A Proposal for a Unified Field Theory

Bruce Rout

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Abstract

A proposal outlining an approach to a unified field theory is presented. A general solution to the time-dependent Schrödinger Equation using an alternative boundary condition is found to derive the Heisenberg uncertainty formulae. A general relativity/quantum mechanical interaction between a photon and a gravitational field is examined to determine the degree of red shifting of light passing through a gravitational field. The Einstein field equations, complete with an arrangement of Faraday tensors, are presented with suggestions to determine the energy of a photon from Einstein's and Maxwell's equations. Schrödingers Equation is coupled with both the Einstein field equations and Maxwells equations to derive a possible foundation for string theory.

1 Derivation of Heisenberg's Uncertainty Equations from a General Solution to the Time Dependent Schrödinger Equation

Consider any particle with mass m and negative charge resulting in a potential energy V which obeys the following:

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\nabla^2\psi - V\psi \tag{1}$$

which is derived from:

$$-\hbar^2\nabla^2\psi = \mathbf{p}^2\psi \tag{2}$$

where $\mathbf{p}$ is the momentum of the particle. It is well known that if there are infinite or semi-infinite boundary conditions, the above equation is commonly solved through a singularity solution. We reject assuming infinite boundary conditions and present a well established and common method, known as the separation of variables, to find a general solution in a finite domain following which the boundary condition becomes obvious.

Rewriting the differential equation we have:

$$i\hbar\frac{\partial}{\partial t}\psi = -\frac{\hbar^2}{2m}\left\{\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right\}\psi - V\psi \tag{3}$$

Let:

$$\psi = T(t)X(x)Y(y)Z(z) \tag{4}$$

where, $T(t)$ is a function of t only, $X(x)$ is a function of x only, $Y(y)$ is a function of y only and $Z(z)$ is a function of z only. Then:

$$i\hbar\frac{\partial}{\partial t}TXYZ = -\frac{\hbar^2}{2m}\left\{\frac{\partial^2}{\partial x^2}TXYZ + \frac{\partial^2}{\partial y^2}TXYZ + \frac{\partial^2}{\partial z^2}TXYZ\right\} - VTXYZ \tag{5}$$

Under the condition that $\psi \neq 0$ we can divide through by $TXYZ$ to yield:

$$i\hbar\frac{T'}{T} = -\frac{\hbar^2}{2m}\frac{X''}{X} - \frac{\hbar^2}{2m}\frac{Y''}{Y} - \frac{\hbar^2}{2m}\frac{Z''}{Z} - V \tag{6}$$

We can see that each term is linearly independent. Since each term is being varied by its independent variable and all variables are linearly independent from each other, and the constant term is also independent from the others, each term must equal a constant. Therefore:

$$i\hbar\frac{T'}{T} = -\alpha^2 \tag{7}$$

$$\frac{\hbar^2}{2m}\frac{X''}{X} = -V - \beta^2 \tag{8}$$

$$\frac{\hbar^2}{2m}\frac{Y''}{Y} = -\gamma^2 \tag{9}$$

$$\frac{\hbar^2}{2m}\frac{Z''}{Z} = -\xi^2 \tag{10}$$

where α, β, γ and ξ are constants, and the equation has been separated. We have placed the constant term, $-V$, with equation 8 since it has been chosen as the direction of travel of the particle.

Then:

$$X = \cos\left(\frac{\sqrt{2m(V+\beta^2)}}{\hbar}x\right) \tag{11}$$

This is, in a way, similar to a term in a Fourier series. We consider a slight re-write as:

$$X = \cos\left(\frac{2\pi\sqrt{2m(V+\beta^2)}}{\hbar}\frac{x}{2\pi}\right) \tag{12}$$

Consider the boundary condition of $X = 1$ which occurs when:

$$x = \frac{2\pi\hbar}{\sqrt{2m(V+\beta^2)}} \tag{13}$$

and

$$x\sqrt{2m(V+\beta^2)} = h \tag{14}$$

since

$$\mathbf{p}^2\psi = -\hbar^2\frac{\partial^2\psi}{\partial x^2} \tag{15}$$

and using boundary condition ...

$$\psi = 1 \tag{16}$$

we get

$$\mathbf{p}^2 = -\hbar^2\frac{\partial^2\psi}{\partial x^2} \tag{17}$$

We also have

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = V + \beta^2 \tag{18}$$

yielding

$$-\hbar^2\frac{\partial^2\psi}{\partial x^2} = 2m(V+\beta^2) \tag{19}$$

Substitution yields:

$$x\mathbf{p} = h \tag{20}$$

at the boundary of the particle. However the "angle" within the cosine goes from 0 to 2π and therefore we have a measure of Δx. Because x varies between the boundaries we have a non-constant $\mathbf{p}$. We therefore have:

$$\Delta x\Delta\mathbf{p} = h. \tag{21}$$

Note that any boundary condition other than $\psi = 1$ substituted into equation 15 invalidates the previous equation.

