

# Cosmic de celebration parameter $q(Z)$ dependence upon gravitons? Implications for the DM rocket/ram jet model

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## Abstract

In the 12th Marcel Grossmann Meeting, July 9<sup>th</sup>, 2009, the author raised the issue of whether early graviton production could affect non-Gaussian contributions to DM density profiles. Another issue to consider, is if or not gravitons with mass affect DM profiles, but could also impact the cosmic acceleration of the universe, leading to an increase of acceleration one billion years ago, in a manner usually attributed to DE. Following Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo, 2009 in an article brought to the attention to the author by Christian Corda, the author, using his modification of Friedman equations, incorporating some brane models to allow for additional dimensions found that there is, for low graviton

mass of the order of  $m_{graviton} \sim 10^{-65}$  grams similar behavior as noted by Alves, et al. 2009. If  $m_{graviton} \sim 10^{-65}$  grams also is reconcilable as to KK dark matter models, which is under investigation, the new modeling super structure could have significant impact upon the DM rocket / ram jet proposal the author, Beckwith, brought up in the AIBEP meeting in Scottsville, Arizona. The author will high light what KK style gravitons, with a slightly different mass profile could mean in terms of his DM rocket proposal brought up in both Christ Church, Dark 2009, and in different form in SPESIF, 2009. I.e. value of up to 5 TeV , as opposed to 400 GeV for DM, which may mean more convertible power for a suitably designed platform.

## Introduction

When at the 12 Marcell Grossman meeting, July 2009 the author talked at length with several of his contemporaries in a section at the Paris Obervatory as to what would happen to DM if hot and cold DM models were mixed together. I.e. the initial model which the author worked with was with WIMPS, and Axions, which are both Cold DM. From what he was told, there would be no structural changes which would occur in galaxy formation, if two cold DM candidates would be partially mixed, as Beckwith (2009) hypothesized and presented in two conferences. It was a very different story if warm and cold DM candidates were mixed together. Karsten Jedamzik, Martin Lemoine and Gilbert Moultaqa (2006), have written that “Stable particle dark matter may well originate during the decay of long-lived relic particles, as recently extensively examined in the cases of the axino, gravitino, and higher-dimensional Kaluza-Klein (KK) graviton” I.e. the axion is a cold DM candidate, whereas the KK graviton is warm DM. The author was drawn to investigating what happens to such a mix as part of his investigation as to why galaxies have an earlier period of formation than what is predicted by the hierarchy model of galaxy formation, represented on figure 1, page eight of this document. Furthermore, the KK graviton has the property that it has as its lower limit the graviton. In what is a departure from usual models of the graviton, the author is considering what happens if there is a tiny mass,  $m_{graviton} \propto 10^{-65}$  grams , as the first KK mode, in contrast to the usual zero mass predicted as to the zeroth mode of the KK graviton. Making sense of this divergence involves eqn 1.12 of this document, page 6 . I.e. a slight modification of the usual KK graviton

mass equation  $m_n(Graviton) = \frac{n}{L} + 10^{-65}$  grams, and  $[\nabla^2 - \partial_\tau^2] \cdot \psi_n = m_n^2(graviton) \cdot \psi_n$  . I.e.

finding a proof of this conjecture is something the author is involved with, and its resolution would also help toward a different path of DM inter relationships with DE, i.e. a way to explain the results forwarded by . P. M. Sutter and P. M. Ricker (2008),. They write “We use high-resolution simulations of large-scale structure formation to analyze the effects of interacting dark matter and dark energy on the evolution of the halo mass function. Using a  $\chi^2$  likelihood analysis, we find significant differences in the mass function between models of coupled dark matter-dark energy and standard concordance cosmology ( $\Lambda$ CDM) out to redshift  $z = 1.5$ .” It so happens that this regime of red shift pre dates the  $Z \sim .55$  point of inflection where

cosmological speed up of expansion occurs, . The author, Beckwith, is convinced that this is not an accident, and may be connected with his page 12  $q(Z)$  graphs of when the  $q(Z)$  decreases its positive value, and becomes negative as of  $Z \sim .45 - .55$  in red shift values.

If gravitons have this sort of behavior as linked to both DM and DE in cosmological evolution, then the question of if gravitons are classical, or quantum comes up as a non trivial exercise, since the answer to the relative classical nature of gravitons / GW would influence the plausibility of the KK graviton dynamics described above. Note that . gravitons are stated conceptually to be akin to photons in light waves. In simple physics analogies. But this simple quantum generalization breaks down, since gravitons are spin two particles with a complex set of interactions not only with themselves, but with evolving space time geometry. Hence the issue of apparently combined sources of planar wave generation of gravitational waves is a precursor to what would happen if squeezed states occurred at the onset of the big bang. I.e., what would happen with multiple superposition of different coherent states? A good reference as to coherent states in cosmology, as in this example, Bianchi I universes, was given by Brett Bolen, Luca Bombelli, Alejandro Corichi (2004) If states are largely coherent, such a small variation/ smoothness of observables will have observational consequences as to relic gravitational wave signals seen in the onset of inflation.. Another is, if or not there are measurable structural consequences as to a small graviton mass. A relevant question to ask, which we will , is can there be macro scopic effect due to what choice of gravitons we pick, which has measurable consequences. The external large structure change picked is one of resumption of DE acceleration increase one billion years ago. Specifically in what is known as a  $q(Z)$  de celebration parameter turning negative one billion years ago. Now, Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo, wrote in their article, "In this work, we explore some cosmological implications of the model proposed by M. Visser in 1998. In his approach, Visser intends to take in account mass for the graviton by means of an additional bimetric tensor in the Einstein's field equations. Our study has shown that a consistent cosmological model arises from Visser's approach." We will re duplicate their procedure as to a graviton with mass, but to do it in the context of theoretic treatment of the Friedman equations with additional dimensions.

