From Ancient Africa

by Frank Dodd (Tony) Smith, Jr.

A little less than 15 billion years ago, our Universe emerged from the Void.

4 billion years ago, our Earth and Moon were orbiting our Sun.
2 billion years ago, bacteria built a nuclear fission reactor in Africa.

100,000 years ago, Humans were expanding from the African home-land to Eurasia and beyond.

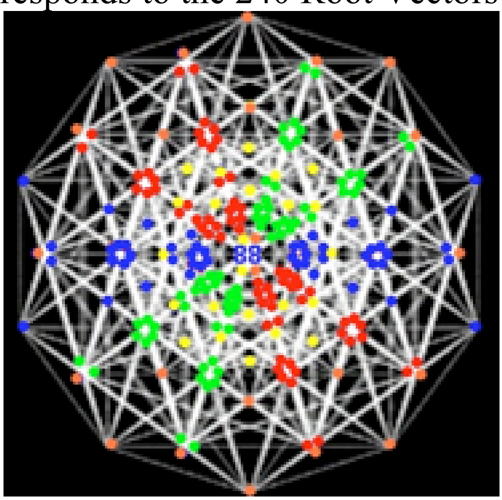
12,000 years ago, Africans developed IFA Oracle divination based on the square of 16 = 16x16 = 256 = = 2^8 corresponding to the vertices of an 8-dimensional hypercube and to the binary 2-choice Clifford algebra Cl(8) and so to related ones such as Cl(8)xCl(8) = Cl(16). Since the number of sub-hypercubes in an 8-dimensional hypercube is 6,561 = 81x81 = 3^8, the IFA Oracle has N=8 ternary 3-structure as well as binary 2-structure:

```
N        2^N                         3^N
0                      1
1                    2   3
2                  4 =2x2  9 = 3x3
3                8           27
4             16 = 4x4         81 = 9x9
5           32                   243
6         64 = 8x8                 729 = 27x27
7      128                           2187
8    256 = 16x16                       6561 = 81x81
```

As ancient African games such as Owari show, binary 2-structure corresponds to static states and ternary 3-structure corresponds to dynamic states. Mathematically, using binary 2-choice static states to define dynamics on 3 ternary neighbor states produces the 256 elements of Elementary Cellular Automata.

The African Oracle patterns spread throughout the Earth, so that by the 13th century parts of them were found in:

India used the 240 parts of the Rig Veda's first sukt. The 240 corresponds to the 240 Root Vectors



of the E8 Lie algebra that is constructed from Cl(8)xCl(8) = Cl(16).

Judaism used the 248 positive Commandments plus the 365 negative Commandments given to Moses during the 50 days from Egypt to Sinai.
The 248 correspond to the 248-dim E8 Lie algebra that is constructed from Cl(8)xCl(8) = Cl(16).
The 365 is constructed by looking at one of the 1792 = 7x256 sub-hypercubes of 6-dim in the 8-dim hypercube, effectively cutting off the ternary 3-structure at N= 6

```
N         2^N                        3^N
0                         1
1                      2    3
2                    4        9 = 3x3
3                 8              27
4             16                    81 = 9x9
5          32                          243
6       64                                729 = 27x27
7    128
8 256
```

and then making a 27x27 Magic Square with (729+1)/2 = 365 as its its central Magic Square number.

Plato, to construct a musical scale, used the full N=8 binary 2-structure but only N=5 of the ternary 3-structure

```
N         2^N                        3^N
0                         1
1                      2    3    =  2 + 1
2                    4        9    =  4 + 4+1
3                 8              27    =  8 + 12+6+1
4             16                    81    = 16 + 32+24+8+1
5          32                          243=32+80+80+40+10+1
6       64
7    128
8 256
```

Plato used the numbers 256 and 243 to form the ratio 256/243, which, along with 9/8,
let him construct the the first octave as:
1   9/8   81/64   4/3   3/2   27/16   243/128   2
by using the multiplicative intervals:
  9/8   9/8   256/243   9/8   9/8   9/8   256/243

China used the 64 possibilities of the binary I Ching and the 81 possibilities of the ternary Tai Hsuan Ching, effectively cutting off the binary 2-structure at N= 6 and the ternary structure at N= 4

N   2^N   3^N

```
0          1
1         2 3
2        4   9 = 3x3
3       8     27
4      16      81 = 9x9
5     32
6    64
```

Japan used the 128 possibilities of Shinto Futomani Divination and the Triad: Jewel-Mirror-Sword, effectively cutting off the binary 2-structure at N = 7 and the ternary 3-structure at N=1

N   2^N   3^N

```
0          1
1         2 3
2        4
3       8
4      16
5     32
6    64
7   128
```

Adding 3 of the ternary 3-structures to 16 of the binary 2-

structures gives the number 19 of the 19x19 board of the game Go (the Chinese version, Wei Qi, may have originally had a 17x17 board).

Mediterranean Africa used the 16 possibilities of the Ilm al Raml, effectively cutting off the binary 2-structures at N = 4 and eliminating the ternary 3-structures

N	2^N	3^N
0	1	
1	2	
2	4	
3	8	
4	16	

As noted about Japan, adding 3 of the ternary 3-structures to 16 of the binary 2-structures gives the number 19, but in Mediterranean Africa the most significant use of the number 19 is in the Quran, which is written in Arabic with a basic message that G-d is ONE, and the Arabic number-value of the Arabic letters of the word ONE is 6+1+8+4 = 19. Rashad Khalifa, whose mathematical analysis of the Quran confirmed the primacy of the number 19, was murdered in 1990 in his Mosque in Arizona because of the results of his analysis.

In the early 13 th century Ibn Arabi (born in Spain, died in Damascus) wrote the “Bezels of Wisdom” describing his Sufi Islamic World-View using terms consistent with the processes of Quantum Theory:

“mumkinat” = quantum possibile Worlds of the Many-Worlds, or Bohmian beables;
“qada” = decoherence of quantum superposition of possibilities, or choice at an Event of a World of the Many-Worlds;
“qadar” = outcome of qada = the World or State that comes into existence as the outcome of an Event;
“al-khalq al-jadid” = the branching of the Worlds of the Many-Worlds that occurs at an Event.

According to the book Sufism and Taoism, by Toshihiko Izutsu (California 1983): “…Ibn Arabi says that … the world in its entirety ... transforms itself kaleidoscopically from moment to moment ... 'new creation' (al-khalq al-jadid) ... ordinary people are not aware of the process … If a man happens to obtain the true knowldege of qadar, the knowledge surely brings him a perfect peace of mind and an intolerable pain at the same time.
The unusual peace of mind arises from the consciousness that everything in the world occurs as it has been determined from eternity. ... Instead of struggling in vain for obtaining what is not in his capacity, he will be happy … He must be tormented, on the other hand, by an intense pain at the sight of all the so-called 'injustices', 'evils', and 'sufferings' that reign rampant around him, being keenly conscious that it is not in his 'preparedness'

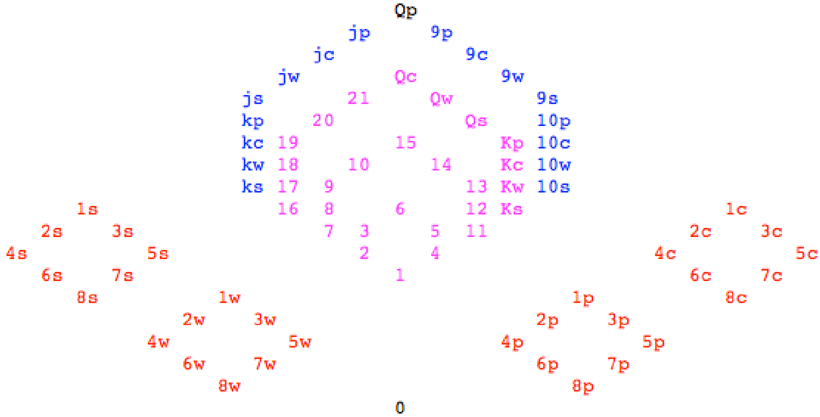
to remove them from the world. ...".

Describing the kaleidoscopic process of qadar in specific physical detail requires, in addition to the processes of Ibn Arabi, a specific mathematical framework.
Near the end of the 13th century, Ramon Llull of Mallorca studied the 16 possibilities of the Ilm al Raml and realized that they had a Fundamental Organizational Principle that he summarized in a Wheel Diagram

with 16 vertices connected to each other by 120 lines.
If the 16 vertices represented a 16-dimensional vector space, then the 120 lines connecting pairs of vectors represented 120-dimensional bivectors of rotations in that 16-dimensional vector space.
That total geometry is described by the Real 16-dimensional Clifford Algebra Cl(16).

Cl(16) not only describes rotations in vector space, but also spinors that describe left-handed and right-handed properties with respect to the space.
Half-spinors of Cl(16) have 128 dimensions.
When you combine the 120 bivectors of Cl(16) with the 128 half-spinors of Cl(16), you get the 248-dimensional object called the exceptional Lie Algebra E8.
In 8-dimensional space, 240 of the 248 generators of E8 form the 240 Root Vectors of E8.
Cl(16) factors into the tensor product of two copies of the Real 8-dimensional Clifford Algebra Cl(8):
Cl(16) = Cl(8)xCl(8)
Since Cl(8), based on an 8-dimensional vector space, has 256 dimensions, and 256 = 16x16,
we have come full circle to the 16x16 possibilities of the ancient African Oracle.

However, Ramon Llull was not as interested in making a physics model by combining his Cl(16) mathematical structure with Ibn Arabi's quantum processes
as he was in using his structure to show the equivalence of Judaism, Christianity, and Islam, an effort that failed, perhaps because the political institutions of those religions were in power struggles with each other and so were in fact hostile to reconciliation/unification.
Ramon Llull's work failed to find institutional acceptance in Judaism or Islam, and Christian Dominican Catholicism suppressed his work,
but his ideas were carried on in the underground world resulting in description of detailed substructures by Tarot



- The magenta 28 are the 28 Spin(8) adjoint bivectors of Cl(8).
- The 16 blue are the 8 vectors of Spin(8) and Cl(8) and their 8 dual/conjugates.
- The 32 red are the 16 spinors (8 +halfspinors and 8 -halfspinors) of Spin(8) and Cl(8) and their 16 dual/conjugates.
- The 2 black are diagonal degrees of freedom in 26-dim traceless J3(O)o part of J3(O) Jordan algebra.

whose 78 cards represent the E6 subalgebra of the 248-dimensional Lie algebra E8 which is constructed from the Clifford algebra Cl(16) = Cl(8)xCl(8) where Cl(8) is the 16x16 = 256-dimensional algebra of IFA.

The rest of this paper gives some further details about:

Earlier Heaven IFA Sequence;
Cl(8) Graded Structure;
Cellular Automata Dynamics;
African Oracle Structures;
88 Equivalence Classes;
256 Elementary Rule Patterns and Physics; and
Past and Future History.

[Earlier Heaven Binary Sequence IFA]

The 16 Tetragrams in Earlier Heaven Binary Number sequence are:

(o = 0 and oo = 1, ordered from top to bottom)

Oludumare (original g-d) creates Earth, the 16 Odu, the Orishas, and humans:

```
   Odu                                Orisha

0
Ogbe (light of creation)     - Orunmila (wisdom Tao)
o      heaven Tao void          (next to Oluddomare)
o
o
o

1
Osa (creativity)             - Oya (wind)
oo
o
o
o

2
Otura (unity of everything)  - Osain (forest spirits)
o
oo
o
o

3
Owonrin (rain)               - Oshun (rivers) (He Xiangu)
oo
oo
o
o

4
Irete (fate)                 - Babalu Aye (healer) (TieGuai Li)
o
o
oo
o

5
Ofun (taboo)                 - Eshu Legba (trickster)
oo
o
oo
o
```

6
Edi (womb) - Obatala (hermaphrodite father of humanity)
o
oo
oo
o

7
Okanran (lightning) - Chango (thunder)
oo
oo
oo
o

8
Ogunda (sword) - Ogun (iron) (Lu Dongbin)
o
o
o
oo

9
Iwori (consciousness) - Eshu Legba (messenger)
oo
o
o
oo

10
Ose (victim of abuse) - Osun (guardian angel)
o - truth protects good and destroys evil
oo
o
oo

11
Oturopon (trap) - Ochosi (hunter)
oo
oo
o
oo

12
Irosun (fire) - Aganyu (volcano)
o
o
oo
oo

13
Ika (forest land) - Yemaya (ocean)

oo

o

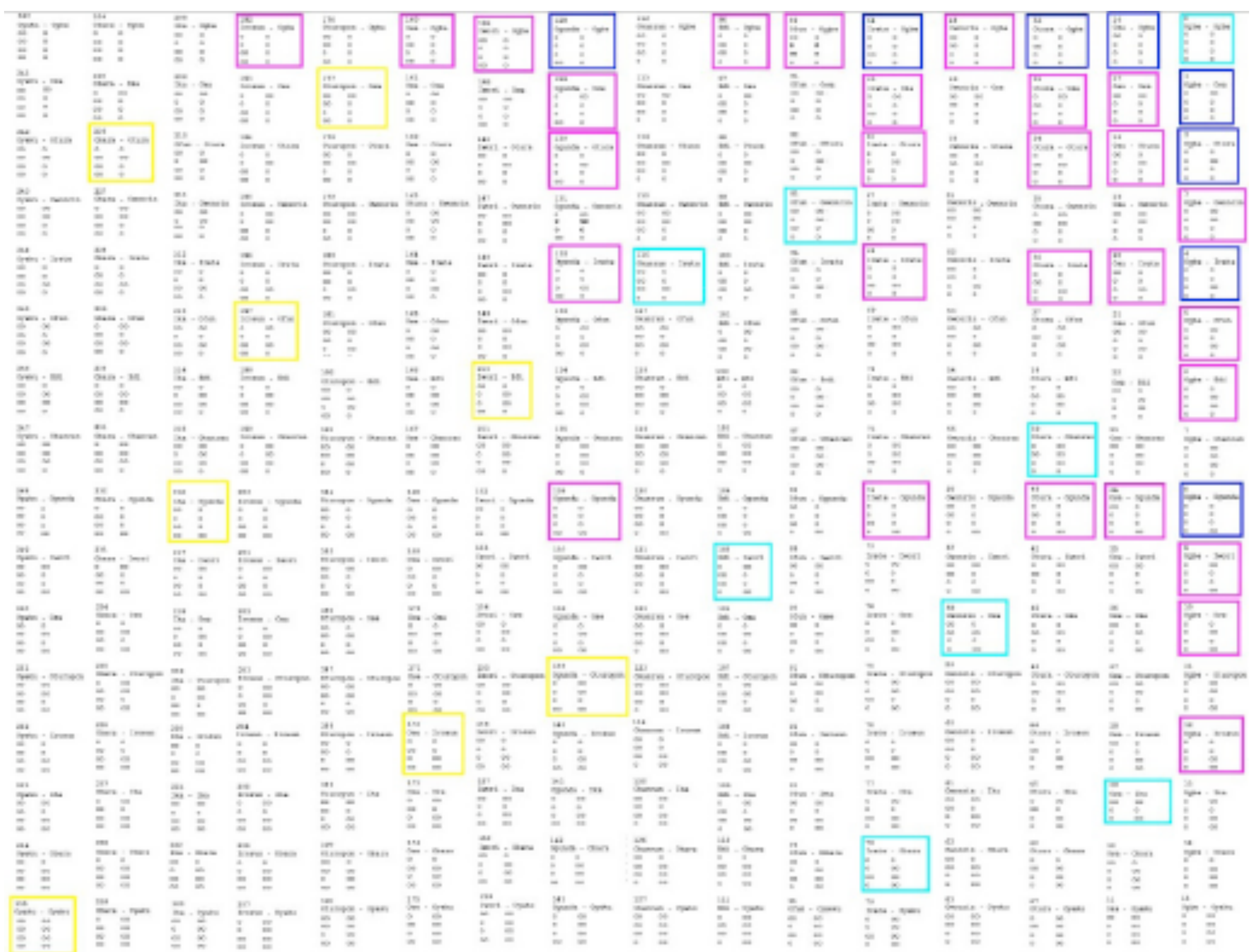
oo

oo

14
Obara (rainbow) - Inle (two intertwined snakes, medicine)
o
oo
oo
oo

15
Oyeku (dark of earth) - Ibeyi (twins Yin-Yang)
oo Tai Ji Yin-Yang
oo
oo
oo

The 256 Pairs of Tetragrams in Earlier Heaven Binary Number Sequence



correspond to the 256 elements of the Cl(8) Clifford Algebra with graded structure

1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 256 = 16x16

Grade:	0	1	2	3	4	5	6	7	8
000	00000000								
001		00000001							
002		00000010							
003			00000011						
004		00000100							
005			00000101						
006			00000110						
007				00000111					
008		00001000							
009			00001001						
010				00001010					
011				00001011					
012			00001100						
013				00001101					
014				00001110					
015					00001111				
016		00010000							
017			00010001						
018			00010010						
019				00010011					
020			00010100						
021				00010101					
022				00010110					
023					00010111				
024			00011000						
025				00011001					
026				00011010					
027					00011011				
028				00011100					
029					00011101				
030					00011110				
031						00011111			
032		00100000							
033			00100001						
034			00100010						
035				00100011					
036			00100100						
037				00100101					
038				00100110					
039					00100111				
040			00101000						
041				00101001					
042				00101010					
043					00101011				
044				00101100					
045					00101101				
046					00101110				

047					00101111	
048		00110000				
049			00110001			
050			00110010			
051				00110011		
052			00110100			
053				00110101		
054				00110110		
055					00110111	
056			00111000			
057				00111001		
058				00111010		
059					00111011	
060				00111100		
061					00111101	
062					00111110	
063						00111111
064	01000000					
065		01000001				
066		01000010				
067			01000011			
068		01000100				
069			01000101			
070			01000110			
071				01000111		
072		01001000				
073			01001001			
074		01001010				
075				01001011		
076			01001100			
077				01001101		
078				01001110		
079					01001111	
080		01010000				
081			01010001			
082			01010010			
083				01010011		
084			01010100			
085				01010101		
086				01010110		
087					01010111	
088			01011000			
089				01011001		
090				01011010		
091					01011011	
092				01011100		
093					01011101	
094					01011110	
095						01011111
096		01100000				
097			01100001			
098			01100010			
099				01100011		
100			01100100			
101				01100101		
102				01100110		
103					01100111	
104			01101000			
105				01101001		

106				01101010			
107					01101011		
108				01101100			
109					01101101		
110					01101110		
111						01101111	
112			01110000				
113				01110001			
114				01110010			
115					01110011		
116				01110100			
117					01110101		
118					01110110		
119						01110111	
120				01111000			
121					01111001		
122					01111010		
123						01111011	
124					01111100		
125						01111101	
126						01111110	
127							01111111
128	10000000						
129		10000001					
130		10000010					
131			10000011				
132		10000100					
133			10000101				
134			10000110				
135				10000111			
136		10001000					
137			10001001				
138			10001010				
139				10001011			
140			10001100				
141				10001101			
142				10001110			
143					10001111		
144		10010000					
145			10010001				
146			10010010				
147				10010011			
148			10010100				
149				10010101			
150				10010110			
151					10010111		
152			10011000				
153				10011001			
154				10011010			
155					10011011		
156				10011100			
157					10011101		
158					10011110		
159						10011111	
160		10100000					
161			10100001				
162			10100010				
163				10100011			
164			10100100				

165			10100101			
166			10100110			
167				10100111		
168		10101000				
169			10101001			
170			10101010			
171				10101011		
172			10101100			
173				10101101		
174				10101110		
175					10101111	
176		10110000				
177			10110001			
178			10110010			
179				10110011		
180			10110100			
181				10110101		
182				10110110		
183					10110111	
184			10111000			
185				10111001		
186				10111010		
187					10111011	
188				10111100		
189					10111101	
190					10111110	
191						10111111
192	11000000					
193		11000001				
194		11000010				
195			11000011			
196		11000100				
197			11000101			
198			11000110			
199				11000111		
200		11001000				
201			11001001			
202			11001010			
203				11001011		
204			11001100			
205				11001101		
206				11001110		
207					11001111	
208		11010000				
209			11010001			
210			11010010			
211				11010011		
212			11010100			
213				11010101		
214				11010110		
215					11010111	
216			11011000			
217				11011001		
218				11011010		
219					11011011	
220				11011100		
221					11011101	
222					11011110	
223						11011111

224				11100000					
225					11100001				
226					11100010				
227						11100011			
228					11100100				
229						11100101			
230						11100110			
231							11100111		
232					11101000				
233						11101001			
234						11101010			
235							11101011		
236						11101100			
237							11101101		
238							11101110		
239								11101111	
240					11110000				
241						11110001			
242						11110010			
243							11110011		
244						11110100			
245							11110101		
246							11110110		
247								11110111	
248						11111000			
249							11111001		
250							11111010		
251								11111011	
252							11111100		
253								11111101	
254								11111110	
255									11111111

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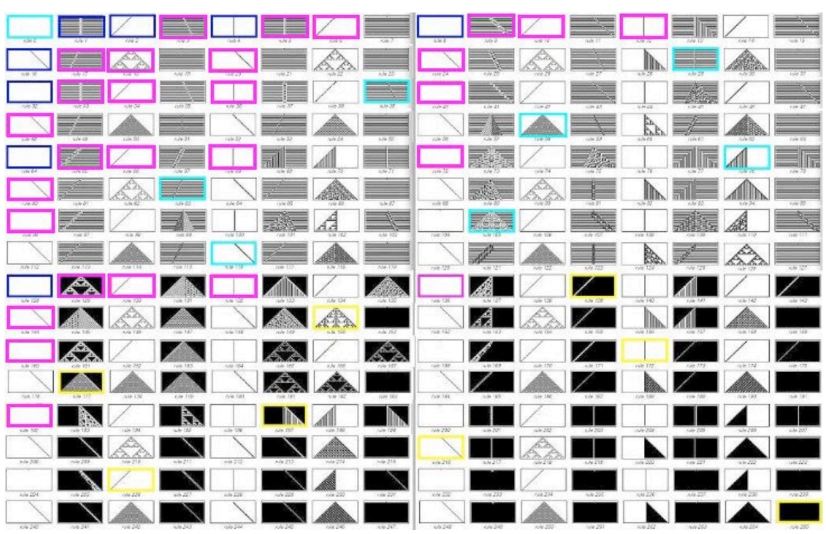
and to the 256 1-dimensional Cellular Automata



IFA and Global Dynamics of Cellular Automata

88 Equivalence Classes -------- 256 Elementary Rules

Images and quoted text are from The Global Dynamics of Cellular Automata by Andrew Wuensche and Mike Lesser (Addison-Wesley 1992 and a scanned version in pdf format downloaded from the web) (unless othewise specified).