We would like to mention here that the boundary would yield a "probability" of one for the particle should ψ represent probability. Inside this boundary this probability would be less than one. At the "centre" of the particle, the probability would be -1 and this is absurd. In the derivation of a solution we had said $\psi \neq 0$. So we will deny the particle to exist inside the boundary and, for that matter, outside the boundary as well. For this particular solution to stand, the particle only exists where $\psi = 1$ and does not exist otherwise. We are stating that the particle does not exist when $\psi < 1$. This is a different case than determining the position or time of the particle. In this case we are determining the existence of the particle itself. We are postulating that if ψ is less than one, then it isn't. We conclude ψ cannot be a measure of probablity. It is a potential. When the potential is 1, the particle exists. From these considerations, the particle can only exist at it's boundary.

From outside the particle we can only measure to an accuracy of:

$$\Delta x \Delta \mathbf{p} \geq h \tag{22}$$

With the time ordered factor, we have an exponential of $i\frac{\alpha^2}{\hbar}t$. Let us now consider α. We note the units of measure. We see that $\hbar$ is in units of joules-sec. We see that t is in seconds and will cancel the time unit of $\hbar$ leaving joules in the denominator. Hence, since the exponential must be unitless, α^2 is in units of joules. To continue the discussion allow α^2 to be some unknown form of energy in joules. We will examine what this means as follows.

Let

$$E = \alpha^2 \tag{23}$$

so the exponential of the time ordered factor becomes

$$\frac{iEt}{\hbar} \tag{24}$$

and we look at the situation where $\psi = 1$. In other words, the particle definitely exists. We have seen that at the boundary, from before, the spacial ordered factor is one. Therefore the time ordered factor is also one for a time ordered boundary. This can only occur should the exponent of the time ordered factor be something like $2\pi i$. In which case we have:

$$\frac{iEt}{\hbar} = 2\pi i \tag{25}$$

rearranging

$$Et = 2\pi\hbar \tag{26}$$

$$Et = h \tag{27}$$

Here we have time going from 0 to some cyclic value yielding an exponent of $2\pi i$. We will then denote this as Δt and ΔE is the magnitude of fluctuation of energy. We now have:

$$\Delta E \Delta t = h \tag{28}$$

and observing from outside the particle in the time dimension, we can only measure to an accuracy of:

$$\Delta E \Delta t \geq h \tag{29}$$

This happens outside some time ordered “boundary” where/when the potential of the existence of the particle yields $\psi = 1$. Combining both time and spacial ordered factors we have the situation where any measurement of the time, location, momentum or energy of the particle must obey the following;

$$\Delta x \Delta \mathbf{p} \geq h \tag{30}$$

and

$$\Delta t \Delta E \geq h \tag{31}$$

because that is determined by the boundary conditions of any particle adhering to Schrödinger’s equation. Since this has been validated by an overwhelming amount of experimental and, now, theoretical evidence, we propose that the Heisenberg Uncertainty Postulate be classified as a theory.

Let us take a closer look at E.

The exponent of the time ordered factor is some sort of phase angle that allows the particle to have a potential of existence equal to one at each cycle.

Let:

$$\frac{E}{\hbar} t = \theta \tag{32}$$

And we differentiate by t on each side to yield:

$$\frac{E}{\hbar} = \frac{d\theta}{dt} \tag{33}$$

or

$$\frac{E}{\hbar} = \omega \tag{34}$$

$$E = \hbar\omega \tag{35}$$

and,

$$E = h\nu \tag{36}$$

So, this energy, E, is not a form of energy coming from the mass of the particle or it’s momentum of motion or even it’s charge generating V. It appears to be an energy that is associated with the time ordered frequency of the particle’s existence. This energy is not associated with mass or charge.

Let us examine α further.

$$\alpha = \frac{\sqrt{2\pi i \hbar}}{\sqrt{t}} \tag{37}$$

and

$$\alpha = \frac{1}{\sqrt{2}} \left(\sqrt{\frac{h}{t}} \right) (1 + i) \tag{38}$$

and it seems that with the presence of $\sqrt{2}$ there is some indication of spin involved.

Continuing, we see that we can also say:

$$\theta_n = n^2 2\pi i, \ n \in \mathbb{N} \tag{39}$$

whenever $\psi = 1$. So this exponential has been quantized by n^2. This can be compared to an orthogonal set of eigenfunctions yielding a complete solution of ψ. There are interesting consequences to the general solution of Schrödinger's equation. We call α an eigenvalue in an eigenspace which we often use to find general solutions. Apparently α^2 is the energy of a photon. We are proposing that the magnitudes of an infinite number of eigenvalues to the general solution of Schrödinger's equation yield the energy values of an infinite number of subatomic particles. The first order temporal eigenvalue yields the energy of a photon and perhaps some value of spin.

From the behaviour of this class of differential equation, ψ can be considered as a conserved scalar potential field. Since electromagnetic radiation can be thought of as a moving disturbance within a scalar potential field, and this field is conserved, there is a slight alteration in the surrounding potential field should any disturbance move through it. We believe there is the possiblility that a bundle of rapidly fluctuating electromagnetic fields moving at the speed of light, commonly known as a photon, would behave as though it had a very small gravitational field. We investigate this possibility by examining the Einstein field equations.

