The Spiral Structure of NGC 3198.

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Abstract

Observations of NGC 3198 show a discrepancy between the rotational velocity and its apparent geometry which defies the predicted behaviour of Keplerian Dynamics. This paper reconciles this anomaly by considering the relativistic effect of gravity on galactic spiral arms over great distances in a rotating reference frame. Keplerian dynamics hold true in this analysis by considering the rotational behaviour of a cloud of stars as more accurate than that of a central mass with satellites at discrete orbits. A re-examination from first principles describes the spiral arms of NGC 3198 as a linear star cloud of near-uniform density which appears, from our local reference frame, as a non-uniform disc due to its rotation. The apparent non-uniform radial distribution of stars is described by delayed gravitational interactions over great distances in an accelerating reference frame whereby a uniform distribution of stars appears to occupy an increasing circumference.

The theory is substantiated by deriving the shape of a linear star cloud of the dimensions and rotational velocity of NGC 3198 as it would appear from Earth, using Einstein's equations and Keplerian dynamics. Since the derived shape is congruent with the observed shape of NGC 3198, the exact shape and size of the resulting spiral can be used to determine its distance from Earth with great accuracy using simple trigonometry.

Begeman[2] has provided an accurate and extensive measurement of the rotational velocities of stars in the spiral arms of NGC 3198. Under the heading "Discussion", Begeman states the following:

A further analysis of the mass distribution in NGC 3198 has been given by van Albada et al (1985)[1]. The main conclusions from that paper are (i) the rotation curve of NGC 3198 can be described by a two-component mass model consisting of an exponential disk and a spherical halo, and (ii) the amount *of dark matter needed to explain the observed rotation curve out to the last measured point is at least a factor 4 larger than the amount of visible matter.*

According to fundamentals of rotational dynamics and Newton's theories of kinematics[6], the unbalanced force acting on any body in circular orbit is directed constantly towards the centre of the body's orbit. We feel it is reasonable to assume circular orbits for the stars in the spiral arms. We consider an equivalent mass function, $M(r)$. The properties of $M(r)$ are that it determines an equivalent source of gravitational attraction at the centre of the galaxy. This is a technique commonly used to simplify the mathematical calculations in gravitational problems involving complex distributions of matter known as finding the centre of mass of the matter distribution. Finding the centre of mass may be somewhat complex, nevertheless, an equivalent centre of mass function, $M(r)$, must exist because stars in the galactic arms orbit around it. We shall now determine this distribution function.

Consider the rotational dynamics formulae of a star of mass m orbiting an equivalent centre of mass $M(r)$ and having a rotational velocity v:

$$\begin{aligned}
\tfrac{v^2}{r} &= a \\
m\tfrac{v^2}{r} &= ma \\
m\tfrac{v^2}{r} &= F \\
m\tfrac{v^2}{r} &= G\tfrac{M(r)m}{r^2} \\
\tfrac{v^2}{r} &= G\tfrac{M(r)}{r^2} \\
v^2 &= G\tfrac{M(r)}{r}\ .
\end{aligned}$$

We can then write:

$$\frac{v^2}{G} = \frac{M(r)}{r}\ . \tag{1}$$

Therefore, since v is constant throughout the galactic arms, as determined by the data, we must have:

$$M(r) = \kappa r \tag{2}$$

where κ is a constant equal to $\frac{v^2}{G}$. In order to attempt to understand the convolutions and complexities of the $M(r)$ mass distribution function of NGC 3198, we graph it as in figure 1.

