Electromagnetic theory without the Lorentz transformations

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Abstract

We consider point magnetic charges as the sources of the magnetostatic fields, like the point electric charges for the electrostatic fields. Forms of the mutual effects of electric and magnetic charges on themselves and on each other are presented in the forms of vectorial relations. Using these relations incorrectness of a usual manner which eventually leads to the deviation from the classical physics and to the rejection of the Galilean transformations and to the resort to the special relativity is proven. Static potential energy of a distribution of electric and magnetic charges is presented with a careful view on the actual essence of each involved term; this itself shows a sample of the usual carelessness existing in the present current electromagnetic theory even in its static discussions. Almost all the fundamental relations in the present current electromagnetic theory are rewritten in new forms by using the fundamental vectorial relations presented at the beginning of the article. In a more detailed argument the proportion of the curl of the dynamic field of one kind (ie magnetodynamic or electrodynamic) to the time derivative of the static field of the other kind (ie electrostatic or magnetostatic) is established; meanwhile the proportion of the current density of one kind to the time derivative of the field of the same kind is also shown. Lenz's law is obtained in its new form. Static and dynamic inductances are presented. By presenting an aspect which views the space full of much tiny electrostatic and magnetostatic dipoles, the possibility of the proportion of the static fields to the dynamic fields is shown.

The way in which the electromagnetic wave propagates through these dipoles is easily explained by using the mentioned fundamental relations, and by obtaining the new form of Maxwell's equations and deducing the wave equations from them, this simple explanation is endorsed. By deducing the dynamic potential energy and explaining its difference with the static potential energy of a set of charges, the Poynting vector is obtained in its new form. It is shown that the fields of an electromagnetic wave are continuous across the boundary interfaces. Fresnel coefficients are obtained in their quite new forms, and it is explained that the coefficient appearing in the fundamental relations showing the relations between two electric and magnetic charges moving relative to each other, μ, must be construed as a world constant. The reflectance and transmittance are introduced in this new approach, and it is shown that sum of them is identical with one.

1 Introduction

We know that the present current electromagnetic theory, so far it concerns the discussion about electrostatic fields, has often the same attractiveness as the logical discussions of the mathematics and the same consistency as the classical mechanics; but from the point where the interference of the magnetic and electric fields commences, it becomes containing not only of some kind of unnatural complexity but also of sudden unexpected contrariety to the classical mechanics, eg easily and in simple forms the law of action and reaction is breached by this theory. It seems that the origin of this problem should be searched in this fact that in spite of this fact that the method of simplifying of problems and in fact the method of subjective modelling of physical problems had been the usual manner of the great theoretical physicists in discovering the physical laws, using of this manner for the electromagnetism has been neglected and instead, much efforts have been made to substantiate some abstract and subsidiary concepts, like field, and to justify deviation from the classical mechanics logic mathematically by which unnecessary complexities of the theory have been increased and consequently the attractiveness of it has been decreased.

When a high school student reads that the magnetic needle of a compass turns beside a wire carrying current and that a force is exerted on a hanging wire carrying current in a fixed magnetic field, the acceptance of this matter that this is the same law of action and reaction (and so there is also some force on the wire due to the compass needle and on the fixed magnet due to the wire) will be much more logical for him or her than ignoring the general validity of this law and attributing the force exerted on the compass needle directly to some vague field around the wire to which he or she should attribute more genuineness than to the agent causing this field! It is much more logical and desirable for him or her to visualize two electric and magnetic point charges moving toward each other on two different parallel lines and then to deduce the form of forces they exert on each other by comparing the situation with the mentioned forces exerted on the compass needle and on the hanging wire. This simple work has been done and in the following section has found its mathematical and in fact vectorial form. (There you can see the form of the force that two moving electric and magnetic charges exert on each other. I recommend and emphasize to study the 15th article of this book (after study of this (13th) article) for probable physical reason for the existence of such form of force.) Just this same simple act has had some interesting consequences to which we proceed in this article. What is certain is that surely many of the scientists will be glad if some way is found through which the electromagnetic theory can be founded totally on the classical physics and in the frame of Galilean transformations, since the reason presented for the deviation from this physics and the choice of other transformations has been the inability to find just such a way.

2 Fundamental relations

We can consider origin in the form of point magnetic charges for the magnetostatic fields as we consider origin in the form of point electric charges for the electrostatic fields. We show the magnetic point charge by "b" and we assume the signs + (referring to N pole) and − (referring to S pole) for the two kinds of magnetic charge. Now with these definitions if q and q' are two point electric charges and b and b' are two point magnetic charges and $\hat{r}$ is the outward radial unit vector for each charge (see Fig. 1), then the following relations will be always true. In these relations the constant values k, k' and k'' are positive.

Electrostatic force arising from q exerted on q':

$$\mathbf{F}_{q'} = kqq'\hat{r}_{q'}/r^2 = q'\mathbf{E}_{q'} \tag{1}$$

Electrostatic field arising from q in the place of q':

$$\mathbf{E}_{q'} = \mathbf{F}_{q'}/q' = kq\hat{r}_{q'}/r^2 \tag{2}$$

Magnetostatic force arising from b exerted on b':

$$\mathbf{F}_{b'} = k'bb'\hat{r}_{b'}/r^2 = b'\mathbf{B}_{b'} \tag{3}$$

Magnetostatic field arising from b in the place of b':

$$\mathbf{B}_{b'} = \mathbf{F}_{b'}/b' = k'b\hat{r}_{b'}/r^2 \tag{4}$$

And when in an inertial reference the point magnetic charge b has a relative velocity $\mathbf{v}_b$ relative to the point electric charge q, then we shall have the following relations. (It is obvious that $\mathbf{v}_q = -\mathbf{v}_b$; see Fig. 2.)

Electrodynamic force arising from b exerted on q:

$$\mathbf{F}^*_q = k''qb\mathbf{v}_q \times \hat{r}_q/r^2 = \frac{k''}{k'}q\mathbf{v}_q \times \mathbf{B}_q \tag{5}$$

Electrodynamic field arising from b in the place of q:

$$\mathbf{E}_q^* = \mathbf{F}_q^*/q = k''b\mathbf{v}_q \times \hat{r}_q/r^2 = \frac{k''}{k'}\mathbf{v}_q \times \mathbf{B}_q = \frac{-k''}{k'}\mathbf{v}_b \times \mathbf{B}_q \qquad (6)$$

Magnetodynamic force arising from q exerted on b:

$$\mathbf{F}_b^* = -k''qb\mathbf{v}_b \times \hat{r}_b/r^2 = \frac{-k''}{k}b\mathbf{v}_b \times \mathbf{E}_b \qquad (7)$$

Magnetodynamic field arising from q in the place of b:

$$\mathbf{B}_b^* = \mathbf{F}_b^*/b = -k''q\mathbf{v}_b \times \hat{r}_b/r^2 = \frac{-k''}{k}\mathbf{v}_b \times \mathbf{E}_b = \frac{k''}{k}\mathbf{v}_q \times \mathbf{E}_b \qquad (8)$$

We determine "C" for the unit of electric charge and "An" for the unit of magnetic charge. We define $k = 1/(4\pi\epsilon)$, $k' = 1/(4\pi\epsilon')$ and $k'' = \mu/(4\pi)$. Since from the units viewpoint we have $k:\ kg.m^3.C^{-2}.s^{-2}$, $k':\ kg.m^3.An^{-2}.s^{-2}$ and $k'':\ kg.m^2.C^{-1}.An^{-1}.s^{-1}$, we have $\epsilon:\ C^2.s^2.kg^{-1}.m^{-3}$, $\epsilon':\ An^2.s^2.kg^{-1}.m^{-3}$ and $\mu: kg.m^2.C^{-1}.An^{-1}.s^{-1}$.

At present it is usual to define $\mathbf{B}$ (and in fact b) from the relation (5), namely $\mathbf{F}_q^* = (k''/k')q\mathbf{v}_q \times \mathbf{B}_q$, instead of reaching the definition of magnetostatic field $\mathbf{B}$ in the manner presented above. In this usual approach the coefficient k''/k' is equal to one for vacuum and is considered without any units, ie the unit $An.s.m^{-1}$ is considered equivalent to the unit C. So the quantity of $(\mu\epsilon)^{-1/2}$ will have the unit of speed, and $\mu\epsilon'$ is equal to one for vacuum and without any units. (This usual approach is unnecessary, I think. This usual definition of $\mathbf{B}$ is equivalent to say that considering the definition of electrostatic charge (and then knowing the quantity of k) and having the other quantities experimentally, we obtain the amount of $k''b$ from (5) ie from $\mathbf{F}_q^* = k''qb\mathbf{v}_q \times \hat{r}_q/r^2$, which we name it (ie $k''b$) as C_1 at present. Now if we apply a pair of the same b experimentally for the relation (3), ie $\mathbf{F} = k'b^2\hat{r}/r^2$, again having the other quantities experimentally, the amount of $k'b^2$ will be obtained which we name it as C_2. Now having $k''b = C_1$ and $k'b^2 = C_2$, if we want to have $k'' = k'$, we shall obtain $b = C_2/C_1$ and $k'' = k' = C_1^2/C_2$. Then we define the amount of applied b equivalent to C_2/C_1 units of An in order that we shall have $k''/k' = 1$. With such a definition, after experimental measuring in vacuum, it is obtained that $k''/k = c^{-2}$ in which c is a quantity as big as the speed of light in vacuum. Now having $k'' = k'$ and $k'' = k/c^2$ we obtain the amount of c^{-2}, which is certainly very small, for the ratio of the coefficient of the relation (8) to the coefficient of the relation (6), ie for $(k''/k)/(k''/k') = k'/k$. It may be thought that in this manner the ratio of the magnitude of the magnetodynamic field to the magnitude of the electrostatic field is much less than the ratio of the magnitude of the electrodynamic field to the magnitude of the magnetostatic field, or in other words if the magnitudes of the electrostatic and magnetostatic fields are almost the same, the magnetodynamic field will be much less than the electrodynamic field; while this is not the case at all and the ratio of these two coefficients, ie $(k''/k)/(k''/k') = k'/k$, with the supposition of having k, depends on the k', which in turn is equal to C_2/b^2 according to the above explanations, and the amount of the magnitude of C_2/b^2 depends on the definition presented for magnetic charge (b) completely. It is obvious that if, in this quantity, b is defined much less than C_2/C_1 units of An, then the ratio k'/k will be increased very much. In fact, what is important in the discussion of electromagnetism is that we see from the relations (5) and (7) that the ratio of the magnitude of $\mathbf{F}_q^*$ to the magnitude of $\mathbf{F}_b^*$ is equal to one, ie the magnitudes of the electrodynamic and magnetodynamic forces are the same. Unfortunately at present it is usual improperly that by using the wrong deductions and confusing the dynamic and static fields with each other, a relation like $F_m/F_e \leq (v/c)(v_1/c)$ is concluded and so what is understood wrongly is that the magnetic field is much insignificant against the electric field (see Foundations of Electromagnetic Theory by Reitz, Milford and Christy, Addison-Wesley, 1979).)

Relation (8) in the form of $\mathbf{B}_b^* = -k''q\mathbf{v}_b \times \hat{r}_b/r^2$ is in fact the same final conclusion obtained from the experiences and experiments of Biot, Savart and Ampere. On the basis of the above eight relations we can present the electromagnetic theory in a simple form by using the logic of the classical physics, as we shall do in this article. Problem, complexity and deviation from the classical physics logic occur when we try to ignore the existent distinction between stared forces and fields (relations (5) to (8)) and nonstared ones (relations (1) to (4)). For example if, as at present is current, we claim that $\mathbf{B}_q = k'b\hat{r}_q/r^2$, which is the magnetostatic field arising from the magnetic charge b in the place of the electric charge q, is in fact the same $\mathbf{B}_q^* = -k''q_1\mathbf{v}_q \times \hat{r}_q/r^2$, which is the magnetodynamic field arising from the electric charge q_1 in the place of the electric charge q, with this condition that the charge q_1 is in the same place of the charge b, and consequently we substitute $\mathbf{B}_q^*$ for $\mathbf{B}_q$ in the relation (5), ie $\mathbf{F}_q^* = (k''/k')q\mathbf{v}_q \times \mathbf{B}_q$, then we shall obtain $\mathbf{F}_q^* = k''^2qq_1/(k'r^2)\mathbf{v}_q \times (\mathbf{v}_{q_1} \times \hat{r}_q)$, and on the other hand in a similar manner we shall have $\mathbf{F}_{q_1}^* = k''^2q_1q/(k'r^2)\mathbf{v}_{q_1} \times (\mathbf{v}_q \times \hat{r}_{q_1})$ which is not identical with $-\mathbf{F}_q^*$, because considering that $\hat{r}_{q_1} = -\hat{r}_q$ it can be seen easily that the expression $\mathbf{v}_{q_1} \times (\mathbf{v}_q \times \hat{r}_q) = \mathbf{v}_q \times (\mathbf{v}_{q_1} \times \hat{r}_q)$ is not an identity; ie the law of action and reaction is breached easily (even without any limit condition); besides, in principle $\mathbf{B}_q$ and $\mathbf{B}_q^*$ cannot indicate only a single vector field, since otherwise it will be necessary that the vectors $\hat{r}_q$ and $\mathbf{v}_q \times \hat{r}_q$ be always parallel with each other which is not the case obviously.

3 Static potential energy

As this section, please study the whole section 3.1 of the paper "Independence of capacitance from dielectric", and then study the following paragraph.