2.1 Cellular Automata

A cellular automaton (CA) is a discrete dynamical system which evolves by the iteration of a simple deterministic rule; as in any dynamical system, the system's variables change as a function of their current values.

An alternative approach is to view a CA as a parallel processing computer[34] where the data is considered to be the initial CA configuration.

Yet a third approach is that a CA is a "logical universe...with its own local physics." [16] Such a CA universe, in spite of its mathematically simple construction, seems to be capable of supporting complex emergent behaviour.

2.1.1 CA Architecture

A CA is constructed as follows: Time is discrete and progresses in steps. A D-dimensional, potentially infinite space is partitioned into discrete "cells" (the CA array or lattice) according to a given geometry. Boundary conditions may be set to define a finite space. Each cell has one attribute (the cell's value) from a limited range of attributes, which may be labelled by an integer. The pattern of values across the whole array is the CA global state at a given time.

Any pattern may be set as an initial condition at time t_0. Each cell of the array simultaneously has its value updated to evolve a new global state at time t_1. The new value of any given cell (the target cell) at t_1 is a function of the values and locations of a set of cells (the neighbourhood) at t_0, typically situated locally in relation to the target cell (see Fig. 2.1). The neighbourhood may be defined by a neighbourhood template or wiring diagram. The CA evolves through a succession of global states (its *trajectory*) by the iteration of this global updating procedure (the *transition function*). Provided that the transition function is constant and the system is closed to noise (the updating is error free), then the evolution of the CA from its initial global state is uniquely determined.

Two types of CA may be distinguished, both deterministic: The more general case may be described as having varying degrees of *disordered* architecture (non-local[21]), where the wiring diagram and/or function at each cell may differ, for example Walker's networks of Boolean functions[27–32] and Kauffman's random Boolean networks.[14,15]

CA with *ordered* architecture are a special case, where the wiring diagram and function are the same over the entire array. In addition, the ordered wiring may be confined to a *local neighbourhood*, an uninterrupted zone of cells typically centred on the target cell. CA of this type will be referred to as having *local architecture*, for example, the architecture of Wolfram's "elementary rules." [33–40] This paper is relevant to deterministic CA in general, however it deals mainly with the simplest possible local CA architecture.

Von Neumann first proposed CA to model self-reproduction.[3,26] His relatively complicated CA architectures were local and two-dimensional with 29 cell values. The tendency since then has been to find simpler architecture that could nonetheless support complex emergent behaviour. For example Conways' "game of life" [2] is a 2-D local

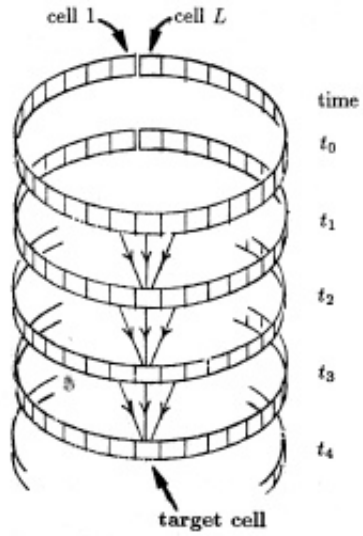


FIGURE 2.1 1-D, local binary CA with periodic boundary conditions, neighbourhood 3 (elementary rules), array length L.

CA with an orthogonal toroidal array, a 9-cell neighbourhood (the target cell and its eight nearest neighbours), and two cell values (binary value range).

2.1.2 Local 1-D Binary CA with Periodic Boundary Conditions

The simplest local CA architecture that we will investigate comprises a 1-D array of a small number of cells, a binary value range and a small local neighbourhood. The array is arranged in a circle; such a circular array is said to have *periodic boundary conditions*. Evolution of the CA may be represented as a sequence of global states on a cylinder, summed up by Fig. 2.1.

2.1.4 CA Dynamics

State space (also called *phase space*) is the set of all possible CA global states. In a finite CA, state space is finite; thus, any trajectory must eventually encounter a repeat of a global state that occurred at an earlier time. Because the system is deterministic, the trajectory will become trapped in this repeating sequence of states, a ***cyclic attractor***, with a specific period of 1 or more.

States are either part of the attractor or belong to a *transient*, a sequence of states leading to the attractor. If transients exist, there must be states at their extremities (*garden-of-Eden states*), unreachable by evolution from any other state. The set of all possible transients leading to an attractor, plus the attractor itself, is the ***basin of***

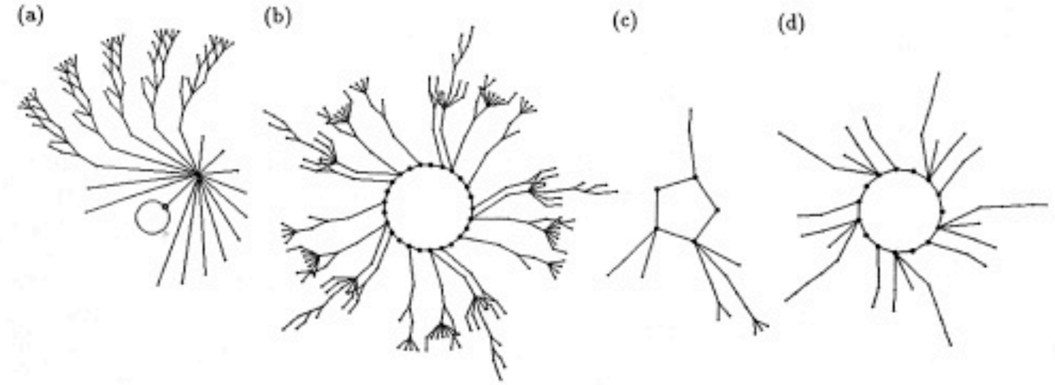


FIGURE 2.3 A basin of attraction field, local 3-neighbour rule 193, L = 10. The number of basins of each type is (a) 1, (b) 2, (c) 10, and (d) 2.

attraction of that attractor. State space is populated by one or more basins of attraction. These basins of attraction constitute the dynamical flow imposed on state space by the CA transition function.

A portrait of this global behaviour is the *basin of attraction field*, a discrete analogue of the familiar basin of attraction field found in the phase space of a continuous dynamical system, known as the system's *phase portrait*.

2.2 The Basin of Attraction Field

The *basin (of attraction) field* of a finite CA is the set of *basins of attraction* into which all possible states and trajectories will be organised by the cellular automaton transition function. The topology, or structure, of a single basin of attraction may be described by a diagram, the *state transition graph*. The set of graphs making up the field specifies the global behaviour of the system. Various other names have been used: state transition fragment,[39] contraction map,[7] topology of behaviour space,[29] and network of attraction.[42]

The notion of basin fields was proposed by Walker,[27] and examples[1] have been given by Martin et al.,[22] Pitsianis et al.,[25] Wolfram,[39,40] Feldberg and Rasmussen,[5] and by the authors[42] in an earlier edition of this atlas.

An example of a basin of attraction field is shown in Fig. 2.3 for the local, binary, 3-neighbour rule 193 (see chapter 3, section 3.3). The array length, L, equals 10, so that state space consists of $2^{10} = 1024$ global states. The CA transition function connects these states into a set of basins, the basin (of attraction) field. In this case there are four different types of basins in the field, some of which occur more than once. The number of each type is indicated.

2.2.1 The State Transition Graph

A state transition graph links up all the states belonging to a single basin of attraction according to their specific evolutionary location; this will typically have a topology of *trees rooted on attractor cycles.*[22] Global states are represented by *nodes* which are linked by *directed arcs*. Each node will have zero or more incoming arcs from nodes at the previous time step (*pre-images*), but because the system is deterministic, exactly one outgoing arc ("out degree") to a single node (the *successor state*), at the next time step. Nodes with no incoming arcs represent garden-of-Eden states. The number of incoming arcs is referred to as the *degree of pre-imaging* ("in degree").

For a given set of CA parameters, state space will, in a sense, crystallise into a set of one or more basins of attraction. The basin of attraction field is the set of state transition graphs representing all the basins in state space.

The make-up of a typical basin of attraction is illustrated by the state transition graph shown in Fig. 2.4 (it is part of the basin field shown in Fig. 2.6). In our graphic convention (see Appendix 2), the length of transition arcs decreases with distance away from the attractor, and the diameter of the graphic representation of the attractor asymptotically approaches an upper limit with increasing period, so that attractor cycles are drawn with approximately the same diameter irrespective of the number of nodes in the attractor. The forward direction of transitions is inward from garden-of-Eden states to the attractor, which is the only closed loop in the basin, and then clockwise around the attractor cycle.

Typically, the vast majority of nodes in a basin field, or a single basin of attraction, lie on *transient trees* outside the attractor cycle. A transient tree is the set of all paths from garden-of-Eden nodes leading to one node on the attractor cycle (an *attractor node*). A branch of the transient tree is termed a *transient branch*, and is the set of all paths from garden-of-Eden nodes leading to a state within a transient tree. A *transient* is one particular path from an arbitrary node in the transient tree leading to the attractor node. In all cases the attractor node itself is excluded from the definition.

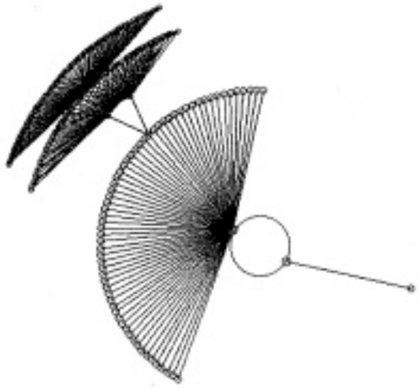


FIGURE 2.8 Example of a basin of the 3-neighbour rule 126, $L = 31$. The seed state is a single 0 followed by six copies of the string 01111.

2.2.6 The Atlas

The program has been used to produce an *atlas* of basin of attraction fields over a range of array lengths, presented in Appendix 2. The Atlas consists of two parts. Part 1 presents all 3-neighbour (elementary) rules. Part 2 presents all 5-neighbour totalistic codes. Selected data is also presented on each basin and basin field. There is an index to rules and codes at the beginning of each part.

A CA rule belongs to a set of up to four *equivalent* rules, the *equivalence class*, that differ only in that they have *negative* and/or *mirror image* space-time patterns, but which have identical basin field topology. Thus, only one rule representing each equivalence class is presented in the Atlas. Pairs of equivalence classes relate in that their rules have *complementary* rule tables, forming a *rule cluster*. The representative rules in the Atlas are presented according to their rule cluster, with complementary equivalence classes shown on facing pages (the relationships between rules are explained in detail in chapter 3, section 3.5)

The 3-neighbour rule clusters belong to one of three *symmetry categories—symmetric*, *semi-asymmetric*, and *fully asymmetric*—and are accordingly presented in three sections. The basin fields for the 88 *equivalence classes* of the 3-neighbour rules are shown for $L = 1$ to 15.

2.2.7 Significance of Basin of Attraction Fields

The ability to represent basin of attraction fields may be significant in a number of areas such as CA theory, complex systems, dynamical systems, computational theory, artificial life, neural networks, and aspects of genetics.

Basin field topology represents a second order of complexity of CA behaviour, where the first order may be said to be the space-time patterns of particular trajectories. Easy access to a systematic presentation of basin fields, for a synoptic, qualitative, as well as explicit view of global behaviour, may provide insights into the dynamical theory of CA, and the structure of CA rule space. It may provide a useful analogue to the global behaviour of CA with more complex architecture, and dynamical systems in general.

The separate basins in a basin of attraction field classify state space. Attractors have been regarded as "memories," [13,14,15] with implications for the mechanism underlying neural networks.

Basin fields may be of interest in genetics because mutations of the CA *rule table* by a small Hamming distance typically result in altered but related basin structures. Analogies have been made between a CA rule table and a DNA sequence.[20] Kauffman and others study CA with non-local architecture as models in biology and genetics.[14,15]

Attractors have been interpreted as "cell types" in ontogeny. Evolution is said to occur in an optimal "fitness landscape" by mutation and selection of the CA "genotype" resulting in adapted dynamics or "phenotype."

A possible approach to basin of attraction fields is to see them as artificial morphologys, analogous to the "biomorphs" proposed by Dawkins.[4] Their explicit morphological form (including the global CA state at each node) is determined by the genetic code, the rule table; a mutation of the rule table results in a mutant morphology. In addition, the genetic code is implicit in the morphology because the rule table can be reconstructed from space-time patterns; this suggests possibilities for reproduction. Such a genotype-phenotype approach to CA may suggests applications in artificial life.

3.3 Rule Clusters

CA parameters may, finally, be restricted as follows: a binary value range ($k = 2$) and local architecture, thus nearest-neighbour wiring, and periodic boundary conditions (a circular array). For a neighbourhood size n, the number of different neighbourhoods equals 2^n, and the number of different rules equals 2^{2^n}.

3.3.1 Rule Numbering System, n=3 (Elementary Rules)

The $k = 2$, $n = 3$ rules have the form,

$$P_i^{t+1} = f(P_{i-1}^t, P_i^t, P_{i+1}^t)$$

where $P_i = 0$ or 1, i is the spatial position between 1 and L, and t is the time. For a circular array, length L, $P_1 = P_{L+1}$.

The system may be represented as follows:

t_0	– – **A B C** –	**B C** – – – **A**	**C** – – – **A B**	The neightbourhood **ABC** at t_0
t_1	– – – **T** – –	**T** – – – – –	– – – – – **T**	defines the cell **T** at t_1.

The $n = 3$ *rule table* with $2^3 = 8$ entries uniquly specifies each individual rule from a total of $2^{2^3} = 256$. Following Wolfram's convention[33] the rule table is ordered in descending values of the binary neighbourhood strings.

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

If the rule table is regarded as a binary number, the rule number, R, is its decimal equivalent; thus, the rule table will range from 00000000 to 11111111, and R from 0 to 255.

3.3.2 Complementary Transformation, n=3

Every rule, R, has a distinct *complementary* rule, R_c, where each entry in the rule table is inverted, so that for a given input line, the next line generated by R_c will be the *negative* of the next line generated by R. R and R_c may have equivalent behaviour (see collapsed clusters below).[1] In any case, their behaviour will be closely related. This will be reflected in the basin field structure. Deterministic structure and symmetry category (see below) will be common to R and R_c. In general, pre-imaging, cycle periods, and transient lengths will be related.

The $n = 3$ rules can be listed in 128 complementary pairs.

$$\begin{array}{rccccc} \text{if}\quad R = & 0, & 1, & 2, & \cdots, & 127 \\ & | & | & | & & | \\ R_c = & 255, & 254, & 253, & \cdots, & 128 \end{array}$$

There are two types of symmetry that relate pairs of neighbourhoods:

Complementary neighbourhood pairs	···	111, 000	T_7, T_0
(0s changed to 1s and vice versa)		110, 001	T_6, T_1
		101, 010	T_5, T_2
		100, 011	T_4, T_3
Reflected neighbourhood pairs	···	110, 011	T_6, T_3
(mirror image)		100, 001	T_4, T_1

3.3.3 Negative Transformation, n=3

For any rule R and input line I, the CA will generate a space-time pattern P. There is a rule, R_n, that, given the negative input line, $\overline{I}$ will generate the negative space-time pattern, $\overline{P}$ (all cell values inverted).

Consider the rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

To find R_n suppose a *negative world* in which the rule table, including neighbourhoods, could be transformed by inverting all values into

	000	001	010	011	100	101	110	111	neighbourhoods
Rule table..	$\overline{T}_7$	$\overline{T}_6$	$\overline{T}_5$	$\overline{T}_4$	$\overline{T}_3$	$\overline{T}_2$	$\overline{T}_1$	$\overline{T}_0$	outputs

The conventional order of neighbourhoods has been altered and must be restored, giving

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	$\overline{T}_0$	$\overline{T}_1$	$\overline{T}_2$	$\overline{T}_3$	$\overline{T}_4$	$\overline{T}_5$	$\overline{T}_6$	$\overline{T}_7$	outputs

The transformed rule table is R_n. Thus the procedure to transform R to R_n is as follows in either order.

1. Take the complement of the rule table, R_c.
2. Swap the output of complementary neighbourhoods.

$$\overline{T}_0 \quad \overline{T}_1 \quad \overline{T}_2 \quad \overline{T}_3 \quad \overline{T}_4 \quad \overline{T}_5 \quad \overline{T}_6 \quad \overline{T}_7$$

For example, rule 193-11000001 is transformed to rule 124-01111100.

3.3.4 Reflection Transformation, n=3

For any rule, R, input line I and space-time pattern P, there is a rule R_r that, given the reflected input line, I_r will generate the reflected (mirror-image) space-time pattern P_r.

Consider the rule table:

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

To find R_r suppose a *mirror image world* in which the rule table, including neighbourhoods would be transformed by reflection into

	000	100	010	110	001	101	011	111	neighbourhoods
Ruletable..	T_0	T_1	T_2	T_3	T_4	T_5	T_6	T_7	outputs

The conventional order of neighbourhoods has been altered and must be restored, giving

	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_3	T_5	T_1	T_6	T_2	T_4	T_0	outputs

The transformed rule table is R_r. Note that only the outputs of asymmetric neighbourhoods are altered. Thus to transform R to R_r, swap the outputs of the two pairs of asymmetric reflected neighbourhoods.

$$T_7 \quad T_6 \quad T_5 \quad T_4 \quad T_3 \quad T_2 \quad T_1 \quad T_0 \quad \text{rule table}$$

For example, rule 193-11000001 is transformed to rule 137-10001001.

3.3.5 Symmetry Categories

The reflection transformation allows rules to be placed into one of three symmetry categories:

1. *symmetric rules* ($R = R_r$), if $T_6 = T_3$ and $T_4 = T_1$.
2. *semi-asymmetric rules*, if either $T_6 \neq T_3$ or $T_4 \neq T_1$, but not both.
3. *fully asymmetric rules*, if $T_6 \neq T_3$ and $T_4 \neq T_1$.

Symmetric rules have space-time patterns whose structures appear to have no bias to move left or right; for semi-asymmetric rules, there is a clear bias towards either the left or right; and for fully asymmetric rules, there is an intersecting bias towards both left and right.

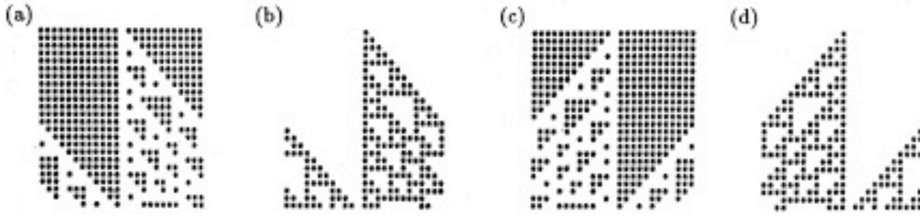


FIGURE 3.5 Equivalent space-time patterns, from equivalent rules. (a) rule R (193). (b) rule R_n (124). (c) rule R_r (137). (d) rule R_{nr} (110).

3.3.6 Bilateral Symmetry of Symmetric Rules

The space-time patterns of symmetric rules, given an input line with *bilateral symmetry* (*bs*), must conserve bilateral symmetry, because the rule acts equivalently on both sides of the axis of symmetry.

As with rotation symmetry (see section 3.2), the degree of bilateral symmetry cannot decrease; in an attractor, bilateral symmetry must remain constant, because any increase would have to be balanced by a decrease as evolution returned around the attractor cycle, and a decrease cannot occur. Bilateral symmetry may increase only in a transient (see Fig. 3.4).

The bilateral axis may divide a circular array, length L, as follows: if L is even, the axis may divide the array into two reflected halves, for instance 0001 − 1000. If the maximum disclosure length is $d_{\max}$, then $d_{\max} \leq 2^{L/2}$. If L is odd, the axis may divide the array into two reflected halves bisecting one cell, for instance 000$\underline{1}$000, then $d_{\max} \leq 2^{((L-1)/2)+1}$. Alternatively, if L is even, there may be more than one axis of bilateral symmetry, for instance 0$\underline{0}$01$\underline{0}$00$\underline{1}$. Such an array has rotation symmetry as well as bilateral symmetry. The interaction between rotational and bilateral symmetry and consequences on behaviour is still unclear. All *totalistic rules* are symmetric (see 3.3.10).

3.3.7 Equivalence Classes

We have seen that a given rule, R, will have two equivalent rules, R_n and R_r. R will also have a third equivalent, R_{nr}, derived by performing the negative and reflection transformations successively, in either order. There will thus be a maximum of four rules grouped in an *equivalence class*; an example is shown in Fig. 3.5, based on rule 193.

The equivalent rules differ only in that they have negative and mirror-image space-time patterns; otherwise, CA behaviour is totally equivalent. There are 88 *equivalence classes* among the 256 $n = 3$ rules.[28,42] Basin field topology for the rules in each class will be identical, though the actual states will differ according to the transformations described.