## Linkage of DM to gravitons and gravitational waves?

Let us state that the object of early universe GW astronomy would be to begin with confirmation of whether or not relic GW were obtainable , and then from there to ascertain is there is linkage which can be made to DM production... Durrer, Massimiliano Rinaldi (2009) , state that there would be probably negligible for this case ( practically non existent ) graviton production in cosmological eras after the big bang.. In fact, they state that they investigate the creation of mass less particles in a Universe which transits from a radiation-dominated era to any other (via an) expansion law. "We calculate in detail the generation of gravitons during the transition to a matter dominated era. We show that the resulting gravitons generated in the standard radiation/matter transition are negligible" This indicated to the author, Beckwith, that it is appropriate to look at the onset of relic GW/ Graviton production.. One of the way to delineating the evolution of GW is the super adiabatic approximation, done for when  $k^2 \ll |a''/a|$  as given by M. Giovannini (page 138), when  $\mu_k \equiv a \cdot h_k$  is a solution to

$$\mu_k'' + \left[ k^2 - \frac{a''}{a} \right] \mu_k = 0 . \quad (0.1)$$

Which to first order when  $k^2 \ll |a''/a|$  leads to a GW solution

$$h_k(\tau) \equiv A_k + B_k \cdot \int_0^\tau \frac{dx}{a(x)} \quad (0.1a)$$

This will be contrasted with a very similar evolution equation for gravitons, of ( i.e. KK gravitons in higher dimensions)

$$h'' - \left[ 4k^2 + \frac{m^2}{a^2(z)} \right] h \equiv 0 \quad (0.1b)$$

One of the models of linkage between gravitons, and DM is the KK graviton, i.e. as a DM candidate. KK gravitons. Note that usual Randal Sundrum brane theory has a production rate of  $\Gamma \sim T^6 / M_{Planck}^2$  as the number of Kaluza Klein gravitons per unit time per unit volume. Note this production rate is for a formula assuming mass for which  $T_* > M_X$ , and that we are assuming that the temperature  $T \sim T_*$ . Furthermore, we also are looking at total production rate of KK gravitons of the form

$$\frac{dn}{dt} \sim \frac{T^6}{M_{Planck}^2} \cdot (T \cdot R)^d \sim T^4 \cdot \left( \frac{T}{M_X} \right)^{2+d} \quad (0.1c)$$

Where R is the assumed higher dimension ‘size’ and , d is the number of dimensions above 4, and typically we obtain  $T \gg 1/R$ . I.e. we can typically assume tiny higher dimensional ‘dimensions’, very high temperatures, and also a wave length for the resulting KK graviton for a DM candidate looking like

$$\lambda_{KK-Graviton} \sim T^{-1} \quad (0.2)$$

If KK gravitons have the same wavelength as DM, this will support Jack Ng’s treatment of DM. All that needs to put this on firmer ground will be to make a de facto linkage of KK Gravitons, as a DM candidate , and more traditional treatments of gravitons, which would assume a steady drop in temperature from  $T \sim T_*$ , to eventually much lower temperature scales. . Note that in a time interval based as proportional to the inverse of the Hubble parameter, we have the total numerical density of KK gravitons ( on a brane? ) as  $n(T) \sim T^2 M_{Planck}^* \cdot (T/M)^{2+d}$ , where  $M_{Planck}^* \sim 10^{18} GeV$  give or take an order of magnitude.

This number density  $n(T)$  needs to be fully reconciled to  $\lambda_{KK-Graviton} \sim T^{-1}$  and can be conflated with

the dimensionality ‘radius’ value  $R \sim 10^{\frac{32}{d}} \cdot 10^{-17}$  centimeters for dimensions above 4 space time GR values, with this value of R being unmanageable for  $d < 2$ . V.A. Rubakov , and others also (2009) makes the claim of the KK graviton obeying the general Yukawa style potential

$$V(r) = -\frac{G_4}{r} \cdot \left( 1 + \frac{const}{k^2 r^2} \right) \quad (0.2a)$$

As well as being related to an overall wave functional which can be derived from a line element  $dS^2 \equiv [a^2(z) \cdot \eta_{uv} + h_{uv}(x, z)] \cdot dx^u dx^v + dz^2$  (0.3)

With  $h'' - \left[ 4k^2 + \frac{m^2}{a^2(z)} \right] h \equiv 0$  (suppressing the u,v coefficients) . This evolution equation for the

KK gravitons is very similar to work done by Baumann, Daniel, Ichiki, Kiyotomo, Steinhardt, Paul J. Takahashi , Keitaro (2007) with similar assumptions, with the result that KK gravitons are a linear combination of Bessel functions. Note that one has for gravitons.

$$h \equiv h_m(z \rightarrow 0) = const \cdot \sqrt{\frac{m}{k}} \quad (0.4)$$

Ruth Gregory, Valery A. Rubakov and Sergei M. Sibiryakov (2000) make the additional claim that for large  $z$  ( the higher dimensions get significant) that there are marked oscillatory behaviors , ie. Rapid oscillations as one goes into the space for branes for massive graviton expansion.

$$h \equiv h_m(z \neq 0) \approx const \cdot \sqrt{a(z)} \cdot \sin\left(\frac{m}{k} \cdot esp(kz) + \varphi_m\right) \quad (0.5)$$

This is similar to what Baumann, Daniel, Ichiki, Kiyotomo, Steinhardt, Paul J. Takahashi , Keitaro (2007) for GW, in a relic setting, with the one difference being that the representation for a graviton is in the  $z$  ( additional dimension) space, as opposed to what Bauman et al did for their evolution of GW, with an emphasis upon generation in over all GR space time.. Furthermore, the equation given in

$$h'' - \left[4k^2 + \frac{m^2}{a^2(z)}\right] h \equiv 0 \text{ for massive graviton evolution as KK gravitons along dS branes is similar to}$$

evolution of GW in more standard cosmology that the author, Beckwith, thinks that the main challenge in clarifying this picture will be in defining the relationship of dS geometry, in overall Randall Sundrum brane world to that of standard 4 space,. We need though, now to look at whether or not higher dimensions are even relevant to GR itself.

## How DM would be influenced by gravitons, in 4 dimensions

We will also discuss the inter relationship of structure of DM, with challenges to Gaussianity. The formula as given by

$$\delta \equiv -\left[\frac{3}{2} \cdot \Omega_m \cdot H^2\right]^{-1} \cdot \nabla^2 \Phi \quad (1.0)$$

Will be gone into. The variation, so alluded to which we will link to a statement about the relative contribution of Gaussianity, via looking at the gravitational potential

$$\Phi \equiv \Phi_L + f_{NL} \cdot \left[\Phi_L^2 - \langle \Phi_L^2 \rangle\right] + g_{NL} \cdot \Phi_L^3 \quad (1.1)$$

Here the expression  $f_{NL} =$  variations from Gaussianity, while the statements as to what contributes, or does not contribute will be stated in our presentation. Furthermore,  $\Phi_L$  is a linear Gaussian potential, and the over all gravitational potential is altered by inputs from the term, presented,  $f_{NL}$ . The author discussed inputs into variations from Gaussianity, which were admittedly done from a highly theoretical perspective with Sabino Matarre, on July 10, with his contributions to non Guassianity being constricted to a reported range of  $-4 < f_{NL} < 80$ , as given to Matarre, by Senatore, et al, 2009.