In a quite similar manner "the magnetostatic potential energy of an elective distribution of the external magnetic charge with the density ρ_B is

$$U_B = 1/2 \int_{V_h} \mathbf{H} \cdot \mathbf{B}_{\rho_B} dv$$

in which $\mathbf{H}$ is the magnetic displacement vector, and we have $\nabla \cdot \mathbf{H} = \rho_B$ in which $\mathbf{H} = \epsilon_0' \mathbf{B} + \mathbf{P}_B$ in which $\mathbf{P}_B$ is an elective distribution of magnetostatic polarization and $\mathbf{B}$ is arising from both $\mathbf{P}_B$ and ρ_B, while $\mathbf{B}_{\rho_B}$ is the field arising only from ρ_B."

4 Modifying and completing the current relations

When the partial charges $dv \sum_i N_i q_i$ (in which N_i is the volume density of the number of the charges q_i's) locating in $\mathbf{r}_1$ are moving with respective velocities $\mathbf{v}_i$'s, according to the relation (8) the partial magnetodynamic field arising from these partial charges in the point $\mathbf{r}_2$ is

$$d\mathbf{B}^*(\mathbf{r}_2) = \frac{\mu}{4\pi}\frac{1}{k}\sum_i (\mathbf{v}_i \times \mathbf{E}_{i\mathbf{r}_2}) = \frac{\mu}{4\pi}\sum_i (\mathbf{v}_i \times dv_1 N_i q_i \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3})$$

$$= \frac{\mu}{4\pi}(\sum_i N_i q_i \mathbf{v}_i) \times \frac{\mathbf{r}_2 - \mathbf{r}_1}{|\mathbf{r}_2 - \mathbf{r}_1|^3} dv_1 = \frac{\mu}{4\pi}\mathbf{J}(\mathbf{r}_1) \times (\mathbf{r}_2 - \mathbf{r}_1)/|\mathbf{r}_2 - \mathbf{r}_1|^3 dv_1,$$

in which $\mathbf{J}$ is the electric current density. In a quite similar manner, by using the relation (6), we can obtain $d\mathbf{E}^*(\mathbf{r}_2) = -\mu/(4\pi)\mathbf{J}'(\mathbf{r}_1) \times (\mathbf{r}_2 - \mathbf{r}_1)/|\mathbf{r}_2 - \mathbf{r}_1|^3 dv_1$, in which $\mathbf{J}'$ is the magnetic current density. Therefore:

$$\mathbf{B}^*(\mathbf{r}_2) = \frac{\mu}{4\pi}\int_V \frac{\mathbf{J}(\mathbf{r}_1) \times (\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3} dv_1, \mathbf{E}^*(\mathbf{r}_2) = -\frac{\mu}{4\pi}\int_V \frac{\mathbf{J}'(\mathbf{r}_1) \times (\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3} dv_1 \tag{9}$$

If we operate the operator $\nabla_2 \cdot$ on both sides of the recent relations, we shall finally obtain

$$\nabla \cdot \mathbf{B}^* = 0, \nabla \cdot \mathbf{E}^* = 0 \tag{10}$$

(Notice that it doesn't mean that $\nabla \cdot \mathbf{B} = 0$ and $\nabla \cdot \mathbf{E} = 0$.)

We have the continuity equations $(\partial\rho_E/\partial t) + \nabla \cdot \mathbf{J} = 0$ and $(\partial\rho_B/\partial t) + \nabla \cdot \mathbf{J}' = 0$. Steady current is a current in which the local charge density is invariant with time, ie the charges are not compressible and then $\partial\rho/\partial t = 0$ and then it is necessary to have $\nabla \cdot \mathbf{J} = 0$ and $\nabla \cdot \mathbf{J}' = 0$ for steady electric and magnetic currents. If we operate the operator $\nabla_2 \times$ on both sides of the relation (9), we shall finally conclude that if the currents are steady, then we shall have:

$$\nabla \times \mathbf{B}^*(\mathbf{r}) = \mu\mathbf{J}(\mathbf{r}), \nabla \times \mathbf{E}^*(\mathbf{r}) = -\mu\mathbf{J}'(\mathbf{r}). \tag{11}$$

The curl of the vectors

$$\mathbf{A}_B(\mathbf{r}_2) = \frac{\mu}{4\pi}\int_{V_1} \frac{\mathbf{J}(\mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|} dv_1, \; \mathbf{A}_E(\mathbf{r}_2) = -\frac{\mu}{4\pi}\int_{V_1} \frac{\mathbf{J}'(\mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|} dv_1 \tag{12}$$

will be equal to $\mathbf{B}^*(\mathbf{r}_2)$ and $\mathbf{E}^*(\mathbf{r}_2)$ respectively; moreover, these vectors have divergences and known normal components on the region boundaries, and then are determined uniquely. We call these unique vectors as magnetic vector potential and electric vector potential respectively.

If we define $\mathbf{A}$ as a vector whose components are the areas enclosed by projections of the supposed directed closed spatial curve C on the yz-, zx-, and xy-plane, then it is obvious that: $\oint_C \mathbf{r} \times d\mathbf{l} = \hat{\imath} \oint_C (ydz - zdy) + \hat{\jmath} \oint_C (zdx - xdz) + \hat{k} \oint_C (xdy - ydx) = \hat{\imath}(A_x + A_x) + \hat{\jmath}(A_y + A_y) + \hat{k}(A_z + A_z) = 2\mathbf{A} \Rightarrow 1/2 \oint_C \mathbf{r} \times d\mathbf{l} = \mathbf{A}$

Now if this curve is in fact a circuit carrying electric current I or magnetic current I' and on definition $\mathbf{m}_B = I\mathbf{A}$ and $\mathbf{m}_E = -I'\mathbf{A}$ are magnetodynamic dipole moment and electrodynamic dipole moment respectively, then we shall have:

$$\mathbf{m}_B = \frac{1}{2} I \oint_C \mathbf{r} \times d\mathbf{l}, \quad \mathbf{m}_E = -\frac{1}{2} I' \oint_C \mathbf{r} \times d\mathbf{l}. \tag{13}$$

By using some mathematical tricks and considering the relations (13) it can be shown that the magnetic vector potential of a point magnetodynamic dipole with the moment $\mathbf{m}_B$ locating in $\mathbf{r}'$ is

$$\mathbf{A}_B(\mathbf{r}) = (\frac{\mu}{4\pi})\mathbf{m}_B \times \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}, \tag{14}$$

and the electric vector potential of a point electrodynamic dipole with the moment $\mathbf{m}_E$ locating in $\mathbf{r}'$ is

$$\mathbf{A}_E(\mathbf{r}) = (\frac{\mu}{4\pi})\mathbf{m}_E \times \frac{(\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3}. \tag{15}$$

$\mathbf{M}_B(\mathbf{r}') = d\mathbf{m}_B/dv'$ and $\mathbf{M}_E(\mathbf{r}') = d\mathbf{m}_E/dv'$ are in turn the magnetodynamic dipole moment density and the electrodynamic dipole moment density (like the static cases $\mathbf{P}_B(\mathbf{r}') = d\mathbf{p}_B/dv'$ and $\mathbf{P}_E(\mathbf{r}') = d\mathbf{p}_E/dv'$). Then considering the relations (14) and (15) we can write

$$d\mathbf{A}_B(\mathbf{r}) = \frac{\mu}{4\pi} \frac{\mathbf{M}_B(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dv', d\mathbf{A}_E(\mathbf{r}) = \frac{\mu}{4\pi} \frac{\mathbf{M}_E(\mathbf{r}') \times (\mathbf{r} - \mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|^3} dv' \tag{16}$$

in which $\mathbf{r}'$ is the location of the point dipole $\mathbf{M}(\mathbf{r}')dv'$.

Considering the manner presented in many of the electromagnetic textbooks we conclude that the magnetodynamic polarization current density is $\mathbf{J}_{M_B} = \nabla \times \mathbf{M}_B$, and in a quite similar manner it can be concluded that the electrodynamic polarization current density is $\mathbf{J}_{M_E} = -\nabla \times \mathbf{M}_E$.

If we make use of μ_0 (related to the free space) instead of μ and integrate the equations (16) over the volume of the dynamic polarized matter, V, to obtain the proper expressions for the respective vector potentials and take the curl of these potentials in order to obtain the respective dynamic fields, then using some mathematical tricks we shall finally obtain the expression

$$-\mu_0 \nabla \frac{1}{4\pi} \int_V \mathbf{M}_B(\mathbf{r}') \cdot \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} dv' + \mu_0 \mathbf{M}_B(\mathbf{r}) \tag{17}$$

for the magnetodynamic field in point $\mathbf{r}$ arising from the distribution of the magnetodynamic polarized matter, and the expression

$$-\mu_0 \nabla \frac{1}{4\pi} \int_V \mathbf{M}_E(\mathbf{r}') \cdot \frac{\mathbf{r}-\mathbf{r}'}{|\mathbf{r}-\mathbf{r}'|^3} dv' + \mu_0 \mathbf{M}_E(\mathbf{r}) \tag{18}$$

for the electrodynamic field in point $\mathbf{r}$ arising from the distribution of the electrodynamic polarized matter. (The reason for using zero subscript is that we want to make distinct the role of the dynamic polarized matter from the role of the free space. So substituting μ_0 for μ, curls of the integrals of the relations (16), which finally lead to the relations (17) and (18), will show the contribution of the dynamic polarized matter, without any regarding to free space, for making the dynamic field.) If we add the role of the nonpolarization conventional current densities, $\mathbf{J}$ and $\mathbf{J}'$, in making the dynamic fields (given by the relations (9)) to the expressions (17) and (18), the following general relations for the dynamic fields, arising from the polarization and nonpolarization currents, will be obtained:

$$\mathbf{B}^*(\mathbf{r}) = \frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}(\mathbf{r}') \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dv' - \mu_0 \nabla \frac{1}{4\pi} \int_V \mathbf{M}_B(\mathbf{r}') \cdot \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dv' + \mu_0 \mathbf{M}_B(\mathbf{r}), \tag{19}$$

$$\mathbf{E}^*(\mathbf{r}) = -\frac{\mu_0}{4\pi} \int_V \frac{\mathbf{J}'(\mathbf{r}') \times (\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dv' - \mu_0 \nabla \frac{1}{4\pi} \int_V \mathbf{M}_E(\mathbf{r}') \cdot \frac{(\mathbf{r}-\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|^3} dv' + \mu_0 \mathbf{M}_E(\mathbf{r}) \tag{20}$$

As we said $\mathbf{J}_{M_B}$ and $\mathbf{J}_{M_E}$ are in turn the curls of $\mathbf{M}_B$ and $-\mathbf{M}_E$, and since the divergence of a curl is zero, the divergences of these dynamic polarization currents are zero (these currents are steady). Therefore, if we suppose that $\mathbf{J}$ and $\mathbf{J}'$ are the (nonpolarization) transport currents and in addition their divergences are zero (they are steady), then according to the relation (11) we shall have $\nabla \times \mathbf{B}^* = \mu_0(\mathbf{J} + \mathbf{J}_{M_B})$ and $\nabla \times \mathbf{E}^* = -\mu_0(\mathbf{J}' + \mathbf{J}_{M_E})$. Considering the curl form of the dynamic polarization currents, we can write the recent relations in the following forms:

$$\nabla \times (\frac{1}{\mu_0}\mathbf{B}^* - \mathbf{M}_B) = \mathbf{J}, \;\; \nabla \times (\frac{1}{\mu_0}\mathbf{E}^* - \mathbf{M}_E) = -\mathbf{J}' \tag{21}$$

We define the magnetic intensity vector, $\mathbf{H}^*$, and the electric intensity vector, $\mathbf{D}^*$, as $\mathbf{H}^* = 1/\mu_0 \mathbf{B}^* - \mathbf{M}_B$ and $\mathbf{D}^* = 1/\mu_0 \mathbf{E}^* - \mathbf{M}_E$ respectively. Then:

$$\nabla \times \mathbf{H}^* = \mathbf{J}, \;\; \nabla \times \mathbf{D}^* = -\mathbf{J}' \tag{22}$$

5 Deduction of Maxwell's equations

We now proceed to the fundamental discussions of this article more. Consider Fig. 3 which shows a piece of a U-shaped wire with a mobile wire on it, set in a uniform magnetostatic field B being perpendicular to the area limited by the wires.

