed of damping coefficient $D$ times

the rotational frequency of the cell. A critical assumption for making this work is that the frequency of rotational movement of the cell , $\omega_{cell}$ , is far smaller than that of the applied AC electric field, $\omega_0$ . If we do this, we have that any time we can set $\theta_{cell} \equiv \theta_0 \exp(\tilde{p}t)$, with $\tilde{p} = q + i \cdot \omega$ we may treat the frequency of the cell as a different quantity than the frequency of the applied electric field. Also ,if $\omega_{cell} << \omega_0$ , we can set $\omega_{cell} = \frac{d\theta_{cell}}{dt}$ and

$$I \cdot \ddot{\theta}_{cell} + D \cdot \dot{\theta}_{cell} = \tau(t) \cdot \sin\theta$$

becomes

$$I \cdot \dot{\omega}_{cell} + D \cdot \omega_{cell} = \tau_c(t, \theta) \qquad (3.2)$$

Should we be not be making this assumption, we would be writing,

$$\frac{d^2\theta}{dt^2} + \frac{D}{I} \cdot \frac{d\theta}{dt} + \frac{\tau_c}{I} \cdot \theta \cong \frac{\tau_c}{I} \cdot \left( \frac{\theta^3}{3!} - \frac{\theta^5}{5!} + \frac{\theta^7}{7!} - ..... \right) \qquad (3.3)$$

This assumes that $\omega_{cell} \approx \omega_0$ in a resonance condition. We are assuming otherwise , here. Equation 3.3 is a de facto driven harmonic oscillator problem with the r.h.s. being a driving force. Now, the right hand side of equation 3.2 has a very different , almost independent angular dependence from the left hand side, primarily because we set $\omega_{cell} << \omega_0$. And :

$$\dot{\omega}_{cell} + \frac{D}{I} \cdot \omega_{cell} = \frac{\tau_c(t, \theta)}{I} \qquad (3.4)$$

This has a general solution ; if we set $\tau_c(t) = \tau_0(\theta) \cdot \exp(-i \cdot \omega_0 \cdot t)$ :

$$\omega_{cell} = \frac{\tau_0}{I} \cdot \frac{\exp(-i \cdot \omega_0 \cdot t)}{F + i \cdot \omega_0} \qquad (3.5)$$

$$\mathrm{Re}\,\omega_{cell}(t) = \frac{\tau_c \cdot \left[ F \cdot \cos(\omega_0 \cdot t) + \omega_0 \cdot \sin(\omega_0 \cdot t) \right]}{I \cdot \left[ F^2 + \omega_0^2 \right]} \tag{3.6}$$

where we are setting $F = \frac{D}{I}$

Furthermore, we set

$$\tau_c = D \cdot \omega \tag{3.7}$$

Here, we may take the time average of the square of equation 3.5 to get:

$$\left\langle \omega_{cell}^2(t) \right\rangle = \frac{\left\langle \tau_c^2 \right\rangle}{I^2} \cdot \frac{1}{\left[ F^2 + \omega_0^2 \right]} \tag{3.8}$$

where we will find spatial averaging of $\left\langle \tau_c^2 \right\rangle$ in the following manner :

First , a ring of cell shell space an angle $\theta$ from an axis of rotation of the cell, with a radius distance a from the center of the sphere , and a thickness of the shell as $d\theta$ leads to a net torque on that particular shell of the cell we may write as :

$$\tau_{ring} = 2 \cdot \pi \cdot \frac{a^3 \cdot \sin^3\theta \cdot \omega}{\lambda} \cdot d\theta \cdot \eta \cdot \tag{3.9}$$