The author, Beckwith, prefers a narrower range along the lines of  $.5 < f_{NL} < 20$  for reasons which will be gone into, in the text. . Needless to state, though, dealing with what we can and cannot measure, what is ascertained as far as DM, via a density profile variation needs to have it reconciled with DM detection values

$$\sigma_{DM-detection} \leq 3 \times 10^{-8} \text{ pb (pico barns)} \quad (1.2)$$

It is note worthy to note that the question of DM/ KK gravitons, and also the mass of the graviton not only has relevance to whether or not, higher dimensions are necessary/ advisable in space time models , but also may be relevant to if massive gravitons may solve / partly fulfill the DE puzzle. To whit, \ KK gravitons would have a combined sum of Bessel equations as a wave functional representation. In fact V. A Rubasov (2009) writes that KK graviton representation as, after using the following

normalization  $\int \frac{dz}{a(z)} \cdot [h_m(z) \cdot h_{\tilde{m}}(z)] \equiv \delta(m - \tilde{m})$ , where  $J_1, J_2, N_1, N_2$  are different forms of

Bessel functions, to obtain the KK graviton/ DM candidate representation along RS dS brane world

$$h_m(z) = \sqrt{m/k} \cdot \frac{J_1(m/k) \cdot N_2([m/k] \cdot \exp(k \cdot z)) - N_1(m/k) \cdot J_2([m/k] \cdot \exp(k \cdot z))}{\sqrt{[J_1(m/k)]^2 + [N_1(m/k)]^2}} \quad (1.3)$$

This allegedly is for KK gravitons having an order of TeV magnitude mass  $M_z \sim k$  (i.e. for mass values at .5 TeV to above a TeV in value) on a negative tension RS brane. What would be useful would be managing to relate this KK graviton, which is moving with a speed proportional to  $H^{-1}$  with regards to the negative tension brane with  $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$  as a possible initial starting value for the

KK graviton mass, before the KK graviton, as a ‘massive’ graviton moves with velocity  $H^{-1}$  along the RS dS brane. If so, and if  $h \equiv h_m(z \rightarrow 0) = \text{const} \cdot \sqrt{\frac{m}{k}}$  represents an initial state, then one may relate the

mass of the KK graviton, moving at high speed, with the initial rest mass of the graviton, which in four space in a rest mass configuration would have a mass many times lower in value, i.e. of at least

$m_{\text{graviton}}(4 - \text{Dim GR}) \sim 10^{-48} \text{ eV}$ , as opposed to  $M_x \sim M_{\text{KK-Graviton}} \sim .5 \times 10^9 \text{ eV}$ . Whatever the range of the graviton mass, it may be a way to make sense of what was presented by Dubovsky, Flauger, Starobinsky, and Thackev (2009) who argue for graviton mass using CMBR measurements, of up

to  $m_{\text{graviton}}(4 - \text{Dim GR}) \sim 10^{-20} \text{ eV}$ . This can be conflated with Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo’s results arguing that non zero graviton mass may lead to acceleration of our present universe, in a manner usually conflated with DE, i.e. their graviton mass would be about

$m_{\text{graviton}}(4 - \text{Dim GR}) \sim 10^{-48} \times 10^{-5} \text{ eV} \sim 10^{65}$  grams, leading to a possible explanation for when the universe accelerated, i.e. the de-acceleration parameter, due to changes in the scale factor, written as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} \quad (1.4)$$

In the case of working with a simpler version of the Friedman equation with no graviton mass, but with pressure and density factored in, we can obtain

$$\frac{\ddot{a}}{a} = \frac{4\pi G}{3} \cdot [(-3p/c^2) - \rho] \quad (1.4a)$$

This will lead to a very simple de celebration parameter value of

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = \left( \frac{4\pi G}{3c^2 H^2} \right) \cdot [3p + \rho] \quad (1.4b)$$

The article will see what happens to insure whether or not the sign of 1.4 and 1.4b goes from positive to negative. Needless to say, if one has a graviton mass  $m_{\text{graviton}} \neq 0$ , then (1.4a) changes, and there will be a way forward to consider whether or not there is a linkage between DM, DE, and structure formation.

Using a modification of GR, with scale factor evolution of, with non zero graviton mass terms added in to obtain

$$\left( \frac{\dot{a}}{a} \right)^2 + \frac{m_g^2 c^4}{2\hbar^2} \cdot (1 - a^2) \equiv \frac{8\pi G}{3c^2} \cdot \rho \quad (1.5)$$

And

$$\frac{\ddot{a}}{a} + .5 \cdot \left( \frac{\dot{a}^2}{a^2} \right) + \frac{m_g^2 c^4}{4\hbar^2} a^2 \cdot (a^2 - 1) = \frac{8\pi G}{3c^2} \cdot p. \quad (1.6)$$

For the matter dominated era, it is important to note that the R.H.S. of (1.6) is zero. This leads to (1.4) having increasingly positive acceleration values as would be definitely be given for masses of

$m_{graviton} (4-Dim GR) \sim 10^{-48} \times 10^{-5} eV \sim 10^{65}$  grams for red shift values  $z \sim .3$  for (1.4) just

becoming  $> 0$  to maximum values of (1.4) today, with  $z = 0$ , all at mass of the order of  $10^{65}$  grams. This increase of (1.4) then leads us to consider how to configure (1.5) and (1.6) and for RS brane world values. There are terms which are added to the first Friedman equation. i.e.. When using ultra low graviton

mass, where  $r_c = \frac{M_P^2}{2M^{(5)3}}$  and, often  $\epsilon = 1$  and  $r_c$  is usually thought of as the separation between branes. I.e. if  $r_c \rightarrow \infty$ , we recover the usual first Friedman equation. For now we write the first Friedman equation for a brane system as.

$$\frac{\dot{a}}{a} \equiv H = \frac{\epsilon}{2r_c} + \sqrt{\frac{8\pi G\rho}{3} + \frac{1}{4r_c^2}} \quad (1.7)$$

As can be related to, if we wish to look at string theory versions of the FRW equation, in Friedman-Roberson – Walker metric space, we can do the following decomposition, with different limiting values of the mass, and other expressions, e.g. as a function of an existing cosmological constant