Notice that in the following discussion there is no necessity that the wires to be good conductors, and they can be, for example, from wood. Suppose that the mobile wire has a velocity $\mathbf{v}$, as in the figure. According to the relation (6) we know that $\mathbf{E}^* = (k''/k')\mathbf{v} \times \mathbf{B}$ is the force exerted on the electric charge unit in the mobile wire. If we choose the unit vector $\hat{n}$ normal to the area surrounded by the wires by using the right-hand rule so that the thumb indicates the direction of $\hat{n}$ while the other four fingers are in the direction of $\mathbf{E}^*$, then depending on the direction of $\mathbf{v}$ we shall have:

$$\Phi_B = \int_S \mathbf{B} \cdot \hat{n} da = \mp BLX \Rightarrow \frac{d\Phi_B}{dt} = \mp BL\frac{dx}{dt} = \mp BL(\pm v) = -BLv$$

or

$$-\frac{d}{dt}\int_S \mathbf{B} \cdot \hat{n} da = BvL.$$

Now we want to find an appropriate expression for BvL. As said, we know $\mathbf{E}^* = (k''/k')\mathbf{v} \times \mathbf{B}$, ie in each point of the mobile wire the magnitude of $\mathbf{E}^*$ is equal to $(k''/k')vB$ and its direction is the same direction of the vector $\mathbf{v} \times \mathbf{B}$ along the mobile wire. So we can write $E^* = (k''/k')vB \Rightarrow (k'/k'')E^*L = BvL$, and since E^* is uniform in all points of the mobile wire, we can write $E^*L = \int_L \mathbf{E}^* \cdot d\mathbf{l}$, and since E^* is zero, in principle, in other points of the loop, we can expand the integral to the whole loop and write $E^*L = \oint_c \mathbf{E}^* \times d\mathbf{l}$. It is obvious that if the wires are conductor, they will cause produce of a velocity for the electric charge unit in the whole loop (even in the mobile wire itself) which , of course, this velocity produces an electrodynamic field normal to the path in the external magnetostatic field and then its respective $\mathbf{E}^* \cdot d\mathbf{l}$ is zero and we again have the same general relation $E^*L = \oint_c \mathbf{E}^* \cdot d\mathbf{l}$. Then we can write $(k'/k'')\oint_c \mathbf{E}^* \cdot d\mathbf{l} = BvL$ and then:

$$-\frac{d}{dt}\int_S \mathbf{B} \cdot \hat{n} da = \frac{k'}{k''}\oint_C \mathbf{E}^* \cdot d\mathbf{l} \Rightarrow \oint_C \mathbf{E}^* \cdot d\mathbf{l} = -\frac{k''}{k'}\frac{d}{dt}\int_S \mathbf{B} \cdot \hat{n} da.$$

With attention to the above discussion and with notice to the right side of the recent relation we see that the quantity being changed with time is the area of integration not $\mathbf{B}$, but obviously it should be presentable that this is equivalent to this task that we assume the integration area is a time-constant and $\mathbf{B}$ changes with time. Thus $\oint_C \mathbf{E}^* \cdot d\mathbf{l} = -k''/k' \int_S \partial\mathbf{B}/\partial t \cdot \hat{n} da$. With some attention and sharp-sightedness we should understand that a mathematical careful analysis should be able to generalize the recent equation for each position and each circuit. (Meanwhile, remembrance of this point that the circuit may be non-conductor is recommended.) By using Stokes' theorem the recent equation will take the form $\int_S \nabla \times \mathbf{E}^* \cdot \hat{n}da = -k''/k' \int_S \partial \mathbf{B}/\partial t \cdot \hat{n}da$, which with its applying to the partial area da we obtain $(\nabla \times \mathbf{E}^* + (k''/k')\partial \mathbf{B}/\partial t) \cdot \hat{n} = 0$, and since this relation must be valid for each $\hat{n}$ without any attention to its direction, $\nabla \times \mathbf{E}^* + (k''/k')\partial \mathbf{B}/\partial t = \mathbf{0}$. (For realizing the validity of Stokes' theorem here, we can suppose that in each point in the space there is an infinitesimal closed circuit perpendicular to $\mathbf{B}$ in that point, which certainly the time-changes of $\mathbf{B}$ cause the induction of some $\mathbf{E}^*$'s, which these $\mathbf{E}^*$'s will cancel each others in the common parts of the adjacent small circuits, or in other words, for small conductor circuits, the inner currents will cancel each others. Thus, in fact, in each given time, $\mathbf{E}^*$ is a well-behaved point vector function in the space.)

Now we return just to the beginning of the above discussion and make some changes in it. So, instead of notice to the relation (6) we consider the relation (8), ie $\mathbf{B}^* = -k''/k\mathbf{v} \times \mathbf{E}$ which is the force exerted on the magnetic charge unit in the mobile wire in Fig. 4.

If we choose $\hat{n}$ normal to the area surrounded by the wires by using the right-hand rule so that the thumb indicates the direction of $\tilde{n}$ while the other four fingers are in the direction of B*, then depending on the direction of $\mathbf{v}$ we shall have:

$$\Phi_E = \int_S \mathbf{E} \cdot \hat{n}da = \pm ELX \Rightarrow \frac{d\Phi_E}{dt} = \pm EL\frac{dX}{dt} = \pm EL(\pm v) = ELv$$

or $d/dt \int_S \mathbf{E} \cdot \hat{n}da = EvL$.

Now we know $\mathbf{B}^* = -(k''/k)\mathbf{v} \times \mathbf{E}$, ie in each point in the mobile wire the magnitude of $\mathbf{B}^*$ is equal to $(k''/k)vE$ and its direction is the same direction of the vector $\mathbf{E} \times \mathbf{v}$ along the mobile wire. Thus $B^* = (k''/k)vE \Rightarrow (k/k'')B^*L = EvL$, and since with a discussion similar to the previous discussion we can write $B^*L = \oint_C \mathbf{B}^* \cdot d\mathbf{l}$, we have $EvL = (k/k'') \oint_C \mathbf{B}^* \cdot d\mathbf{l}$, and then

$$\frac{d}{dt}\int_S \mathbf{E} \cdot \hat{n}da = (k/k'') \oint_C \mathbf{B}^* \cdot d\mathbf{l} \Rightarrow \oint_C \mathbf{B}^* \cdot d\mathbf{l} = (k''/k) \int_S \frac{\partial \mathbf{E}}{\partial t} \cdot \hat{n}da,$$

and with a discussion similar to the previous one, we shall obtain $\nabla \times \mathbf{B}^* - (k''/k)\partial \mathbf{E}/\partial t = \mathbf{0}$. (Notice that existence of magnetic current is not necessary, because as we said the circuit could be non-conductor.)

We said before that if the currents were steady, ie the charge densities were not being changed with time, we would have the relations (11), and now we have shown in fact that under the same condition, ie if the currents are steady, we have $\nabla \times \mathbf{B}^* = (k''/k)\partial \mathbf{E}/\partial t$ and $\nabla \times \mathbf{E}^* = -(k''/k)\partial \mathbf{B}/\partial t$. This condition is implied by perceiving this fact that as the charge density hasn't been changed during the displacement of the mobile wire, the charge density won't also be changed in the position of the point loop, which is in fact a point of the line of the current being in fact the same movement of the sources, during the displacement of the sources causing the time variation of the static fields. (The movement of the sources is in fact the same currents in the relations (9) which ultimately lead to the relations (11).) Thus we have

$$\mathbf{J}' = \epsilon' \frac{\partial \mathbf{B}}{\partial t}, \;\; \mathbf{J} = \epsilon' \frac{\partial \mathbf{E}}{\partial t}, \tag{23}$$

and in any case:

$$\nabla \times \mathbf{B}^* = \mu\epsilon \frac{\partial \mathbf{E}}{\partial t}, \;\; \nabla \times \mathbf{E}^* = -\mu\epsilon \frac{\partial \mathbf{B}}{\partial t}. \tag{24}$$

Now again we return to the beginning of the previous discussions related to the loop circuit. Depending on the kind of material used for the wires (including the mobile wire) the force exerted on the unit charge, ie $\mathbf{E}^*$, can move the charges in the circuits and cause them to reach the final state of equilibrium in motion very soon and thus produce an electric current. The work which $\mathbf{E}^*$ does in transferring the unit electric charge from one end of the mobile wire to its other end is E^*L or $(k''/k')BvL$. We can attribute this work, $\mathcal{E}$, to some potential difference formed between the two ends of the mobile wire, $\mathcal{V}$. Consequently we have

$$(\oint_C \mathbf{E}^* \cdot d\mathbf{l} = \textit{The work done on the unit electric charge in a complete cycle})$$

$$= (k''/k')BvL \quad \& \quad (k'/k'') \oint_C \mathbf{E}^* \cdot d\mathbf{l} = (-\frac{d}{dt}\int_S \mathbf{B} \cdot \hat{n} da = -\frac{d\Phi_B}{dt})$$
$$\Rightarrow \mathcal{V} = \mathcal{E} = -(k''/k')\frac{d\Phi_B}{dt},$$

and then, considering the circulation direction on the loop for showing $\hat{n}$, which is in agreement with the right-hand rule mentioned at the beginning of this section, Lenz's law is obtained which states: "If, passing the time, Φ_B is increased in the circuit, $\mathcal{E}$ has such form that as a result of the electric current arising from it, some magnetodynamic flux is produced that wants to decrease Φ_B. And if, passing the time, Φ_B is decreased in the circuit, $\mathcal{E}$ has such form that as a result of the electric current arising from it, some magnetodynamic flux is produced that wants to increase Φ_B."

And referring to the next loop discussion we see that the work done by $\mathbf{B}^*$ in transferring the unit magnetic charge from one end to the other end of the mobile wire is B^*L or $(k''/k)EvL$, which we show it by $\mathcal{B}$, and we can attribute it to a potential difference, $\mathcal{V}'$, formed between the two ends of the mobile wire. Consequently we have

$$(\oint_C \mathbf{B}^* \cdot d\mathbf{l} = \textit{The work done on the unit magnetic charge in a complete cycle})$$

$$=(k''/k)EvL \;\; \& \;\; (k/k'')\oint_C \mathbf{B}^* \cdot d\mathbf{l} = (\frac{d}{dt}\int_S \mathbf{E} \cdot \hat{n} da = \frac{d\Phi_E}{dt}) \Rightarrow \mathcal{V}' = \mathcal{B} = (k''/k)\frac{d\Phi_E}{dt}.$$

Here with attention to this fact that the electrodynamic field arising from a magnetic current is obtained by using the left-hand rule, no right-hand rule, (so that the thumb of the left hand indicates the direction of the magnetic current while its other four fingers are in the direction of the electrodynamic field; pay attention to the relation (8) in comparison with the relation (6)) and considering the mentioned circulation direction on the loop for showing $\hat{n}$, we have Lenz's law again which states: "If, passing the time, Φ_E is increased in the circuit, $\mathcal{B}$ has such form that as a result of the magnetic current arising from it, some electrodynamic flux is produced that wants to decrease Φ_E. And if, passing the time, Φ_E is decreased in the circuit, $\mathcal{B}$ has such form that as a result of the magnetic current arising from it, some electrodynamic flux is produced that wants to increase Φ_E."

The above two results can also be shown as:

$$\mathcal{E} = -\mu\epsilon' \frac{d\Phi_B}{dt}, \quad \mathcal{B} = \mu\epsilon \frac{d\Phi_E}{dt} \tag{25}$$

With attention to the relation (9) and by using the identity $\mathbf{J}dv = Id\mathbf{l}$, in the case of a rigid and motionless isolated circuit we have:

$$\mathbf{B}^*(\mathbf{r}_2) = \frac{\mu}{4\pi} I_1 \int_1 \frac{d\mathbf{l}_1 \times (\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3} \ \ \& \ \ (The\ circuit\ is\ rigid\ and\ motionless.)$$

$$\Rightarrow \frac{d\mathbf{B}^*(\mathbf{r}_2)}{dI_1} = \mathbf{B}^*(\mathbf{r}_2)/I_1, \tag{26}$$

$$\Phi_B^* = \int_S \mathbf{B}^*(\mathbf{r}_2) \cdot \hat{n} da \Rightarrow \frac{d\Phi_B^*}{dI_1} = \int_S \frac{d\mathbf{B}^*(\mathbf{r}_2)}{dI_1} \cdot \hat{n} da, \tag{27}$$

$$(26) \ \ \& \ \ (27) \ \ \& \ \ (The\ circuit\ is\ rigid\ and\ motionless.) \Rightarrow$$

$$\frac{d\Phi_B^*}{dI_1} = \frac{\Phi_B^*}{I_1}, \tag{28}$$

$$(The\ circuit\ is\ rigid\ and\ motionless.) \ \ \& \ \ \Phi_B^* = \int_S \mathbf{B}^*(\mathbf{r}_2) \cdot \hat{n} da$$

$$\Rightarrow \frac{d\Phi_B^*}{dt} = \int_S \frac{d\mathbf{B}^*(\mathbf{r}_2)}{dt} \cdot \hat{n} da = \int_S \frac{d\mathbf{B}^*(\mathbf{r}_2)}{dI_1} \frac{dI_1}{dt} \cdot \hat{n} da = \frac{d\Phi_B^*}{dI_1} \frac{dI_1}{dt} \ \ \& \ \ (28) \Rightarrow$$

$$\frac{d\Phi_B^*}{dt} = \frac{\Phi_B^*}{I_1} \frac{dI_1}{dt} \tag{29}$$

In a similar manner we have $d\Phi_E^*/dt = (d\Phi_E^*/dI_1')(dI_1'/dt)$ or $d\Phi_E^*/dt = (\Phi_E^*/I_1')(dI_1'/dt)$. We define $L_B^* = d\Phi_B^*/dI$ and $L_E^* = d\Phi_E^*/dI'$ as magnetodynamic and electrodynamic inductances respectively. The magnetodynamic inductance, L_B^*, is in fact not useful so much, and its chief role is in fact giving order to the magnetostatic dipoles of the medium and so producing the magnetostatic inductance $L_B = d\Phi_B/dI$. Probably in many cases we have $\Phi_B = a'\Phi_B^*$ practically which itself is arising from $\mathbf{B} = a'\mathbf{B}^*$ which we prescribe this as the definition of the medium being "magnetolinear" in which a' is the proportion constant which is nonnegative. By presenting similar matters we can attain to the relation

$\mathbf{E} = a\mathbf{E}^*$ which we prescribe it as the definition of the medium being "electrolinear" in which a is the proportion constant that is nonnegative. Trying to justify these proportions we suppose (and probably posterity will show) that the space of any substance is full of some much tiny electrostatic and magnetostatic dipoles that have almost random orientations in general state. By exerting a dynamic field in this space some of the relevant dipoles which have weaker bonds will be put into order in the direction of the field, and it is natural that they will produce a relevant static field in the same direction of the dynamic field in a point of the space between the dipoles. It is obvious that if for instance the exerted dynamic field becomes twofold, in addition to the oriented previous group of the relevant static dipoles another equinumber group of these dipoles, which of course have stronger bonds, will be put into order in the direction of the field, and therefore the static field will become twofold in the same point between the dipoles. In this same manner the proportion of the exerted dynamic field to the static field produced in the space between the dipoles as a result of the orientation of the dipoles in the direction of the dynamic field will become meaningful. We should know that these tiny dipoles don't belong to the substance in question, but have penetrated it.