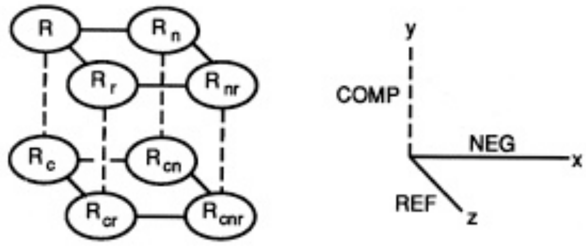


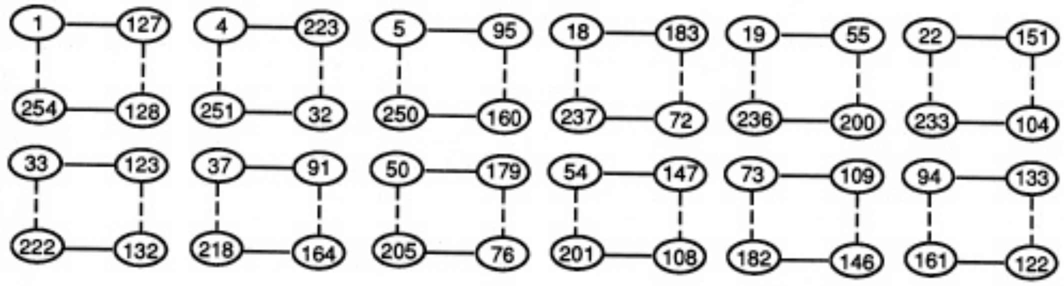
FIGURE 3.6 A rule cluster, two complementary sets of four equivalent rules.

3.3.8 Rule Clusters

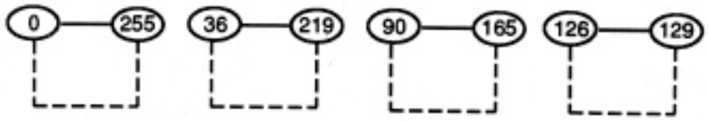
Every rule R has a distinct complement R_c. The 4-rule equivalence class relates to a complementary 4-rule equivalence class, resulting in an 8-rule cluster. We depict the cluster as a box with the rules at each corner (Fig. 3.6), with complementary links along the y axis (dashed line), negative links along the x axis, and reflection links along the z axis.

The rules in a rule cluster are described by two basin fields, one for the top and one for the bottom layer. The outcome of transformations may produce the same rule, resulting in more than one occurrence of that rule in the rule cluster. In this case the cluster is shown collapsed, so that each rule only occurs once. For the purpose of reference, the lowest rule number identifies the cluster, and is positioned in the top left-hand corner. The rule clusters for all $n = 3$ (elementary rules) are set out below.

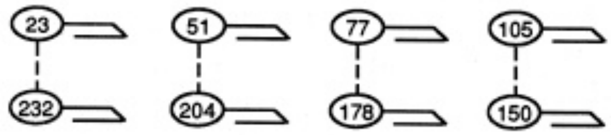
SYMMETRIC RULE CLUSTERS ($T_6 = T_3$ and $T_4 = T_1$). By definition, $R = R_r$, so the reflection links (z axis) will collapse.



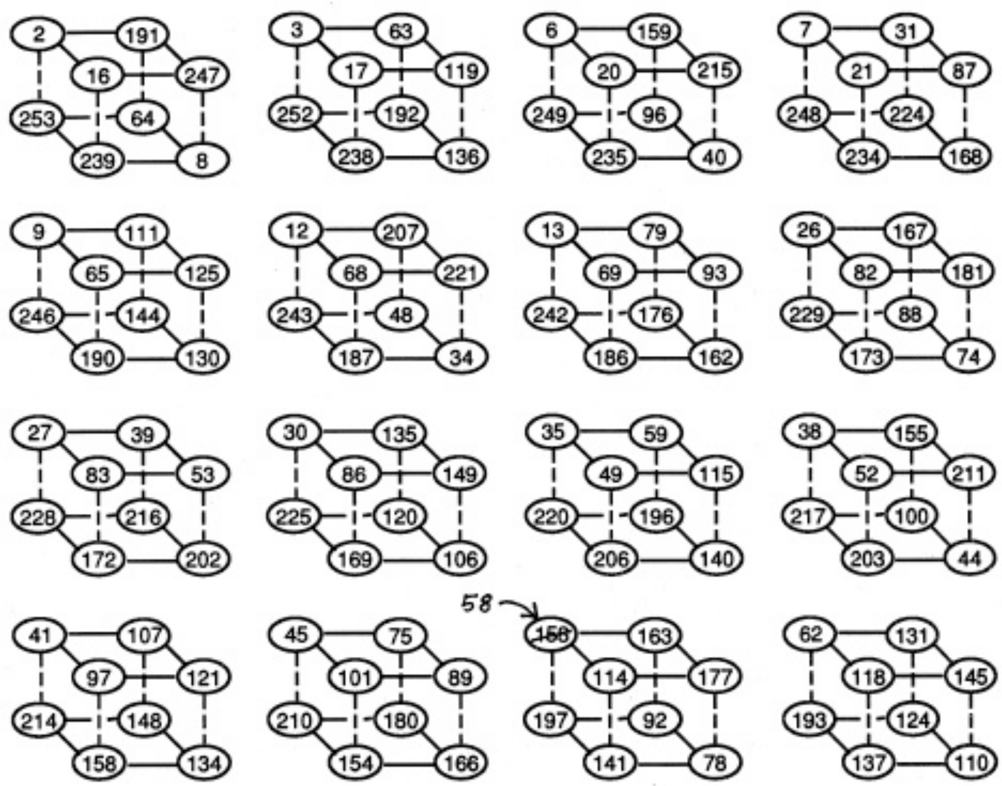
The cluster will collapse further, if for a given rule R, $R_c = R_n$,



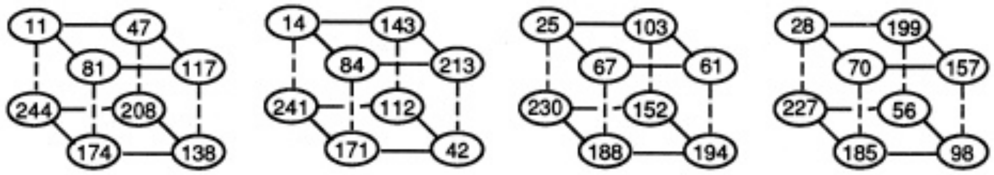
and also if $R = R_n$,



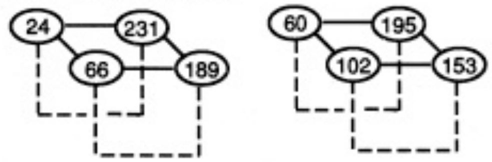
SEMI-ASYMMETRIC RULE CLUSTERS (either $T_6 \neq T_3$ or $T_4 \neq T_1$). There are no collapsed clusters among the semi-asymmetric rules.



FULLY ASYMMETRIC RULE CLUSTERS ($T_6 \neq T_3$ and $T_4 \neq T_1$).



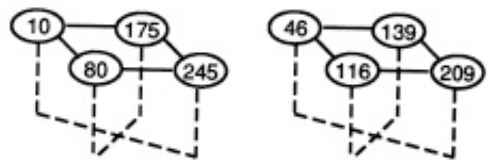
The cluster will collapse if, for a given rule R, $R_c = R_n$,



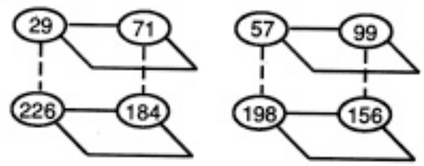
(continued)

Fully asymmetric rule clusters (continued):

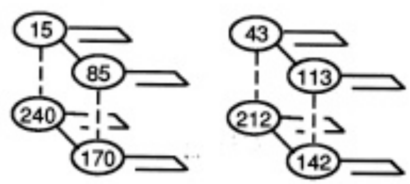
And if, for a given rule R, $R_c = R_{nr}$,



And if, for a given rule R, $R_n = R_r$,



And also if $R = R_n$,



88 Equivalence Classes

To summarise, the 256 rules collapse into 88 equivalence classes[20,28,42] and 48 rule clusters in Table 3.1.

TABLE 3.1

	rules	equiv. classes	clusters
Symmetric rules	64	36	20
Semi-asymmetric rules	128	32	16
Fully asymmetric rules	64	20	12
Total	256	88	48

The 256 rules have been tabulated on a 16 × 16 "rule-space matrix" (Appendix 4). Manipulations of the matrix simulate the clustering transformations.

Rule-Space Matrix, 256 Rules

The 256 $n = 3$ rules (0 to 255) may be set out on a 16×16 matrix. Rows i and columns j are numbered 0 to 15 as shown on the next page, and equivalently by the 4-bit binary numbers 0000 to 1111. Each entry in the matrix, a_{ij}, is a function of its position, and is assigned the decimal equivalent of the 8-bit binary number formed by the concatenation (denoted by the symbol +) of its 4-bit row and column binary expressions. If bin\$ (x) = the 4-bit binary string equivalent of x, and dec(x\$) = the decimal equivalent of the binary string x\$, then

$$a_{ij} = \mathrm{dec}(\mathrm{bin\$}(i) + \mathrm{bin\$}(j));$$

for example, to establish the entry $a_{5,6}$

$$\mathrm{bin\$}(5) = \text{"0101"}, \text{ and } \mathrm{bin\$}(6) = \text{"0110"}$$
$$\mathrm{bin\$}(5) + \mathrm{bin\$}(6) = \text{"0101"} + \text{"0110"} = \text{"01010110"}, \ \mathrm{dec}(\text{"01010110"}) = 86$$

Conversely, given a rule number, 0 to 255, its position on the matrix is found by the separating its 8-bit binary expression into two equal parts. The left 4 bits denotes the row i and the right 4 bits the column j. The resulting matrix is set out on the next page.

i \ j	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
0	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	21
2	32	33	34	35	36	37	38	39	40	41	42	43	44	45	46	47
3	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63
4	64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79
5	80	81	82	83	84	85	86	87	88	89	90	91	92	93	94	95
6	96	97	98	99	100	101	102	103	104	105	106	107	108	109	110	111
7	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	127
8	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143
9	144	145	146	147	148	149	150	151	152	153	154	155	156	157	158	159
10	160	161	162	163	164	165	166	167	168	169	170	171	172	173	174	175
11	176	177	178	179	180	181	182	183	184	185	186	187	188	189	190	191
12	192	193	194	195	196	197	198	199	200	201	202	203	204	205	206	207
13	208	209	210	211	212	213	214	215	216	217	218	219	220	221	222	223
14	224	225	226	227	228	229	230	231	232	233	234	235	236	237	238	239
15	240	241	242	243	244	245	246	247	248	249	250	251	252	253	254	255

The position of rules on the matrix depends on the rule numbering convention implicit in the sequence of the rule table entries. The conventional sequence as described in section 3.3 is as follows:

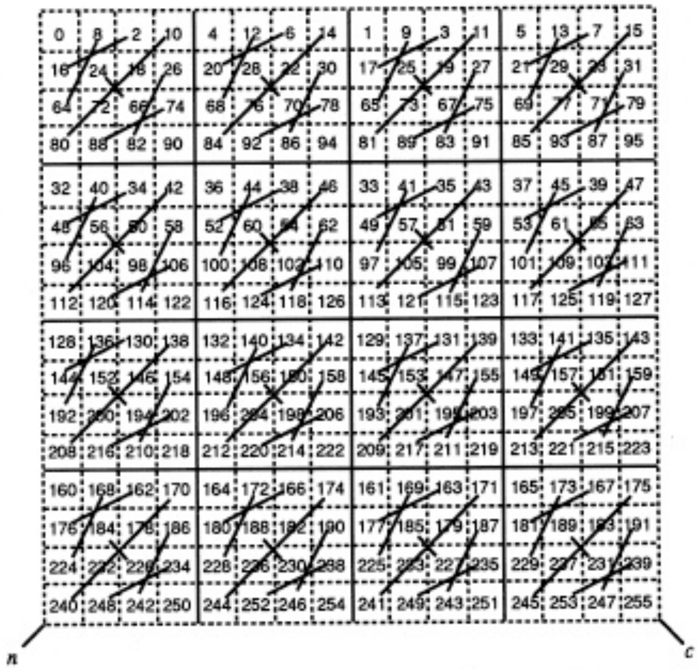
	111	110	101	100	011	010	001	000	neighbourhoods
Rule table..	T_7	T_6	T_5	T_4	T_3	T_2	T_1	T_0	outputs

Thus $T_7T_6T_5T_4T_3T_2T_1T_0$ is the conventional binary expression of the rule. Other sequences of the rule table would be equally valid; indeed, there are $8! = 13440$ permutations, and thus the same number of possible alternative numbering conventions.

If the equivalence relationships are indicated by drawing lines between equivalent rules on the matrix, a systematic pattern is apparent, however the clarity of this pattern varies for different numbering conventions. A limited search of alternatives has turned up a permutation that results in a pattern of exceptional clarity; this is the rearranged sequence:

	111	101	110	100	000	010	001	011	neighbourhoods
Rule table..	T_7	T_5	T_6	T_4	T_0	T_2	T_1	T_3	outputs

giving the alternative binary expression of the rule $T_7T_5T_6T_4T_0T_2T_1T_3$. In the matrix shown on the next page, rules are *positioned* according to the alternative numbering system, but are still numbered according to the conventional system. The diagonals, labeled c (complement) and n (negative), are indicated.



The matrix shows reflection equivalent rules linked by lines, and provides a graphic demonstration of rule categories and relationships as follows:

1. Simulates all the rule cluster transformations.
2. Distinguishes between symmetric, semi-asymmetric, and fully asymmetric rules.
3. Identifies special status rules resulting in collapsed clusters.
4. Accounts for the numbers of rules in equivalence classes and symmetry categories.

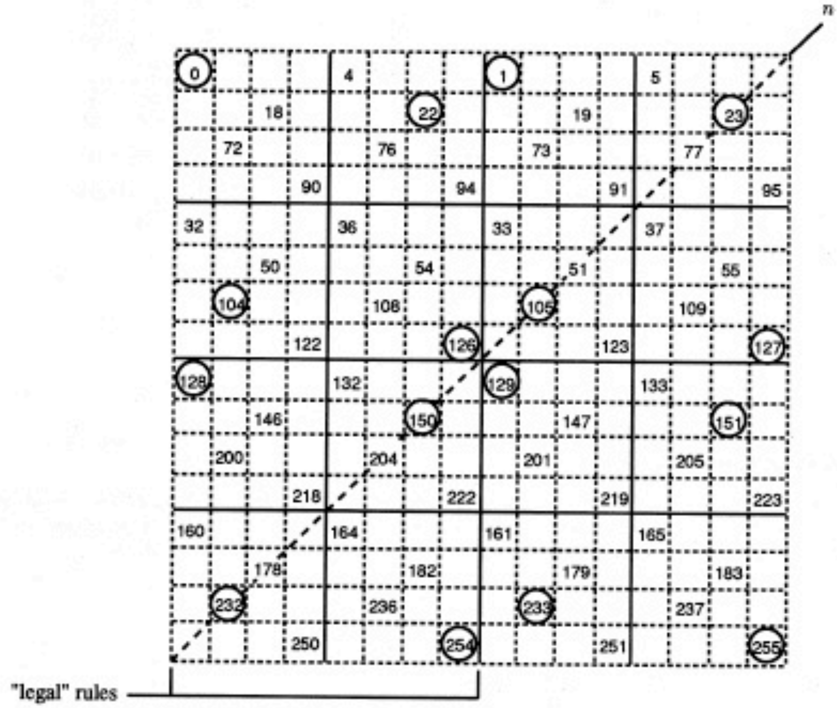
Rules that lie on the c diagonal have the property, for a given rule R, $R_c = R_n$. Fully asymmetric rules whose reflection link is bisected by the c diagonal, have the property, for a given rule R, $R_c = R_{nr}$. As stated earlier, R and R_n will be superimposed if the matrix is folded across the n diagonal; rules that lie on this diagonal (on the fold) have the property, $R = R_n$. Fully asymmetric rules whose reflection link is bisected by the n diagonal have the property that, for a given rule R, $R_n = R_r$.

Semi-asymmetric rules are unrelated to the diagonals in the ways described above, and therefore have no collapsed clusters.

A4.5 Symmetric Rules

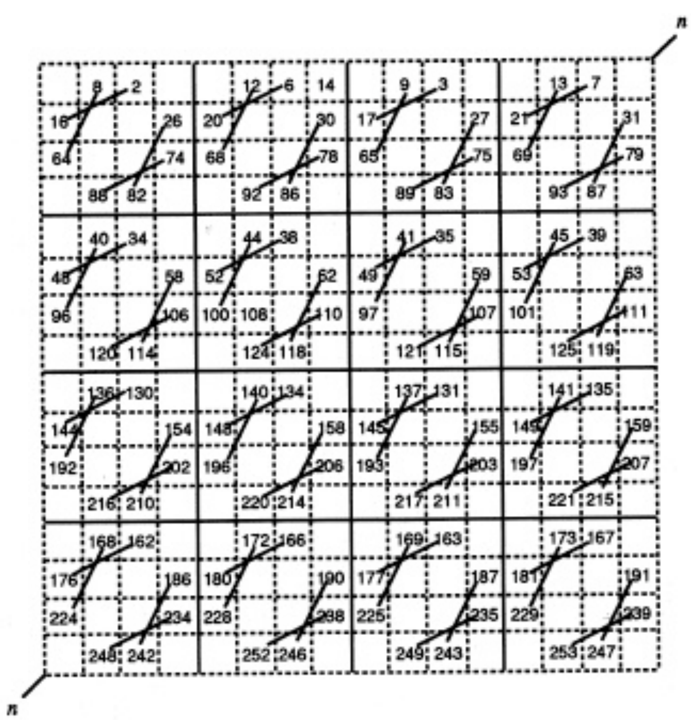
The equivalence classes among the symmetric rules (total 36) consist of the rules which are superimposed when the matrix is folded across the n diagonal.

The 32 rules that have sometimes been designated as "legal"[33] (both symmetric and even) are located in the left half of the matrix. The 16 totalistic rules among the $n = 3$ rules are circled.



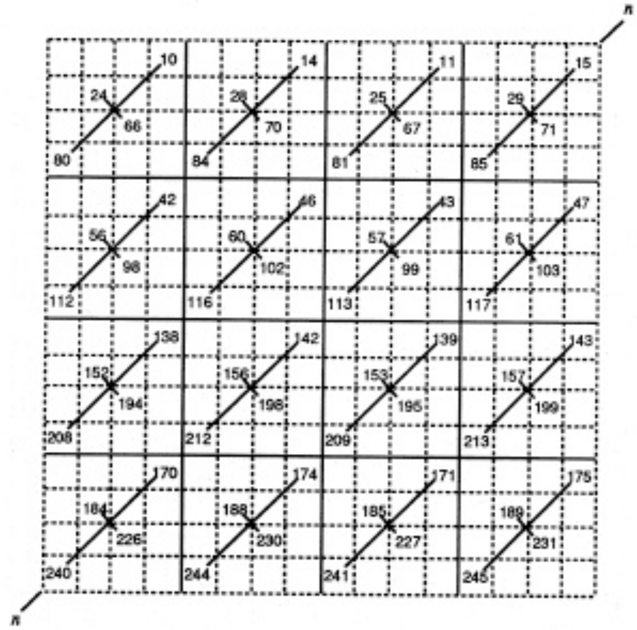
A4.6 Semi-Asymmetric Rules

The equivalence classes among the fully asymmetric rules (total 32) consist of the linked rules which are superimposed when the matrix is folded across the n diagonal.



A4.7 Fully Asymmetric Rules

The equivalence classes among the fully asymmetric rules (total 20) consist of the linked rules which are superimposed when the matrix is folded across the *n* diagonal.



Wolfram CA Rule Classes

4.1 Basin Field Topology and Rule Space

In this section, some implications of the emerging basin field landscape on the current perception of the structure of rule space are examined. Wolfram has proposed that all CA rules belong to one of four universality classes.[34,36,39] These classes are essentially phenomenological,[8] based on the characteristic appearance of typical space-time patterns. Wolfram's classes, and their analogues in continuous dynamical systems, are said to exhibit the behaviour shown below.

CA behaviour	*dynamical systems analogue*
class 1 evolves to a fixed, homogeneous state	limit points
class 2 evolves to separated periodic regions	limit cycles
class 3 evolves to *chaotic*, aperiodic patterns	strange attractors
class 4 evolves to *complex*, localised structures.........	long transients, no analogue

It is accepted that many rules show "intermediate" behaviour,[36] and there is also scope for defining subclasses,[19,20] but the discussion below uses a loose definition of the four classes listed. Of these classes, it is conjectured that the supposedly rare[16] *complex rules* (class 4) may in some cases be capable of supporting *universal computation*.[17,34] The space-time patterns of the complex rules contain interacting, static, and propagating structures, sometimes called *information structures*, similar to those illustrated in Figs. 2.2 and 4.1.

Langton proposed that class 4 rules are located at a *phase transition* in rule space. He suggested that the long transients typical of these rules have potential for information processing, and implications for understanding the origin and evolution of life.[17]

4.1.1 The λ Parameter

Rule space has been characterised by the λ parameter,[16,17] the proportion of *non-zero* entries in the rule table. If the value range is k, with possible values $0, 1, 2, \ldots, k-1$, then a particular value, say 0, is selected as the *quiescent* value. The λ parameter is the proportion of rule table entries other than 0.

Langton and others[17,20,41] have shown that the rule classes can be roughly selected by adjusting λ between 0 and $1-(1/k)$. A rule table may be constructed by assigning one of the k values to each entry with equal probability, so that the density of all values, including the quiescent value, will be roughly equal, and $\lambda \simeq 1-(1/k)$. At this value of λ, space-time patterns are most likely to appear chaotic.

Rules may be selected according to other values of λ. This is typically done by assigning the quiescent value with a selected probability x, so that $\lambda = 1 - x$. The remaining values are then assigned with equal probability $\lambda/(k-1)$.

For binary rules where $k = 2$, λ is simply the density of 1s in the rule table. Maximum chaos in the appearance of space-time patterns is likely to occur at $\lambda = 1/2$, a rule table with an equal number of 0s and 1s.