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho_{Total}}{3M_{Planck}^2} - \frac{k}{a^2} + \frac{\Lambda}{3} \quad (1.8)$$

As well as

$$\left(\frac{\ddot{a}}{a}\right) = -\frac{(\rho_{Total} + 3p_{Total})}{6M_{Planck}^2} + \frac{\Lambda}{3} \quad (1.9)$$

Not only this, if looking at the brane theory Friedman equations as presented by / for Randall Sundrum theory, it would be prudent working with

$$\dot{a}^2 = \left[ \left( \frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{\rho^2}{36M_{Planck}^2} \right) a^2 - \kappa + \frac{C}{a^2} \right] \quad (1.10)$$

For the purpose of Randal Sundrum brane worlds, (1.10) is what will be differentiated with respect to  $d/d\tau$ , and then terms from (1.5) will be used, and put into a derivable equation which will be for a RS

brane world version of  $q = -\frac{\ddot{a}a}{\dot{a}^2}$ . Several different versions of what q should be will be offered as far as

what the time dependence of terms in 1.10 actually is. Note that Roy Maartens has written as of 2004 that KK modes (graviton) satisfy a 4 Dimensional Klein – Gordon equation, with an effective 4 dim mass,

$m_n(Graviton) = \frac{n}{L}$ , with  $m_0(Graviton) = 0$ , and L as the stated ‘dimensional value’ of higher

dimensions. The value  $m_0(Graviton) \sim 10^{-65} - 10^{-60}$  gram in value picked is very small, but ALMOST zero. Grossing has shown how the Schrodinger and Klein Gordon equations can be derived from classical lagrangians, i.e. using a version of the relativistic Hamilton-Jacobi- Bohm equation, with a wave functional  $\psi \sim \exp(-iS/\hbar)$ , with S the action, so as to obtain working values of for a tier of purported masses of a graviton from the equation, for 4 D of  $[g^{\alpha\beta} \partial_\alpha \partial_\beta \xrightarrow{FLAT-SPACE} \nabla^2 - \partial_\tau^2]$ , and

$[\nabla^2 - \partial_\tau^2] \cdot \psi_n = m_n^2(graviton) \cdot \psi_n$  If one is adding, instead the small mass of

$m_n(Graviton) = \frac{n}{L} + 10^{-65}$  grams, with  $m_0(Graviton) \approx 10^{-65}$  grams, then the problem being

worked with is a source term problem of the form given by Peskins as of the type

$$\psi_n(x) \equiv \int d^3 p \cdot \frac{1}{(2\pi)^3} \cdot \frac{1}{\sqrt{2E_p}} \cdot \left\{ \left( a_p + \frac{i}{\sqrt{2E_p}} \cdot FT(m_0(graviton)) \right) \exp(-ipx) + H.C. \right\} \quad (1.11)$$

This is, using the language V.A. Rubakob (2009) put up equivalent to writing, using (1.3)

$$\psi_m(x) \approx h_m(x) + \int d^3 p \cdot \frac{1}{(2\pi)^3} \cdot \left( \frac{1}{\sqrt{2E_p}} \right)^2 \cdot \{(i \cdot FT(m_0(\text{graviton}))) \exp(-ipx) + H.C.\} \quad (1.12)$$

I.e. how to interpret the quantity  $FT(m_0(\text{graviton}))$  being the issue which will be covered in this document. If  $m_0(\text{graviton})$  is a constant, then the expression (1.12) has delta functions. This goes into evaluating, then, momentum, appropriately, We will do a time differentiation of \*(1.10) in this document, and compare it term by term with what arises if there is a suitable graviton mass, and comment as to what would be needed to have graviton mass in a brane version of \*(1.7), and its time derivative, and do a similar analysis as to what was done to recover the positive acceleration, for \*(1.4) using brane equivalents to (1.5) as well as inputs from (1.6). Now why is this important? This datum may especially show up about modification of the typical galaxy models, as follows

## Controversies of DM/ DE applications to cosmology. How HFGW may help resolve them.

The following is meant as a travelogue as to current problems in cosmology which will require significant revision of our models. Exhibit A as to what to consider is the ‘cosmic void hypothesis’. See Timothy Clifton, Pedro G. Ferreira and Kate Land. I.e. Clifton raises the following question- can HFGW and detectors permit cosmologist to get to the bottom of this? “Solving Einstein’s equations for an averaged matter distribution is NOT the same as solving for the real matter distribution and then averaging the resultant geometry”(“We average, then solve when in effect we should solve, then average”).

Next, let us look at a recently emerging conundrum of DM feeding into the structure of new galaxies and their far earlier than expected development, i.e. 5 billion years after the big bang. Galaxy formation issues.... Hierarchical Galaxy Formation theory at a glance usually proceeds as follows. I.e. What happens when the following diagram of simple addition of new structure no longer holds? This is very significant, since when the significant formation of galaxies occurs, as of about  $z \sim .2$  is before the turn up in the expansion rate for the universe, which will be referenced as of occurring about  $z \sim .5 - .55$ . What do we do if, as an example, find that the initial start of galaxy formation occurred five billion years ago, at, say  $z \sim .5$ . What could cause the earlier clumping?

Several scenarios which will be investigated. First of all, note the formula of variation of DM density which exists has, among other things a Hubble parameter  $H$ , and also the 2<sup>nd</sup> derivative of the gravitational potential  $\nabla^2 \Phi$ , where  $\rho_0, a_0$  are today’s values for density and ‘distance’. Note that if the

$$H^2 = \left( \frac{\dot{a}}{a} \right)^2 = \left[ \left( \frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{\rho^2}{36M_{Planck}^2} \right) - \frac{\kappa}{a^2} + \frac{C}{a^4} \right] \xrightarrow[\Lambda_4 \rightarrow 0]{\kappa \rightarrow 0} \left[ \frac{\rho}{3M_4^2} + \frac{\rho^2}{36M_{Planck}^2} + \frac{C}{a^4} \right],$$

$$\rho \rightarrow \rho(z) \equiv \rho_0 \cdot (1+z)^3 - \left[ \frac{m_g}{8\pi G} \right] \cdot \left( \frac{a_0^4}{14 \cdot (1+z)^4} + \frac{2a_0^2}{5 \cdot (1+z)^2} - \frac{1}{2} \right), \text{ and } 1+z = a_0/a, \text{ then the}$$

contribution of large  $z$ , i.e. large contributions from red shift, that a significant early contributions will be for non zero contributions from  $1/\rho^\beta$  terms, for [ **large number** ]  $> \beta \geq 1$  in the DM density variation parameters. So long as  $m_{graviton} \neq 0$ , even if  $m_{graviton}$  is very small. In addition, if the following is true

$$\Phi \equiv \Phi_L + f_{NL} \cdot \left[ \Phi_L^2 - \langle \Phi_L^2 \rangle \right] + g_{NL} \cdot \Phi_L^3 \text{ then there are contributions from terms to be considered.}$$