We suppose that Φ_B to be a point function of I (what that I don't think that is valid, for instance, for the ferromagnetic materials). Therefore, $d\Phi_B/dI = a' d\Phi_B^*/dI$ or $L_B = a' L_B^*$. (We should know that we can say $d\Phi_B/dI = \Phi_B/I$ still, because $\Phi_B = a'\Phi_B^*$ and $d\Phi_B^*/dI = \Phi_B^*/I$, and I think that the necessity of distinguishing between Φ_B/I and $d\Phi_B/dI$ which is necessary sometimes is because of the deviation from linearity of substances (pay attention to the definition of linearity presented above).) Thus

$$\frac{d\Phi_B^*}{dt} = \frac{d\Phi_B^*}{dI}\frac{dI}{dt} = L_B^*\frac{dI}{dt} = \frac{L_B}{a'}\frac{dI}{dt} \;\;\&\;\; \mathcal{E} = -\mu\epsilon'\frac{d\Phi_B}{dt}\frac{dI}{dt} = -\mu\epsilon' L_B\frac{dI}{dt}$$

$$\Rightarrow \mathcal{E} = -\mu\epsilon' a' L_B^*\frac{dI}{dt}.$$

(The relation that is under consideration instead of this relation in the present electromagnetic books has the form $\mathcal{E} = -L dI/dt$ in which $L = d\Phi/dI$, and of course the magnetodynamic flux is in mind from Φ, without any coefficient like a' that can show the intervention of the role of the medium in $\mathcal{E}$. Now since $\mathcal{E}$ is measurable practically, if we make the conditions so that all the effective parameters remain unaltered except that the mediums of experiment are made altered in the magnetic respect, then if $\mathcal{E}$ will be altered with altering the mediums, the aspect presented in this article will be confirmed practically.) In a quite similar manner we obtain $L_E = aL_E^*$ and $\mathcal{B} = \mu\epsilon a L_E^* dI'/dt$ for an electrolinear medium.

So far only isolated circuits have been under study, so that the totality of the flux passing through the circuit was due to the current of the circuit itself. We can eliminate this limitation by supposing that there exist n circuits with respective numbers $1, 2, 3, \cdots, n$. In this state, if we suppose that the medium is full-linear (ie both magnetolinear and electrolinear), then we shall have

$$\mathcal{E}_i = -\mu\epsilon' \sum_{j=1}^{n} \frac{d\Phi_{B_{ij}}}{dt} = -\mu\epsilon' \sum_{j=1}^{n} \frac{d\Phi_{B_{ij}}}{dI_j}\frac{dI_j}{dt} = -\mu\epsilon' \sum_{j=1}^{n} M_{B_{ij}}\frac{dI_j}{dt}$$

and

$$\mathcal{B}_i = \mu\epsilon \sum_{j=1}^{n} \frac{d\Phi_{E_{ij}}}{dt} = \mu\epsilon \sum_{j=1}^{n} \frac{d\Phi_{E_{ij}}}{dI'_j}\frac{dI'_j}{dt} = \mu\epsilon \sum_{j=1}^{n} M_{E_{ij}}\frac{dI'_j}{dt}.$$

It is obvious that $M_{E_{ii}} = L_{E_i}$ and $M_{B_{ii}} = L_{B_i}$. We know that $M_{B_{ij}} = d\Phi_{B_{ij}}/dI_j = a' d\Phi^*_{B_{ij}}/dI_j = a'\Phi^*_{B_{ij}}/I_j$ and since $M^*_{B_{ij}} = d\Phi^*_{B_{ij}}/dI_j$, we have $M^*_{B_{ij}} = \Phi^*_{B_{ij}}/I_j$ and also $M_{B_{ij}} = a' M^*_{B_{ij}}$; and in a similar manner $M^*_{E_{ij}} = \Phi^*_{E_{ij}}/I'_j$ and also $M_{E_{ij}} = aM^*_{E_{ij}}$. Now suppose that $i = 2$ and $j = 1$, then

$$\Phi^*_{B_{21}} = \int_{S_2} \mathbf{B}^*_{21} \cdot \hat{n} da_2 = \int_{S_2} [\frac{\mu}{4\pi} I_1 \oint_{C_1} \frac{d\mathbf{l}_1 \times (\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3}] \cdot \hat{n} da_2$$
$$= \frac{\mu}{4\pi} I_1 \int_{S_2} (\oint_{C_1} \frac{d\mathbf{l}_1 \times (\mathbf{r}_2 - \mathbf{r}_1)}{|\mathbf{r}_2 - \mathbf{r}_1|^3}) \cdot \hat{n} da_2;$$

and by using the mathematical relation $\oint_{C_1} (d\mathbf{l}_1 \times (\mathbf{r}_2 - \mathbf{r}_1)/|\mathbf{r}_2 - \mathbf{r}_1|^3) = \nabla_2 \times \oint_{C_1} (d\mathbf{l}_1/|\mathbf{r}_2 - \mathbf{r}_1|)$ we have the relation $M^*_{B_{21}} = \Phi^*_{B_{21}}/I_1 = \mu/(4\pi) \int_{S_2} \nabla_2 \times (\oint_{C_1} (d\mathbf{l}_1/|\mathbf{r}_2 - \mathbf{r}_1|)) \cdot \hat{n} da_2$ and so, by using Stokes' theorem, we have $M^*_{B_{21}} = \mu/(4\pi) \oint_{C_2} \oint_{C_1} (d\mathbf{l}_1 \cdot d\mathbf{l}_2/|\mathbf{r}_2 - \mathbf{r}_1|)$, which is in fact the same Neumann's formula for the mutual inductance. In a similar manner we shall have $M^*_{E_{21}} = -\mu/(4\pi) \oint_{C_2} \oint_{C_1} (d\mathbf{l}_1 \cdot d\mathbf{l}_2/|\mathbf{r}_2 - \mathbf{r}_1|)$. ($d\mathbf{l}_1$ and $d\mathbf{l}_2$ are in the directions of the respective currents, and as it is seen we have $M^*_{B_{ij}} = M^*_{B_{ji}}$ and $M^*_{E_{ij}} = M^*_{E_{ji}}$.)

We want to see in a simple manner how an electromagnetic wave propagates through the space carrying the wave with attention to the supposition of the existence of the much tiny static dipoles filling up this supposed full-linear space. This matter can be perceived by observing Fig. 5 in which it is supposed that the + enclosed with the square is the representative of the accelerated electric charge or in fact the oscillator producing the electromagnetic wave. (Use properly the right and left hand rules in each point for finding the next movements.) Meanwhile we see easily why $\mathbf{E}^*$ and $\mathbf{B}^*$ are perpendicular to each other. This aspect shows that how beautiful the electric and magnetic fields are complementaries to each other or in fact producers of each other, not as two elements as if being quite distinct from each other that have been set by the side of each other by chance, ie the aspect pursued in the present electromagnetic texts! It also shows a produced current due to the turn or in fact orientation of the static dipoles. With attention to the proportion of the static field to the dynamic one in the above supposed full-linear medium, these currents are obviously unidirectional with the fields generating them (compare with the relations (23)). It is obvious that the time changes of the wave, for instance the sinusoidal changes of the wave, are related to the changes of the source movement. More explanations about this aspect will be presented in the next section. The confirmation for this aspect and, in principle, the confirmation of this matter that the space through which the wave passes should contain some static charges as wave carrier is that like the other cases in physics that we reach to a wave equation, the equations by which we reach to the wave equation should be valid for each section of the carriers making the wave path, and this matter requires existence of some static charges in the whole wave path in order that the fundamental equations can be executed for them. We now proceed to these fundamental equations. We have the following equations instead of Maxwell's equations with this supposition that the electromagnetic wave carrier mediums are full-linear and considering the discussions presented so far.

$$\nabla \cdot \mathbf{B}^* = 0 \ \ \& \ \ \nabla \cdot \mathbf{E}^* = 0 \ \ \& \ \ \nabla \times \mathbf{B}^* = \mu\epsilon a \frac{\partial \mathbf{E}^*}{\partial t} \ \ \& \ \ \nabla \times \mathbf{E}^* = -\mu\epsilon' a' \frac{\partial \mathbf{B}^*}{\partial t} \tag{30}$$

It may be said that while considering the discussions about the static fields we know that the curl of the static fields must be zero, considering the proportion of the static to the dynamic field in a full-linear medium we can deduce from the two last equations of (30) that the curl of the static fields of one kind is proportional to the time derivative of the static fields of the other kind or in other words to the respective generated currents (see (23)). The answer to this problem should be searched in the potential discontinuity in a continuous distribution of the static point dipoles (eg it is clear that in passing through a dipole layer, the potential finds a jump, which is in fact the same potential changing in passing across the infinitesimal thickness of the layer (see Classical Electrodynamics by Jackson, John Wiley & Sons, 1962)). After accepting the existence of such a discontinuity, it is natural that we should also accept that the curl of the static field is no longer identical with zero in each point of this distribution, because otherwise the existence of a continuous potential would be necessary.

By using the relation $\mathbf{B}^*(\mathbf{r}_2) = \mu/(4\pi)\int_L (Id\mathbf{l} \times (\mathbf{r}_2 - \mathbf{r}_1)/|\mathbf{r}_2 - \mathbf{r}_1|^3)$, which is the same relation of (9) in which we have made use of the relation $\mathbf{J}dv = Id\mathbf{l}$ for a wire carrying electric current I, the magnetodynamic field arising from the electric current I in a long straight wire, lying along the x-axis from minus infinity to plus infinity, will be $\mathbf{B}^* = \mu I\hat{k}/(2\pi a)$ in a distance "a" from the wire on the y-axis. Now we proceed to another different problem. We consider an infinite long wire along the x-axis which carries the current I from minus infinity to plus infinity. We suppose that there is a uniform magnetostatic field $\mathbf{B}$ along the whole length of the wire. We proceed to accounting the electrodynamic force exerted on the unit length of the wire by using the relation (5):

$$d\mathbf{F}^*_{dq} = (k''/k')dq\mathbf{v}_{dq} \times \mathbf{B} = (k''/k')dq(d\mathbf{l}/dt) \times \mathbf{B} = (k''/k')Id\mathbf{l} \times \mathbf{B}$$

$$\Rightarrow (d\mathbf{F}^*_{dq}/dl) = (k''/k')I\hat{i} \times \mathbf{B}$$

This means that if, for example, $\mathbf{B}$ is in the positive or negative direction of the z-axis, the force will be in the negative or positive direction of the y-axis respectively. Another usual mistake is replacing $\mathbf{B}^*$, arising from another current-carrying wire parallel to this wire, instead of $\mathbf{B}$ in the recent relation and, with this contrivance, justifying the attractive force between two parallel wires with same directions for currents and the repulsive force between them with different directions for their currents; while we should say that as we said the magnetodynamic field $\mathbf{B}^*$ causes production of the magnetostatic field $\mathbf{B}$ in the magnetolinear medium around the wire, ie we have $\mathbf{B} = a'\mathbf{B}^*$. And then we should substitute $a'\mathbf{B}^*$ for $\mathbf{B}$ in the recent formula.

6 Wave equations and Poynting vectors

By using the equations (30) we have:

$$(\nabla \times \nabla \times \mathbf{B}^* = -\nabla^2\mathbf{B}^*) = (\mu\epsilon a\frac{\partial}{\partial t}(\nabla \times \mathbf{E}^*) = \mu\epsilon a\frac{\partial}{\partial t}(-\mu\epsilon' a'\frac{\partial \mathbf{B}^*}{\partial t})$$

$$= -\mu^2\epsilon\epsilon' aa'\frac{\partial^2\mathbf{B}^*}{\partial t^2}) \Rightarrow \nabla^2\mathbf{B}^* = \mu^2\epsilon\epsilon' aa'\frac{\partial^2\mathbf{B}^*}{\partial t^2} \tag{31}$$

and

$$(\nabla \times \nabla \times \mathbf{E}^* = -\nabla^2\mathbf{E}^*) = (-\mu\epsilon' a'\frac{\partial}{\partial t}(\nabla \times \mathbf{B}^*) = -\mu\epsilon' a'\frac{\partial}{\partial t}(\mu\epsilon a\frac{\partial \mathbf{E}^*}{\partial t})$$

$$= -\mu^2\epsilon\epsilon' aa'\frac{\partial^2\mathbf{E}^*}{\partial t^2}) \Rightarrow \nabla^2\mathbf{E}^* = \mu^2\epsilon\epsilon' aa'\frac{\partial^2\mathbf{E}^*}{\partial t^2} \tag{32}$$

The equations (31) and (32) are wave equations indicating electromagnetic wave propagating with the speed $v = (\mu^2\epsilon\epsilon' aa')^{-1/2}$ in the full-linear medium of a and a'.

We have these cases: With attention to Lenz's law and Kirchhoff's loop law, immediately after closing the circuit in Fig. 6(a) we have $\mathcal{V} - (-\mathcal{E}) - IR = 0 \Rightarrow \mathcal{V} = IR - \mathcal{E}$; and immediately after opening the circuit in Fig. 7(a) we have $\mathcal{V} + \mathcal{E} - IR = 0 \Rightarrow \mathcal{V} = IR - \mathcal{E}$.