2 The Field Equations

Consider an equation which partially comes from Minkowski and also quoted by Einstein [7]:

$$G_{\mu\nu} = 8\pi T_{\mu\nu} - F_{\mu\alpha} F_{\nu}{}^{\alpha} + \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \tag{40}$$

From equation 40 we denote $T_{\mu\nu}$ as a material stress energy tensor and the Faraday tensor terms as a field stress energy tensor.

We can see that if there is no charge present we have the formula:

$$G_{\mu\nu} = 8\pi T_{\mu\nu} \tag{41}$$

and we have the usual Einstein Field Equations.

Should there be no mass, but charge is present, we have:

$$G_{\mu\nu} = -F_{\mu\alpha}F_{\nu}{}^{\alpha} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \tag{42}$$

Equation 40 is the complete field equation resulting from the presence of both mass and charge in boundless space. Equation 41 is the gravitational field equation and equation 42 describes spacetime under Maxwell's Equations. Note that it was probably Minkowski who developed the tensor equation for Maxwell's electromagnetic theory and Eistein developed the tensor equation for gravity. For the sake of clarity, allow:

$$\Omega_{\mu\nu} = -F_{\mu\alpha}F_{\nu}{}^{\alpha} + \frac{1}{4}g_{\mu\nu}F_{\alpha\beta}F^{\alpha\beta} \tag{43}$$

and

$$\Xi_{\mu\nu} = 8\pi T_{\mu\nu} \tag{44}$$

So we have:

$$G_{\mu\nu} = \Xi_{\mu\nu} + \Omega_{\mu\nu} \tag{45}$$

and consider the following situation.

In the case of a photon passing by the sun, the mass of the sun yields the material stress energy tensor already described. The fluctuations of the electromagnetic fields from the photon have to be derived from equation 40. The $\Omega_{\mu\nu}$ tensor is a microscopic view of the actions of a stationary object having electrostatic charge and magnetic properties. However, to describe the photon itself from these equations we need to take a macroscopic average of the sress energy generated by the $\Omega_{\mu\nu}$ tensor.

A photon is a region of rapidly fluctuating electric and magnetic fields moving at the speed of light. Overall, there is a stress-energy tensor within the region of the photon in which:

$$G_{\mu\nu} = \rho \mathbf{l}_{\mu}\mathbf{l}_{\nu} \tag{46}$$

where ρ is the energy density, or $h\nu$, per unit volume of the photon, and $\mathbf{l}$ is the four-velocity of the particle of light known as a photon. In this way, it can be seen that the bundle of rapidly fluctuating electric and magnetic fields appears to behave on a macroscopic level as a particle having mass and momentum. Therefore, if the appropriate differentiation is applied to the tensors describing a wave bundle moving at the speed of light, it's energy can be derived from equation 40 which must be equal to $h\nu$. In this way Plank's constant enters the field equations.

We know that $G_{\mu\nu}$ describes the curvature of a local region, in this case the local region of a photon. The $(0,0)$ component is the localized energy density. Momentum, pressure and shear stress densities are also contained in the Einstein tensor (i.e. $G_{\mu\nu}$), which has been well known for nearly a hundred years. In this way, the local energy and momentum densities of a localized region of space-time undergoing rapid fluctuations of electric and magnetic fields and moving at the speed of light, can be calculated from well understood mathematical principles and procedures. Obviously, from equation 40, the energy and momentum of a photon can be obtained. The photon has momentum.

As the photon travels deeper into the gravitational well of the sun, we see that the energy of the photon increases; it blue shifts. As the photon bypasses the sun and climbs back out of the gravitational well, it red shifts. In the reference frame of the sun, assuming the sun's position is unaffected, the red shift equals the blue shift as the photon follows the geodesic described by the gravitational field of the sun. However, there is one small problem with this approach. We are working in the reference frame of the sun. We are assuming the sun is not moving in our laboratory universe having only the sun and a particular photon. If such were the case, the sun being treated as an inertial reference frame, then there would be no resultant red shift of the photon bypassing the sun. But such is not the case.

The photon has momentum. We must take this into consideration. We must move to an inertial reference frame utilizing the total momentum of the sun and photon. The total momentum of the system must remain constant. In this reference frame, there is a slight change in the photon's momentum due to its change in direction, this change in momentum must be subtracted from the sun's change in momentum so that the total momentum remains constant. This takes energy which comes from the photon, which red shifts to make up for the gain in kinetic energy of the sun. As a result, the bypassing of the photon causes the sun to very slightly shift which in turn alters the region of the photon's local value of $G_{\mu\nu}$ and thereby, the value of $\rho\mathbf{l}_\mu\mathbf{l}_\nu$ for the photon. We see that the slope of the gravitational well is not the same as when the photon exits the well of the sun as when it enters it. This is because the sun very slightly shifts toward the photon as it passes. The well is steeper coming out of it than when entering it.