When using the formula,  $\nabla^2 \Phi$  consider the contributions to the expression  $f_{NL}$ . To do this consider first what Licia Verde (2000) put up about  $\Phi$  considered to be the gravitational potential, and  $\Phi_L$  its linear

Gaussian contribution. P. Chingabam, C. Park (2009) improved upon the simulation done by Verde (2003), who worked with  $f_{NL}$  bounded as:  $10^{-4} < f_{NL} < 10^{-2}$ , whereas the Chingabam, Park (2009)  $-4 < f_{NL} < 80$  at a confidence level of 95%. One of the simpler suppositions a person could use is what would be involved if,  $\lambda_{Graviton} = (\hbar/m_{graviton} \cdot c) \equiv 1/m_{graviton}$

$$\left(1 + \frac{f_{NL} \cdot [\Phi_L^2 - \langle \Phi_L \rangle^2]}{\Phi_L}\right) \propto \left[1 - \frac{r}{\lambda_{graviton}}\right] \sim \exp(-r/\lambda_{graviton}) \quad (1.12a)$$

Alternately, if the brane theory model of a gravitational potential were used, with KK graviton modes, then

$$\left(1 + \frac{f_{NL} \cdot [\Phi_L^2 - \langle \Phi_L \rangle^2]}{\Phi_L}\right) \propto \left[1 + \frac{const.}{r^2 k^2}\right] \quad (1.12b)$$

Now for some sort of bounds as to what may be acceptable bounds in error, based upon CMB data

$$|f_{NL} \cdot [\Phi_L^2 - \langle \Phi_L^2 \rangle]| \leq 10^{-5} \cdot |f_{NL}| < ? \text{ up to } 10^{-3} \quad (1.12c)$$

Depending upon which model is used for describing  $\Phi_L$  i.e. as a perturbation of a gravitational potential, this eqn. (1.12c) may allow us to obtain a good guess as to what dimensions are crucial for the formation of a graviton, i.e. how much spread may be permitted. In addition, one can, as a crude approximation write to first order  $\Phi_L \sim 1/r$ . Also the parameter  $f_{NL}$  is usually, often with partly sinusoidal variation, taken from primordial non gaussianity traces taken from the CMBR itself. Also, White and Hu (1996), also have a convenient way to link the gravitational potential  $\Phi$  to temperature fluctuations, and do it as

$$\frac{\Delta T}{T} \Big|_{Final} - \frac{\Delta T}{T} \Big|_{Initial} = -\Phi_{Initial} \quad (1.12d)$$

A simple way to understand what is said by equation (1.12d) is to consider if or not it is linkable to the Sachs-Wolfe effect. Here, the Sachs-Wolfe effect (ISW) occurs when the Universe is dominated in its density by something other than matter. If the Universe is dominated by matter, then large-scale gravitational potential wells and hills do not evolve significantly. If the Universe is dominated by radiation, or by dark energy, though, those potentials do evolve, subtly changing the energy of photons passing through them. If there is a major difference in the initial and final ratios  $\Delta T/T$  of temperature variations are for different red shift values, and for the Friedman model, to good approximation,  $\delta T/T = \phi/3(c^2=1)$ .

$$(\delta T/T) \cong (1/3) \cdot [\Phi_L + f_{NL} \cdot (\Phi_L^2 - \langle \Phi_L \rangle^2)] \quad (1.12e)$$

It is possible to construct good semi classical physical states by such a procedure in this model.

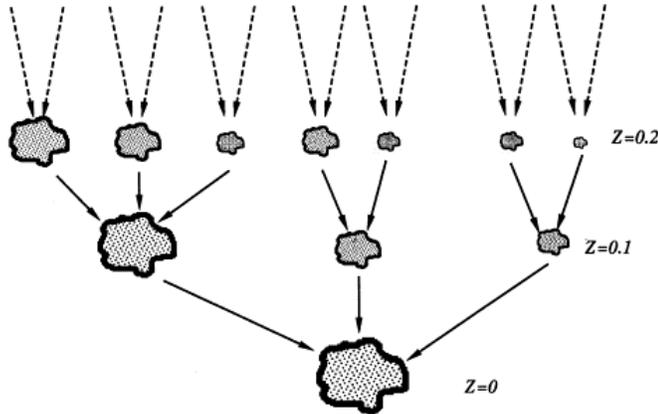
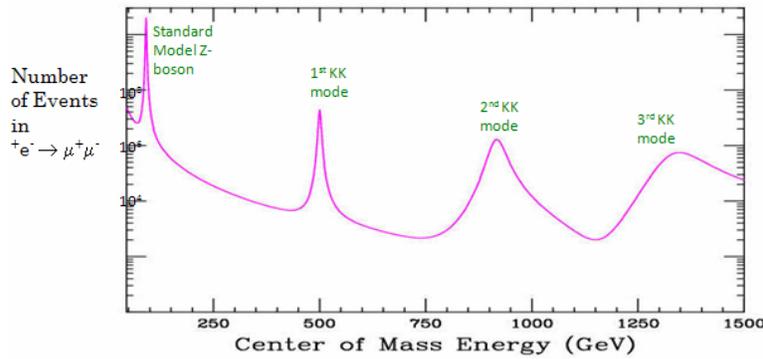


Figure 1. A schematic representation of a halo merging history 'tree'.