Similarly with attention to Lenz's law and Kirchhoff's loop law, immediately after closing the circuit in Fig. 6(b) we have $\mathcal{V}' - (-\mathcal{B}) - I'R' = 0 \Rightarrow \mathcal{V}' = I'R' - \mathcal{B}$; and immediately after opening the circuit in Fig. 7(b) we have $\mathcal{V}' + \mathcal{B} - I'R' = 0 \Rightarrow \mathcal{V}' = I'R' - \mathcal{B}$. Thus if a source $\mathcal{V}$ is exerted to a circuit, we have $\mathcal{V} = IR - \mathcal{E}$ in which $\mathcal{E}$ is the induced electromotive force, and if a source $\mathcal{V}'$ is exerted to a circuit, we have $\mathcal{V}' = I'R' - \mathcal{B}$ in which $\mathcal{B}$ is the induced magnetomotive force.

In the following discussion we suppose that none of the circuits are in any external field. Now we have:

$$\mathcal{V} = IR - \mathcal{E} \ \ \& \ \ dq = Idt \ \ \& \ \ \mathcal{E} = -\mu\epsilon'\frac{d\Phi_B}{dt} \Rightarrow \mathcal{V}dq = \mu\epsilon' Id\Phi_B + I^2Rdt \tag{33}$$

$$\mathcal{V}' = I'R' - \mathcal{B} \ \ \& \ \ db = I'dt \ \ \& \ \ \mathcal{B} = \mu\epsilon\frac{d\Phi_E}{dt} \Rightarrow \mathcal{V}'db = -\mu\epsilon I' d\Phi_E + I'^2 R' dt \tag{34}$$

The left side terms in the relations (33) and (34) are the work done on the partial charge, and the second terms on the right are that part of this work which is wasted in the circuit in the form of heat, and the first terms on the right are the work done in the circuit for opposing the induced motive forces and in fact in the case of rigidity and immobility of the circuit this work will be spent totally for making the field around the circuit and naturally will be stored in it as the potential energy. For better understanding of this matter we should say that it is only after ordering the tiny static dipoles of the medium by the dynamic field arising from the current, and so producing the static field, that the recent work will be stored in this static field as the potential energy (because in the expressions $\mathcal{E} = -\mu\epsilon' d\Phi_B/dt$ and $\mathcal{B} = \mu\epsilon d\Phi_E/dt$ we see the static fluxes, not the dynamic ones). Of course attention to this point is important that this potential energy is only that portion of the static potential energy presented in the section 3 that easily, ie without any necessity to changing the structural composition of the mentioned dipoles, is changeable to other forms of energy particularly heat (notice that if it is supposed that the static potential energy of a nonzero charge to be changeable to heat (or other forms of energy) totally, it will be required that all the differential parts of the charge to disintegrate from one another and then the structural composition of the charge to be changed). Suppose that our system is full-linear and includes n stationary circuits. For accounting the magnetic or electric potential energy (mentioned above) arising from the currents of the system, it is sufficient to integrate the expressions $\mu\epsilon' \sum_{i=1}^{n} I_i d\Phi_{B_i}$ and $-\mu\epsilon \sum_{i=1}^{n} I'_i d\Phi_{E_i}$ from the zero flux situation to the final values of the fluxes. (In these two expressions Φ_{B_i} and Φ_{E_i} are arising from all the currents, not only arising from the current in the circuit i.) Since this energy is independent of the way in which the currents are brought to their final set of values, we choose a way in which at any instant of time all the currents will be at the same fraction of their final values. We call this fraction as α. Now in the first degree, before all, we have:

$$\Phi^*_{B_i} = \int_{S_i} \mathbf{B}^*_i \cdot \hat{n} da \Rightarrow \frac{d\Phi^*_{B_i}}{dI_j} = \int_{S_i} \frac{d\mathbf{B}^*_i}{dI_j} \cdot \hat{n} da \quad : (1-35)$$

$$\mathbf{B}^*_i = \sum_{j=1}^{n} \mathbf{B}^*_{ij} \Rightarrow \frac{d\mathbf{B}^*_i}{dI_j} = \frac{d\mathbf{B}^*_{ij}}{dI_j} \quad : (2-35)$$

$$\frac{d\mathbf{B}^*_{ij}}{dI_j} = \frac{\mathbf{B}^*_{ij}}{I_j} \quad : (3-35)$$

$$(2-35) \ \ \& \ \ (3-35) \Rightarrow \frac{d\mathbf{B}^*_i}{dI_j} = \frac{\mathbf{B}^*_{ij}}{I_j} \quad : (4-35)$$

$$(1-35) \ \ \& \ \ (4-35) \Longrightarrow \frac{d\Phi^*_{B_i}}{dI_j} = \frac{\Phi^*_{B_{ij}}}{I_j} \tag{35}$$

(we are attentive that each of the expressions of both sides of the equality (35) is related only to the structural shape of the circuit and that it is unimportant that $\mathbf{B}^*_{ij}$ and I_j belong to which instant of time and then we suppose that in the right side of this relation I_j and $\mathbf{B}^*_{ij}$ are the final values of these quantities.)

Now we show the electric and magnetic currents which have not reached to their final values and are related to the circuit j as $(I_j)'$ and $(I'_j)'$ respectively. We have:

$$\frac{d\Phi^*_{B_i}}{d(I_j)'} = \frac{\Phi^*_{B_{ij}}}{I_j} \Rightarrow d\Phi^*_{B_i} = \Phi^*_{B_{ij}} \frac{d(I_j)'}{I_j} \qquad : (1-36)$$

$$\Phi^*_{B_{ij}} = \frac{\Phi_{B_{ij}}}{a'} \ \ \& \ \ \Phi^*_{B_i} = \frac{\Phi_{B_i}}{a'} \qquad : (2-36)$$

$$(1-36) \ \ \& \ \ (2-36) \Rightarrow d\Phi_{B_i} = \Phi_{B_{ij}} \frac{d(I_j)'}{I_j} \qquad : (3-36)$$

$$(I_j)' = \alpha I_j \Rightarrow d(I_j)' = I_j d\alpha \Rightarrow \frac{d(I_j)'}{dI_j} = d\alpha \quad (4-36)$$

$$(3-36) \ \ \& \ \ (4-36) \Longrightarrow d\Phi_{B_i} = \Phi_{B_{ij}} d\alpha \qquad (36)$$

Now we know that $d\Phi_{B_i} = \sum_{j=1}^n \partial\Phi_{B_i}/\partial(I_j)' d(I_j)'$ and the expression (36) is in fact the same $\partial\Phi_{B_i}$ in the recent relation and so $d\Phi_{B_i} = \sum_{j=1}^n \Phi_{B_{ij}} \partial\alpha/\partial(I_j)' d(I_j)'$ and since α is related to each $(I_j)'$ in the one-variable form of $\alpha = (I_j)'/I_j$, the sign ∂ will become changed into d and we shall have $d\Phi_{B_i} = \sum_{j=1}^n \Phi_{B_{ij}} d\alpha = \Phi_{B_i} d\alpha$ in which Φ_{B_i} is arising from all the currents. In a similar manner we have $d\Phi_{E_i} = \Phi_{E_i} d\alpha$. Thus

$$\int_{\alpha=0}^1 \mu\epsilon' \sum_{i=1}^n \alpha I_i \Phi_{B_i} d\alpha = \mu\epsilon' \sum_{i=1}^n I_i \Phi_{B_i} \int_{\alpha=0}^1 \alpha d\alpha = \mu\epsilon'/2 \sum_{i=1}^n I_i \Phi_{B_i} = U^*_B$$

and

$$\int_{\alpha=0}^1 -\mu\epsilon \sum_{i=1}^n \alpha I'_i \Phi_{E_i} d\alpha = -\mu\epsilon \sum_{i=1}^n I'_i \Phi_{E_i} \int_{\alpha=0}^1 \alpha d\alpha = -\mu\epsilon/2 \sum_{i=1}^n I'_i \Phi_{E_i} = U^*_E.$$

We know $\mathbf{B}^* = \nabla \times \mathbf{A}_B$ and $\mathbf{E}^* = \nabla \times \mathbf{A}_E$ and

$$\Phi_{B_i} = \int_{S_i} \mathbf{B}_i \cdot \hat{n} da = \int_{S_i} a' \mathbf{B}^* \cdot \hat{n} da = a' \int_{S_i} \nabla \times \mathbf{A}_{B_i} \cdot \hat{n} da = a' \oint_{C_i} \mathbf{A}_{B_i} \cdot d\mathbf{l}_i$$

and

$$\Phi_{E_i} = \int_{S_i} \mathbf{E}_i \cdot \hat{n} da = \int_{S_i} a\mathbf{E}^* \cdot \hat{n} da = a \int_{S_i} \nabla \times \mathbf{A}_{E_i} \cdot \hat{n} da = a \oint_{C_i} \mathbf{A}_{E_i} \cdot d\mathbf{l}_i.$$

(We have attention that the direction of $\hat{n}$ is determined by considering the direction of the current in the circuit i and by using the right-hand rule, and so the circulation is also in the direction of the current.) And so

$U_B^* = 1/2\mu\epsilon' a' \sum_i \oint_{C_i} I_i \mathbf{A}_{B_i} \cdot d\mathbf{l}_i$ and $U_E^* = -1/2\mu\epsilon a \sum_i \oint_{C_i} I_i' \mathbf{A}_{E_i} \cdot d\mathbf{l}_i$. In a general state each "circuit" is a closed path in a given linear medium which traces a current density line. So we can set $\mathbf{J}dv$ instead of $I_i d\mathbf{l}_i$ and $\mathbf{J}'dv$ instead of $I_i' d\mathbf{l}_i$ and $\int_V$ instead of $\sum_i \oint_{C_i}$. So we have $U_B^* = 1/2\mu\epsilon' a' \int_V \mathbf{J} \cdot \mathbf{A}_B dv$ and $U_E^* = -1/2\mu\epsilon a \int_V \mathbf{J}' \cdot \mathbf{A}_E dv$. Now using the vector identity $\nabla \cdot (\mathbf{F} \times \mathbf{G}) = (\nabla \times \mathbf{F}) \cdot \mathbf{G} - \mathbf{F} \cdot (\nabla \times \mathbf{G})$ and using the equations (11) we obtain

$$U_B^* = 1/2\epsilon' a' \int_V (\nabla \times \mathbf{B}^*) \cdot \mathbf{A}_B dv = 1/2\epsilon' a' (\int_V \mathbf{B}^* \cdot (\nabla \times \mathbf{A}_B) dv - \oint_S (\mathbf{A}_B \times \mathbf{B}^*) \cdot \hat{n} da)$$

and

$$U_E^* = 1/2\epsilon a \int_V (\nabla \times \mathbf{E}^*) \cdot \mathbf{A}_E dv = 1/2\epsilon a (\int_V \mathbf{E}^* \cdot (\nabla \times \mathbf{A}_E) dv - \oint_S (\mathbf{A}_E \times \mathbf{E}^*) \cdot \hat{n} da).$$

We take the volume V and the closed surface S infinitely large (an infinite sphere about the center of which our system has been set). Now we know that $\mathbf{B}^*$ and $\mathbf{E}^*$ fall off as fast as r^{-2} (r is the radius of this sphere) and $\mathbf{A}_B$ and $\mathbf{A}_E$ fall off as fast as r^{-1} and S is proportional to r^2. Thus the second integrals, surface integrals, in both the recent relations fall off as fast as $r^{-2}r^{-1}r^2 = r^{-1}$ so that when the volume and surface go to infinity these integrals will become zero, and considering that $\nabla \times \mathbf{A}_B = \mathbf{B}^* = \mathbf{B}/a'$ and $\nabla \times \mathbf{A}_E = \mathbf{E}^* = \mathbf{E}/a$ we obtain

$$U_B^* = \frac{1}{2}\epsilon' a' \int_{V_h} \mathbf{B}^* \cdot \frac{\mathbf{B}}{a'} dv = \frac{1}{2}\epsilon' \int_{V_h} \mathbf{B}^* \cdot \mathbf{B} dv$$

$$\& \qquad (37)$$

$$U_E^* = \frac{1}{2}\epsilon a \int_{V_h} \mathbf{E}^* \cdot \frac{\mathbf{E}}{a} dv = \frac{1}{2}\epsilon \int_{V_h} \mathbf{E}^* \cdot \mathbf{E} dv$$

in which V_h is the whole space. As we said before, the dynamic potential energies, U_B^* and U_E^*, are not separate from the static potential energies, U_B and U_E, but they are only those parts of the static potential energies which are related to the energy spent for orienting the tiny static dipoles and do not include the energy needed for gathering the partial ingredient parts of the dipoles themselves from infinity differential by differential. These dynamic energies are easily changeable to other forms of energy especially heat. Now we return to the equations (30). We have

$$\nabla \times \mathbf{B}^* = \mu\epsilon \frac{\partial \mathbf{E}}{\partial t} \Rightarrow \mathbf{E}^* \cdot \nabla \times \mathbf{B}^* = \mu\epsilon \mathbf{E}^* \cdot \frac{\partial \mathbf{E}}{\partial t} = \frac{1}{2}\mu\epsilon \frac{\partial}{\partial t}(\mathbf{E}^* \cdot \mathbf{E}) \quad : (1-38)$$

$$\nabla \times \mathbf{E}^* = -\mu\epsilon' \frac{\partial \mathbf{B}}{\partial t} \Rightarrow \mathbf{B}^* \cdot \nabla \times \mathbf{E}^* = -\mu\epsilon' \mathbf{B}^* \cdot \frac{\partial \mathbf{B}}{\partial t} = -\frac{1}{2}\mu\epsilon' \frac{\partial}{\partial t}(\mathbf{B}^* \cdot \mathbf{B}) \quad : (2-38)$$

$$\nabla \cdot (\mathbf{E}^* \times \mathbf{B}^*) = \mathbf{B}^* \cdot \nabla \times \mathbf{E}^* - \mathbf{E}^* \cdot \nabla \times \mathbf{B}^* \quad : (3-38)$$

$$(1-38)\ \&\ (2-38)\ \&\ (3-38) \Longrightarrow \nabla\cdot(\frac{1}{\mu}\mathbf{E}^*\times\mathbf{B}^*) = -\frac{\partial}{\partial t}(\frac{1}{2}\epsilon'\mathbf{B}^*\cdot\mathbf{B}+\frac{1}{2}\epsilon\mathbf{E}^*\cdot\mathbf{E}) \tag{38}$$

With attention to the equations (37) we show the right side of the relation (38) by $-\partial u/\partial t$ in which u is the energy density consisting of sum of the densities of the electrodynamic and magnetodynamic potential energies (which are changeable to heat). The left side of this relation can be written as $\nabla \cdot \mathbf{S}$ in which $\mathbf{S}$ is the same $1/\mu\mathbf{E}^* \times \mathbf{B}^*$ and we call it as Poynting vector. Thus

$$\nabla \cdot \mathbf{S} + \frac{\partial u}{\partial t} = 0. \tag{39}$$

This equation is a continuity equation and states that if the energy density is decreased (or increased) in a point, then necessarily some energy has gone out of this point (or some energy has entered into this point). Therefore, $\mathbf{S}$ is the local current of energy per unit time per unit area (as unitary investigation of $\mathbf{S}$ shows this matter too).