General Relativity meets Quantum Mechanics.

Since the Faraday tensors describing the local spacetime of a photon affect $G_{\mu\nu}$, a photon therefore adds to the curvature of local space-time and therefore interacts gravitationally with local objects, however, extremely slightly.

We can figure out the red shift of the photon by treating the interaction as a collision using Plank's formula or we can figure it out with momentum considerations from rather difficult and complex operations and differentiations on the photon's quickly moving locality and the interaction it has in changing the sun's momentum. There are two ways to solve the problem and both should be equal.

Furthermore, we can also see the derivation of the very slight red shift of light by-passing the sun has been approximated as linear. Over great distances and with very large masses, this effect becomes more pronounced and non-linear. There is a cosine factor involved which comes into play the more and more the light is "bent". At great distances this would not behave as a linear function and may well match the observations that have formed the basis of an inflationary universe. Any determination concerning an expanding universe must take into account the red shifting of light passing through gravitational fields.

We next work out the red shift using the approach of a collision.

3 A Particle/Photon Gravitational Interaction

If a photon having momentum $\frac{h\nu}{c}$ bounces off a sphere "at rest" having mass M and is deflected by a small angle θ, then the sphere would gain momentum [5]. This would mean that the sphere would, in an ideal situation, move very, very slowly. Since the sphere has gained kinetic energy, according to the law of conservation of energy, the photon would lose energy equivalent to the kinetic energy gained by the sphere. As a result, the photon must red shift.

Let us examine the interaction of light passing through the gravitational field of the sun as a gravitational slingshot between a photon and the sun. A slingshot can be modeled as an elastic collision such as a collision between billiard balls on a frictionless pool table. If the sun is considered an incompressible billiard ball and a photon is considered as an incompressible cue ball barely touching the sun in a so-called "kiss shot", then we can calculate a possible change in frequency as follows:

Let θ be the angle of the photon coming off the "kiss" compared to travelling in a "straight" line as if missing the hit. If the sun has mass M and recoils with velocity V, the conservation of momentum demands:

$$\frac{h\nu}{c} = MV\sin(\frac{\theta}{2}) + \frac{h\nu'}{c}\cos(\theta) \tag{47}$$

for the "x" direction and:

$$MV\cos(\frac{\theta}{2}) = -\frac{h\nu'}{c}\sin(\theta) \tag{48}$$

for the "y" direction where ν' is the frequency of the photon after the interaction.

Substituting for MV from equation 2 into equation 1, we have:

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos(\theta) - \frac{h\nu'}{c}\frac{\sin(\theta)\sin(\frac{\theta}{2})}{\cos(\frac{\theta}{2})} \tag{49}$$

which reduces to:

$$\nu = \nu'(\frac{\sin(\theta)\sin(\frac{\theta}{2})}{\cos(\frac{\theta}{2})} + \cos(\theta)) \tag{50}$$

For very small θ:

$$\nu \simeq \nu'(\frac{\theta^2}{2} + 1) \tag{51}$$

Let $\nu - \nu' = \Delta\nu$. Then, subtracting ν' from both sides:

$$\Delta\nu \simeq \nu'\frac{\theta^2}{2} \tag{52}$$

Since $\nu' \simeq \nu$ we have:

$$\Delta\nu \simeq \nu\frac{\theta^2}{2} \tag{53}$$

The first approximation of a solution to a solar/photon interaction was done by Einstein [5] in which the following equation was derived:

$$\theta_{rad} = \frac{4M}{R} \tag{54}$$

in which θ_{rad} is the angle coming off the interaction in radians, M is the mass of the sun in meters, (Schwartzchild radius). R is the distance from the centre of the sun to the point of perihelion of the hyperbolic orbit of the photon in the path around the sun following the geodesic, also in meters. To convert R to the angular separation from the star to the sun in radians, divide R by an astronomical unit.

The formula derived previously is:

$$\Delta\nu = \frac{\theta_{deg}^2}{2}\nu \tag{55}$$

where θ_{deg} is the angle coming off the interaction in degrees, $\Delta\nu$ is the change in frequency and ν is the frequency of the signal.

Combine both formulae to yield:

$$\Delta\nu = \frac{M^2\pi^2}{2025R^2}\nu \tag{56}$$

or:

$$\Delta E = \frac{M^2\pi^2}{2025R^2}E \tag{57}$$

or:

$$\Delta p = \frac{M^2\pi^2}{2025R^2}p \tag{58}$$

By-passing the sun, a photon is red-shifted by a factor of about 10^{-7}.

4 String Theory

We have looked at Schrödinger's Equation, which is a diffusion equation with a linear term added on the end. Diffusion is an interesting mathematical phenomena. Mathematically, effects occur instantly throughout the media into which diffusion is penetrating. That means things happen faster than the speed of light; they happen instantly.

Consider three classes of differential equations. The diffusion equation, of which Schrödinger is included, the harmonic equation and the biharmonic equation.