We have that $\lambda$ is the thickness of the 'shell' . For our purposes, we set $\lambda = .2 \cdot a$ . Also we have that $\frac{a \cdot \sin\theta \cdot \omega}{\lambda}$ = gradient of the 'velocity' of the ring 'surface' of the cell, and that the surface area of the 'ring' is given by $2 \cdot \pi \cdot a^2 \cdot \sin\theta \cdot d\theta$ . Also, the

viscosity of the ring as its net ‘ friction’ with respect to the medium the cell is in colloidal suspension with is given by $\eta$ . Here, by dimensional analysis, we have that $\eta = A \cdot \frac{\Delta v}{\lambda}$ . The area $A$ is times a net velocity change, divided by the assumed shell thickness of the cell. . We can then get , if we integrate over the entire sphere , a total torque of

$$\langle \tau_{cell} \rangle = 2 \cdot \pi \cdot \frac{a^3}{\lambda} \cdot \eta \cdot \omega \cdot \int_0^{\pi} \sin^3 \theta \cdot d\theta = \frac{8}{3} \cdot \pi \cdot \frac{a^3}{\lambda} \cdot \eta \cdot \omega \qquad (3.10)$$

So, if we have

$$\langle \tau_{cell} \rangle = D \cdot \omega \qquad (3.11)$$

and

$$F = \frac{D}{I} \qquad (3.12)$$

we shall put these last four equations directly into equation 3.8 above. In doing so, we may then use the net cell frequency as given by equation 3.8 as a function of an AC electric field wave frequency $\omega_0$ and then go directly from this to construct how we measure the cell dielectric constant value as given by equation 2.7 with the cell frequency behavior given by equation 3.8. In an example given in our next section, this leads to quite high cell dielectric values, and an overall value of equation 2.3 at least four orders of magnitude higher than given by prior numerical simulations , for peak values of biological cells in colloidal suspension in the low end of applied AC electric field frequencies ( approaching actual measured dielectric experimental values in the process ).

We will, to help our visualization of the examples given in the next section refer to explicit formulations of dipole moment of a cell, torque, and the damping coefficient, $D$ ,of a cell experiencing electromotive rotation in a fluid. These will lead to a dimensional analysis description of a general coefficient for dielectric values of a cell , which will experience specific functional variation of parameters leading to answering such question as when we can expect an inflection point , signifying the onset of $\alpha$ dispersion effects when we plot cell dielectric constant values as a function of the frequency of an applied electric field to biological cells in colloidal suspension.

As an example, we derived , for a general dipole moment of the cell

$$P\left(r=a,\tilde{\alpha}\right)\equiv\frac{3}{4}\cdot\pi\cdot a^{3}\cdot\left(1-\cos\frac{\tilde{\alpha}}{2}\right)\cdot\frac{\left(1+\cos\frac{\tilde{\alpha}}{2}\right)^{2}}{\left(2+\cos\frac{\tilde{\alpha}}{2}\right)} \tag{3.13}$$

where $\tilde{\alpha}$ is the angle we can make from the center of a spherical cell to the region of surface charge which is either positive or negative.

$$\tilde{\alpha}\left(n\right)=n\cdot\frac{\pi}{180} \tag{3.13a}$$

We found it useful to set $\tilde{\alpha}\left(n\right)=n\cdot\frac{\pi}{180}$ in order to represent the range of the angles of a cone facing the charged area on the surface of the cell. In addition, we have that we can conveniently write a cell torque expression as:

$$\tau_{c}\left(\theta,a,\tilde{\alpha}\right)=\frac{1}{2}\cdot P\left(r=a,\tilde{\alpha}\right)\cdot\sin\theta \tag{3.14}$$

where angle $\theta$ is between the applied electric field to the cell and the net dipole value written up in equation 3.13. And, $a=$ cell radius which can be varied as one sees fit.

Furthermore we have that the rotational velocity of the cell has a counter acting 'drag' factor we can write up as our damping coefficient, namely

$$D(r) = \frac{8}{3} \cdot \pi \cdot \frac{r^3}{\lambda} \cdot \eta \tag{3.15}$$

so if we wrote individual cell volume represented by

$$V_{IndC}\ (r) = \frac{4}{3} \cdot \pi\ \cdot (1 + \lambda)^3 \cdot r^3 \tag{3.16}$$

as well as consider the cell individual moment of inertia we can represent by

$$I(r) = \frac{2}{5} \cdot \rho \cdot V_{IndC}(r) \cdot (1+\lambda)^2 \cdot r^2 \tag{3.17}$$