What we deduce from the above discussion is that $\mathbf{S} = 1/\mu\mathbf{E}^*\times\mathbf{B}^*$ also shows the direction of propagation of the wave, the wave that its energy is easily changeable to other forms of energy especially heat. Now we want as far as possible to present a simple physical interpretation about this matter that $\mathbf{E}^*\times\mathbf{B}^*$ shows the direction of propagation of the electromagnetic wave considering the tiny particle (or in fact dipole) model about which we have explained before. For simplicity consider the orientational movement of only the positive charges (ie N-poles and positive electric charges) of the dipoles. Suppose that some electric current is flowing in the positive direction of a straight wire set along the x-axis as in Fig. 8. This current causes the moving (or in fact the orientation) of the positive magnetic charges of the magnetostatic tiny dipoles of the medium in a range the extent of which is determined by the physical conditions of the medium, indicated here by A, toward the negative direction of the y-axis, according to the right-hand rule. This movement causes the moving of the positive electric charges of the electrostatic tiny dipoles of the medium in a range which is again determined by the same physical conditions, indicated here by B, toward the positive direction of the x-axis, according to the left-hand rule. This recent movement of the electric particles causes not only the returning of A to its initial equilibrium state but also the moving of the positive magnetic particles in the range A' toward the negative direction of the y-axis, and this recent movement of the magnetic particles, in turn, not only restores B to its initial equilibrium state but also causes the moving of the positive electric particles in the range B', and in this manner the wave movement will be transferred. Now we may encounter with this question that whether this wave movement doesn't have continuity as in Fig. 8. The answer to this question is that this is only one half of the work and the other half is that the current in the wire also causes the moving of the magnetic particles of the medium in the range A_1 according to the right-hand rule, and this movement, which is not restored to its initial equilibrium state as a result of a similar agent, causes the moving of the positive electric particles in the range A in the negative direction of the x-axis according to the left-hand rule, and this recent movement, in turn, not only restores A_1 to its initial equilibrium state but also causes the moving of the positive magnetic particles in the range B toward the positive direction of the y-axis, both according to the right-hand rule. Again in this manner the wave movement will be transferred. Consequently on the whole we have the wave movement of Fig. 9.

The physical interpretation presented above is raw but properly shows that why in each point of the medium of the wave propagation $\mathbf{E}^*$ and $\mathbf{B}^*$ are perpendicular to each other in such a way that $\mathbf{E}^* \times \mathbf{B}^*$ shows the direction of propagation of the wave. From the above discussions we can deduce that the maximums and minimums of $\mathbf{E}^*$ and $\mathbf{B}^*$ occur simultaneously, because otherwise at some part of a period the direction of propagation of the wave would be reversed and this is not possible (or at least this is not the case in the discussions presented in this article, on supposition).

7 Sinusoidal waves and Fresnel coefficients

We can rewrite the wave equations (31) and (32) as

$$\begin{gathered}\frac{\partial^2 \mathbf{E}^*}{\partial t^2} = v^2 \nabla^2 \mathbf{E}^* \\ \& \\ \frac{\partial^2 \mathbf{B}^*}{\partial t^2} = v^2 \nabla^2 \mathbf{B}^*\end{gathered} \tag{40}$$

in which $v = (\mu^2 \epsilon \epsilon' a a')^{-1/2}$ is the speed of propagation of the wave. (In this article we suppose that the mediums of propagation of the electromagnetic wave are isotropic such that the speed of the wave in all the directions is the same.) A particular solution to these wave equations for, for example, the x component of the vector $\mathbf{E}^*$ is $E^*_x = f_1(\hat{u} \cdot \mathbf{r} - vt) + f_2(\hat{u} \cdot \mathbf{r} + vt)$ which exhibits a plane wave moving in the sense of the unit vector $\hat{u}$ with the absolute speed $|v|$. Then $E^*_x(\mathbf{r}, t) = E^*_{x_0} \cos k(\hat{u} \cdot \mathbf{r} - vt)$ can be a solution, and since $\cos k(\hat{u} \cdot \mathbf{r} - vt) = \cos k(\hat{u} \cdot \mathbf{r} + (2\pi/k) - vt)$, we conclude that $\lambda = 2\pi/k$ represents the spatial period of the wave or the wave length, and $k = 2\pi/\lambda$, which is the number of the wave lengths in distance 2π, is called as the wave number, and with the definition of the angular frequency of the wave, ω, and the period of the wave,T, and the frequency of the wave, ν, by the relation $\omega = kv = 2\pi v/\lambda = 2\pi/(\lambda/v) = 2\pi/T = 2\pi\nu$ and with the definition of $k\hat{u} = \mathbf{k}$ (as the wave vector) we shall have $E^*_x = E^*_{x_0} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)$ (which is also equal to the real part of the $E^*_{x_0} e^{-i(\omega t - \mathbf{k} \cdot \mathbf{r})}$). Then we suppose that we have

$$\mathbf{B}^* = B^*_{x_0} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)\hat{\imath} + B^*_{y_0} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)\hat{\jmath} + B^*_{z_0} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)\hat{k}$$

and

$$\mathbf{E}^* = E^*_{x_0} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)\hat{\imath} + E^*_{y_0} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)\hat{\jmath} + E^*_{z_0} \cos(\mathbf{k} \cdot \mathbf{r} - \omega t)\hat{k}$$

From the equations (30), by relevant differentiations, it is easily obtained that

$$\begin{gathered} \mathbf{k}\cdot\mathbf{E}^* = 0 \quad \& \quad \mathbf{k}\cdot\mathbf{B}^* = 0 \\ \& \\ \mathbf{k}\times\mathbf{E}^* = \mu\epsilon' a'\omega\mathbf{B}^* \quad \& \quad \mathbf{k}\times\mathbf{B}^* = -\mu\epsilon a\omega\mathbf{E}^* \end{gathered} \tag{41}$$

which indicates that the orthogonal system ($\mathbf{E}^*$, $\mathbf{B}^*$, $\mathbf{k}$) is right-handed. We consider a medium (generally the vacuum (we should notice that the vacuum is not empty of the mentioned tiny dipoles, while the free space is so)) as the reference medium and we show the speed of the electromagnetic wave in it by c. We show the ratio of the c to the speed of the electromagnetic wave in each medium by n and we call it as the refractive index of that medium ($n = c/v$ and then $v = c/n$). Therefore, we have $k = \omega/v = n\omega/c$. Considering this matter, from the last two relations of (41) we shall have:

$$\begin{gathered} \mathbf{B}^* = \frac{n}{\mu\epsilon' a' c}\hat{u}\times\mathbf{E}^* \\ \& \\ \mathbf{E}^* = -\frac{n}{\mu\epsilon a c}\hat{u}\times\mathbf{B}^* \end{gathered} \tag{42}$$

It is obvious that $[n/(\mu\epsilon' a' c)]\cdot[n/(\mu\epsilon a c)] = 1$. It is proper here to see the following deduction:

$$(42) \Rightarrow \frac{B^*}{E^*} = \frac{1}{\mu\epsilon' a' v} = \frac{\sqrt{\mu^2\epsilon\epsilon' a a'}}{\mu\epsilon' a'} = \frac{\sqrt{\mu\epsilon a}}{\sqrt{\mu\epsilon' a'}} \Rightarrow \sqrt{\mu\epsilon' a'}B^* - \sqrt{\mu\epsilon a}E^* = 0$$

$$\Rightarrow \mu\epsilon' a' {B^*}^2 + \mu\epsilon a {E^*}^2 = 2\sqrt{\mu^2\epsilon\epsilon' a a'}E^* B^*$$

$$\Rightarrow \mathbf{E}^*\times\mathbf{B}^* = (\frac{1}{2}\mu\epsilon' a'\mathbf{B}^*\cdot\mathbf{B}^* + \frac{1}{2}\mu\epsilon a\mathbf{E}^*\cdot\mathbf{E}^*)\frac{\hat{u}}{\sqrt{\mu^2\epsilon\epsilon' a a'}}$$

$$\Rightarrow \frac{1}{\mu}\mathbf{E}^*\times\mathbf{B}^* = (\frac{1}{2}\epsilon'\mathbf{B}^*\cdot\mathbf{B} + \frac{1}{2}\epsilon\mathbf{E}^*\cdot\mathbf{E})v\hat{u}$$

$$\Longrightarrow \mathbf{S} = u\mathbf{v}, \tag{43}$$

and we know that the relation (43) is an expectable relation, comparable with the relation $\mathbf{J} = \rho_E\mathbf{v}$.

Now we proceed to the boundary conditions. We have $\nabla\cdot\mathbf{B}^* = 0$. Considering a pillbox-shaped surface, as in Fig. 10, which its height is infinitesimal, and applying the divergence theorem over the volume enclosed by this surface we obtain

$$\int_V \nabla\cdot\mathbf{B}^* dv = \int_{S_1}\mathbf{B}^*\cdot\hat{n}_1 da + \int_{S_2}\mathbf{B}^*\cdot\hat{n}_2 da \quad : (1')$$

$$\nabla\cdot\mathbf{B}^* = 0 \quad : (2')$$

$$(1')\ \&\ (2') \Longrightarrow \int_{S_1}\mathbf{B}^*\cdot\hat{n}_1 da = \int_{S_2}\mathbf{B}^*\cdot(-\hat{n}_2)da$$

and since $S_1 = S_2$ and $\hat{n}_1 = -\hat{n}_2$, considering $\hat{n}_1$ as $\hat{n}$ we shall have $B^*_{1n} = B^*_{2n}$.

In a similar manner we shall obtain $E^*_{1n} = E^*_{2n}$. Then the normal components of the fields are continuous across the boundary interface. Now from the equation $\nabla \times \mathbf{B}^* = \mu\epsilon a \partial \mathbf{B}^*/\partial t$ we have

$$\int_S \nabla \times \mathbf{B}^* \cdot \hat{n} da = \int_S \mu\epsilon a \frac{\partial \mathbf{E}^*}{\partial t} \cdot \hat{n} da$$

in which S is the area enclosed by the loop in Fig. 11.

If the width of this rectangular loop is infinitesimal, by applying Stokes' theorem and with attention to this fact that the flat area enclosed by the loop approaches zero and therefore the second integral approaches zero, we shall obtain $B^*_{1t} = B^*_{2t}$. In a similar manner we shall obtain $E^*_{1t} = E^*_{2t}$. (It is necessary to notice that the orientation of this loop is optional.) Then the tangential components of the fields are continuous across the boundary interface too, and then altogether we should say that the dynamic fields connected to an electromagnetic wave are continuous across the boundary interfaces.