Because of the great distances involved, the time it takes for the gravitational influence of the inner region of this mass distribution function to reach

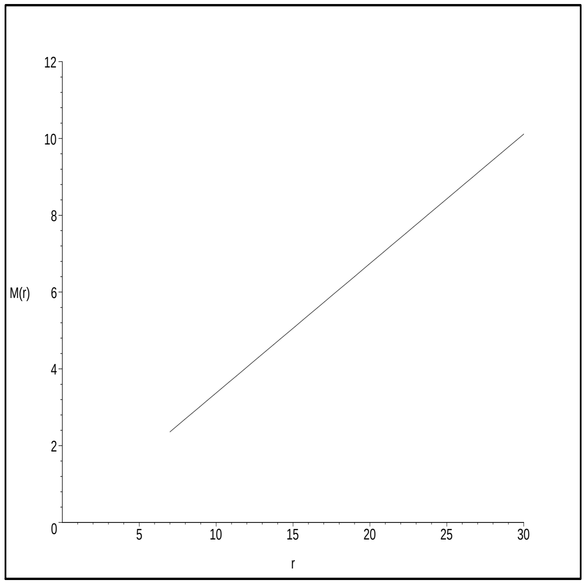

Figure 1: Plot of $M(r)$ vs r.

outer stars must be considered. Considering that changes in gravitational influence travel at the speed of light, the orbital displacement of the distribution function is proportional to the radial distance from the centre of the galaxy. We have mathematically modeled this phenomenon and show it in figure 2.

All matter arranged in this linear distribution, orbiting about the centre of mass, will have the same orbital velocity. This object appears as a spiral because it is very large and the speed of light is finite. This is precisely what we see in the data. Any presence of exotic material, such as dark matter, will invalidate the given data. Note the geometry of the matter distribution of the galaxy determines the rotational velocity profile. A spherical matter distribution will result in a rotational velocity profile proportional to r^2 while a disk will result in a rotational velocity profile proportional to r. Since the rotational velocity profile is constant, the only matter distribution possible is one which is proportional to r. This can only be the result of a linear geometry.

Furthermore, it is a well established fact, and one I have observed myself on many occasions, that our own galaxy does not have spiral arms as we look directly into the centre of our galaxy from a vantage point in the southern hemisphere. All the stars of the spiral arm appear to be in a line of sight from Earth into the centre. It seems our spiral arm is straight while the arms of other galaxies are curved. In essence, we are looking into the nucleus of our galaxy straight down a geodesic. In our local reference frame, the inner stars pull us towards the centre of the galaxy because they all appear lined up that way. However, were we to travel millions of light years above our galaxy, it

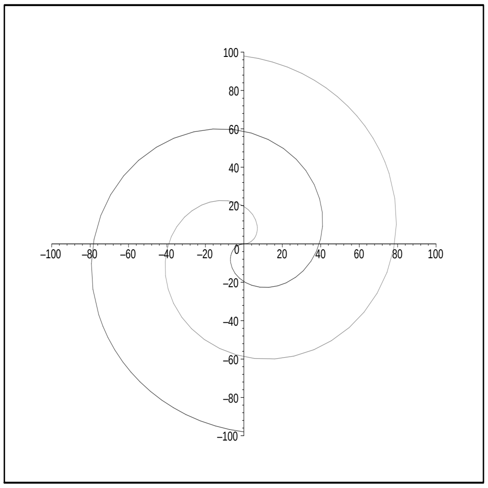

Figure 2: A spiral generated from the effects of the general theory of relativity on a galaxy having a radius of 46 kpc and whose stars all have an orbital velocity of 151 Km s^{-1}. The scale is in thousands of light years.

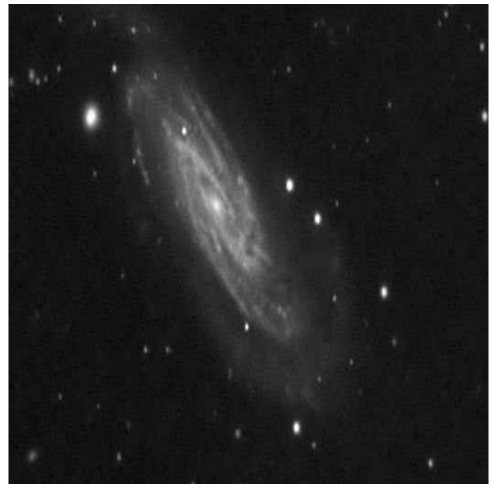

Figure 3: NGC 3198 in Ursa Major. NGC 3198 is classified SBc(R).[9]

would look like a spiral just like very many other galaxies. The inner stars have moved along in their orbit by the time their light and gravitational influence have reached us. See figure 7.

