Odd algebraic bases, and their utility in computer science

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Abstract

It is shown that we can take all numbers to odd bases such that we require only about half of the digits required in the current method provided that we introduce negative mark for each digit. Most probably this method will have various applications in the computer technology.

1 Odd algebraic bases

As we know we can write a number in a natural base greater than or equal to two such that it is equal to the sum of the multiples of the intiger powers of this base while these multiples are zero or natural numbers less than the base. We show here that we can write a number in an odd natural base greater than one such that it is equal to the algebraic sum of the multiples of the integer powers of this base while these multiples are zero or (positive or negative) integer numbers the magnitude of each being less than the half of the base. We show this fact by a simple example.

Suppose that we want to write numbers in algebraic base of 3 as defined above. Zero and natural numbers less than half of 3 are 0 and 1. (Presently nonnegative) integer powers of 3 are $3^0, 3^1, 3^2, 3^3, \cdots$. We can write each (zero, negative or positive) integer as an exclusive algebraic sum of zero or one algebraic multiples of these powers. For example 208 (in the ordinary base of 10) is written as $10\bar{1}\bar{1}01$ in the algebraic base of 3, because $1 \times 3^5 + 0 \times 3^4 + (-1) \times 3^3 + (-1) \times 3^2 + 0 \times 3^1 + 1 \times 3^0 = 208$. A way for taking a number to the algebraic base of 3 is that at first we take it to the ordinary base of 3 by successive divisions. For example through

$$\begin{array}{l} 208\underline{|3} \\ \quad 1\ 69\underline{|3} \\ \qquad 0\ 23\underline{|3} \\ \qquad\quad 2\ \ 7\underline{|3} \\ \qquad\qquad 1\ 2\underline{|3} \\ \qquad\qquad\quad 2\ 0 \end{array}$$

we obtain $(208)_{10} = (21201)_3$. Then we should subtract the base, here 3, from any digit which is greater than half of the base, and add one to the digit on its left instead. For example for $(21201)_3$ we have

$$(21201)_3 = (2\overbrace{2}^{1+1}\underbrace{\bar{1}}_{2-3}01)_3 = (\overbrace{3}^{2+1}\underbrace{\bar{1}}_{2-3}\bar{1}01)_3 = (\overbrace{1}^{0+1}\underbrace{0}_{3-3}\bar{1}\bar{1}01)_3$$

(If necessary, we can also add 3 to a digit, and subtract 1 from the digit on its left instead. For example:

$$\underbrace{(1021)_3}_{(11\bar{1}1)_3} = (1\overbrace{\bar{1}}^{0-1}\underbrace{5}_{2+3}1)_3 \text{ or } (\bar{1}\bar{1}0)_3 = (\overbrace{\bar{2}}^{-1-1}\underbrace{2}_{-1+3}0)_3)$$

Thus we saw $(208)_{10} = (10\bar{1}\bar{1}01)_3$. As we can easily see, this kind of base includes, in a natural manner, negative numbers. Algebraic sum of these numbers are done easily (and no separate method for subtraction is necessary). For example for the algebraic base of 3 we have

$$\begin{array}{r} 1\bar{1}0 \\ \underline{+10\bar{1}1} \\ 11\bar{2}1 \end{array} = 1\overbrace{0}^{1-1}\underbrace{1}_{-2+3}1 \qquad \text{or} \qquad \begin{array}{r} \bar{1}\bar{1}0 \\ \underline{+101} \\ \bar{1}1 \end{array} \quad .$$

Their multiplications are also done easily. For example for the algebraic base of 3 we have:

$$\begin{array}{r} 11\bar{1}0 \\ \underline{\times 1\bar{1}1} \\ 11\bar{1}0 \\ \bar{1}\bar{1}10\ \ \\ 11\bar{1}0\ \ \ \ \\ \hline 10\bar{1}2\bar{1}0 \end{array} = 1\overbrace{0}^{-1+1}\underbrace{0}_{2-3}\bar{1}\bar{1}0$$

Their divisions are also done easily. For example for the algebraic base of 3 we have:

$$
\begin{array}{ll} 10\bar{1}0 & |\underline{1\bar{1}1} \\ 1\bar{1}1 & \;1 \end{array}
\rightarrow
\begin{array}{ll} 10\bar{1}0 & |\underline{1\bar{1}1} \\ \underline{\bar{1}1\bar{1}} & \;1 \\ 01\bar{2} = 001 & \end{array}
\rightarrow
\begin{array}{ll} 10\bar{1}0 & |\underline{1\bar{1}1} \\ \underline{\bar{1}11} & \;1 \\ 0010 & \end{array}
\rightarrow
\begin{array}{ll} 10\bar{1}0 & |\underline{1\bar{1}1} \\ \underline{\bar{1}11} & \;10 \\ \;\;10 & \\ \;\;\underline{00} & \\ \;\;10 & \end{array}
$$

$$
\begin{array}{ll} 1\bar{1}\bar{1}0 & |\underline{10} \\ 10 & \;1 \end{array}
\rightarrow
\begin{array}{ll} 1\bar{1}\bar{1}0 & |\underline{10} \\ \underline{\bar{1}0} & \;1 \\ \overline{0\bar{1}\bar{1}} & \end{array}
\rightarrow
\begin{array}{ll} 1\bar{1}\bar{1}0 & |\underline{10} \\ \underline{\bar{1}0} & \;1\bar{1} \\ \;\overline{\bar{1}\bar{1}} & \\ \;\bar{1}0 & \end{array}
\rightarrow
\begin{array}{ll} 1\bar{1}\bar{1}0 & |\underline{10} \\ \underline{\bar{1}0} & \;1\bar{1} \\ \;\overline{\bar{1}\bar{1}} & \\ \;10 & \\ \;\overline{0\bar{1}0} & \end{array}
\rightarrow
\begin{array}{ll} 1\bar{1}\bar{1}0 & |\underline{10} \\ \underline{\bar{1}0} & \;1\bar{1}\bar{1} \\ \;\overline{\bar{1}\bar{1}} & \\ \;\underline{10} & \\ \;\;\bar{1}0 & \\ \;\;\underline{\bar{1}0} & \\ \;\;00 & \end{array}
$$

In this manner, instead of successive divisions and transformations done above to obtain $(208)_3 = (10\bar{1}\bar{1}01)_3$ we could directly do as in the following:

$$
\begin{array}{llllll}
208|\underline{3} & & & & & \\
1 & 69|\underline{3} & & & & \\
 & 0 & 23|\underline{3} & & & \\
 & & \bar{1} & 8|\underline{3} & & \\
 & & & \bar{1} & 3|\underline{3} & \\
 & & & & 0 & 0
\end{array}
$$

Such a situation cannot exist for an even base as an algebraic base, because any power of an even number is an even number, and this even number multiplied by any (odd or even) number makes an even number, and this means that we cannot make odd numbers in this manner.

As we saw in the above example of the algebraic base of 3, we only handle the digits 0 and 1 in this base (of course algebraically, ie using negative mark for each digit). This fact probably will be useful in making computers to work advantageously based on the algebraic base of (the greater number of) 3 rather than the base of 2. Also, similarly we will handle 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 (of course algebraically) in the algebraic base of 19, and this fact that we are able to use only all the one-digit numbers, using minus mark for each when necessary, for faster processing and more storing of the numbers by using the algebraic base of (the big number of) 19 will probably be useful in the computer technology.