Setting these out:

$$\begin{aligned} &\text{The diffusion equation:} & \tfrac{\partial\psi}{\partial t} &= \nabla^2\psi \\ &\text{The harmonic equation:} & \tfrac{\partial^2\psi}{\partial t^2} &= \nabla^2\psi \\ &\text{The biharmonic equation:} & 0 &= \nabla^4\psi \end{aligned} \tag{59}$$

These three cover a great deal within the field of applied mathematics. There are, of course, many variations. ψ is almost always defined as some unknown potential.

The second formula above, the harmonic equation, has the property that alterations propogate at a particular speed. That the speed of propogation within the media described by the differential equation has some definite finite value such as the speed of light. However, space-time is a tensor field and the harmonic equation can only describe a vector field. This has possibilities for electromagnetism but not for gravity.

The third formula above is the biharmonic equation and describes the world of elasticity. It is used in geophysics to describe movements of plate techtonics. It contains various stress tensors of an elastic media.

Consider the Heisenberg shell previously derived. Consider a potential ψ within a shell bounded by $r = \frac{h}{2mc}$ and $t = \frac{h}{mc^2}$. If we use the Schrödinger Equation as follows:

$$\left\{ \frac{\partial}{\partial t} \right\} \psi = \left\{ \frac{i\hbar^2 \nabla^2}{2m} - iV \right\} \psi \tag{60}$$

as a "Schrödinger operator". In order to find a measure of acceleration or force, or second time ordered differential, we re-apply the operator within the shell to obtain:

$$\frac{\partial^2 \psi}{\partial t^2} = \frac{\hbar^4}{4m^2} \nabla^4 \psi - \frac{\hbar^2 V}{m} \nabla^2 \psi + V^2 \psi \tag{61}$$