As the λ parameter is varied from 0 to 1/2, the various classes of behaviour are traversed. Complex, class 4 behaviour occurs at a *phase transition* between periodic, class 2 and chaotic, class 3 behaviour, reordering Wolfram's classes as follows[16,17]:

$$
\begin{array}{rccccccccc}
 & & & & & \text{complex} & & & \\
 & & & & & \text{rules} & & & \\
 & & & & & | & & & \\
\text{class}: & 1 & \longrightarrow & 2 & \longrightarrow & \mathbf{4} & \longrightarrow & & 3 \\
\text{in general, } \lambda: & 0 & \ldots & \ldots & \ldots & \ldots & & \ldots & 1-(1/k) \\
\text{binary, } k=2,\ \lambda: & 0 & \ldots & \ldots & \ldots & \ldots & & \ldots & 1/2
\end{array}
$$

As λ increases from 1/2 to 1 the sequence is reversed, so that the complex rules occur in two limited regions on either side of $\lambda = 1/2$.

4.1.2 Convergence of State Space

It is suggested that the λ parameter is modulating the same aspect of CA global behaviour as the Z parameter (introduced in chapter 3, section 3.6)—the *maximum pre-imaging* (mp) as a function of array length. In general this will reflect the *degree of pre-imaging* typical of a rule. The degree of pre-imaging is apparent in the topology of a rule's basin of attraction field, and reflects the *convergence of state space*,[14] which may be equivalently measured as the density of garden-of-Eden nodes in state space (or in one basin).

Low pre-imaging implies low convergence and a low density of garden-of-Eden nodes; high pre-imaging implies high convergence and high density of garden-of-Eden nodes. The degree of pre-imaging and the density of garden-of-Eden nodes are available in the Atlas (or program).

4.1.3 The Z Parameter and the λ Parameter

The Z parameter is the probability that the *next cell* of a partial pre-image has a unique value (see chapter 3, section 3.6). For binary rules, when $Z = 1$, the quantity of 0s and 1s in the rule table is equal, $\lambda = 1/2$; when $Z = 0$, $\lambda = 0$. As Z varies between 0 and 1, λ varies between 0 and 1/2 or conversely between 1 and 1/2.

The Z parameter may possibly be the mechanism underlying the operation of the λ parameter, because the probability of a rule table having a given value of Z depends on the proportions of 0s and 1s assigned at random to the rule table according to the setting of λ . Thus λ seems to be a measure of the *probability* of a particular value of Z. The Z parameter is concerned not only with the numbers of 0s and 1s in the rule table, but also their position and thus might be expected to modulate behaviour more closely.

For a more direct comparison between the values of λ and Z for a given binary rule table, the λ parameter may be modified as follows. The quiescent state is taken as the majority value in the rule table; the minority value is taken as the active state. The modified λ parameter, referred to as the λ *ratio*, is the ratio of the active values to 1/2 of the rule table (the potential maximum of active values). For example, in an $n = 5$ rule table with 32 entries, say 10 entries in the rule table are 0, then the λ ratio equals 10/16.

$$
\begin{aligned}
&\text{If } \lambda \leq 1/2, \text{ then } \lambda \text{ ratio } = 2\lambda. \\
&\text{If } \lambda > 1/2, \text{ then } \lambda \text{ ratio } = 2(1-\lambda).
\end{aligned}
$$

The value of the λ ratio varies between 0 and 1 roughly in line with, but never smaller than, the Z parameter. $Z \leq \lambda$ ratio, and $Z = \lambda$ ratio $= 1$ only for limited pre-image rules.

As an example of the λ ratio and the Z parameter, consider the space-time patterns with complex structures that were illustrated in Figs. 2.2(a–c). The relevant rules have the following values for the λ ratio and Z:

a. rule 3112581872 λ ratio $= 1$, $Z = .671875$
b. rule 2334561936 λ ratio $= .8125$, $Z = .6875$
c. rule 3583552890 λ ratio $= .875$, $Z = .75$

There are many examples of rules where the λ ratio equals 1 or close to 1, suggesting a chaotic space-time pattern, when in fact the pattern appears complex. In such cases the value of Z will typically be between .6 and .8.

	HIGH convergence and garden of Eden density		*phase transition* mp and (mc, ml, nb) *in balance*		*LOW* convergence and garden of Eden density
class	1	→ 2 →	4	→	3
Z :	0	… … …	…	…	1
λ ratio :	0	… … …	…	…	1
λ :	0	… … …	…	…	$1-(1/k)$
mp :	diverges exponentially with L	… … …	…	…	fixed upper limit
(mc, ml, nb) :	fixed lower limit	… … …	…	…	diverges exponentially with L

FIGURE 4.2 Relationship between the Z parameter (and λ) and the change in basin of attraction field topology as L increases, reflecting convergence of state space.

4.1.5 Examples of Typical Basin Topology in Relation to Rule Class

To illustrate the relationships in Fig. 4.2, an example of a typical basin of attraction for $n = 3$ rules characteristic of each rule class is given in Figs. 4.3–4.6, including the following data: mp, maximum pre-imaging; *g-density*, density of garden-of-Eden nodes; *period*, of attractor cycle; and ml, maximum transient length.

In these examples, rotation equivalent transients are suppressed for the sake of clarity. General examples of the complete basin field over a range of array length L are presented in the Atlas.

CLASS 1. Rules where $Z = .25$ will have a large proportion of ambiguous permutations in the rule table, so mp will diverge exponentially with L; mc, ml, and nb will either remain fixed or relate arithmetically to L. Most states are locked into pre-image fans flowing into point attractors, or two-state attractors, especially the *uniform* states (all 0s or 1s). The few states outside the influence of these attractors will only have enough scope to form basins with short cycles and transients. (See Fig. 4.3.)

CLASS 2. Rules where $Z = .5$ may have many basins separated from the uniform attractor. mp will diverge exponentially with L, but to a lesser extent than for class 1. mc, ml, and nb will relate arithmetically to L. Again many states are locked up in pre-image fans leaving enough scope for only short cycles or transients. (See Fig. 4.4.)

CLASS 4. The complex, class 4 rules occur typically at $Z = .75$. mp and some combination of mc, ml, and nb will be finely balanced and will diverge by some intermediate function with L. Controlled pre-imaging allows enough scope for moderately long cycles and transients. (See Figure 4.5.)

CLASS 3. The chaotic, class 3 rules are limited pre-image rules where $Z = 1$. mp is fixed irrespective of L. Some combination of mc, ml, and nb diverge exponentially with increasing L. For instance, clusters 30 and 45, where $mp = 3$, have the greatest values of mc and ml among the $n = 3$ rules. (See Fig. 4.6.)

In general, the space-time patterns of the $n = 3$ and $n = 5$ rules have been found to correlate with the Z parameter; however, as with the λ parameter, there are exceptions. It has been noted that the transition to chaotic behaviour may occur at different λ values, although there is a well-defined distribution around a mean value.[17,19] Chaotic space-time patterns may also occur at low values of Z. For example, the $n = 3$ rules 18 and 126 have a value of $Z = .5$. Although the typical basin topology of these rules correlates with Z, their space-time patterns appear chaotic, but are confined to only a subset of possible configurations of neighbourhoods. This is clearly seen when colors are assigned to cells in space-time patterns according to the neighbourhood which determined the cell's value, implemented in the program Space1 following a suggestion by Warrell.[43] The process whereby active neighbourhoods are eliminated requires further investigation.

FIGURE 4.3 Class 1, rule 251. $L = 12$, seed 111111111111. $mp = 853$, *g-density* = .92, period = 1, $ml = 6$. mp diverges exponentially with L; $Z = .25$; (λ ratio = .25). Equivalent transients above level 1 are suppressed.

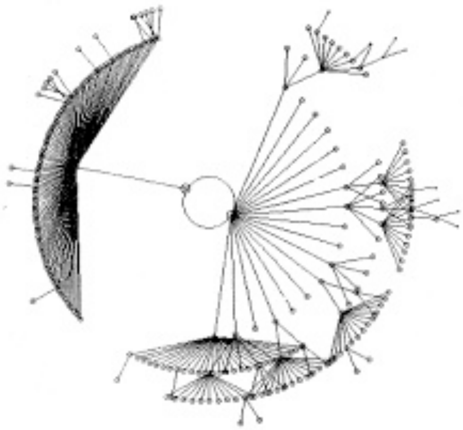


FIGURE 4.4 Class 2, rule 33. $L = 16$, seed 0111111111100000. $mp = 97$, *g-density* = .84, period = 2, $ml = 5$. mp diverges exponentially with L; $Z = .5$; (λ ratio = .5).

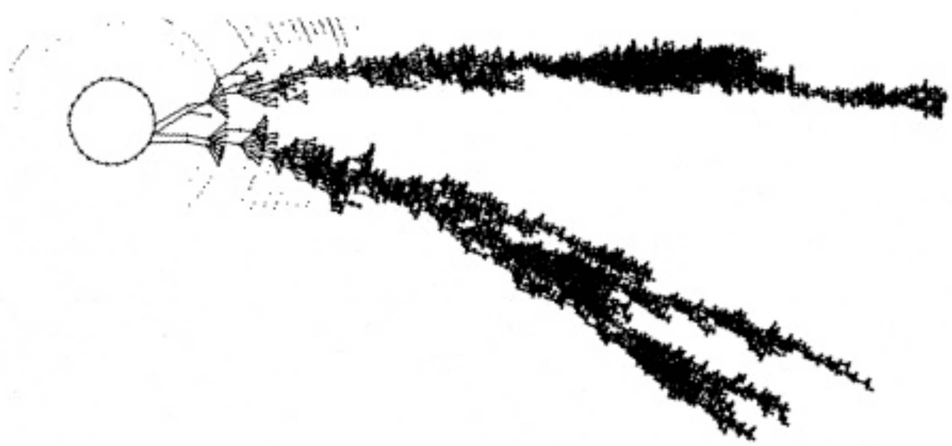


FIGURE 4.5 Class 4, rule 193. $L = 18$, seed 011010001110000010. $mp = 70$, *g-density* = .61, period = 27, $ml = 120$. mp, mc, and ml diverge with L; $Z = .75$; (λ ratio = .75). Equivalent transients are suppressed.

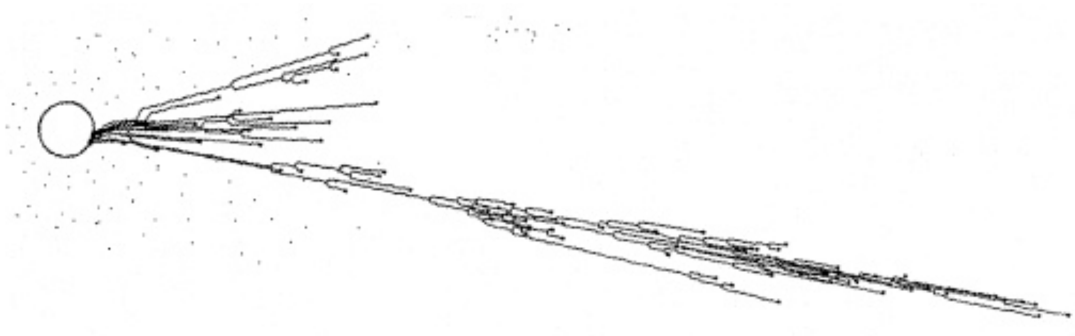


FIGURE 4.6 Class 3, rule 30. L = 15, seed 110110111000000. mp = 2, g-$density$ = .04, period = 1455, ml = 321. mp and ml diverge exponentially with L; Z = 1; (λ ratio = 1). Equivalent transients and pre-image nodes are suppressed; angle between pre-image arcs is increased.

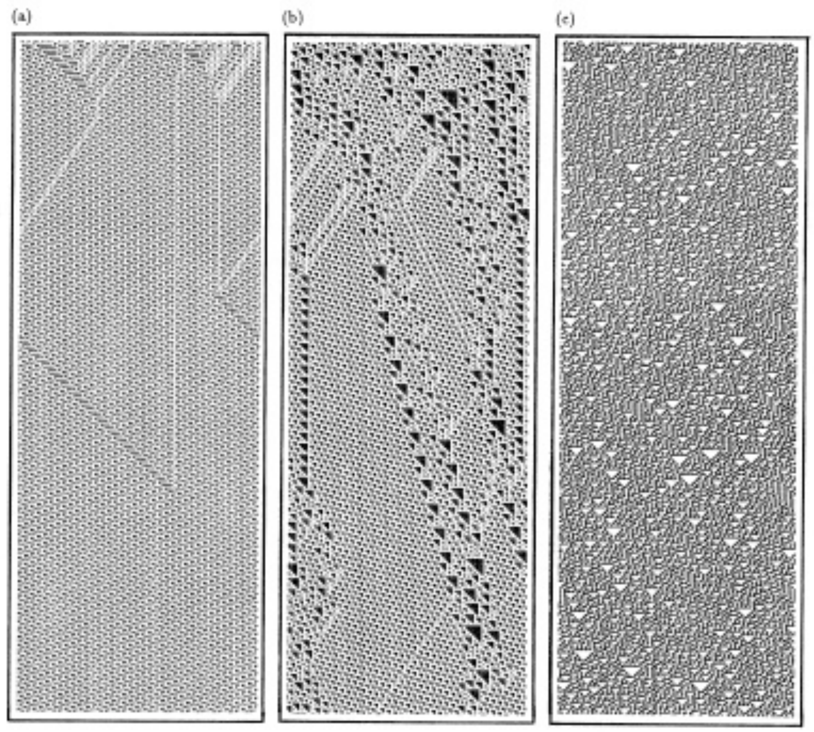


FIGURE 4.7 Space-time patterns for n = 3 rules. Array size is 150, 420 time steps from a random initial state. Rule numbers and Z parameter are: (a) Class 2: rule ~~25~~ 9, Z = .5; (b) Class 4: rule 193, Z = .75; (c) Class 3: rule 30, Z = 1.

Parameters for 48 n=3 Clusters

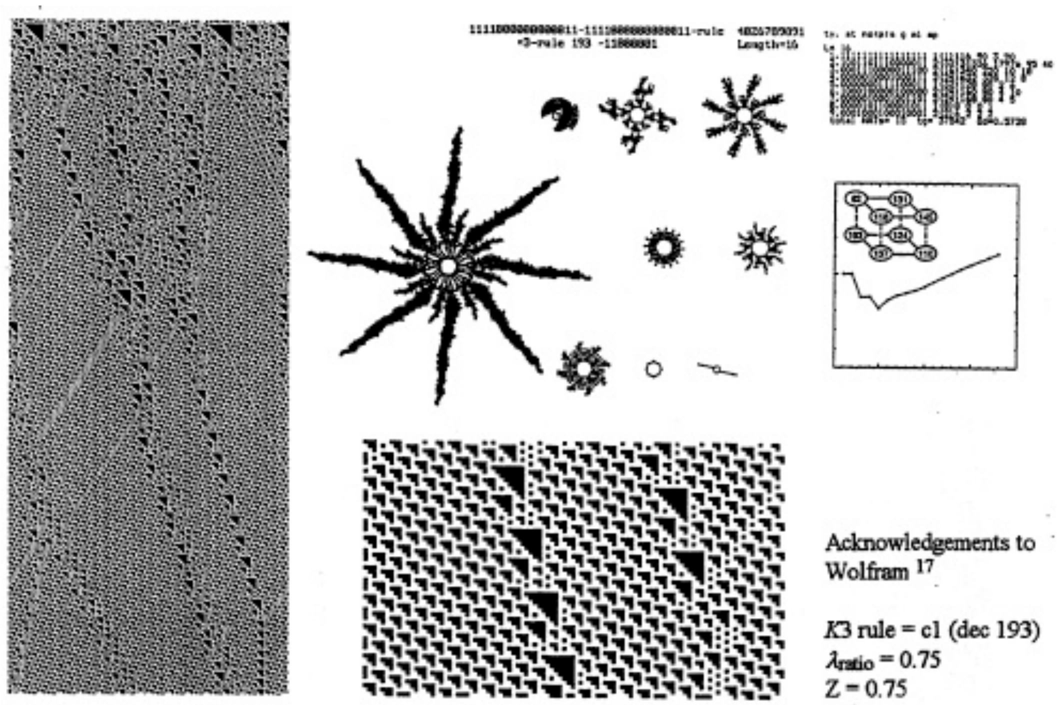
TABLE 3.3 $n = 3$ rules

	rule number	λ parameter	λ ratio	Z parameter
symmetric rules	0	0/8	0	0
	1	1/8	0.25	0.25
	4	1/8	0.25	0.25
	5	2/8	0.5	0.5
	18	2/8	0.5	0.5
	19	3/8	0.75	0.625
	22	3/8	0.75	0.75
	23	4/8	1	0.5
	33	2/8	0.5	0.5
	36	2/8	0.5	0.5
	37	3/8	0.75	0.75
	50	3/8	0.75	0.625
	51	4/8	1	1
	54	4/8	1	0.75
	73	3/8	0.75	0.75
	77	4/8	1	0.5
	90	4/8	1	1
	94	5/8	0.75	0.75
	105	4/8	1	1
	126	6/8	0.5	0.5
semi-asymmetric rules	2	1/8	0.25	0.25
	3	2/8	0.5	0.5
	6	2/8	0.5	0.5
	7	3/8	0.75	0.75
	9	2/8	0.5	0.5
	12	2/8	0.5	0.5
	13	3/8	0.75	0.75
	26	3/8	0.75	0.75
	27	4/8	1	0.75
	30	4/8	1	1
	35	3/8	0.75	0.625
	38	3/8	0.75	0.75
	41	3/8	0.75	0.75
	45	4/8	1	1
	58	4/8	1	0.75
	62	5/8	0.75	0.75
fully asymmetric rules	10	2/8	0.5	0.5
	11	3/8	0.75	0.75
	14	3/8	0.75	0.75
	15	4/8	1	1
	24	2/8	0.5	0.5
	25	3/8	0.75	0.75
	28	3/8	0.75	0.75
	29	4/8	1	0.5
	43	4/8	1	0.5
	46	4/8	1	0.5
	57	4/8	1	0.75
	60	4/8	1	1

In his 1996-97 D.Phil. Thesis at the University of Sussex, Andrew Wuensche said:

"... Glider Dynamics ... corresponds to Wolfram's class 4 behaviour ... , to notions of emergent computation on the edge of chaos, and to a phase transition between order and chaos ... Gliders are analagous to autocatalytic sets of polymers in the sense of Kauffmann ... members of such sets have a survival advantage in occupying space, anad the set acquires its own identity as an observed object at a higher level. Gliders are also discrete examples of Prigogine's far-from-equilibrium dissipative structures ... in CA the process of formation, persistence and interaction of gliders and other dissipative structures can be traced at the lowest level of the system's basic components and their local interactions which are completely defined. This ability to see two levels of behaviour simultaneously, the underlying and emergent, may lead to insights into the mechanics of self-organization. ... Among the 256 binary 3-neighbor rules, the "elementary rules" ... , an exhaustive search reveals two sets of glider rules, rule 54 and 110, and their equivalents. ...

Appendix 2.4 [re rule 193] ...

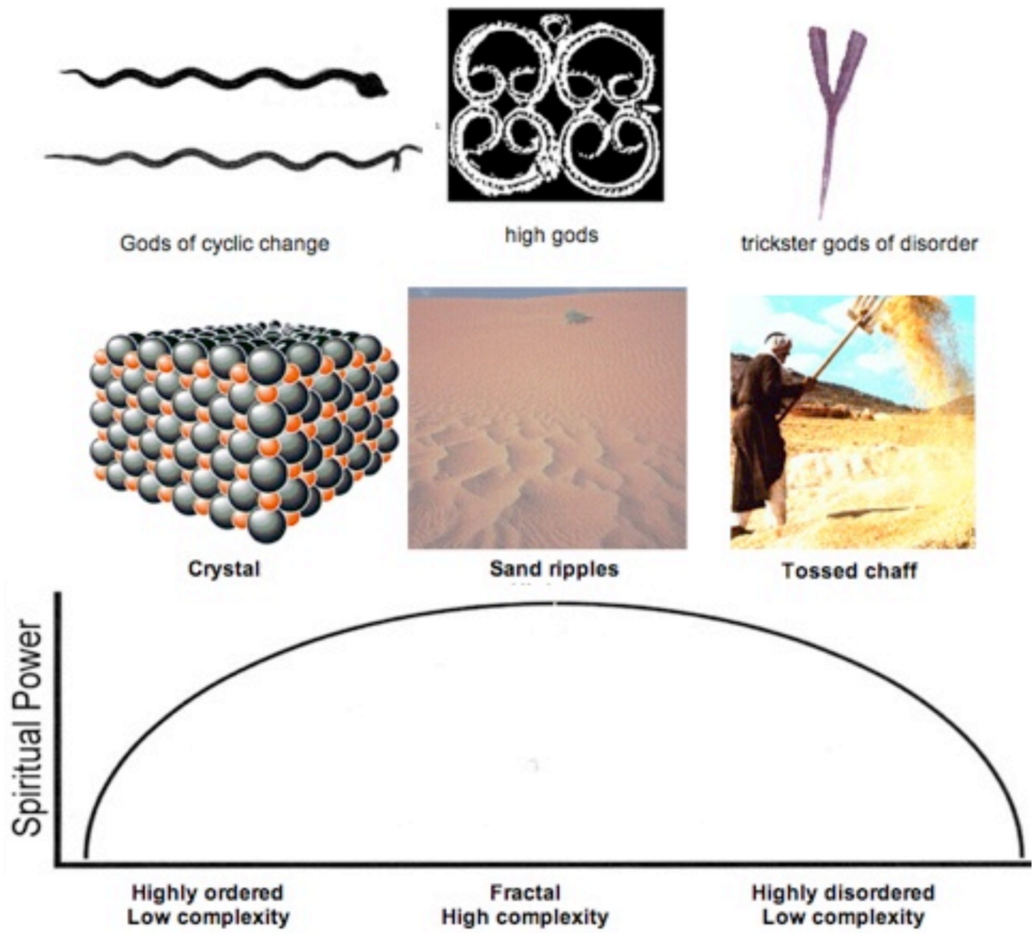


...[The]... spectrum of CA behaviour ... rang[es] progressively through ordered, complex and chaotic dynamics, corresponding to Wolfram's ... classes

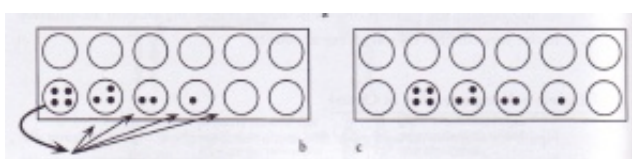
(1 and 2) [ordered] - (4) [complex, including 54 and 193] - (3) [chaotic]

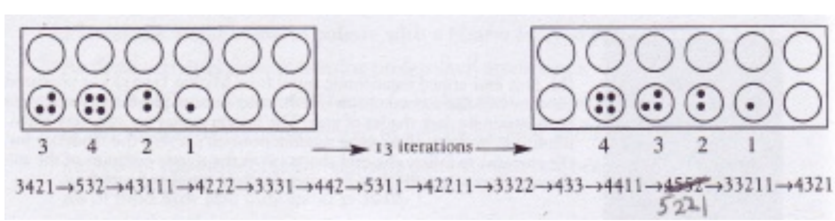
...".