Although the data invalidates the presence of any mass distribution function other than that given by figure 1, we do accept the validity of Begeman's measurements of rotational velocity of the component stars within NGC 3198 as well as Begeman's error analysis. This data is invaluable in obtaining an accurate measure of the distance to NGC 3198.

A spiral, in polar coordinates, is a remarkably simple function. It is:

$$r(\theta) = k\theta. \tag{3}$$

Any two spirals which can be laid one on top of the other so that they coincide are said to be congruent. Congruent figures are said to be equal in all respects. If the parameters of two congruent spirals are equal, then the scales of the rectilinear sizes of the two spirals are also equal. The only parameter of any spiral is the constant k in equation 3. We show that k depends only on the rotational velocity of the galaxy to determine its absolute size. We can then match its spiral morphology, such as that displayed by a photograph, to an independently generated spiral determined by NGC 3198's stellar rotational velocity and the general theory of relativity. The essence and foundation of the general theory of relativity is simply that gravitational influences travel at the speed of light[4]. This is all we require of the theory for this measurement. Note that the photograph has a rectilinear scale of minutes of arc while the generated spiral has a rectilinear scale in thousands of light years. Matching the two spirals by scaling them so that they coincide allows us to determine the distance to the galaxy.

We have determined that the galaxy subtends 7.33 $\pm .1$ minutes of arc for every 136 ± 4 kpc and have therefore obtained the following data:

1. Distance $= 19.6 \pm 1.2$ Mpc.
2. Radius = 180,000 ly. (55 kpc.) $\pm 10\%$

This is done as follows: we first manipulate the position data of stars within NGC 3198 using a geometric projection algorithm to obtain figure 4. To do this we rotate an image of the galaxy 46° so that the major axis of the galaxy is horizontal and will not be altered by further manipulations of the image. The minor axis is measured and compared to the length of the major axis. By inspection and using a spiral overlay oriented with the rotated picture of the galaxy, we measure that a vertical stretching factor of 2.65 will result in a geometric translation equivalent to seeing the galaxy from above. The image is a time lapse photo of the galaxy with a high degree of resolution. Even after applying the required vertical stretch to the image, individual stars can still be seen as a background dusting of stars along the lines of the visible spiral arms of the galaxy. Also, there is very little pixilation. We obtain a printout of the resultant figure 4 and set it aside.

We then turn to generating a spiral overlay. To construct the spiral we note the rotational velocity of the points of the spiral is a constant. Using equation 3 we need to find the parameter k.

Consider an observer at the centre of the galaxy observing a star at one light year radial distance. Then $r =$ one light year. However, the star has moved for one year in its orbit and has traversed v/c light years of arc length. Since we are using radians, we have $\theta = (v/c)$. Furthermore, our scaling parameter to draw the spiral in polar coordinates requires θ to be scaled 2π per complete revolution and r to be scaled as one light year per revolution for a series of concentric circles marking the scale of the spiral to be drawn. We therefore have:

$$r(\theta) = \frac{2\pi\theta}{\left(\frac{v}{c}\right)} \tag{4}$$

To have thousands of light years for the radial scale, we have used:

$$r(\theta) = \frac{2\pi\theta}{1000 \times \left(\frac{v}{c}\right)} \tag{5}$$

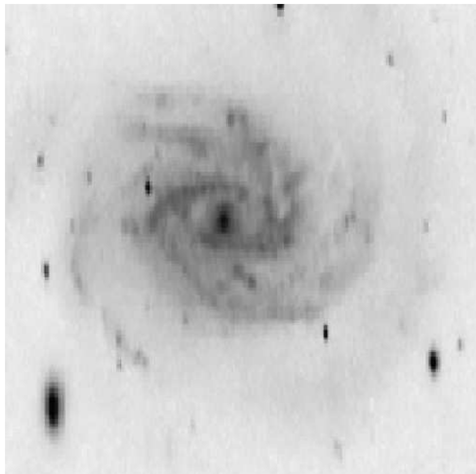

Figure 4: NGC 3198 with the positions of stars adjusted through a geometric projection algorithm in order to view the galaxy from a reference point directly above the galaxy.