which lead to us determining where the initial frequency drops in half , i.e. about half way after we have $\alpha$ dispersion start:

$$f_D(r) = \frac{D(r)}{2 \cdot \pi \cdot I(r)} \tag{3.18}$$

which as we will see in the next section varies wildly as we change the radius of cells in the colloidal suspension . Furthermore, we have that we may set up a maximum value for the cell dielectric constant which is dependent upon the radius of the cell and the angle $\tilde{\alpha}$ which is measuring the impact of charge distribution on the cell ends, i.e. :

$$\varepsilon_{\max-cell}(r, \tilde{\alpha}) = \left( \frac{2}{5} \cdot \frac{\rho \cdot r^2}{\varepsilon_0} \right) \cdot \tau_c^2 \left( \theta = \frac{\pi}{2}, r, \tilde{\alpha} \right) \cdot (D(r))^{-2} \tag{3.19}$$

Furthermore , with the above formalism set up, we can re write equation 3.8 as, then,

$$\omega_{cell}^2\left(f,r,\tilde{\alpha}\right)=\frac{\tau_c^2\left(\theta=\frac{\pi}{2},r,\tilde{\alpha}\right)}{D^2(r)\cdot\left(1+\left(\frac{f}{f_D}\right)^2\right)} \tag{3.20}$$

Here, the variable $f$ refers to the frequency of the applied AC electric field impinging directly upon the cell in colloidal suspension. We also can take this angular cellular velocity and then put it directly into a given dielectric constant of the cell , as

$$\varepsilon_{cell}\left(f,r,\tilde{\alpha}\right)=\frac{2}{5}\cdot\frac{\rho\cdot r^2}{\varepsilon_0}\cdot\omega_{cell}^2\left(f,r,\tilde{\alpha}\right) \tag{3.21}$$

**IV. Basic results from the above relationships of section III.**

We should now discuss some of the basic implications of our model and what we can expect experimentally, if the following predictions are true. First of all, we managed to find a way to duplicate the curve for $\alpha$ dispersion as a function of AC applied electric field frequency rates. This is assuming $\tilde{\alpha}=33.3^0$ for the angle of a cone facing the charged area of the surface of the cell and a cell radius of 10 microns in value. Then, we can show how we can expect the dispersion relationship involving frequency plotted against net cell dielectric values. This dispersion curve matches almost exactly the experimental condition, for the $\alpha$ dispersion region, except that the maximum value of a colloidal suspensions would be by necessity about $10^7$, if we assume a very dense colloidal mix of cells with fluids with $p\cong.1$ .Still though, this is four orders of magnitude greater than simulations given which are primarily with peak dielectric values of about $10^3$ for cells in a colloidal suspension. This means that formula 3.19 has some direct applicability . I.e. a classical model actually may work .

Next, we managed to predict the relative mid – point of $\alpha$ dispersion by determining how the dielectric value drops to half in a plot of dielectric values versus applied electric field frequency values . Interestingly enough, we found that if we fixed $\tilde{\alpha} = 33.3\%$ in our value of $f_D(r)$ as given in equation 3.16 and varied the radius of the cell, that we observed that the $\alpha$ dispersion inflection point occurred at a high point of about $10^3$ Hertz for cell radius about four microns in radius value, to a low of about 7-8 Hertz for cell radius approximately about 30 microns in radius.

We next observed how the maximum dielectric value of a cell dielectric is affected by an increase in $\tilde{\alpha}$ values. Unsurprisingly, if the $\tilde{\alpha}$ angle increases, which indicates a spreading out of charges on the surface of a cell, we have that the maximum possible value of dielectric constant of the cell increases. We also see in that as the radius of the cell increases that there is a monotonic increase in the cell dielectric constant. This is attributable to how polarization in the cell is affected by charge mobility on the surface as well as other effects, potentially one of them being flexoelectric variations of the cell membrane [6] in ways affecting the distribution of charges on the surface region of the cell.