**Figure 1. I.e. how we obtain from the ‘bottom up’ development of galactic super structure.**

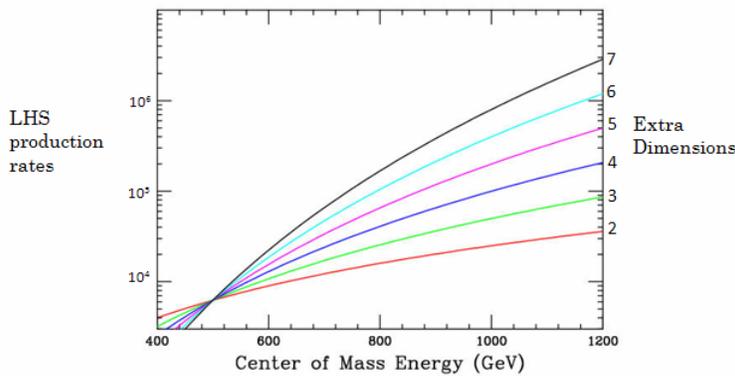
What is actually observed, contradicts this halo emerging history ‘tree’, i.e. Just ONE little problem: DM appears to be fattening up young galaxies, allowing for far-earlier-than-expected creation of early galaxies. “A clutch of massive galaxies that seem to be almost fully-formed just 5 billion years after the big bang challenge models that suggest galaxies can only form slowly. Tendrils of dark matter that fed the young galaxies on gas could be to blame (NASA/CXC/ESO/P Rosati et al)” <http://www.newscientist.com/article/dn16912-overweight-galaxies-forcefed-by-dark-matter-tendrils.html>. Needless, to say though, an analysis of the influence of DM on structure formation takes into consideration the datum as to the relative super abundance of DM in early universe conditions. The following is a KK tower model for gravitons, with the zeroth KK mode being approximately the 4 dimensional graviton.

**Kaluza Klein modes in detector simulations for / as a DM candidate.**



**Figure 2a:** Figure 2a. **Example:** Number of Events in  $e+e- \rightarrow \mu+\mu-$  For a conventional braneworld model with a single curved extra dimension of size  $\sim 10^{-17}$  cm. Numbers range from  $10^4$  to about  $10^8$  for the number of events in scattering. First peak is for KK zero mode, a.k.a. the standard Z boson, ending with the 4<sup>th</sup> peak for the 3<sup>rd</sup> KK mode,

**Production rate for  $e+e- \rightarrow \gamma +$  Graviton**



**Figure 2b: Example:** Production of Graviton Kaluza-Klein modes in flat extra dimensions, probes gravity at distances of  $\sim 10$  to the  $-18$  power cm. The LHS of the graph have production rates ranging from a low point of  $10^4$  at about 600 GeV, to values of about  $10^6$  at about 1000 GeV. Understanding the KK gravitons as a DM candidate may permit us to understand how DM and DE are inter related. See as given below. The discussion of such will involve coherent state of gravitons as contributors to GW.

## 2. Creating an analysis of how graviton mass, assuming branes, can influence expansion of the universe

Following development of \*(1.13) as mentioned above, with inputs from Friedman eqns. To do this,, the following normalizations will be used, i.e.  $\hbar = c = 1$  , so then

$$q = A1 + A2 + A3 + A4 \quad (2.1)$$

Where

$$A1 = \frac{C}{a^3} \cdot \left[ 1 / \sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left( \frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_p^6} \right)} \right] \quad (2.2)$$

$$A2 = - \left( \frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_p^6} \right) / \left[ \frac{C}{a^4} - \frac{\kappa}{a^2} + \left( \frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_p^6} \right) \right] \quad (2.3)$$

$$A3 = - \frac{1}{2} \cdot \left[ \frac{d\rho/d\tau}{3M_4^2} + \frac{d\Lambda_4/d\tau}{3} + \frac{1}{18} \cdot \frac{\rho \cdot (d\rho/d\tau)}{M_p^2} \right] / \left[ \frac{C}{a^4} - \frac{\kappa}{a^2} + \left( \frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_p^6} \right) \right]^{3/2} \quad (2.4)$$

$$A4 = \frac{\kappa}{a^3} \cdot \left[ \frac{da/d\tau}{3} \right] / \left[ \frac{C}{a^4} - \frac{\kappa}{a^2} + \left( \frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_p^6} \right) \right]^{3/2} \quad (2.5)$$

Furthermore, if we are using density according to whether or not 4 dimensional graviton mass is used, then

$$\rho \equiv \rho_0 \cdot \left( \frac{a_0}{a} \right)^3 - \left[ \frac{m_g c^6}{8\pi G \hbar^2} \right] \cdot \left( \frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2} \right) \quad (2.6)$$

So, then one can look at  $d\rho/d\tau$  obtaining

$$d\rho/d\tau = - \left( \frac{\dot{a}}{a} \right) \cdot \left[ 3 \cdot \rho_0 \cdot \left( \frac{a_0}{a} \right)^3 + 4 \cdot \left( \frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left( \frac{m_g c^6}{8\pi G \hbar^2} \right) \right] \quad (2.7)$$

Here, use,  $\left( \frac{\dot{a}}{a} \right) = \sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left( \frac{\rho}{3M_4^2} + \frac{\Lambda_4}{3} + \frac{1}{36} \cdot \frac{\rho^2}{M_p^6} \right)}$ , and assume eqn. (2.6) covers  $\rho$ , then

If  $\hbar \equiv c \equiv 1$  Now, if, to first order,  $d\Lambda_4/d\tau \sim 0$ , and, also, we neglect  $\Lambda_4$  as of being not a major contributor

$$d\rho/d\tau \equiv - \sqrt{\frac{C}{a^4} - \frac{\kappa}{a^2} + \left( \frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_p^6} \right)} \cdot \left[ 3 \cdot \rho_0 \cdot \left( \frac{a_0}{a} \right)^3 + 4 \cdot \left( \frac{a^4}{14} + \frac{a^2}{5} \right) \cdot \left( \frac{m_g}{8\pi G} \right) \right] \quad (2.9)$$

Also, then, set the curvature equal to zero. i.e.  $\kappa = 0$ , so then  $A4 = 0$ , and

Pick, here,  $\rho \equiv \rho_0 \cdot \left( \frac{a_0}{a} \right)^3 - \left[ \frac{m_g}{8\pi G} \right] \cdot \left( \frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2} \right)$ , after  $\hbar = c = 1$ , and also set

$$\Phi(\rho, a, C) = \frac{C}{a^4} + \left( \frac{\rho}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho^2}{M_p^6} \right) \quad (2.14)$$