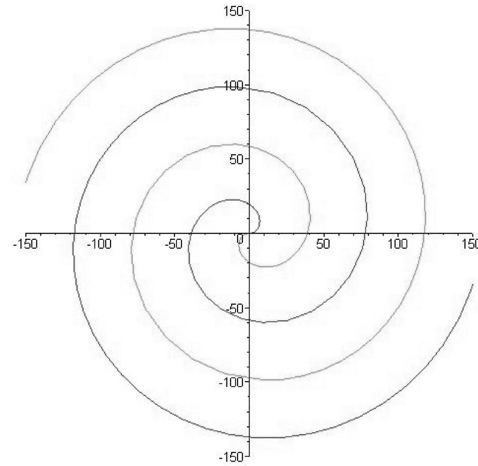

Figure 5: A spiral ruler with size scale included. The axes are in thousands of light years.

We draw a spiral, as in figure 5, which results in a coordinate axis with a scale of thousands of light years using Maple, which we shall refer to as a spiral ruler. We then size figure 5, including coordinate axis, to match the size of the photograph, and print it out on a sheet of clear plastic. Placing the spiral ruler on top of the printout of figure 4 and rotating it about the mutual centres of the two printouts, we achieve a remarkably good match. See figure 6. Since the rotational velocity of the galaxy as measured from a local reference point has been used to create both spirals, that is: one from the general theory of relativity to determine the shape of NGC 3198 from the rotational velocity of its component stars and the other from the constraints of the general theory of relativity on the galaxy itself; they match. We thereby determine the ratio of the angular separation given by figure 4 to absolute distance as determined by the coordinate axis of the spiral ruler and the rest is straightforward. Finding the distance to the galaxy is demonstrated. We have determined the ratio of 136×10^3 ly to $7.33'$ of arc. This is 41.7 kpc per $0.122°$. Then:

$$R = \frac{41.7 \times 360}{2\pi \times .122} \tag{6}$$

where R is the distance to the galaxy, or 19.6 Mpc. We estimate each of the arms to be about $180,000$ ly long. Included is figure 6 to demonstrate the technique more clearly.

Conclusions

We conclude:

1. NGC 3198 is a spiral galaxy by inspection. There is no bar present in the galaxy.

2. NGC 3198 is gravitationally self bound in a linear mass distribution function of constant linear density.

3. NGC 3198 orbits about its centre of mass in a single plane of orbit.

4. The stars of NGC 3198 all orbit with a constant rotational velocity independent of their respective distance from the centre of rotation.

5. From overwhelming evidence in the observations of our own galaxy, the linear mass distribution of NGC 3198 consists of stars and interstellar dust and gas. There is no requirement for exotic material in the linear mass distribution. Any material, exotic or otherwise, having a geometry other than that given by a linear mass distribution would invalidate a constant rotational velocity profile of material making up the galaxy. We therefore conclude there is no “dark matter” in NGC 3198.

6. Although in the reference frame of the stars of NGC 3198, the galaxy appears as a linear mass distribution function, in the reference frame of the Earth, the galaxy appears as a spiral.

7. NGC 3198 is 19.6 ± 1.2 Mpc. away from us.

8. Einstein’s general theory of relativity and Newton’s principles of gravitational attraction hold over very great distances.