In spherical coordinates this is: $\frac{\partial^2}{\partial t^2}\psi\left(t,r,\theta,\phi\right) = \frac{\hbar^4}{4m^2}$ ($2\ r \sin(\theta)$ (- 2 ($2\ r \sin(\theta) \frac{\partial}{\partial r}\psi(t,r,\theta,\phi) + r^2 \sin\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right) + \cos\left(\theta\right) \frac{\partial}{\partial \theta}\psi\left(t,r,\theta,\phi\right) + \sin\left(\theta\right) \frac{\partial^2}{\partial \theta^2}\psi\left(t,r,\theta,\phi\right) + \frac{\frac{\partial^2}{\partial \phi^2}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) r^{-3} ($\sin\left(\theta\right)$) $^{-1}$ + ($2 \sin\left(\theta\right) \frac{\partial}{\partial r}\psi\left(t,r,\theta,\phi\right) + 4\ r \sin\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right) + r^2 \sin\left(\theta\right) \frac{\partial^3}{\partial r^3}\psi\left(t,r,\theta,\phi\right) + \cos\left(\theta\right) \frac{\partial^2}{\partial r \partial \theta}\psi\left(t,r,\theta,\phi\right) + \sin\left(\theta\right) \frac{\partial^3}{\partial \theta \partial r \partial \theta}\psi\left(t,r,\theta,\phi\right) + \frac{\frac{\partial^3}{\partial \phi^2 \partial r}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) r^{-2} $\left(\sin\left(\theta\right)\right)^{-1}$) + $r^2 \sin(\theta)$ (6 ($2\ r \sin\left(\theta\right) \frac{\partial}{\partial r}\psi\left(t,r,\theta,\phi\right) + r^2 \sin\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right) + \cos\left(\theta\right) \frac{\partial}{\partial \theta}\psi\left(t,r,\theta,\phi\right) + sin\left(\theta\right) \frac{\partial^2}{\partial \theta^2}\psi\left(t,r,\theta,\phi\right) + \frac{\frac{\partial^2}{\partial \phi^2}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) r^{-4} $\left(\sin\left(\theta\right)\right)^{-1}$ - 4 ($2 \sin\left(\theta\right) \frac{\partial}{\partial r}\psi\left(t,r,\theta,\phi\right) + 4\ r \sin\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right) + r^2 \sin\left(\theta\right) \frac{\partial^3}{\partial r^3}\psi\left(t,r,\theta,\phi\right) + \cos\left(\theta\right) \frac{\partial^2}{\partial r \partial \theta}\psi\left(t,r,\theta,\phi\right) + \sin\left(\theta\right) \frac{\partial^3}{\partial \theta \partial r \partial \theta}\psi\left(t,r,\theta,\phi\right) + \frac{\frac{\partial^3}{\partial \phi^2 \partial r}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) $r^{-3} \left(\sin\left(\theta\right)\right)^{-1}$ + ($6 \sin\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right) + 6\ r \sin\left(\theta\right) \frac{\partial^3}{\partial r^3}\psi\left(t,r,\theta,\phi\right) + r^2 \sin\left(\theta\right) \frac{\partial^4}{\partial r^4}\psi\left(t,r,\theta,\phi\right) + \cos\left(\theta\right) \frac{\partial^3}{\partial r^2 \partial \theta}\psi\left(t,r,\theta,\phi\right) + \sin\left(\theta\right) \frac{\partial^4}{\partial \theta \partial r^2 \partial \theta}\psi\left(t,r,\theta,\phi\right) + \frac{\frac{\partial^4}{\partial r \partial \phi^2 \partial r}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) r^{-2} $\left(\sin\left(\theta\right)\right)^{-1}$) + $\cos\left(\theta\right)$ (- ($2\ r \sin\left(\theta\right) \frac{\partial}{\partial r}\psi\left(t,r,\theta,\phi\right) + r^2 \sin\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right) + \cos\left(\theta\right) \frac{\partial}{\partial \theta}\psi\left(t,r,\theta,\phi\right) + \sin\left(\theta\right) \frac{\partial^2}{\partial \theta^2}\psi\left(t,r,\theta,\phi\right) + \frac{\frac{\partial^2}{\partial \phi^2}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) $\cos\left(\theta\right)$ r^{-2} $\left(\sin\left(\theta\right)\right)^{-2}$ + ($2\ r \cos\left(\theta\right) \frac{\partial}{\partial r}\psi\left(t,r,\theta,\phi\right) + 2\ r \sin\left(\theta\right) \frac{\partial^2}{\partial r \partial \theta}\psi\left(t,r,\theta,\phi\right) + r^2 \cos\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right) + r^2 \sin\left(\theta\right) \frac{\partial^3}{\partial r^2 \partial \theta}\psi\left(t,r,\theta,\phi\right)$ - $\sin\left(\theta\right) \frac{\partial}{\partial \theta}\psi\left(t,r,\theta,\phi\right) + 2 \cos\left(\theta\right) \frac{\partial^2}{\partial \theta^2}\psi\left(t,r,\theta,\phi\right) + \sin\left(\theta\right) \frac{\partial^3}{\partial \theta^3}\psi\left(t,r,\theta,\phi\right)$ - $\frac{\left(\frac{\partial^2}{\partial \phi^2}\psi(t,r,\theta,\phi)\right)\cos(\theta)}{\left(\sin(\theta)\right)^2} + \frac{\frac{\partial^3}{\partial \phi^2 \partial \theta}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) r^{-2} ($\sin\left(\theta\right)$) $^{-1}$) + $\sin\left(\theta\right)$ (2 ($2\ r \sin\left(\theta\right) \frac{\partial}{\partial r}\psi\left(t,r,\theta,\phi\right) + r^2 \sin\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right) + \cos\left(\theta\right) \frac{\partial}{\partial \theta}\psi\left(t,r,\theta,\phi\right) + \sin\left(\theta\right) \frac{\partial^2}{\partial \theta^2}\psi\left(t,r,\theta,\phi\right) + \frac{\frac{\partial^2}{\partial \phi^2}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) $\left(\cos\left(\theta\right)\right)^2$ r^{-2} ($\sin\left(\theta\right)$) $^{-3}$ -2 ($2\ r \cos\left(\theta\right) \frac{\partial}{\partial r}\psi\left(t,r,\theta,\phi\right) + 2\ r \sin\left(\theta\right) \frac{\partial^2}{\partial r \partial \theta}\psi\left(t,r,\theta,\phi\right) + r^2 \cos\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right) + r^2 \sin\left(\theta\right) \frac{\partial^3}{\partial r^2 \partial \theta}\psi\left(t,r,\theta,\phi\right)$ - $\sin\left(\theta\right) \frac{\partial}{\partial \theta}\psi\left(t,r,\theta,\phi\right) + 2 \cos\left(\theta\right) \frac{\partial^2}{\partial \theta^2}\psi\left(t,r,\theta,\phi\right) + \sin\left(\theta\right) \frac{\partial^3}{\partial \theta^3}\psi\left(t,r,\theta,\phi\right)$ - $\frac{\left(\frac{\partial^2}{\partial \phi^2}\psi(t,r,\theta,\phi)\right)\cos(\theta)}{\left(\sin(\theta)\right)^2} + \frac{\frac{\partial^3}{\partial \phi^2 \partial \theta}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) $\cos\left(\theta\right) r^{-2}$ ($\sin\left(\theta\right)$) $^{-2}$ + ($2\ r\ sin\left(\theta\right) \frac{\partial}{\partial r}\psi\left(t,r,\theta,\phi\right) + r^2 \sin\left(\theta\right) \frac{\partial^2}{\partial r^2}\psi\left(t,r,\theta,\phi\right)$