For what it is worth, the above can have the shift to red shift put in by the following substitution. I.e. use  $1+z = a_0/a$  .. Assume also that  $C$  is the dark radiation term which in the brane version of the Friedman equation scales as  $a^{-4}$  and has no relationship to the speed of light.  $a_0$  is the value of the scale

factor in the present era, when red shift  $z=0$ , and  $a \equiv a(\tau)$  in the past era, where  $\tau$  is an interval of time after the onset of the big bang.  $(a_0/a)^3 = (1+z)^3$ , and  $a \equiv a_0/(1+z)$ , Then

$$\rho(z) \equiv \rho_0 \cdot (1+z)^3 - \left[ \frac{m_g}{8\pi G} \right] \cdot \left( \frac{a_0^4}{14 \cdot (1+z)^4} + \frac{2a_0^2}{5 \cdot (1+z)^2} - \frac{1}{2} \right) \quad (2.15)$$

$$A1(z) \equiv \frac{C \cdot (1+z)^3}{a_0^3} \cdot \left[ 1/\sqrt{\Phi(\rho(z), a_0/(1+z), C)} \right] \quad (2.16)$$

$$A2(z) \equiv - \left( \frac{\rho(z)}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho(z)^2}{M_p^6} \right) / \left[ \Phi(\rho(z), a_0/(1+z), C) \right] \quad (2.17)$$

$$A3(z) \equiv \frac{1}{2} \left( \left[ \frac{1}{3M_4^2} + \frac{1}{18} \cdot \frac{\rho(z)}{M_p^6} \right] / \left[ \Phi(\rho(z), a_0/(1+z), C) \right]^{1/2} \right). \quad (2.18)$$

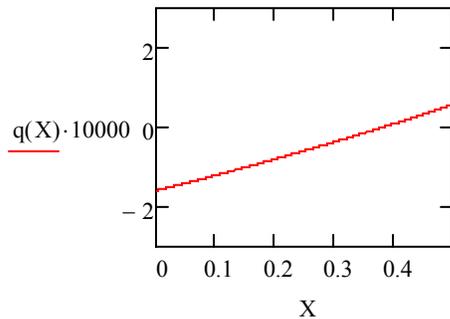
$$\left[ 3 \cdot \rho_0 \cdot (1+z)^3 + 4 \cdot \left( \frac{a_0^4/(1+z)^4}{14} + \frac{a_0^2/(1+z)^2}{5} \right) \cdot \left( \frac{m_g}{8\pi G} \right) \right] \Phi(\rho(z), a_0/(1+z), C) = \frac{C \cdot (1+z)^4}{a_0^4} + \left( \frac{\rho(z)}{3M_4^2} + \frac{1}{36} \cdot \frac{\rho(z)^2}{M_p^6} \right) \quad (2.19)$$

So, for  $4 < z \leq 0$ , i.e. not for the range, say  $z \sim 1100$  380 thousand years after the big bang, it would be possible to model, here

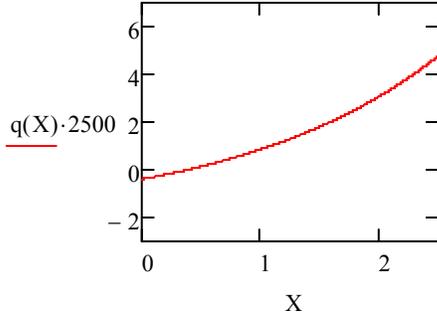
$$q(z) = A1(z) + A2(z) + A3(z) \quad (2.20)$$

Easy to see though, that to first order,  $q(z) = A1(z) + A2(z) + A3(z)$  would be enormous when  $z \sim 1100$ , and also that for  $Z=0$ ,  $q(0) = A1(0) + A2(0) + A3(0) > 0$ . Negative values for Eqn. (2.20) appear probable at about  $z \sim 1.5$ , when Eqn. (2.17) would dominate, leading to  $q(z \sim 1.5)$  with a negative expression/ value. The positive value conditions rely upon, the  $C$  dark radiation term,

And here are the results! Assume X is red shift, Z.  $q(X)$  is De - Celebration. Here we have a graph of De celebration parameter due to small  $m_{graviton} \propto 10^{-65}$  grams, with one additional dimension added



**Figure 4 a: re duplication of basic results of** Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo, 2009, using their parameter values, with an additional term of C for Dark flow added, corresponding to one KK additional dimensions



**Figure 4 b: re duplication of basic results of** Marcio E. S. Alves, Oswaldo D. Miranda, Jose C. N. de Araujo, 2009, using their parameter values, with an additional term of C for ‘Dark flow’ added, corresponding to one KK additional dimensions. Results show asymptotic ‘collapse’ of de celebration parameter as one comes away from the red shift  $Z=1100$  of the CMBR ‘turn on’ regime for de coupling of photons from ‘matter’, in end of ‘dark ages’ Figures 4a, and 4b suggest that additional dimensions are permissible. They do not state that the initial states of GW/ initial vacuum states have to form explicitly due to either quantum or semi classical processes.

### 3. Unanswered questions, and what this suggests for future research endeavors

First of all, what can researchers expect if KK gravitons exist, and exist in inter stellar space with axions? Cembranos, Jose A. R.; Feng, Jonathan L.; Strigari, Louis E. (2007) give a partial answer, i.e. that “The instability of dark matter may produce visible signals in the spectrum of cosmic gamma-rays. We consider this possibility in frameworks with additional spatial dimensions”, i.e. the predicted cosmic gamma ray spectrum may need to be revisited, if there are more than just KK gravitons. It is not just the gamma ray spectrum which may be altered. I.e. [Alexey Boyarsky](#), [Julien Lesgourgues](#), [Oleg Ruchayskiy](#) and [Matteo Viel](#) (2009) have strict Bayesian statistical limits as to what sort of warm to cold dark matter mixes are allowed.

One of their basic result, which is put here,  $\rho_{Baryons}$ ,  $\rho_{Cold-Dark-Matter}$ ,  $\rho_{Warm-Dark-Matter}$  refer to density profiles, of the respective baryons, CDM, and WDM candidates, whereas, the density fluctuations  $\delta_{Baryons}$ ,  $\delta_{Cold-Dark-Matter}$ ,  $\delta_{Warm-Dark-Matter}$  are with regards to the fluctuations of these density values. So

$$\left(\frac{\delta\rho}{\rho}\right) \equiv \frac{\rho_{Baryons}\delta_{Baryons} + \rho_{Cold-Dark-Matter}\delta_{Cold-Dark-Matter} + \rho_{Warm-Dark-Matter}\delta_{Warm-Dark-Matter}}{\rho_{Baryons} + \rho_{Cold-Dark-Matter} + \rho_{Warm-Dark-Matter}} \quad (3.1)$$