Ron Eglash (in his book "African Fractals" (Rutgers 1999) and on his web site at www.csdt.rpi.edu) says: "...



... the owari marching-group system can be used as a one-diemnsional cellular automaton ...





... transients of many different lengths can be produced. ... the constant pattern is called a "point attractor", and the transients would be said to lie in the "basin of attraction".

The marching group rule can also produce periodic behavior (a "limit cycle" or "periodic attractor" ...). Here is a period-3 system using only four conters:

211 -> 22 -> 31 -> 211

Total number of counters	Behavior (after transients)
1	Marching
2	Period 2
3	Marching
4	Period 3
5	Period 3
6	Marching
7	Period 4
8	Period 4
9	Period 4
10	Marching
11	Period 5
12	Period 5
13	Period 5
14	Period 5
15	Marching

... The numbers which lead to marching groups - 1 , 3 , 6 , 10 , 15 ... - ...[are]... the triangular numbers ...

[the triangular numbers correspond to the dimension of the grade-2 bivectors in Clifford Algebras -

- for the case of the 2^8 = 256 Elementary CA Rules, there are 28 grade-2 CA Rules]

... One-dimensional versions can ... be used as a kind of parallel computer. Consider, for example, a rule that in each iteration the number of counters in a cup is replaced by the sum of itself and its left neighbor. Starting with one:

0100000 -> 0110000 -> 0121000 -> 0133100 -> 0146410

This fourth iteration gives the us the binomial coefficients for expansion of (a+b)^4 ,

which equals to a^4 + 4 a^3 b + 6 a^2 b^2 + 4 a b^3 + b^4 .

[Such a rule reproduces at each step succeeding rows of the Yang Hui triangle.]

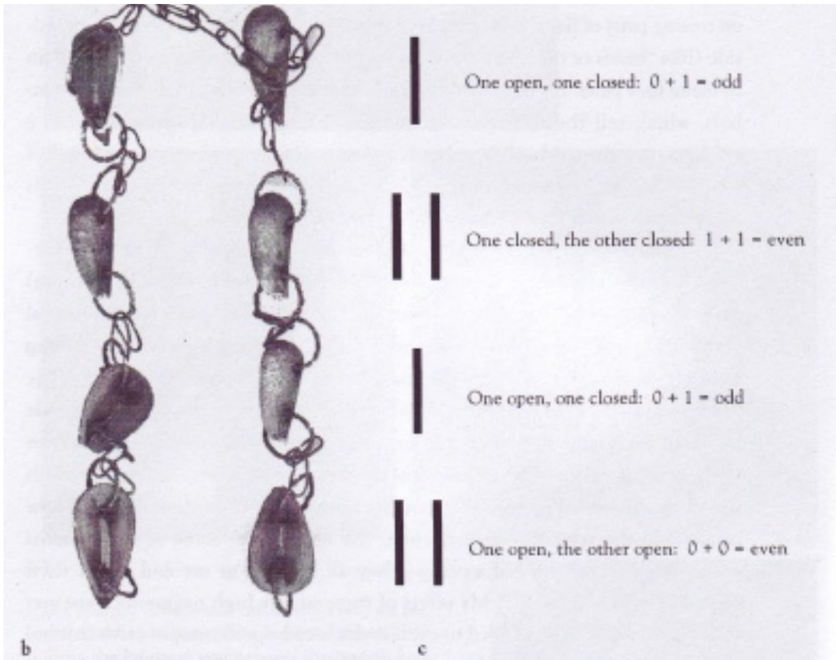


FIGURE 7.6

Binary codes in divination

(a) This Nigerian priest is telling the future by Ifa divination, in which pairs of flat shells or seeds split in two are tossed with each landing open-side or closed-side. They are connected by a doubled chain to make four pairs, giving a total of 16 divination symbols. In this version of Ifa (used in the Abigba region of Nigeria) they use two doubled chains and consider the cast more accurate if there is a correlation between the two sets. **(b)** Here we see a chain using split seeds. Each half lands either "closed" (meaning we see the rounded outside) or "open" (meaning we see the interior). By using open to represent 0 (double lines), and closed to represent 1 (single line), we can see how the divination symbol is obtained. **(c)** The divination chain is interpreted as pairs summing to odd (one stroke) or even (two strokes).

*(**a**, photo by E. M. McClelland, courtesy Royal Anthropological Institute.)*

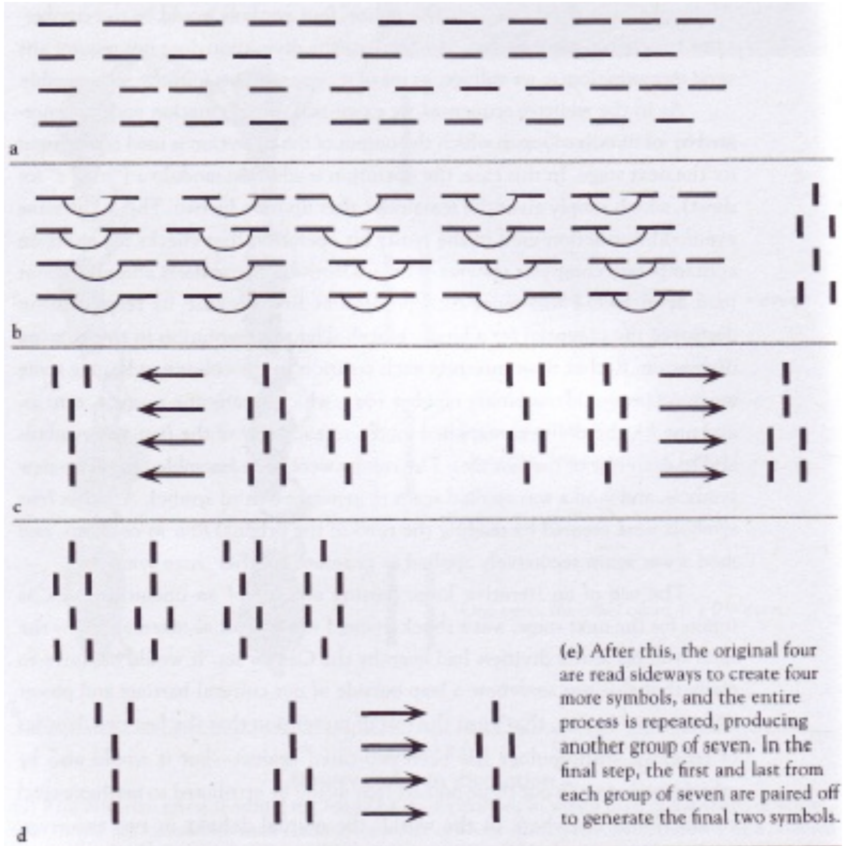


FIGURE 7-7
Bamana sand divination
(a) Four sets of random dashes are drawn. (b) Each of the dashes is paired, and the odd/even results are recorded. (c) The process is repeated four times, resulting in four symbols. Each row of the first two symbols and the last two symbols are paired off to generate two new symbols. (d) The two newly generated symbols, now placed below the original four, are again paired off to generate a seventh symbol.

... If we think of the two-strokes as zero and single stroke as one, the Bamana divination system is almost identical to the process of pseudorandom number generation used by digital circuits called "shift registers". Here the circuit takes mod 2 of the last two bits in the register and places the result in the first position. The other bits are shifted to the right, with the last discarded. ...

```
1111
0111
0011
0001
1000
0100
0010
1001
1100
0110
1011
0101
1010
1101
1110
```

... This four-bit register will only produce 15 binary words before the cycle starts over, but the period of the cycle increases with more bits ... For the entire 16 bits ... that begin the Bamana divination, 65,535 binary words can be produced before repeating the cycle. ...

Skinner ... "Terrestrial Astrology: Divination by Geomancy". London: Routledge and Kegan Paul, 1980 ... provides a well-documented history of the diffusion evidence ... for ... Arabic, European, West African, and East African ... "geomancy" ...divination technique. ...

[Such diffusion seems also to have extended

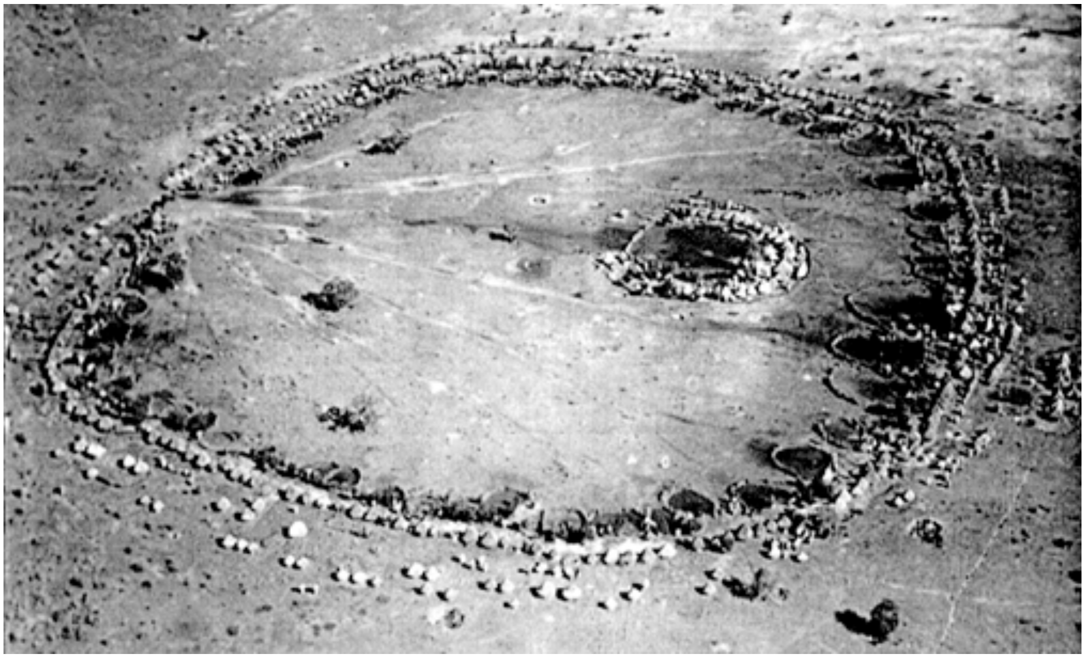
from Africa to India, China, and Japan.]

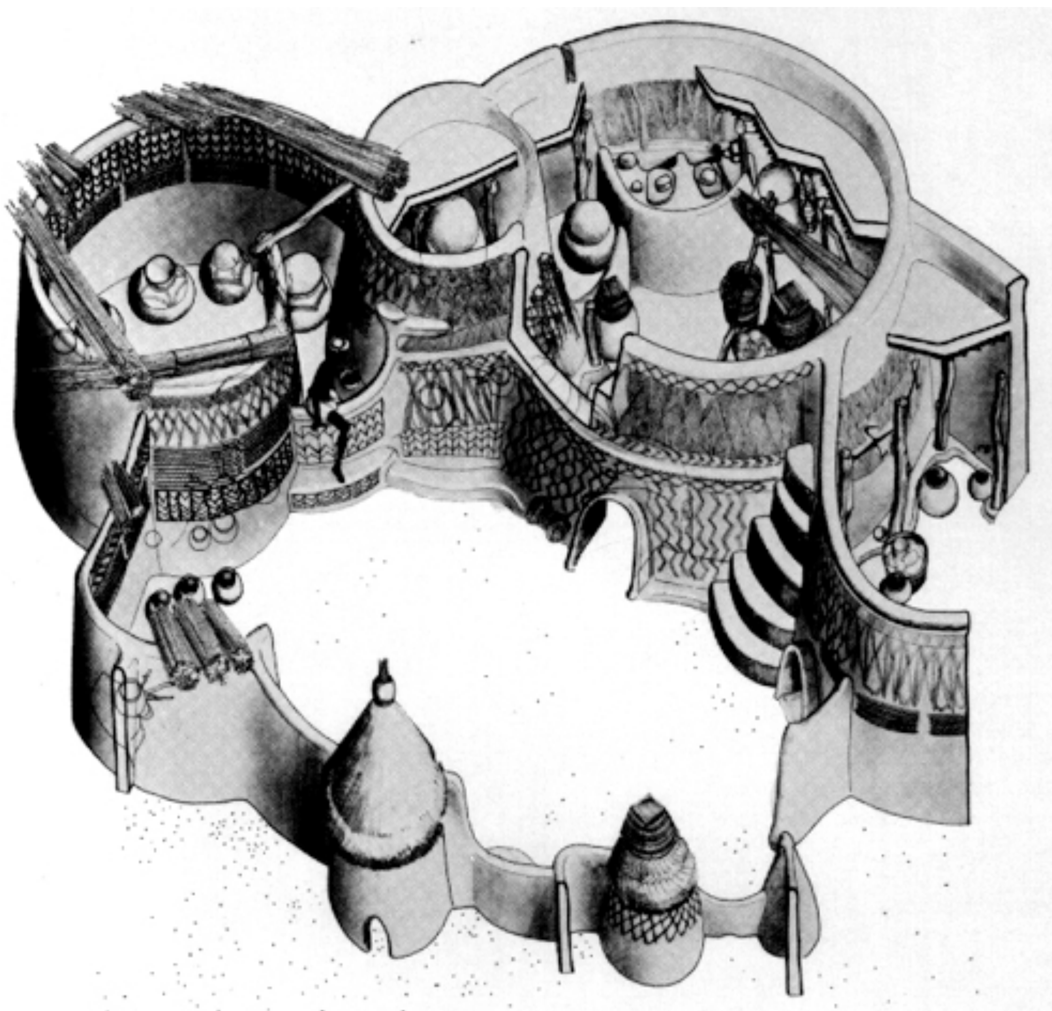
... The implications of this trajectory - from sub-Saharan Africa to North Africa to Europe - are quite significant for the history of mathematics.

... a historical path for base-2 calculation ... begins with African divination, runs through the geomancy of European alchemists, and is finally transformed into binary calculation, where it is now applied in every digital circuit ...

... Following the introduction of geomancy to Europe by Hugo of Santalla in twelfth-century Spain ... European geomancers ... Ramon Lull ... and others ... persistently replaced the deterministic aspects of the system with chance. By mounting the 16 figures on a wheel and spinning it, they maintained their society's exclusion of any connections between determinism and unpredictability. The Africans, on the other hand, seem to have emphasized such connections ...[with]]... a "trickster" god, one who is both deterministic and unpredictable. ...

The fractal settlement patterns





of Africa stand in sharp contrast to the Cartesian grids of Euro-American settlements. ... Euro-American cultures are organized by ... "top-down" organization. Precolonial African cultures included ... societies that are organized "bottom-up" rather than "top-down". ... African architecture tends to be fractal because that is a prominent design theme in African culture ... most of the indigenous African societies were neither utterly anarchic, nor frozen in static order; rather they utilized an adaptive flexibility ... African traditions of decentralized decision making could ... be combined with new information technologies, creating new forms that combine democratic rule with collective informaiton sharing ... what is needed is not ... "small is beautiful", but rather a self-organized approach to changes in the relations between scale and the socioenvironmental systems - not just appropriate technology, but appropriate scaling. ...

we are trapped between the periodic stasis of the preservationists' limit cycle,

and the white noise of the profiteering positive feedback loop.

... both are lacking in flexible interactions with memory;

the ... preservationists' ... limit cycle being too tied to it, and the ... profiteering ... white noise being too free from it. ...".

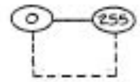
During the decade since Ron Eglash published his book in 1999, the USA/UK global financial system is collapsing due to grossly excessive profiteering.

Perhaps "African traditions of decentralized decision making" could produce a successor global finanancial system that will better serve the needs of human civilization.

Atlas of Basin of Attraction Fields

for 88 n=3 Equivalence Classes

0000000000000000-0000000000000000-rule
=3-rule 0 -00000000



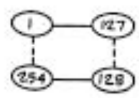
00000000 0
11111111 255

Length=8

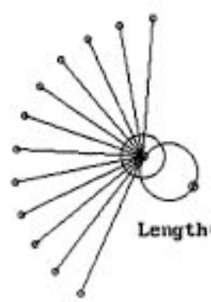
3-rule 0 =(
ty. at no(p

$Z = .25$

0000000000000011-0000000000000011-rule
=3-rule 1 -00000001



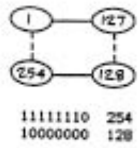
00000001 1
01111111 127

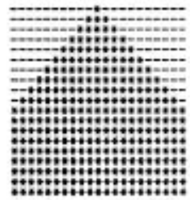


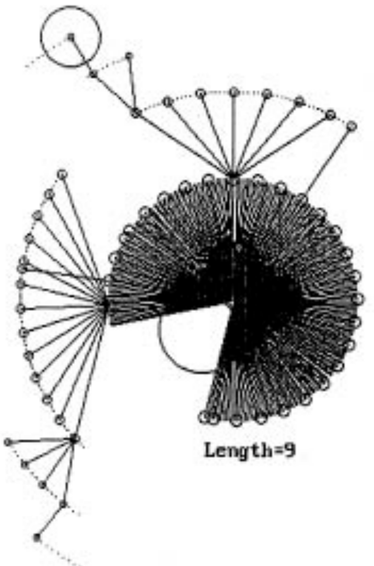
Length=9

3-rule 1 =000(
ty. at no(p)s

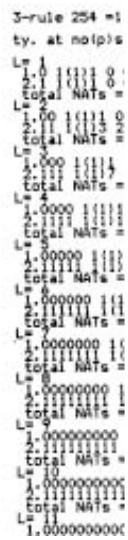
1111111111111100-1111111111111100-rule
=3-rule 254 -11111110





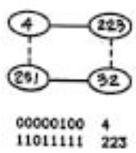


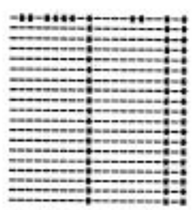
Length=9

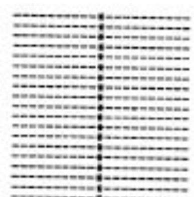


0000000000110000-0000000000110000-rule 3145776
=3-rule 4 -00000100
Length=1 -13

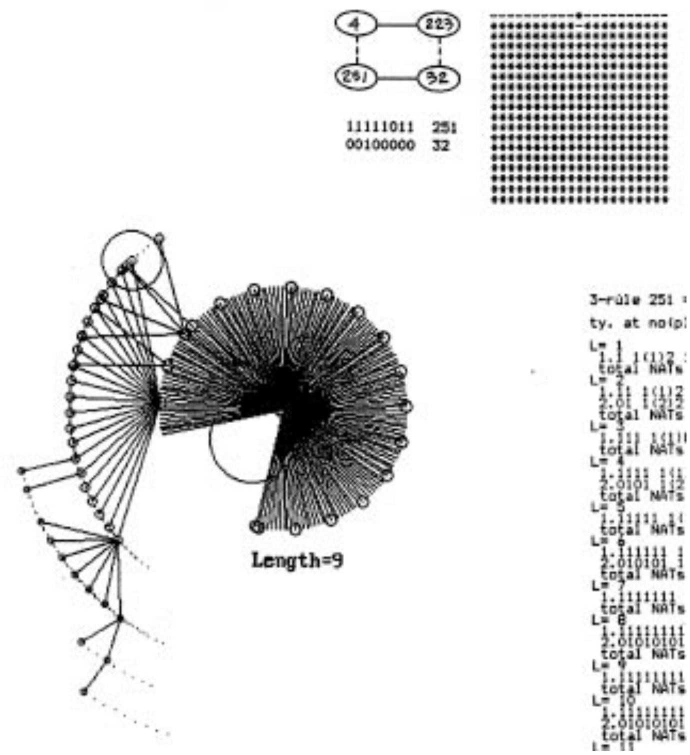
Length=9



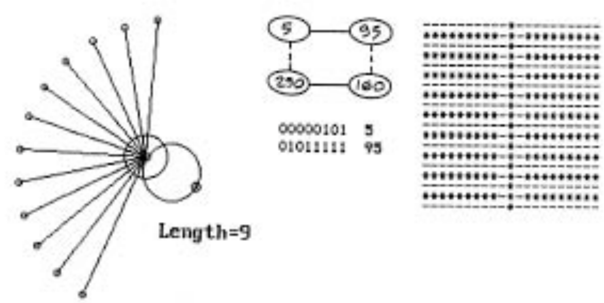




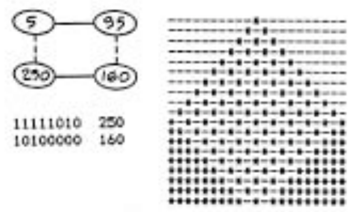
.25 1111111111001111-1111111111001111-rule
=3-rule 251 -11111011