$+ \cos(\theta) \frac{\partial}{\partial\theta}\psi(t,r,\theta,\phi) + \sin(\theta) \frac{\partial^2}{\partial\theta^2}\psi(t,r,\theta,\phi) + \frac{\frac{\partial^2}{\partial\phi^2}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) r^{-2} $(\sin(\theta))^{-1}$ + (- 2 r $\sin(\theta)$ $\frac{\partial}{\partial r}\psi(t,r,\theta,\phi)$ + 4 r $\cos(\theta)$ $\frac{\partial^2}{\partial r\partial\theta}\psi(t,r,\theta,\phi)$ + 2 r $\sin(\theta)$ $\frac{\partial^3}{\partial\theta\partial r\partial\theta}\psi(t,r,\theta,\phi)$ - r^2 $\sin(\theta)$ $\frac{\partial^2}{\partial r^2}\psi(t,r,\theta,\phi)$ + 2 r^2 $\cos(\theta)$ $\frac{\partial^3}{\partial r^2\partial\theta}\psi(t,r,\theta,\phi)$ + r^2 $\sin(\theta)$ $\frac{\partial^4}{\partial\theta\partial r^2\partial\theta}\psi(t,r,\theta,\phi)$ - $\cos(\theta)$ $\frac{\partial}{\partial\theta}\psi(t,r,\theta,\phi)$ - 3 $\sin(\theta)$ $\frac{\partial^2}{\partial\theta^2}\psi(t,r,\theta,\phi)$ + 3 $\cos(\theta)$ $\frac{\partial^3}{\partial\theta^3}\psi(t,r,\theta,\phi)$ + $\sin(\theta)$ $\frac{\partial^4}{\partial\theta^4}\psi(t,r,\theta,\phi)$ + 2 $\frac{\left(\frac{\partial^2}{\partial\phi^2}\psi(t,r,\theta,\phi)\right)(\cos(\theta))^2}{(\sin(\theta))^3}$ - 2 $\frac{\cos(\theta)\frac{\partial^3}{\partial\phi^2\partial\theta}\psi(t,r,\theta,\phi)}{(\sin(\theta))^2}$ + $\frac{\frac{\partial^2}{\partial\phi^2}\psi(t,r,\theta,\phi)}{\sin(\theta)}$ + $\frac{\frac{\partial^4}{\partial\theta\partial\phi^2\partial\theta}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) r^{-2} $(\sin(\theta))^{-1}$) + (2 $\sin(\theta)$ $\frac{\partial^3}{\partial\phi^2\partial r}\psi(t,r,\theta,\phi)$ + r^2 $\sin(\theta)$ $\frac{\partial^4}{\partial r\partial\phi^2\partial r}\psi(t,r,\theta,\phi)$ + $\cos(\theta)$ $\frac{\partial^3}{\partial\phi^2\partial\theta}\psi(t,r,\theta,\phi)$ + $\sin(\theta)$ $\frac{\partial^4}{\partial\theta\partial\phi^2\partial\theta}\psi(t,r,\theta,\phi)$ + $\frac{\frac{\partial^4}{\partial\phi^4}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) r^{-2} $(\sin(\theta))^{-2}$) r^{-2} $(\sin(\theta))^{-1}$ - $\frac{\hbar^2 V}{m}$ (2 r $\sin(\theta)$ $\frac{\partial}{\partial r}\psi(t,r,\theta,\phi)$ + r^2 $\sin(\theta)$ $\frac{\partial^2}{\partial r^2}\psi(t,r,\theta,\phi)$ + $\cos(\theta)$ $\frac{\partial}{\partial\theta}\psi(t,r,\theta,\phi)$ + $\sin(\theta)$ $\frac{\partial^2}{\partial\theta^2}\psi(t,r,\theta,\phi)$ + $\frac{\frac{\partial^2}{\partial\phi^2}\psi(t,r,\theta,\phi)}{\sin(\theta)}$) r^{-2} ($\sin(\theta)$) $^{-1}$ + V^2 $\psi(t,r,\theta,\phi)$

From the general solution to the above equation we can apply boundary conditions: $r = \frac{h}{2mc}$ and initial condition $t = \frac{h}{mc^2}$ to show that the resultant Bessel functions and their zeros along with Legendre polynomials lead to zeta functions appropriate to develop string theory.

First we resolve the harmonic equation, which also solves the biharmonic, as follows:

$$\begin{array}{l}
\psi^*(t,r,\theta,\phi) = T(t)\,R(r)\,\Theta(\theta)\,\Phi(\phi)\,, \\
\frac{d^2}{dt^2}T(t) = -\alpha^2 T(t)\,, \\
\frac{d^2}{dr^2}R(r) = -\alpha^2 R(r) + \beta^2\frac{R(r)}{r^2} - 2\,\frac{\frac{d}{dr}R(r)}{r}, \\
\frac{d^2}{d\theta^2}\Theta(\theta) = -\Theta(\theta)\,\beta^2 + \frac{\Theta(\theta)\gamma}{(\sin(\theta))^2} - \frac{\cos(\theta)\frac{d}{d\theta}\Theta(\theta)}{\sin(\theta)}, \\
\frac{d^2}{d\phi^2}\varPhi(\phi) = -\gamma^2\Phi(\phi)
\end{array}$$