In rough details, if axions are CDM, and KK gravitons are for WDM, then up to a point,  $\rho_{Warm-Dark-Matter}$  would dominate Eqn. (3.1) in earlier times, ie. Up to  $Z\sim 1000$ . However, Boyarsky, et al (2009) also stress that as of the recent era, i.e. probably for  $Z\sim .55$  to  $Z\sim 0$  today, they would expect to see the following limiting behavior

$$\begin{aligned} \delta_{Baryons} &\equiv \delta_{CDM}, \\ \delta_{WDM} &\ll \delta_{CDM} \end{aligned} \quad (3.2)$$

In earlier times, what is put in, with regards to eqn. (3.2) would be probably far different. However, up in the present era, the denominator of Eqn (3.1) would be dominated by KK DM, whereas there would be rough equality in the contributions  $\rho_{Cold-Dark-Matter}\delta_{Cold-Dark-Matter}$ ,  $\rho_{Warm-Dark-Matter}\delta_{Warm-Dark-Matter}$ , with the baryon contribution to the numerator being ignorable, due to how small baryon values would be for  $Z\sim .55$  to  $Z\sim 0$  today. Somehow, contributions as to Eqn (3.1) should be compared with.

$$\left(\frac{\delta\rho}{\rho}\right)_{\text{Horizon}} \cong \frac{k^{3/2}|\delta_k|}{\sqrt{2\pi}} \propto \frac{k^{(3/2)+3\alpha-3/2}}{\sqrt{2\pi}} \approx (1/\sqrt{2\pi}) \cdot k^{3\alpha} \quad (3.3)$$

, where  $-0.1 < \alpha < 0.2$ , and  $\alpha \equiv 0 \Leftrightarrow n_s \equiv 1$  and to first order,  $k \cong Ha$ . The values, typically of

$n_s \neq 1$  If working with  $H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \left[\left(\frac{\rho}{3M_4^2} + \frac{\rho^2}{36M_{\text{Planck}}^2}\right) + \frac{C}{a^4}\right]$ , and with a density value

$$\rho \equiv \rho_0 \cdot \left(\frac{a_0}{a}\right)^3 - \left[\frac{m_g c^6}{8\pi G \hbar^2}\right] \cdot \left(\frac{a^4}{14} + \frac{2a^2}{5} - \frac{1}{2}\right) \text{ where } m_g \approx 10^{-65} \text{ grams, and } \alpha < 0.2 \text{ is usually}$$

picked to avoid over production of black holes, a very complex picture emerges. Furthermore, if working with  $\alpha < 0.2$  and  $\alpha \neq 0$ . The following limits as to what is picked as of Eqn. (3.1) in early and later times should be reconciled with.

$$\left(\frac{\delta\rho}{\rho}\right)_{\text{Horizon}} \cong (1/\sqrt{2\pi}) \cdot k^{3\alpha} \sim \frac{H^2}{\dot{\phi}} \propto 10^{-4} - 10^{-5} \quad (3.4)$$

The above equation gives inter relationships between the time evolution of a pop up inflaton field  $\phi$ , and a Hubble expansion parameter  $H$ , and a wave length parameter  $\lambda = (2\pi/k) \cdot a(t)$  for a mode given as  $\delta_k$ .

What should be considered is the inter relationship of the constituent components of (3.6) and  $\lambda \leq H^{-1}$ . What the author thinks is of particular import is to look at whether or not the more general expression, as given by Steinhardt.

$$\left(\frac{\delta\rho}{\rho}\right) \cong Ak^{\left(\frac{n_s-1}{2}\right)} \propto 10^{-4} - 10^{-5} \quad (3.5)$$

Understanding the inter relationships of Eqn.(3.1) to Eqn. (3.5) may allow resolution to Figure 1. of page eight no longer working, i.e. how galaxies could form far earlier than expected.

### **Now for the DM rocket / ram jet problem, as proposed a year ago, a brief review. As put in , in a discussion by Beckwith, 2009, as referenced for SPESIF, 2009**

Quoting from the conference paper : ”. So, we can only talk about perhaps a ram jet engineering construction, i.e., scooping up Axions /DM from the interstellar void and using that as a fuel source. So how do we get around this ? It so happens that the mass values as ascertained above in the authors IDM 2008 meeting presentation, of perhaps up to several hundred GeV is the only way possible to get high frequency

As can be inferred from P. Sikivie (1983), “Every axion which is converted to a photon with the same total energy and going in the same direction produces a momentum kick of

$$\Delta p = mc \times \gamma \cdot (1 - \beta) \quad (4)$$

where  $m$  is the axion rest mass.” What is the rest mass of a KK DM graviton candidate ? It is up to a *mass* of 5 TeV. The conversion factor to be considered is 5 TeV versus the upper limit of 13.5 MeV, tops, for an axion ( it is usually a lot LESS) as reported by A. Bischoff-Kim, M. H. Montgomery and D. E. Winget (2008) wrote, “our analysis yields strong limits on the DFSZ axion mass. Our thin hydrogen solutions place an upper limit of 13.5 meV on the axion, while our thick hydrogen solutions relaxes that limit to 26.5 meV”. For this result, I am picking the 13.5 meV as the upper limit for axion mass analysis. i.e. values as low as 1 eV have been figured as to axion mass, 5 TeV corresponds to  $5.0 \times 10^{12}$  electron volts, Whereas 13.5 MeV is = 13 500 000 electron volts At the high of the energy scale for axions, there is still roughly  $10^5 - 10^6$  times more energy in a DM from KK gravitons, as opposed to axions, and the disparity can be

$10^{12}$  times greater. Contrasting this with the 400 GeV value for WIMPS specified as of being 400 000 000 000 eV, then it is apparent from inspection that the KK graviton would yield a far higher amount of energy ~ mass value than the WIMP.

The implication may be that Eqn (4) has a far stronger change in momentum contribution as to the DM ram jet / rocket problem, than expected. That has to be considered via engineering studies in the future

## Conclusion

Looking at the KK graviton as an enabler to adding more momentum kick to Eqn (4) seems to be a reasonable thought experiment. Of greater concern is the relative distribution of mass/ DM distributions as presented in Eqns (3.1) and (3.5). That has huge implications as to what concentration of DM/ energy scoop up could be configured as to an interstellar probe. Left unsaid here is the necessary datum of a suitable power boost of a ram net, to sufficient speed to work at all. Ultimately, that involves lasers, a topic which the author will present in a AIBEP conference, in November, 2009. In addition, the density profile of DM and of fuel to the rocket engine has to be mapped out. WMAP techniques will not get that for us. Unfortunately, like many scientific endeavors, it will require test flights in the solar system itself, and not just theory to obtain realistic data as to what to expect.

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