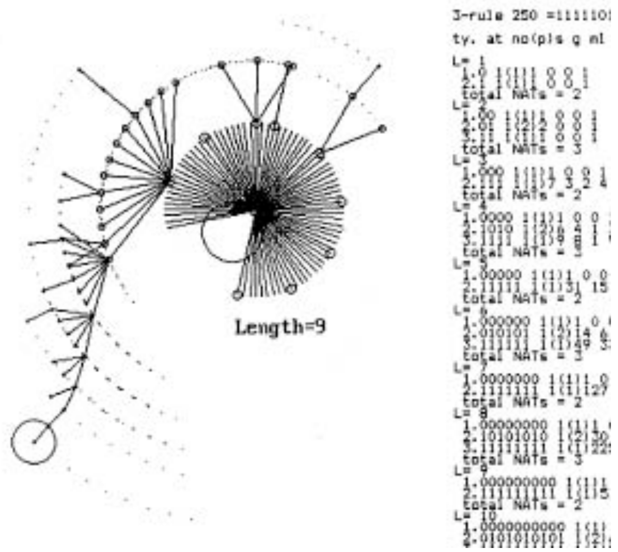


0000000000110011-0000000000110011-rule
=3-rule 5 -00000101

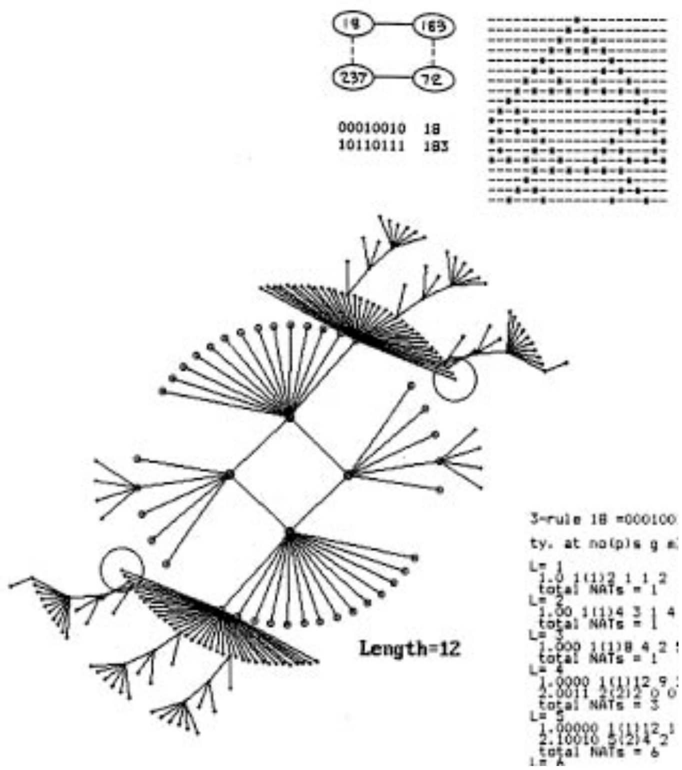


1111111111001100-1111111111001100-rule
=3-rule 250 -11111010

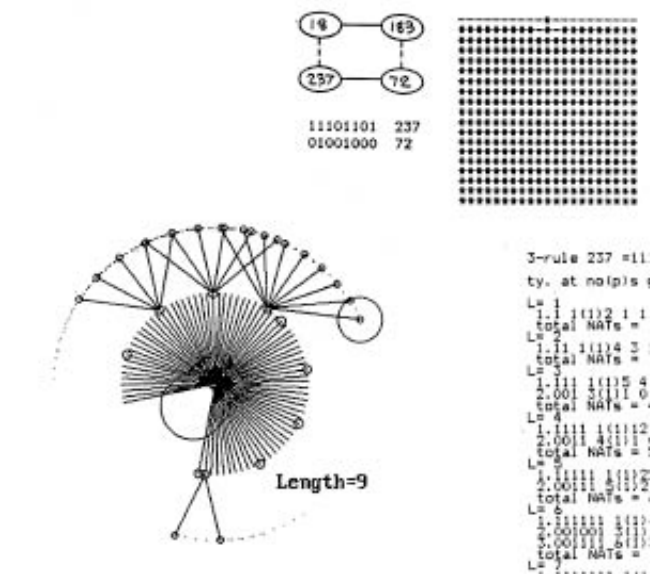




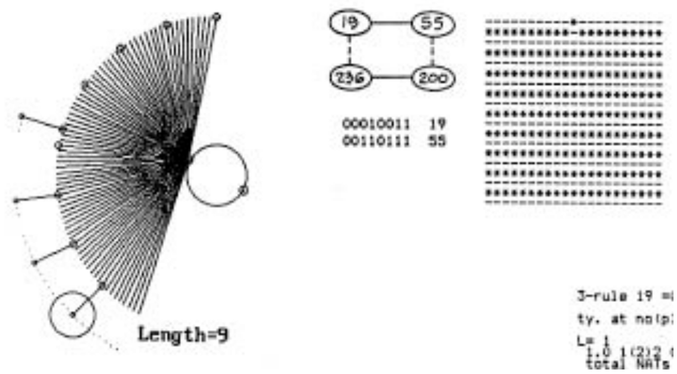
0000001100001100-0000001100001100-rule
=3-rule 18 -00010010



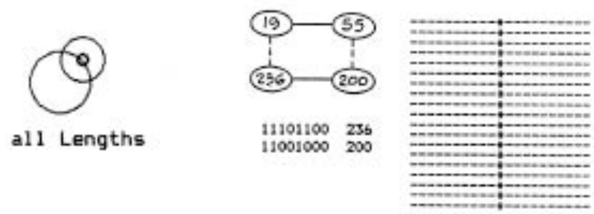
1111110011110011-1111110011110011-rule
=3-rule 237 -11101101



0000001100001111-0000001100001111-rule
=3-rule 19 -00010011

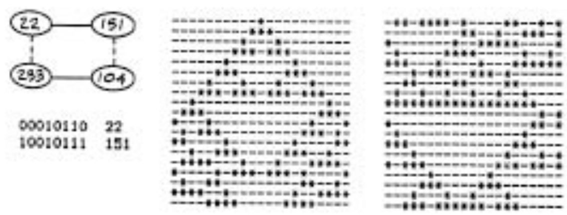


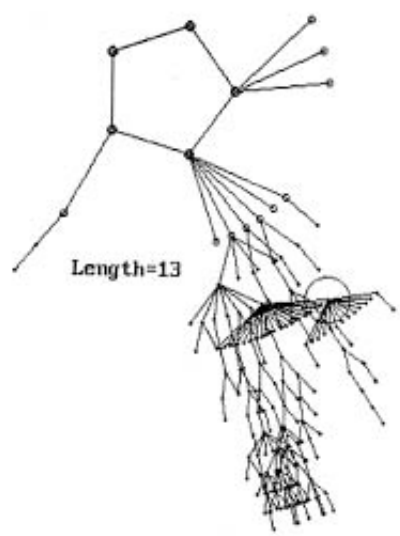
1111110011110000-1111110011110000-rule
=3-rule 236 -11101100



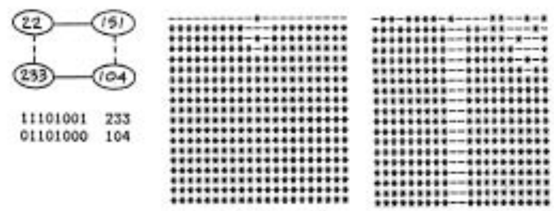
=3-rule 22 -00010110

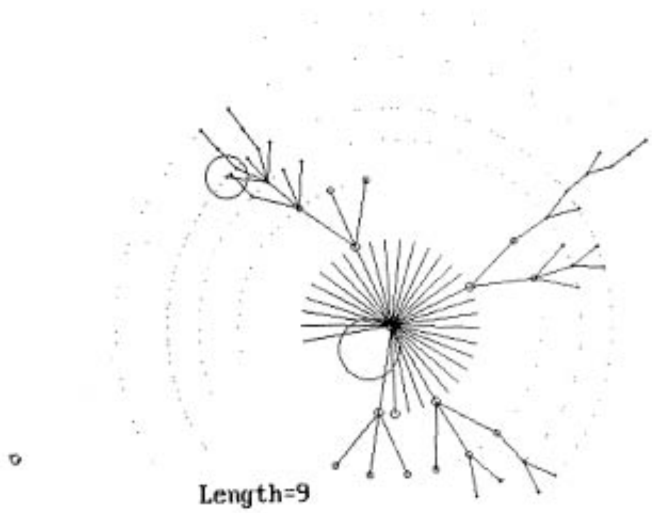
Length=1 -15



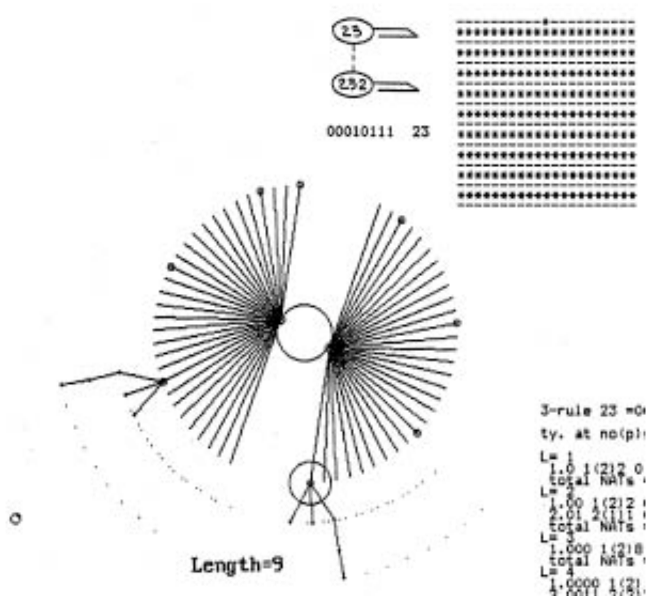


=3-rule 233 -11101001 Length=1 -15

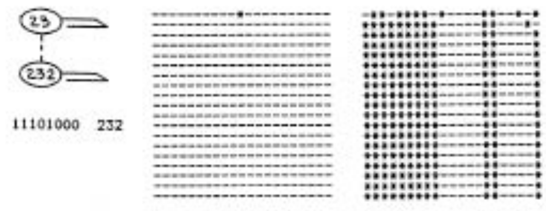


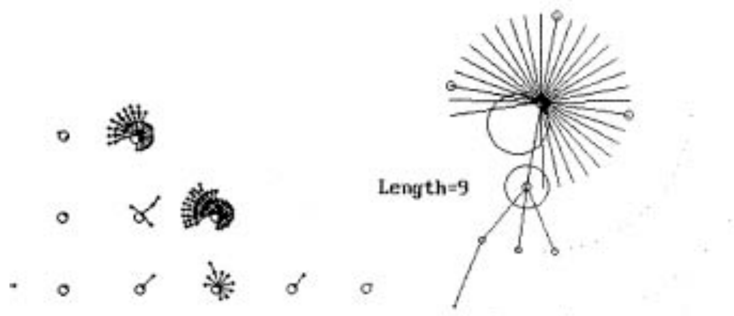


0000001100111111-0000001100111111-rule
=3-rule 23 -00010111

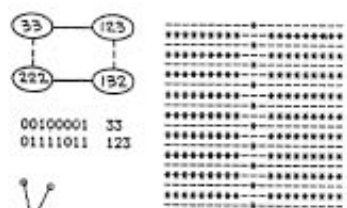


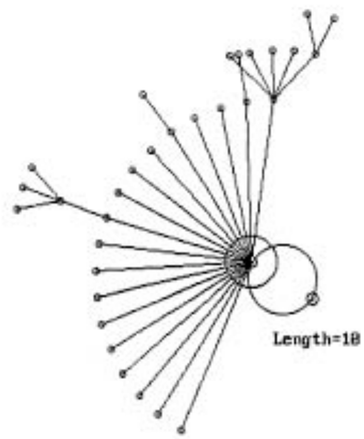
=3-rule 232 -11101000 Length=1 -14





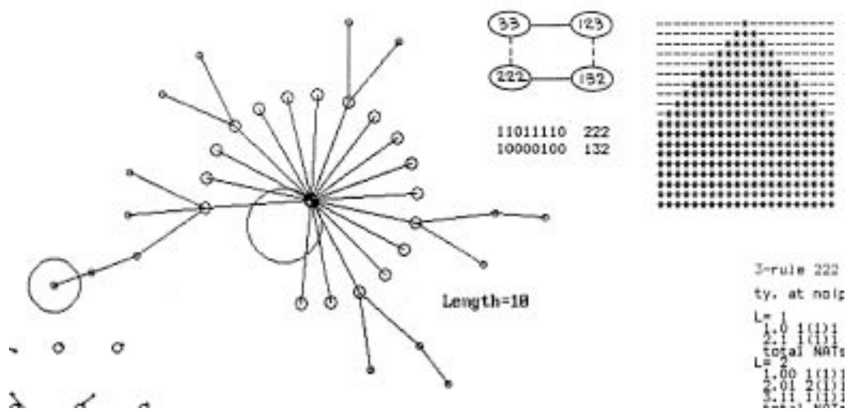
00001100000000011-00001100000000011-rule
=3-rule 33 -00100001



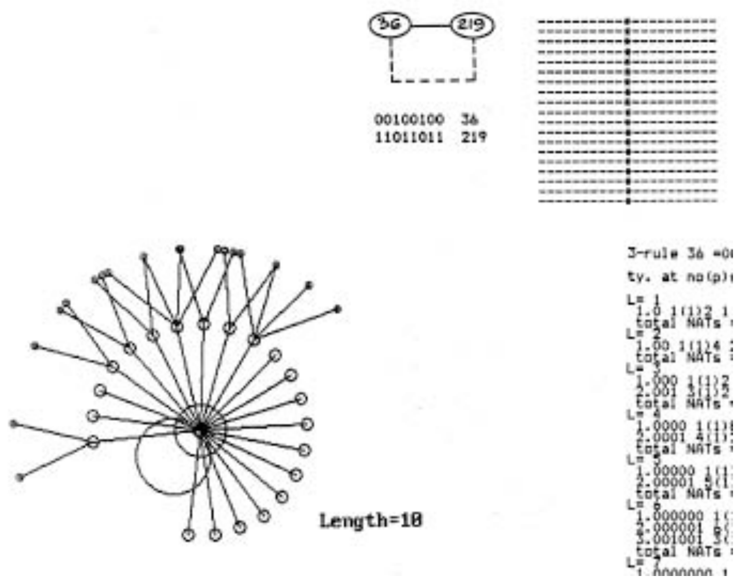


3-rule 33 =0
ty. at no(p)
L= 1
1.0 1(2)2 0
total NATs
L= 2
1.00 1(2)2
2.01 1(2)2
total NATs
L= 3
1.000 1(2)8
total NATs
L= 4

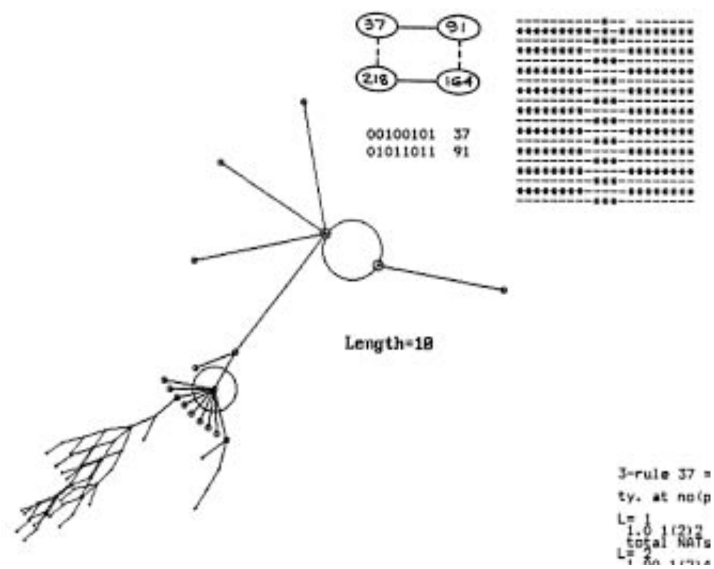
$Z = .5$ 1111001111111100-1111001111111100-rule
=3-rule 222 -11011110



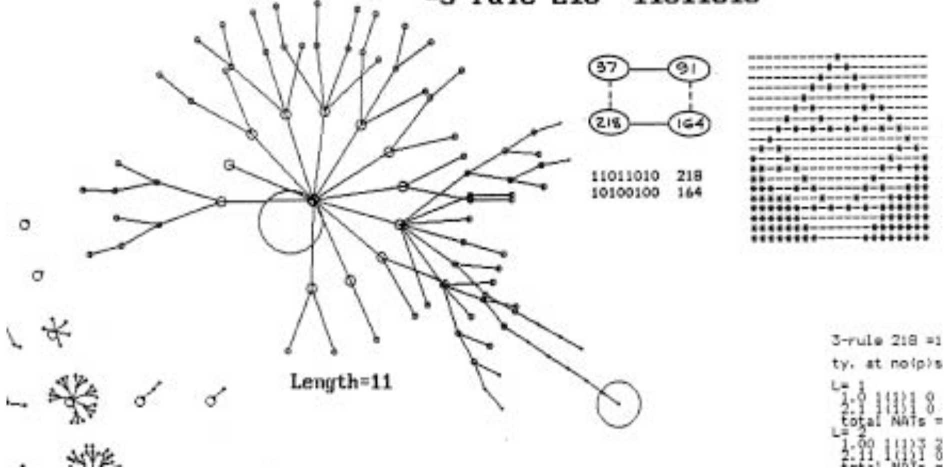
$Z = .5$ 0000110000110000-0000110000110000-rule
=3-rule 36 -00100100



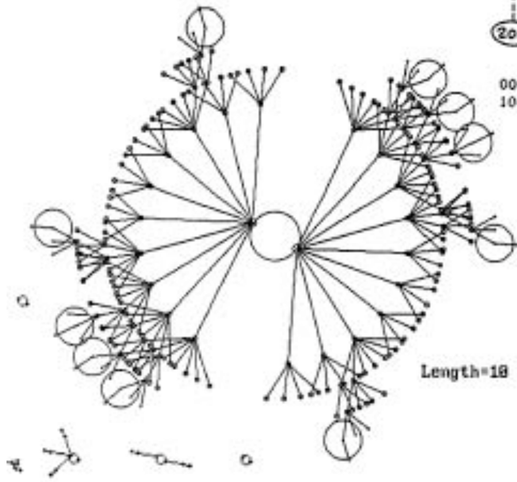
75 0000110000110011-0000110000110011-rule
=3-rule 37 -00100101

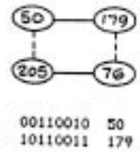


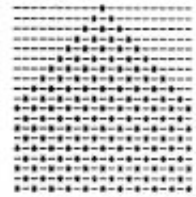
λ ratio = .75 Z = .75 1111001111001100-1111001111001100-rule
=3-rule 218 -11011010



.75 $Z = .625$ 0000111100001100-0000111100001100-rule
=3-rule 50 -00110010



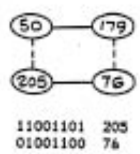


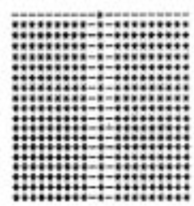


3-rule 50 =001100
ty. at no(p)s g a
L= 1
1.0 1(1)2 1 1 2
total NATs = 1
L= 2
1.00 1(1)2 1 1 2
2.01 1(2)2 0 0 1
total NATs = 2
L= 3
1.000 1(1)2 1 1

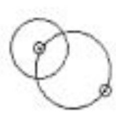
1111000011110011-1111000011110011-rule
=3-rule 205 -11001101

all lengths

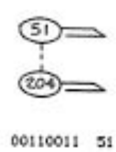


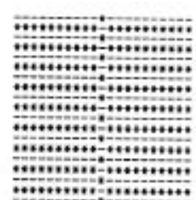


$Z = 1$ 0000111100001111-0000111100001111-rule
=3-rule 51 -00110011

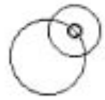


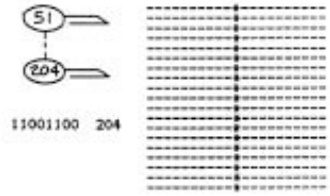
all lengths





1111000011110000-1111000011110000-rule
=3-rule 204 -11001100

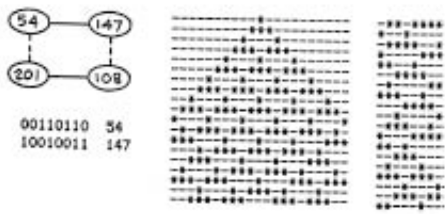


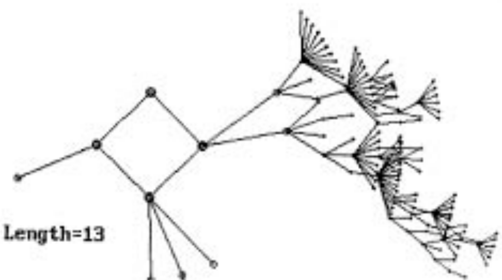


all lengths

=3-rule 54 -00110110

Lengt



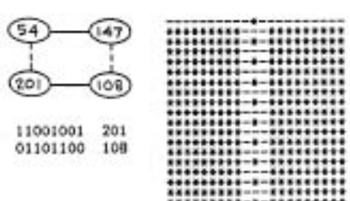


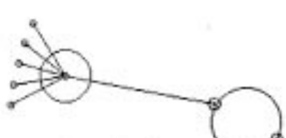
Length=13

```
3-rule 54 =00110110
ty. at no(p)s g ml mp
L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.000 1(1)8 3 3 3
 total NATs = 1
L= 4
 1.0000 1(1)4 2 2 2
 2.0001 2(8)4 0 0 1
 3.0011 2(2)2 0 0 1
 total NATs = 5
L= 5
 1.00000 1(1)32 10 6 5
 total NATs = 1
L= 6
 1.000000 1(1)34 17 5 5
 2.000001 6(4)5 1 1 2
 total NATs = 7
L= 7
```