Finally, we have an idea of how an increase in cell radius size ( $\mu m$ ) will affect maximum dielectric values for cells in colloidal suspension, if we have low frequency values for the applied AC electric field. The main point is that if the angle $\tilde{\alpha}$ increases due to a less pronounced , or at least a less focused dipolar charge concentration in the biological cell, that the net maximum dielectric value of the cell decreases. Also, as the

cell size INCREASES, we also see a drop in dielectric response .This most likely can be interpreted as a geometric measure of how polarization affects dielectric values. A full simulation of the processes inherent is this will probably require sophisticated finite element modeling of piezoelectric phenomena in biological cells..Ulrich Zimmerman et al [7] in 1998 gave a more typical representation of an electro rotational spectra of biological cells in colloidal suspension. We shall write it here and compare it with what we wrote for equation 3.7 above. In addition, we shall also refer to some issues affecting the onset of electrorotation which will be to show how non uniform charge distribution in cell structure will lead to torque allowing us to consider a rotational model along the lines we wrote above .We should note that the electro rotation we are working with is not the same as discussed by prior authors. Note that Ulrich Zimmerman et al in 1998 wrote the 'typical' electrorotational spectra of biological cells as linked to ' to their effective polarizability $(f_{CM})$ ' via

$$\Omega = -\frac{\varepsilon_e \cdot E^2 \,\mathrm{Im}\left( f_{CM} \right)}{2 \cdot \eta} \qquad (4.1)$$

where $(f_{CM})$ is the Clausius-Mosotti factor which is for $\beta$ dispersion effects.. This is simply not useful in lower frequency $\alpha$ dispersion which is what our problem is doing. There is an additional problem, that electrorotation normally would need a four electrode system to split an AC electric field into four 'signals of particular phase relations'. We will answer this by saying that if the cells were perfectly spherical with symmetric distribution of charges and with a net polarization parallel ( anti parallel ) to the direction of an applied electric field to the colloidal suspension of biological cells,

that there would be no torque and hence no spin to the cells in colloidal suspension. This simply does not happen. Numerous effects tend to keep cells non uniform in both distribution of charges, and cell shape. This is why a net torque was used in the derivation of $\left\langle \omega_{cell}^2(t,\theta) \right\rangle$ . The assumption of spherical cells was used to simply what would otherwise be a messy calculation of moment of inertia. In addition, non spherical cell calculations of moment of inertia will only change the value by less than an order of magnitude.

## V. Conclusion

Asami [2] and other authors actually calculated realistic dielectric values for cells in colloidal suspension for the $\beta$ region of dispersion values. Those papers correctly calculate the *electromagnetic* contribution to the low frequency dielectric constant (as well as conductivity!). But cell anisotropies and inhomogeneities result in a polarization vector that is not parallel to E. Note in our calculation we assumed that P was PERPENDICULAR to E. Of course this represents and extreme case and in general the angle between P and E can vary. So our model is somewhat idealized. Also we need to mention in the discussion that brownian motion and the elastic energy stored in some cells may also give a significant contribution to the low frequency dielectric constant. (The elastic contribution may be significant in tissue for example) Also we hope that this work will motivate experiments to investigate mechanical contribution to the dielectric response in the alpha range

Our paper gives a useful start in outlining the importance of what we refer to as electromechanical effects in the calculation of a net dielectric value $\varepsilon_{sus}$ when we are

considering when we have applied an electric field to cells in suspension in the low hertz limit for $\alpha$ dispersion effects. This approach gives order of magnitude agreement with some experimental data sets. . One paper actually claims to be able to link both $\alpha$ and $\beta$ dispersion [8], by use of charge mobility. This may be appropriate for some biological systems, but it neglects what we think is an unexplored effect which has been seen experimentally. . Another paper [9] is interesting, but is heavily weighed toward adjustment of what they call geometrical parameters in order to obtain dielectric values for biological cells considerably below our maximal values. Both of these mentioned approaches have been extensively utilized in $\beta$ dispersion , but do not make sense when very low frequency AC electric fields are applied to biological cells in colloidal suspension. Additional work needs to be done to consider a range of effects , i.e. possible interaction effects between biological cells in low frequency AC electric fields . However, we believe that the methodology outlined is a necessary beginning to start a systematic investigation of $\alpha$ dispersion effects with biological cells .

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