With attention to Fig. 12 we suppose that the electrodynamic fields of the incident, reflected, and transmitted waves of a plane electromagnetic wave at the boundary of a dielectric with the refractive index n_2, set in a space with the refractive index n_1, are as follows:

$$\begin{gathered} \mathbf{E}^*_1 = \mathbf{E}^*_{10p} \cos(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t) + \mathbf{E}^*_{10s} \cos(\mathbf{k}_1 \cdot \mathbf{r} - \omega_1 t) \quad \& \\ \mathbf{E}^*_{1'} = \mathbf{E}^*_{1'0p} \cos(\mathbf{k}_{1'} \cdot \mathbf{r} - \omega_{1'} t) + \mathbf{E}^*_{1'0s} \cos(\mathbf{k}_{1'} \cdot \mathbf{r} - \omega_{1'} t) \quad \& \\ \mathbf{E}^*_2 = \mathbf{E}^*_{20p} \cos(\mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t) + \mathbf{E}^*_{20s} \cos(\mathbf{k}_2 \cdot \mathbf{r} - \omega_2 t) \end{gathered} \tag{44}$$

These three fields must be in phase in $\mathbf{r} = \mathbf{0}$ in different times, then $\omega_1 = \omega_{1'} = \omega_2 = \omega$; and they must be also in phase in each point on the boundary interface, $z = 0$, at $t = 0$, so we have the relations $\mathbf{k}_1 \cdot \mathbf{r} = \mathbf{k}_{1'} \cdot \mathbf{r} = \mathbf{k}_2 \cdot \mathbf{r}$ on the boundary, from which the three consequences of the coplanarity of the wave vectors, the law of reflection, and Snell's law are easily obtained in the manner followed by many of the electromagnetic and optics books. Now we want to obtain the Fresnel coefficients. Substituting the first relation of (42) into the boundary condition $\mathbf{B}^*_1 + \mathbf{B}^*_{1'} = \mathbf{B}^*_2$ we obtain $n_1/(\mu_1\epsilon'_1 a'_1)(\hat{u}_1 \times \mathbf{E}^*_1 + \hat{u}_{1'} \times \mathbf{E}^*_{1'}) = n_2/(\mu_2\epsilon'_2 a'_2)\hat{u}_2 \times \mathbf{E}^*_2$ which with the cross product of the unit vector $\hat{n} = \hat{k}$, which is normal to the interface, multiplied by the two sides of the recent relation we shall have $n_1/(\mu_1\epsilon'_1 a'_1)\hat{n} \times (\hat{u}_1 \times \mathbf{E}^*_1 + \hat{u}_{1'} \times \mathbf{E}^*_{1'}) = n_2/(\mu_2\epsilon'_2 a'_2)\hat{n} \times (\hat{u}_2 \times \mathbf{E}^*_2)$. Expanding the vector products in the recent relation, and considering only the components normal to the plane of incidence (ie the plane of $\hat{k}_1$ and $\hat{n}$), which are distinguished by the subscript s, and with attention to this fact that $\theta_1 = \theta_{1'}$, we conclude that

$$n_1/(\mu_1\epsilon'_1 a'_1) \cos\theta_1 (\mathbf{E}^*_{1s} - \mathbf{E}^*_{1's}) = n_2/(\mu_2\epsilon'_2 a'_2) \cos\theta_2 \mathbf{E}^*_{2s}. \tag{45}$$

The other boundary condition, ie $\mathbf{E}_1^* + \mathbf{E}_{1'}^* = \mathbf{E}_2^*$, also necessitates having separately the relation:

$$\mathbf{E}_{1s}^* + \mathbf{E}_{1's}^* = \mathbf{E}_{2s}^*. \tag{46}$$

By solving the equations (45) and (46) for $\mathbf{E}_{1's}^*$ and $\mathbf{E}_{2s}^*$, we shall have

$$\mathbf{E}_{1's}^* = r_{12s}\mathbf{E}_{1s}^* \quad \& \quad \mathbf{E}_{2s}^* = t_{12s}\mathbf{E}_{1s}^* \tag{47}$$

in which

$$r_{12s} = \frac{n_1/(\mu_1\epsilon_1' a_1')\cos\theta_1 - n_2/(\mu_2\epsilon_2' a_2')\cos\theta_2}{n_1/(\mu_1\epsilon_1' a_1')\cos\theta_1 + n_2/(\mu_2\epsilon_2' a_2')\cos\theta_2} \tag{48}$$

$$\& \quad t_{12s} = \frac{2n_1/(\mu_1\epsilon_1' a_1')\cos\theta_1}{n_1/(\mu_1\epsilon_1' a_1')\cos\theta_1 + n_2/(\mu_2\epsilon_2' a_2')\cos\theta_2}. \tag{49}$$

And also by substituting the second relation of (42) into the boundary condition $\mathbf{E}_1^* + \mathbf{E}_{1'}^* = \mathbf{E}_2^*$ and performing similar operations and considering the other boundary condition, ie $\mathbf{B}_1^* + \mathbf{B}_{1'}^* = \mathbf{B}_2^*$, we shall obtain

$$\mathbf{B}_{1's}^* = r_{12p}\mathbf{B}_{1s}^* \quad \& \quad \mathbf{B}_{2s}^* = t_{12p}\mathbf{B}_{1s}^* \tag{50}$$

in which

$$r_{12p} = \frac{n_1/(\mu_1\epsilon_1 a_1)\cos\theta_1 - n_2/(\mu_2\epsilon_2 a_2)\cos\theta_2}{n_1/(\mu_1\epsilon_1 a_1)\cos\theta_1 + n_2/(\mu_2\epsilon_2 a_2)\cos\theta_2} \tag{51}$$

$$\& \quad t_{12p} = \frac{2n_1/(\mu_1\epsilon_1 a_1)\cos\theta_1}{n_1/(\mu_1\epsilon_1 a_1)\cos\theta_1 + n_2/(\mu_2\epsilon_2 a_2)\cos\theta_2}. \tag{52}$$

Also, for the components parallel to the plane of incidence, which are distinguished by the subscript p, we have

$$\begin{aligned}
\mathbf{E}_{2p}^* &= -n_2/(\mu_2\epsilon_2 a_2 c)\hat{u}_2 \times \mathbf{B}_{2s}^* = -n_2/(\mu_2\epsilon_2 a_2 c)\hat{u}_2 \times t_{12p}\mathbf{B}_{1s}^* \\
&= n_2 n_1 t_{12p}/(\mu_2\epsilon_2 a_2 c\mu_1\epsilon_1' a_1' c)\hat{u}_2 \times (\hat{u}_1 \times \mathbf{E}_{1p}^*) \\
&\Rightarrow E_{2p}^* = n_2 n_1/(\mu_2\epsilon_2 a_2 c\mu_1\epsilon_1' a_1' c)|t_{12p}|E_{1p}^*, \qquad (53)\\
\mathbf{B}_{2p}^* &= n_2/(\mu_2\epsilon_2' a_2' c)\hat{u}_2 \times \mathbf{E}_{2s}^* = n_2/(\mu_2\epsilon_2' a_2' c)\hat{u}_2 \times t_{12s}\mathbf{E}_{1s}^* \\
&= -n_2 n_1 t_{12s}/(\mu_2\epsilon_2' a_2' c\mu_1\epsilon_1 a_1 c)\hat{u}_2 \times (\hat{u}_1 \times \mathbf{B}_{1p}^*) \\
&\Rightarrow B_{2p}^* = n_2 n_1/(\mu_2\epsilon_2' a_2' c\mu_1\epsilon_1 a_1 c)|t_{12s}|B_{1p}^*, \qquad (54)
\end{aligned}$$

$$\begin{aligned}
\mathbf{E}_{1'p}^* &= \frac{-n_1}{\mu_1\epsilon_1 a_1 c}\hat{u}_{1'} \times \mathbf{B}_{1's}^* = \frac{-n_1}{\mu_1\epsilon_1 a_1 c}\hat{u}_{1'} \times r_{12p}\mathbf{B}_{1s}^* = -r_{12p}\hat{u}_{1'} \times (\hat{u}_1 \times \mathbf{E}_{1p}^*) \\
&\Longrightarrow E_{1'p}^* = |r_{12p}|E_{1p}^*, \qquad (55)\\
\mathbf{B}_{1'p}^* &= \frac{n_1}{\mu_1\epsilon_1' a_1' c}\hat{u}_{1'} \times \mathbf{E}_{1's}^* = \frac{n_1}{\mu_1\epsilon_1' a_1' c}\hat{u}_{1'} \times r_{12s}\mathbf{E}_{1s}^* = -r_{12s}\hat{u}_{1'} \times (\hat{u}_1 \times \mathbf{B}_{1p}^*) \\
&\Longrightarrow B_{1'p}^* = |r_{12s}|B_{1p}^*. \qquad (56)
\end{aligned}$$

For obtaining more confidence to the correctness of the above relations and expressions obtained for Fresnel coefficients, it is proper to examine them when $\theta_1 = \theta_2 = 0$, because the isotropy of the mediums, which is our supposition in this article, necessitates having $(E_{1'p}^*/E_{1p}^*) = (E_{1's}^*/E_{1s}^*)$ or $|r_{12p}| = |r_{12s}|$ in this state, ie we should have an identity in the form of $r_{12p} = r_{12s}$ or $r_{12p} = -r_{12s}$ when $\theta_1 = \theta_2 = 0$. A simple investigation shows that only the relation $r_{12p} = -r_{12s}$ is an identity when $\theta_1 = \theta_2 = 0$ (in this investigation we need the obvious relation $n_1 v_1 = n_2 v_2$). Likewise, the isotropy of the mediums necessitates having $(E^*_{2p}/E^*_{1p}) = (E^*_{2s}/E^*_{1s})$ or $(n_2 n_1/(\mu_2\epsilon_2 a_2 c\mu_1\epsilon'_1 a'_1 c))|t_{12p}| = |t_{12s}|$ in the case of $\theta_1 = \theta_2 = 0$, which again a simple investigation shows that we have the identity $t_{12s} = (n_2 n_1/(\mu_2\epsilon_2 a_2 c\mu_1\epsilon'_1 a'_1 c))t_{12p}$ when $\theta_1 = \theta_2 = 0$. The isotropy of the mediums in the case of $\theta_1 = \theta_2 = 0$ for $\mathbf{B}^*$ also necessitates the identities $|r_{12s}| = |r_{12p}|$ and $(n_2 n_1/(\mu_2\epsilon'_2 a'_2 c\mu_1\epsilon_1 a_1 c))|t_{12s}| = |t_{12p}|$, but as said before we have the identity $r_{12s} = -r_{12p}$ for the first and a simple investigation shows that we have the identity $t_{12p} = (n_2 n_1/(\mu_2\epsilon'_2 a'_2 c\mu_1\epsilon_1 a_1 c))t_{12s}$ for the second, both when $\theta_1 = \theta_2 = 0$.

With attention to the relation $n/(\mu\epsilon a) = c^2\mu\epsilon' a'/n$ we can write

$$r_{12p} = \frac{(\mu_1\epsilon'_1 a'_1/n_1)\cos\theta_1 - (\mu_2\epsilon'_2 a'_2/n_2)\cos\theta_2}{(\mu_1\epsilon'_1 a'_1/n_1)\cos\theta_1 + (\mu_2\epsilon'_2 a'_2/n_2)\cos\theta_2} \tag{57}$$

$$t_{12p} = \frac{2(\mu_1\epsilon'_1 a'_1/n_1)\cos\theta_1}{(\mu_1\epsilon'_1 a'_1/n_1)\cos\theta_1 + (\mu_2\epsilon'_2 a'_2/n_2)\cos\theta_2}. \tag{58}$$

Now with definition of $n_E = n/(\mu\epsilon' a')$ we have

$$r_{12s} = \frac{n_{E1}\cos\theta_1 - n_{E2}\cos\theta_2}{n_{E1}\cos\theta_1 + n_{E2}\cos\theta_2}, t_{12s} = \frac{2n_{E1}\cos\theta_1}{n_{E1}\cos\theta_1 + n_{E2}\cos\theta_2} \tag{59}$$

$$\& \quad r_{12p} = \frac{n_{E2}\cos\theta_1 - n_{E1}\cos\theta_2}{n_{E2}\cos\theta_1 + n_{E1}\cos\theta_2}, \quad t_{12p} = \frac{2n_{E2}\cos\theta_1}{n_{E2}\cos\theta_1 + n_{E1}\cos\theta_2}. \tag{60}$$

And with attention to the relation $n/(\mu\epsilon' a') = c^2\mu\epsilon a/n$ and definition of $n_B = n/(\mu\epsilon a)$ we have

$$r_{12s} = \frac{n_{B2}\cos\theta_1 - n_{B1}\cos\theta_2}{n_{B2}\cos\theta_1 + n_{B1}\cos\theta_2}, t_{12s} = \frac{2n_{B2}\cos\theta_1}{n_{B2}\cos\theta_1 + n_{B1}\cos\theta_2} \tag{61}$$

$$\& \quad r_{12p} = \frac{n_{B1}\cos\theta_1 - n_{B2}\cos\theta_2}{n_{B1}\cos\theta_1 + n_{B2}\cos\theta_2}, \quad t_{12p} = \frac{2n_{B1}\cos\theta_1}{n_{B1}\cos\theta_1 + n_{B2}\cos\theta_2}. \tag{62}$$

and also

$$n_2 n_1/(\mu_2\epsilon'_2 a'_2 c\mu_1\epsilon_1 a_1 c) = \frac{n_{B1}n_{E2}}{c^2}$$

and

$$n_2 n_1/(\mu_2\epsilon_2 a_2 c\mu_1\epsilon'_1 a'_1 c) = \frac{n_{E1}n_{B2}}{c^2}.$$

Before continuing I should mention a point. Attention to this fact that in the relations (5) and (7) of the fundamental relations k'' which is proportional to μ shows the relation between two electric and magnetic charges only when they are moving relative to each other (and the situation is not like the static situation in which two electric charges or two magnetic charges, which are stationary relative to each other, can polarize the space between themselves and depending on this polarization of the medium can change k and k' in fact) states that μ should be the same for all the mediums and in fact it should be a world constant. Now considering this matter and in fact accepting it and with attention to the relation $n_1 \sin\theta_1 = n_2 \sin\theta_2$, if the magnetic characteristics of the two mediums are the same, ie if we have $a'_1 = a'_2$ and $\epsilon'_1 = \epsilon'_2$, then we shall see that

$$r_{12s} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = \frac{\sin(\theta_2 - \theta_1)}{\sin(\theta_2 + \theta_1)}, \tag{63}$$

$$t_{12s} = \frac{2n_1 \cos\theta_1}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = \frac{2\cos\theta_1 \sin\theta_2}{\sin(\theta_1 + \theta_2)}, \tag{64}$$

$$r_{12p} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}, \tag{65}$$

$$t_{12p} = \frac{2n_2 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{\sin 2\theta_1}{\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)}, \tag{66}$$

and since $(\epsilon a)^{-1} = c^2\mu^2 a'\epsilon'/n^2$, $\mu_1 = \mu_2$, $\epsilon'_1 = \epsilon'_2$ and $a'_1 = a'_2$, we conclude that $n_2 n_1/(\mu_2\epsilon_2 a_2 c\mu_1\epsilon'_1 a'_1 c) = n_1/n_2$ and $n_2 n_1/(\mu_2\epsilon'_2 a'_2 c\mu_1\epsilon_1 a_1 c) = n_2/n_1$ and $(n_1/n_2)t_{12p} = 2\cos\theta_1 \sin\theta_2/(\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2))$ and $(n_2/n_1)t_{12s} = \sin(2\theta_1)/\sin(\theta_1 + \theta_2)$ too. In a similar manner we have the following relations for the condition in which the electric characteristics of the two mediums are the same, ie when $a_1 = a_2$ and $\epsilon_1 = \epsilon_2$:

$$r_{12s} = \frac{n_2 \cos\theta_1 - n_1 \cos\theta_2}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{\tan(\theta_1 - \theta_2)}{\tan(\theta_1 + \theta_2)}, \tag{67}$$

$$t_{12s} = \frac{2n_2 \cos\theta_1}{n_2 \cos\theta_1 + n_1 \cos\theta_2} = \frac{\sin 2\theta_1}{\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2)}, \tag{68}$$

$$r_{12p} = \frac{n_1 \cos\theta_1 - n_2 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = \frac{\sin(\theta_2 - \theta_1)}{\sin(\theta_2 + \theta_1)}, \tag{69}$$

$$t_{12p} = \frac{2n_1 \cos\theta_2}{n_1 \cos\theta_1 + n_2 \cos\theta_2} = \frac{\sin 2\theta_2}{\sin(\theta_1 + \theta_2)}, \tag{70}$$

and also $n_2 n_1/(\mu_2\epsilon_2 a_2 c\mu_1\epsilon'_1 a'_1 c) = n_2/n_1$ and $n_2 n_1/(\mu_2\epsilon'_2 a'_2 c\mu_1\epsilon_1 a_1 c) = n_1/n_2$ and then $(n_2/n_1)t_{12p} = 2\sin\theta_1 \cos\theta_2/\sin(\theta_1+\theta_2)$ and $(n_1/n_2)t_{12s} = 2\cos\theta_1 \sin\theta_2/(\sin(\theta_1 + \theta_2)\cos(\theta_1 - \theta_2))$.