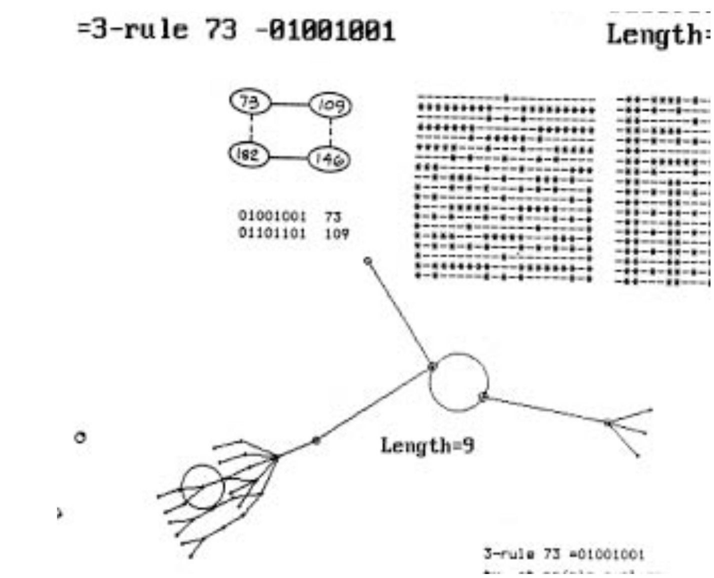
$Z = .75$

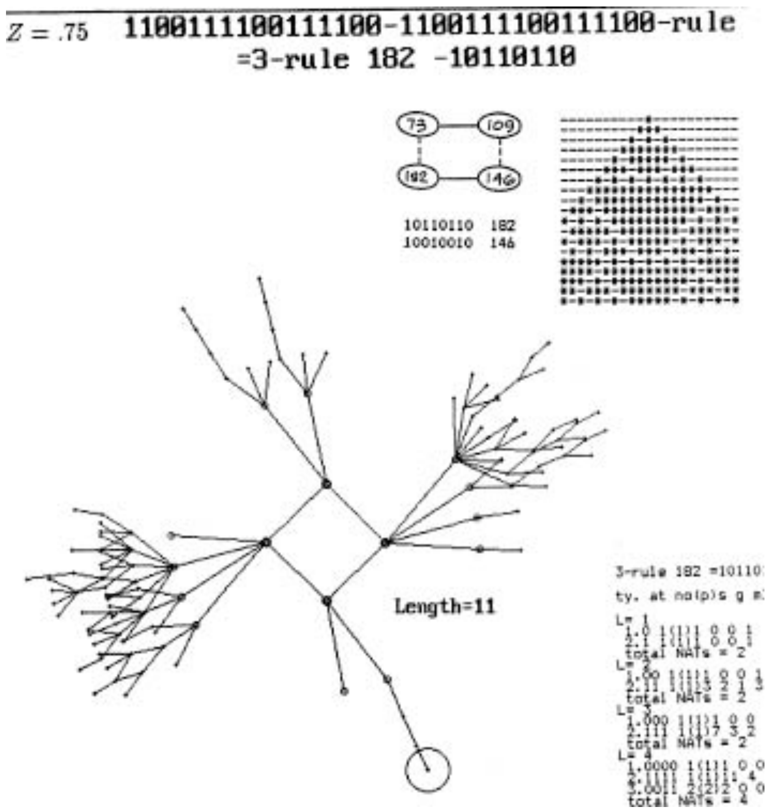
1111000011000011-1111000011000011-rule
=3-rule 201 -11001001



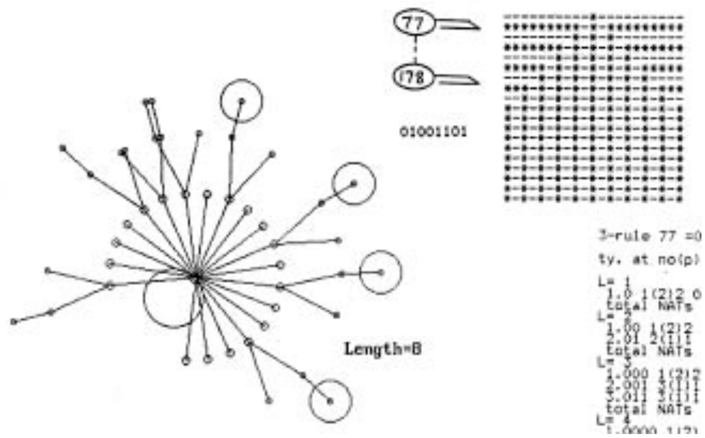


Length=13

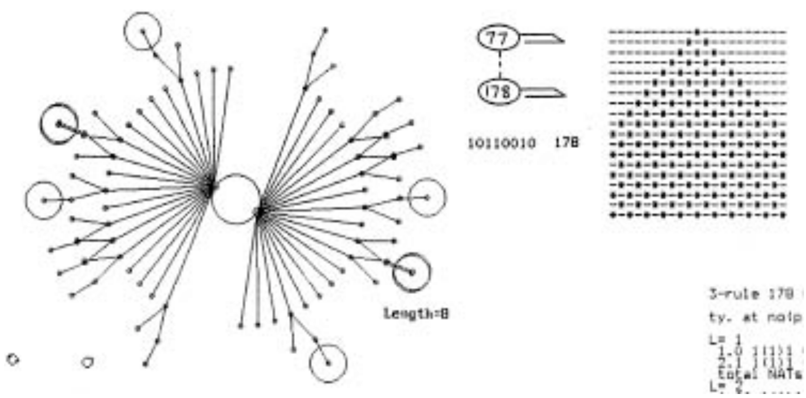

=3-rule 73 -01001001
Length:
73
109
182
146
01001001 73
01101101 109
Length=9
3-rule 73 =01001001


$Z = .75$ 1100111100111100-1100111100111100-rule
=3-rule 182 -10110110
73
109
182
146
10110110 182
10010010 146
Length=11
3-rule 182 =10110

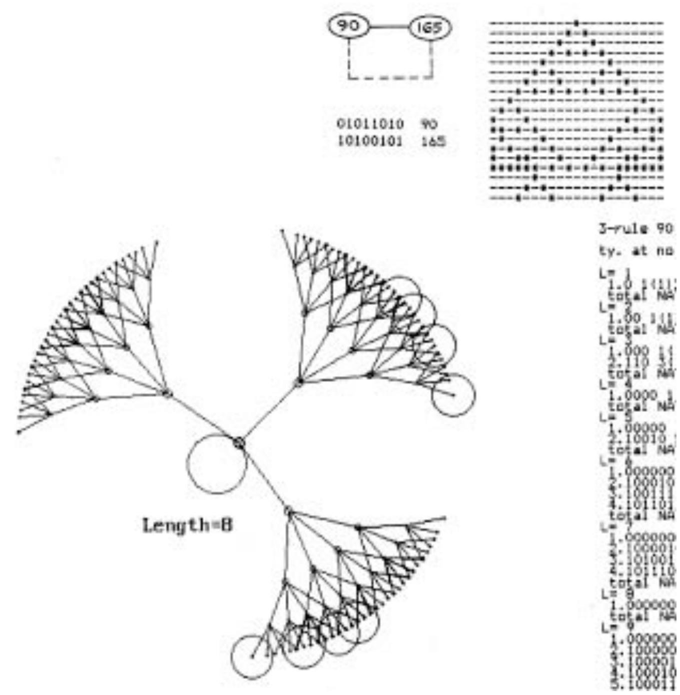
0011000011110011-0011000011110011-rule
=3-rule 77 -01001101



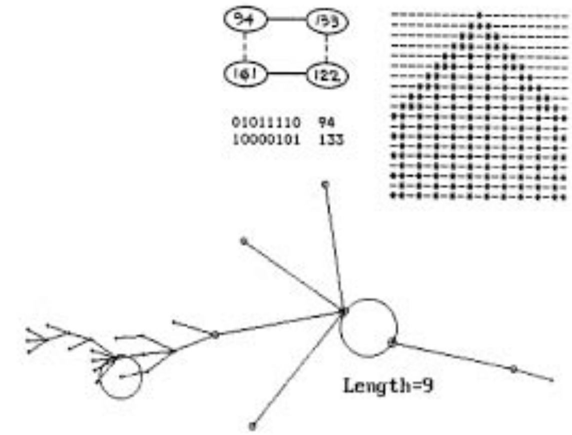
$Z = .5$ 1100111100001100-1100111100001100-rule
=3-rule 178 -10110010



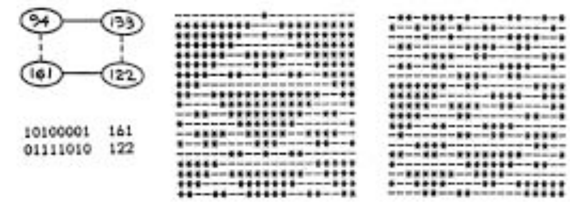
0011001111001100-0011001111001100-rule
=3-rule 90 -01011010

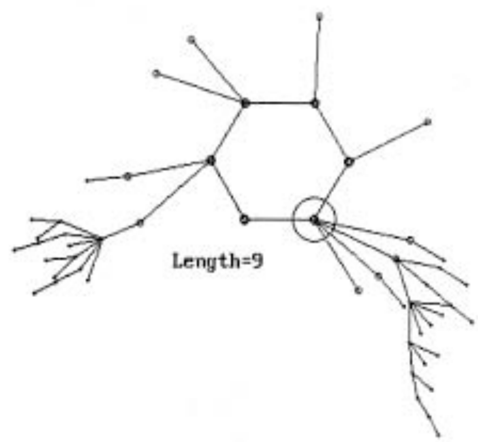


.1001111111100-00110011111111100-rule
=3-rule 94 -01011110

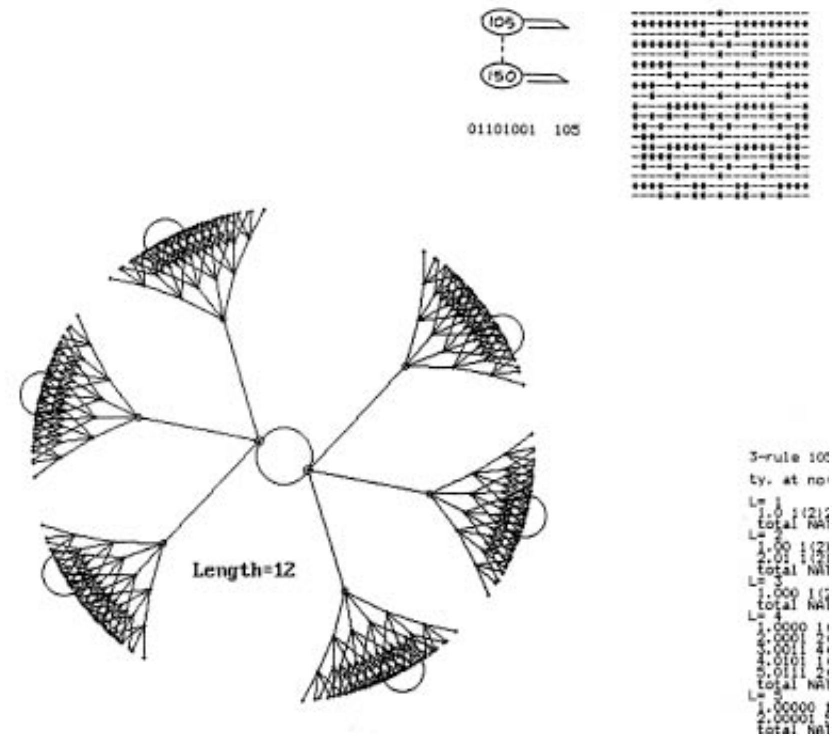


=3-rule 161 -10100001 Length=1 -15

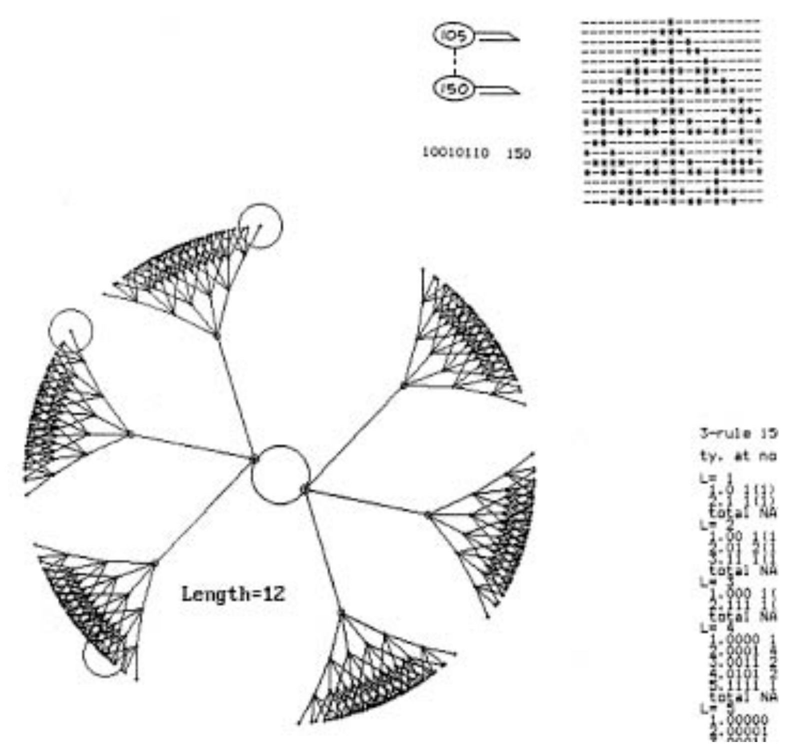




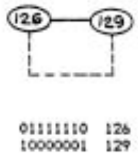
$Z = 1$ 0011110011000011-0011110011000011-rule =3-rule 105 -01101001

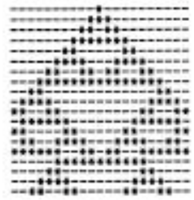


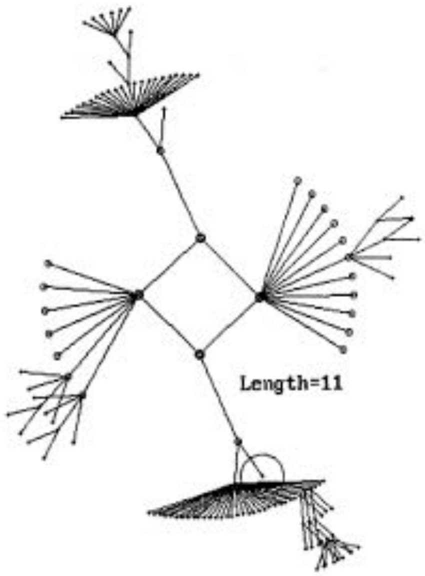
$Z = 1$ 1100001100111100-1100001100111100-rule =3-rule 150 -10010110



0011111111111100-0011111111111100-rule
=3-rule 126 -01111110

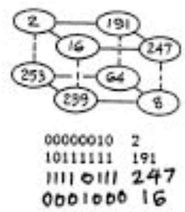


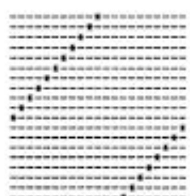




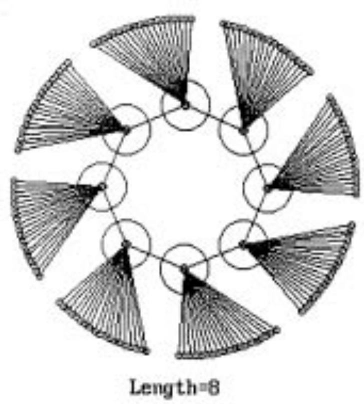

```
3-rule 126 =01111
ty. at no(p)s g n
L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 2 2 2
 total NATs = 1
L= 3
 1.000 1(1)8 6 2
 total NATs = 1
L= 4
```

.25 0000000000001100-0000000000001100-rule
=3-rule 2 -00000010



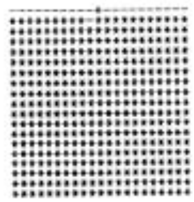


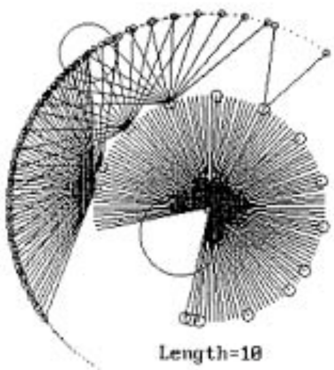
```
3-rule 2 =000000
ty. at no(p)s g
L= 1
 1.0 1(1)2 1 1 2
 total NATs = 1
L= 2
 1.00 1(1)4 3 1
 total NATs = 1
L= 3
 1.000 1(1)5 4 1
 2.001 1(3)3 0 0
 total NATs = 2
L= 4
 1.0000 1(1)8 7
 2.0001 1(4)8 4
 total NATs = 2
L= 5
 1.00000 1(1)12
 2.00001 1(5)20
 total NATs = 2
L= 6
 1.000000
 2.000001
 3.001001
 total NATs = 3
L= 7
 1.0000000
 2.0000001
 3.0001001
 total NATs = 3
L= 8
 1.00000000 1(1)
 2.00000001 1(8)
 3.00001001 1(8)
 4.00010001 1(4)
 total NATs = 4
L= 9
 1.000000000 1(1
 2.000000001 1(9
 3.000001001 1(9
 4.000010001 1(9
```



.25 1111111111110011-1111111111110011-rule
=3-rule 253 -11111101

11111101	253
01000000	64
00001000	8
11101111	239

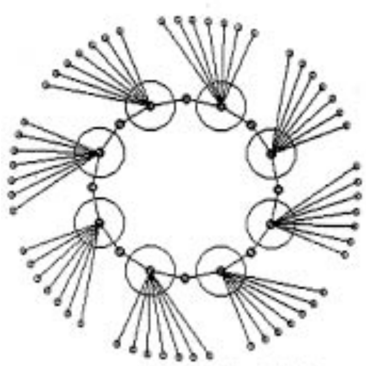




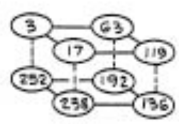
Length=10

3-rule 253 =11111101
ty. at no(p|s g nl mp
L= 1
1.1 1(1)2 1 1 2
total NATs = 1
L= 2
1.11 1(1)4 3 1 4
total NATs = 1

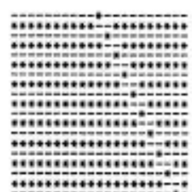
$Z = .5$ 0000000000001111-0000000000001111-rule
=3-rule 3 -00000011



Length=8

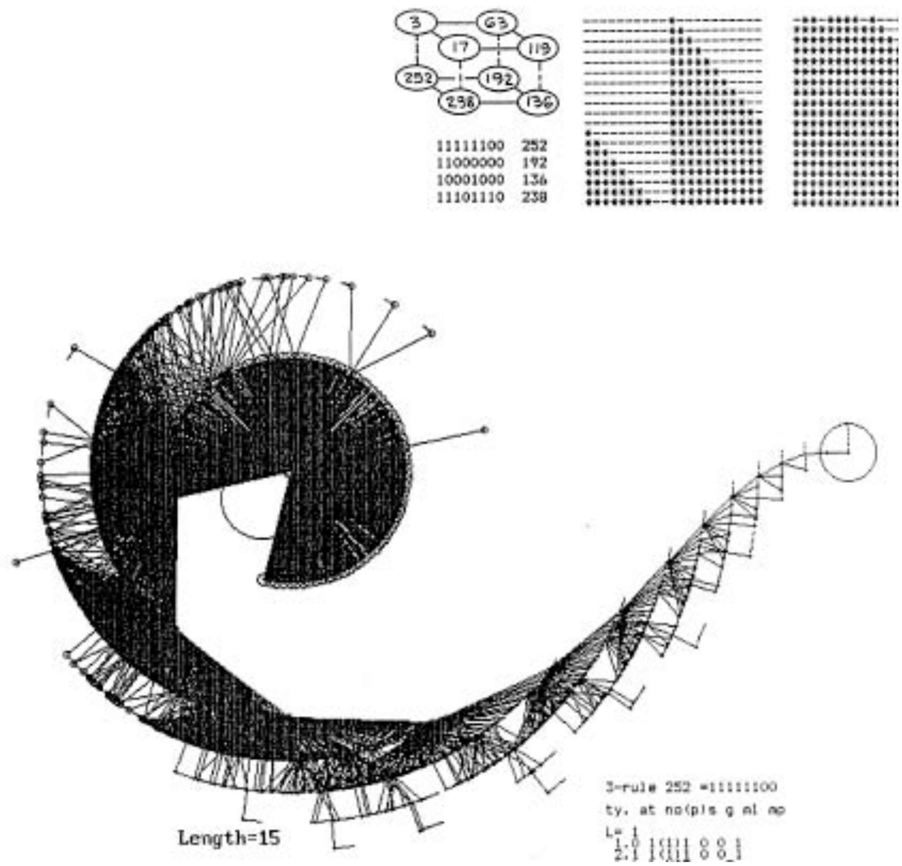


00000011	3
00111111	63
01110111	119
00010001	17

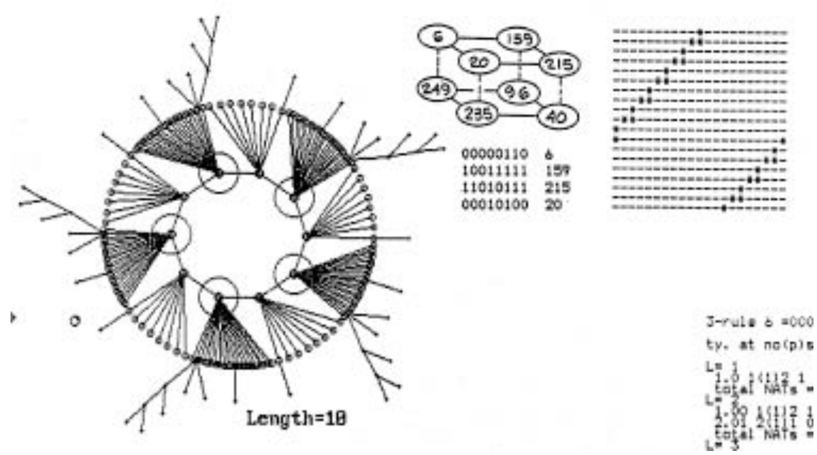


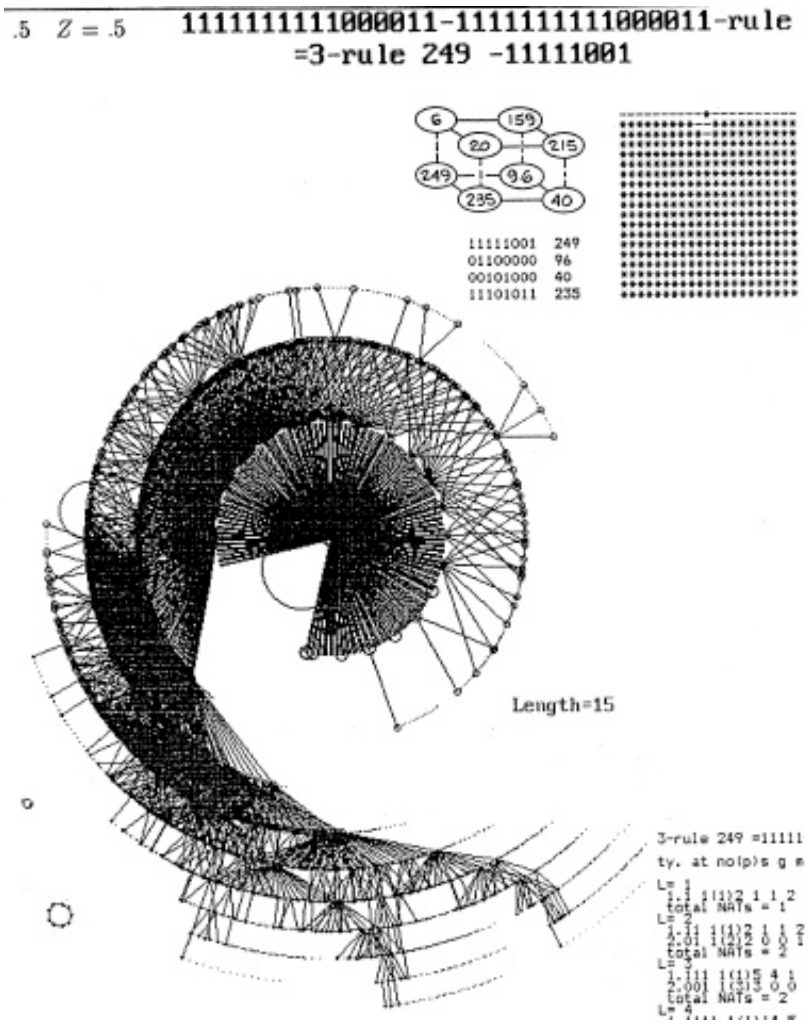
3-rule 3 =0
ty. at no(p
L= 1
1.0 1(2)2
total NATs
L= 2
1.00 1(2)4
total NATs
L= 3
1.000 1(2)
2.001 1(3)
total NATs
L= 4
1.0000 1(2