Where:

$$\frac{\partial^2}{\partial t^2}\psi^*(t,r,\theta,\phi) = \left(\begin{array}{l}
2\,r\sin(\theta)\,\frac{\partial}{\partial r}\psi^*(t,r,\theta,\phi) \\
+r^2\sin(\theta)\,\frac{\partial^2}{\partial r^2}\psi^*(t,r,\theta,\phi) \\
+\cos(\theta)\,\frac{\partial}{\partial\theta}\psi^*(t,r,\theta,\phi) \\
+\sin(\theta)\,\frac{\partial^2}{\partial\theta^2}\psi^*(t,r,\theta,\phi) \\
+\frac{\frac{\partial^2}{\partial\phi^2}\psi^*(t,r,\theta,\phi)}{\sin(\theta)}
\end{array}\right)\frac{1}{r^2(\sin(\theta))} \qquad (62)$$

and

$$
\begin{aligned}
T\,(t) = &\quad A\,\sin\,(\alpha\,t) + B\,\cos\,(\alpha\,t)\\
R\,(r) = &\quad \tfrac{C}{\sqrt{r}}\,\mathit{BesselJ}\left(1/2\,\sqrt{1+4\,\beta^2}, \alpha\,r\right) + \tfrac{D}{\sqrt{r}}\,\mathit{BesselY}\left(1/2\,\sqrt{1+4\,\beta^2}, \alpha\,r\right)\\
\Theta\,(\theta) = &\quad E\,\mathit{LegendreP}\left(1/2\,\sqrt{1+4\,\beta^2} - 1/2, \sqrt{\gamma}, \cos\,(\theta)\right)\\
&\quad + F\,\mathit{LegendreQ}\left(1/2\,\sqrt{1+4\,\beta^2} - 1/2, \sqrt{\gamma}, \cos\,(\theta)\right)\\
\Phi\,(\phi) = &\quad G\,\sin\,(\gamma\,\phi) + H\,\cos\,(\gamma\,\phi)
\end{aligned}
\tag{63}
$$

Now we resolve the linear part which is:

$$
\frac{\partial^2\psi(t,r,\theta,\phi)}{\partial t^2} = k\psi(t,r,\theta,\phi) \tag{64}
$$

having solution:

$$
\psi(t,r,\theta,\phi) = f_1(r,\theta,\phi)e^{\sqrt{k}t} + f_2(r,\theta,\phi)e^{-\sqrt{k}t} \tag{65}
$$

Now, we will show a little of the biharmonic part in Cartesian coordinates. Note that if:

$$
\begin{aligned}
&\tfrac{\partial^4\psi(x)}{\partial x^4} = \alpha^4\psi(x)\\
&(D^4 - \alpha^4)\psi(x) = 0\\
&(D-\alpha)(D+\alpha)(D^2+\alpha^2)\psi(x) = 0\\
&(D-\alpha)(D+\alpha)(D-i\alpha)(D+i\alpha)\psi(x) = 0\\
&\psi(x) = Ae^{\alpha x} + Be^{-\alpha x} + Ce^{i\alpha x} + De^{-i\alpha x}\\
&\psi(x) = F\cos(\alpha x) + G\sin(\alpha x) + H\cosh(\alpha x) + I\sinh(\alpha x)
\end{aligned}
\tag{66}
$$

So, both the real and the imaginary parts to the harmonic equation work as a solution to the biharmonic equation in Cartesian coodinates. In this case, we can use both x and ix in the harmonic term for a solution to the biharmonic term which doubles the number of solutions for each dimension; however, only half can be used at a time. This is because all the reals have to be equal on both sides of the equation and all the imaginaries must also be equal on both sides.

You can then apply the boundary conditions to the equation in spherical coordinates to solve for the arbitrary constants resulting in the appearance of zeta functions.

It may be interpreted that in eigenspace each of the eigenvalues, α, β, etc. are summed over an infinite number of values in such a way that each term of the solution is orthogonal in order to match the boundary conditions. In Cartesian coodrdinates we could have α_n, β_n, γ_n and ξ_n and possibly sum n^2 over the general solution as n goes from one to infinity. Each value of n results in an "eigenset" and each set of eigenvalues forms a vector space containing orthogonal eigenvectors. These eigenvalues therefore form a multidimensional orthogonal space. There are twelve spacial for the biharmonic term, six for the harmonic term and two for the temporal term. That makes a 20-dimensional eigenspace. However, only half the eigenspace can be used at a time as previously explained. We therefore have a ten-dimensional eigenspace.

This forms an interesting approach which may possibly be used in string theory.

5 Conclusions

Firstly, ψ is the potential of existence.

Secondly, the space-time continuum is the infinite array of a four dimensional coordinate system.

Thirdly, the lines of the space-time continuum, the coordinate system at the root of the Einstein/Minkowski equations, is made of ψ. The potential of existence as determined by Schrödinger, ψ, forms the coordinate system which is the Universe itself. The coordinate system, ψ, can be bent or "curved". We propose that the second time ordered differential of ψ and the biharmonic of ψ are measures of this curvature. The more the coordinate system is bent or curved, the greater the potential of existence. This coordinate system can only be bent so far. The limit of bending is determined by the boundary conditions of Schrödinger's equation as calculated by Heisenberg. At that limit, the lines of the coordinate system crimp and become "knotted" into a particle. The energy of creating the particle from the bending of space-time is equal to its mass times the speed of light squared.

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