In Brewster's incident angle $\theta_1 = \theta_B$ in which $\theta_B + \theta_2 = \pi/2$ and then $1/\tan(\theta_B + \theta_2) = 0$ if the magnetic characteristics of the two mediums are the same, we shall have $r_{12p} = 0$ and so in the reflection we can only have an electromagnetic wave which the vibration of its $\mathbf{E}^*$ is perpendicular to the plane of incidence; and if the electric characteristics of the two mediums are the same, we shall have $r_{12s} = 0$ and so in the reflection we can only have an electromagnetic wave which the vibration of its $\mathbf{E}^*$ lies in the plane of incidence. If the laboratory works show that in Brewster's angle we have the first state in the mediums under experimentation, we should conclude that in these mediums the magnetic characteristics are the same and are not changed from a medium to another medium practically.

Finally we pay our attention to the energy or the intensity of the electromagnetic wave. We have

$$\mathbf{S} = (\mathbf{E}^* \times \mathbf{B}^*)/\mu = (\mathbf{E}^* \times (\frac{n}{\mu\epsilon' a' c}\hat{u} \times \mathbf{E}^*))/\mu = \frac{n}{\mu^2\epsilon' a' c}\mathbf{E}^{*^2}\hat{u}$$

$$\Rightarrow \langle\mathbf{S}\rangle_{av} = \frac{1}{2}\frac{n}{\mu^2\epsilon' a' c}\mathbf{E}_0^{*^2}\hat{u},$$

$$\mathbf{S} = (\mathbf{E}^* \times \mathbf{B}^*)/\mu = ((\frac{-n}{\mu\epsilon a c}\hat{u} \times \mathbf{B}^*) \times \mathbf{B}^*)/\mu = \frac{n}{\mu^2\epsilon a c}\mathbf{B}^{*^2}\hat{u}$$

$$\Rightarrow \langle\mathbf{S}\rangle_{av} = \frac{1}{2}\frac{n}{\mu^2\epsilon a c}\mathbf{B}_0^{*^2}\hat{u}.$$

On definition we have the following relations:

$$R_s = \frac{(-\hat{n})\cdot\langle\mathbf{S}_{1's}\rangle_{av}}{\hat{n}\cdot\langle\mathbf{S}_{1s}\rangle_{av}}, T_s = \frac{\hat{n}\cdot\langle\mathbf{S}_{2s}\rangle_{av}}{\hat{n}\cdot\langle\mathbf{S}_{1s}\rangle_{av}}, \tag{71}$$

$$R_p = \frac{(-\hat{n})\cdot\langle\mathbf{S}_{1'p}\rangle_{av}}{\hat{n}\cdot\langle\mathbf{S}_{1p}\rangle_{av}}, T_p = \frac{\hat{n}\cdot\langle\mathbf{S}_{2p}\rangle_{av}}{\hat{n}\cdot\langle\mathbf{S}_{1p}\rangle_{av}}, \tag{72}$$

and in terms of the Fresnel coefficients we shall have

$$R_s = r_{12s}^2, T_s = \frac{1/2(n_2/(\mu_2^2\epsilon_2' a_2' c))\cos\theta_2}{1/2(n_1/(\mu_1^2\epsilon_1' a_2' c))\cos\theta_1}t_{12s}^2, \tag{73}$$

$$R_p = r_{12p}^2,\ T_p = \frac{1/2(n_2/(\mu_2^2\epsilon_2' a_2' c))\cos\theta_2}{1/2(n_1/(\mu_1^2\epsilon_1' a_1' c))\cos\theta_1}\cdot\frac{n_2^2 n_1^2}{\mu_2^2\epsilon_2^2 a_2^2 c^2\mu_1^2\epsilon_1'^2 a_1'^2 c^2}t_{12p}^2. \tag{74}$$

By using some simple algebraic operations and accepting that μ is a world constant it is easily seen that we have always

$$R_s + T_s = 1, \quad R_p + T_p = 1. \tag{75}$$

8 Conclusion

As it was seen, by using an obvious and simple supposition, mentioned in the introduction and formed mathematically in the fundamental relations, all of the relations of the electromagnetic theory were modified and perfected in a sense consistent with the classical physics, and in addition to this, new relations and theories were presented for showing themselves in practice and experiment.

What is perhaps more important than the carefulness in the details of the presented discussions is attention to the validity and stability of the front opened for defending the classical physics and mathematical logic of the classical mechanics.

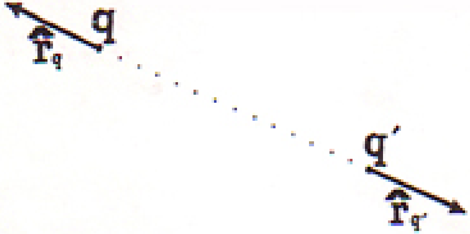

Fig. 1

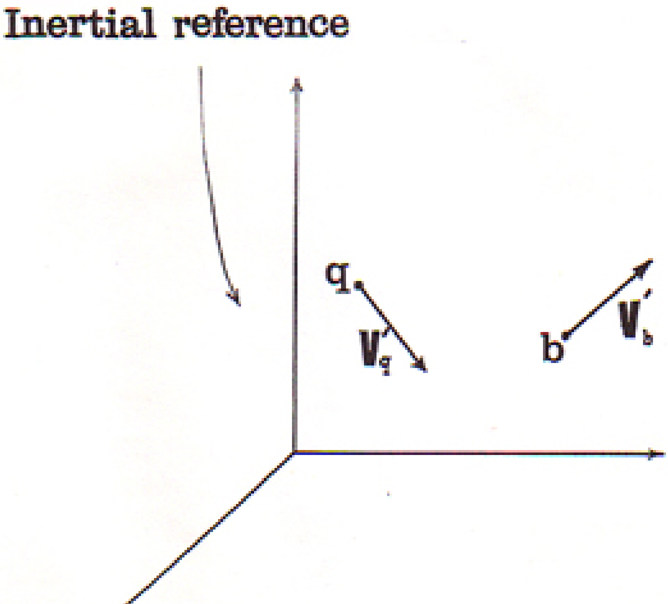

Fig. 2

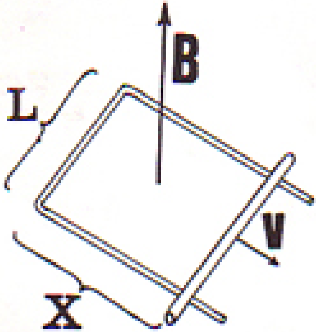

Fig. 3

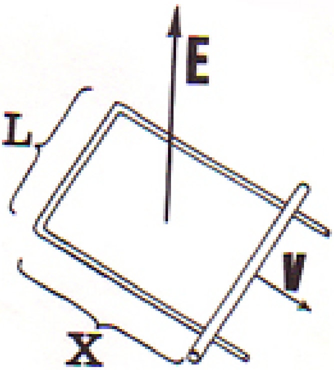

Fig. 4

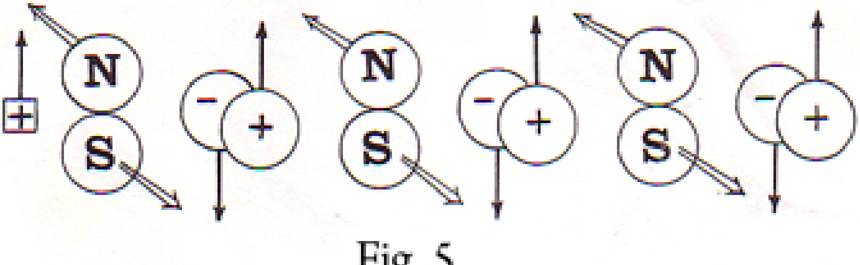

Fig. 5

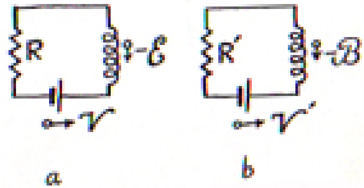

Fig. 6

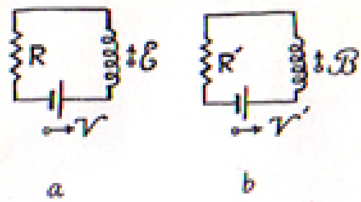

Fig.7

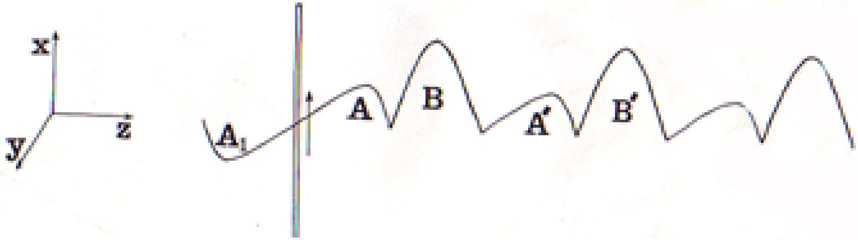

Fig. 8

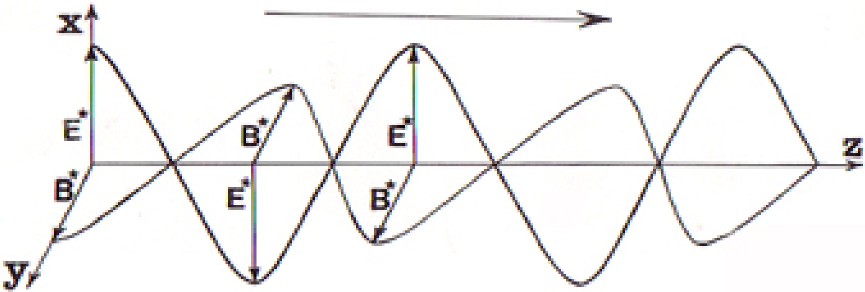

Fig. 9

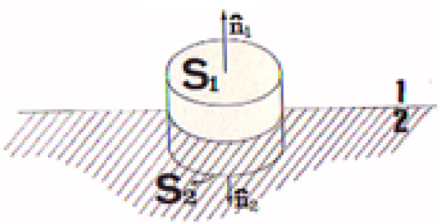

Fig. 10

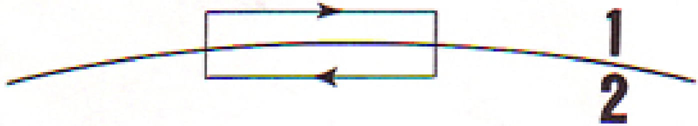

Fig. 11

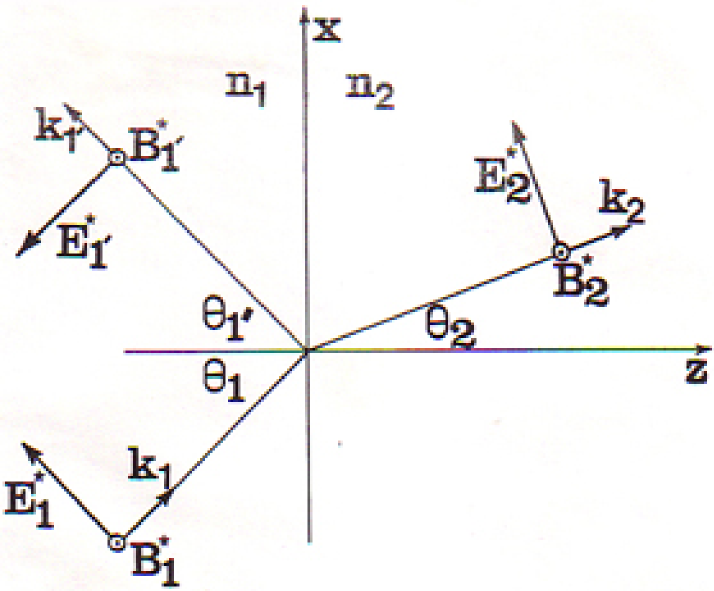

Fig. 12