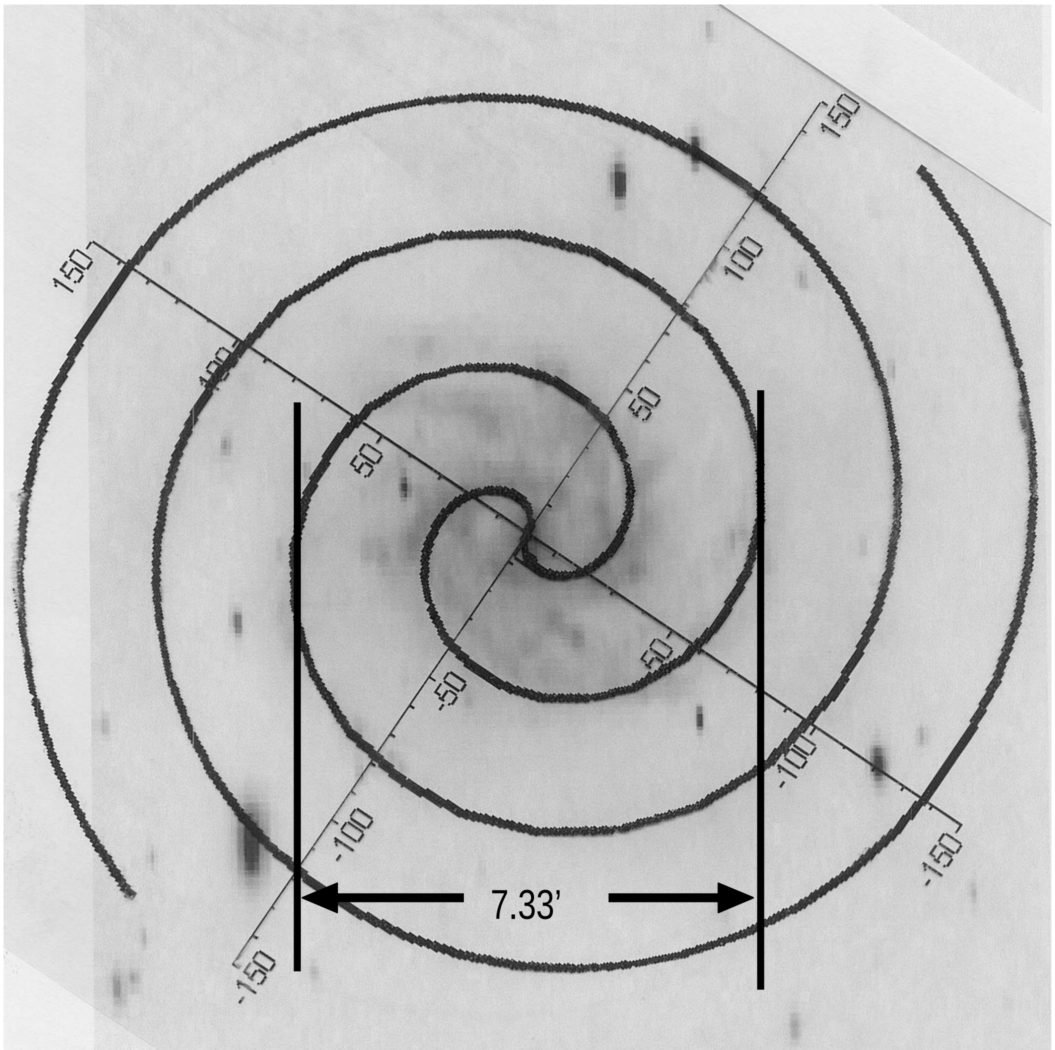

Figure 6: Overlay of figure 4 with figure 5. The angular separation across the relatively brighter ends of the spiral galaxy is indicated. The scale on the axis is in thousands of light years.

Figure 7: The central region of our galaxy. There is no spiral arm structure visible since we are looking down a geodesic into the centre. The geodesic is a line of sight straight down into the photograph like looking into a deep well. The central region of the galaxy here appears at the bottom of the well. By the time the light and gravitational influence of the stars in the central region have reached us they have all moved in their respective orbits. In our reference frame, there appears to be a ball of stars surrounding the central region. This ball of stars is pulling us towards the centre of the galaxy while our tangential rotational velocity keeps us in a circular orbit.[8]

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[9] Photo credit: Glen Youman, Penryn, California, www.astrophotos.net, by permission.