$Z = .5$ 1111111111110000-1111111111110000-rule 4293984
=3-rule 252 -11111100 Length=

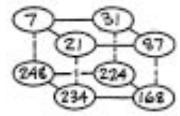


$= .5$ $Z = .5$ 0000000000111100-0000000000111100-rule
=3-rule 6 -00000110




.5 Z = .5 1111111111000011-1111111111000011-rule
=3-rule 249 -11111001
6
159
20
215
249
96
235
40
11111001 249
01100000 96
00101000 40
11101011 235
Length=15
3-rule 249 =11111

0000000000111111-0000000000111111-rule
=3-rule 7 -00000111



00000111	7
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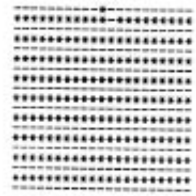
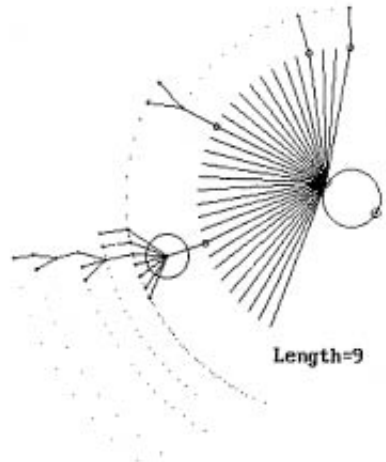
01010111	87
00010101	21





Length=9

```
3-rule 7 =00000111
ty. at no(p)s g al
L= 1
 1.0 1(2)2 0 0 1
 total NATs = 1
L= 2
 1.00 1(2)2 0 0 1
 2.01 2(1)1 0 0 1
 total NATs = 3
L= 3
 1.000 1(2)8 3 2 4
 total NATs = 1
L= 4
 1.0000 1(2)14 4 3
 2.0101 2(1)1 0 0
 total NATs = 3
L= 5
 1.00000 1(2)27 1(
 2.00011 1(5)5 0 (
 total NATs = 2
L= 6
 1.000000 1(2)44 1
 2.000011 1(12)18
 3.010101 2(1)1 0
 total NATs = 4
L= 7
 1.0000000 1(2)79
 2.0000011 1(14)2
 3.0000111 1(7)28
 total NATs = 3
L= 8
 1.00000000 1(2)1
 2.00000011 1(16)
 3.00000111 1(16)
 4.01010101 2(1)1
 total NATs = 5
L= 9
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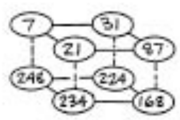
1111000000-1111111111000000-rule
=3-rule 248 -11111000



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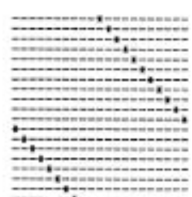
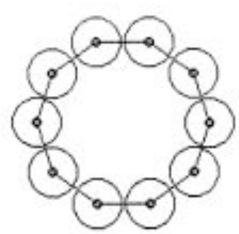
10101000	168
11101010	234





Length=10

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3-rule 248 =
ty. at no(p)
L= 1
 1.0 1(1)1
 2.1 1(1)1
 total NATs
L= 2
 1.00 1(1)1
 2.01 1(2)2
 3.11 1(1)1
 total NATs
L= 3
 1.000 1(1)
 2.001 1(3)
 3.111 1(1)
 total NATs
L= 4
 1.0000 1(1
 2.0001 1(4
 3.1111 1(1
 4.0101 1(2
 total NATs
L= 5
 1.00000 1(
 2.00001 1(
 3.11111 1(
 4.00101 1(
 total NATs
```
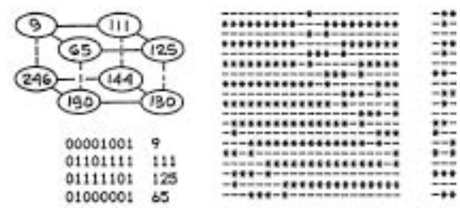
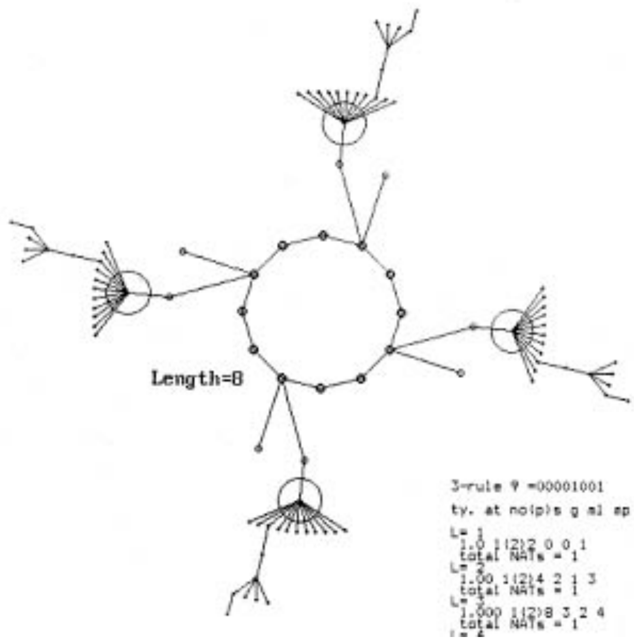

=3-rule 9 -00001001 Le





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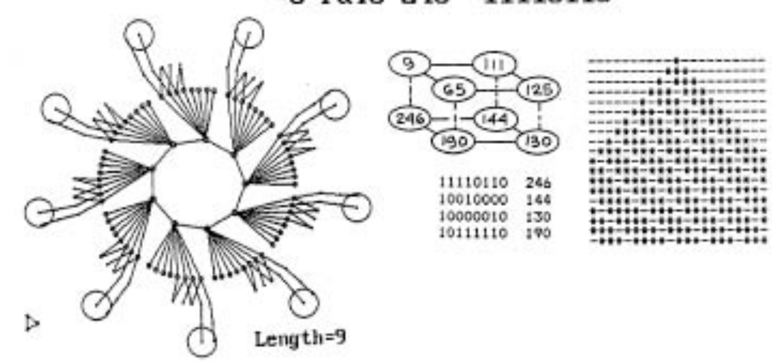
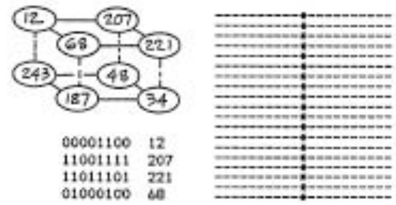
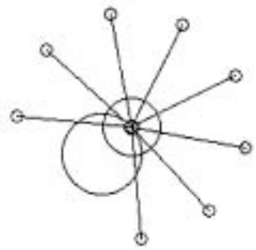
=3-rule 246 -11110110

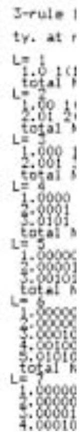


0000000011110000-0000000011110000-rule
=3-rule 12 -00001100





all lengths



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1111111100001111-1111111100001111-rule
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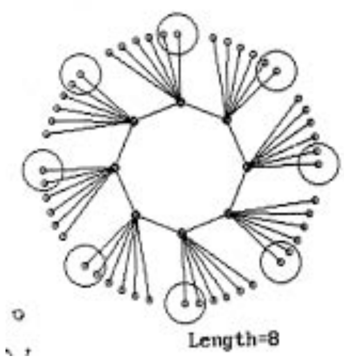

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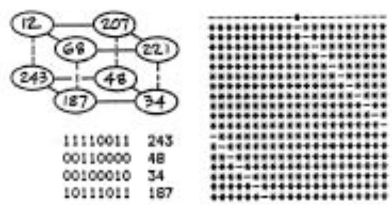

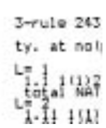

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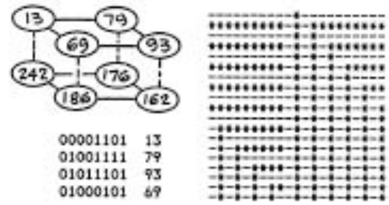
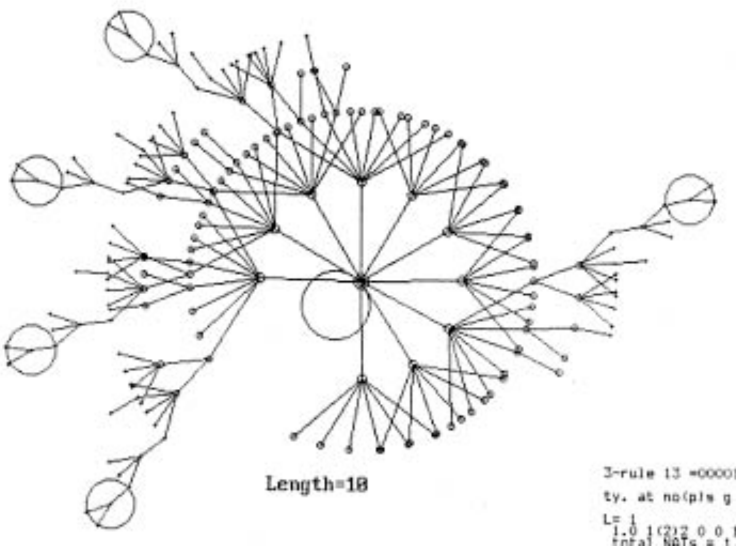
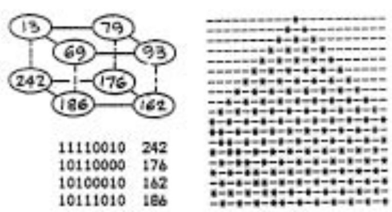
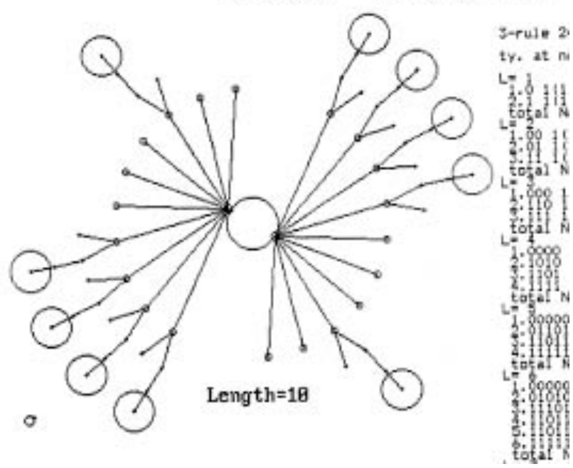
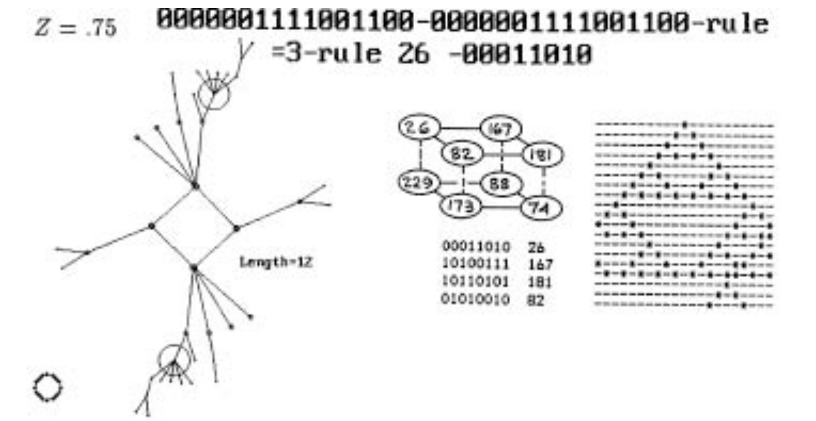




1111100001100-1111111100001100-rule
=3-rule 242 -11110010



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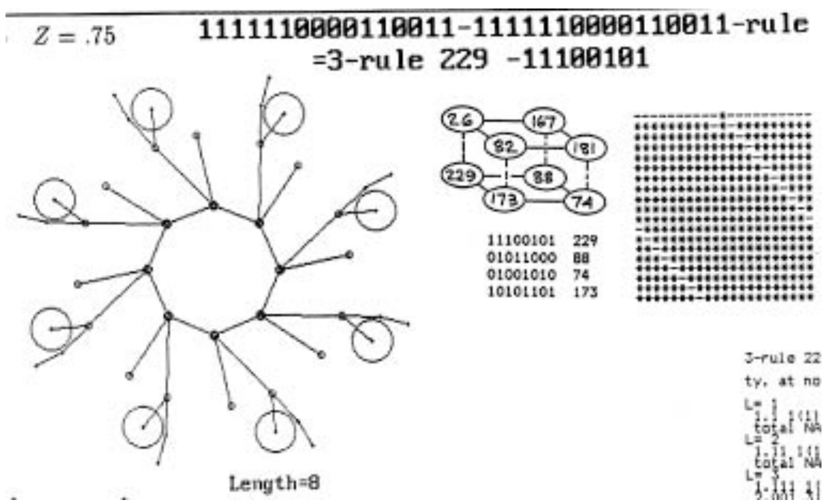
=3-rule 26 -00011010
Length=12
00011010 26
10100111 167
10110101 181
01010010 82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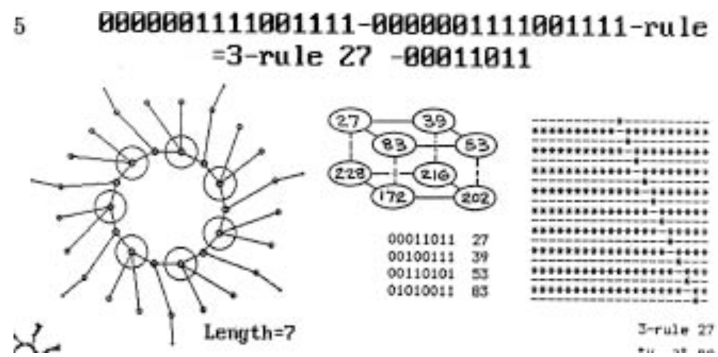
1111110000110011-1111110000110011-rule
=3-rule 229 -11100101
Length=8
11100101 229
01011000 88
01001010 74
10101101 173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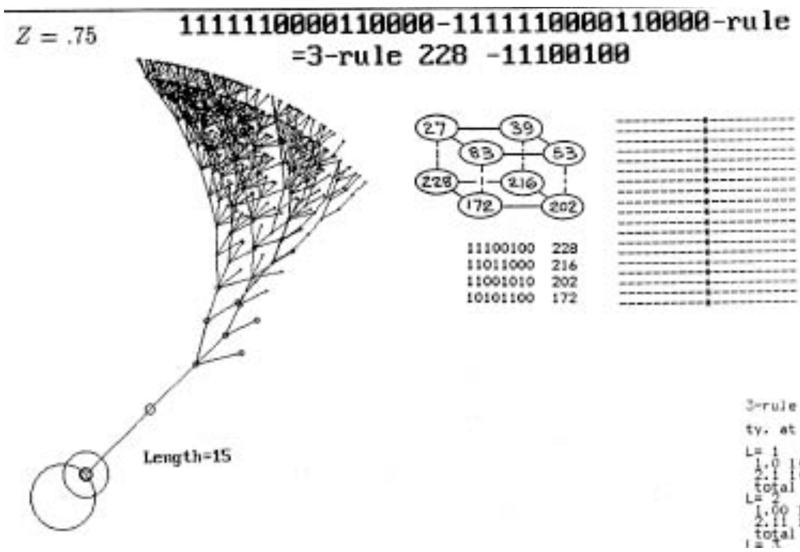
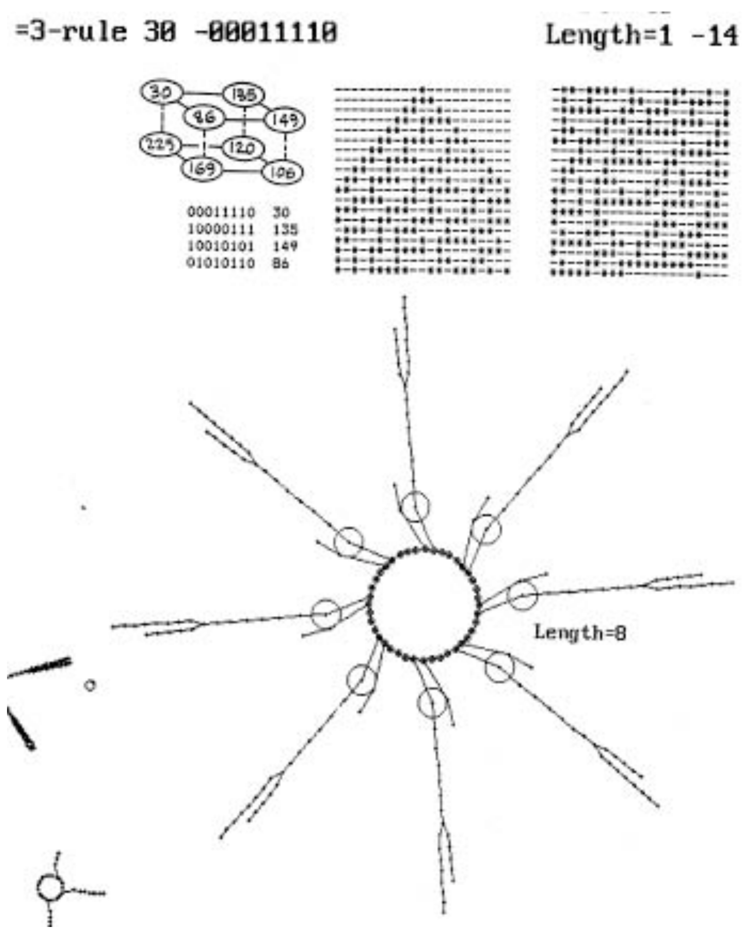
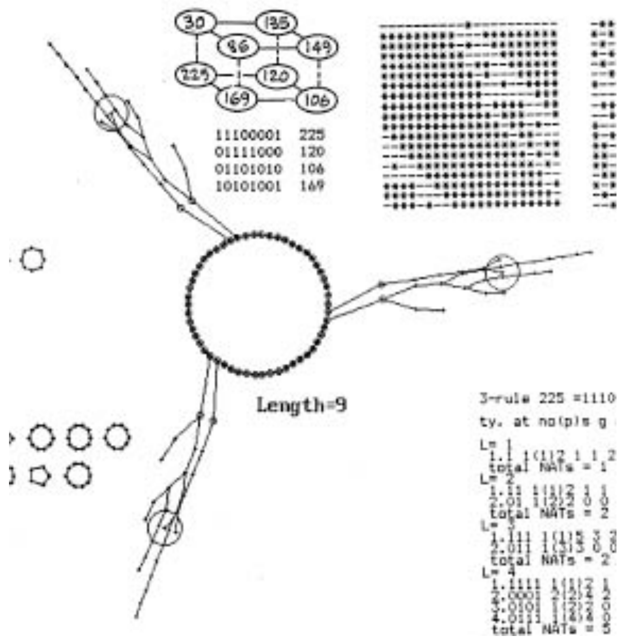
0000001111001111-0000001111001111-rule
=3-rule 27 -00011011
Length=7
00011011 27
00100111 39
00110101 53
01010011 83


Z = .75
1111110000110000-1111110000110000-rule
=3-rule 228 -11100100
27
39
83
53
228
216
172
202
11100100 228
11011000 216
11001010 202
10101100 172
Length=15


=3-rule 30 -00011110
Length=1 -14
30
135
86
149
225
120
169
106
00011110 30
10000111 135
10010101 149
01010110 86
Length=8

=3-rule 225 -11100001 Le



625 0000110000001111-0000110000001111-rule
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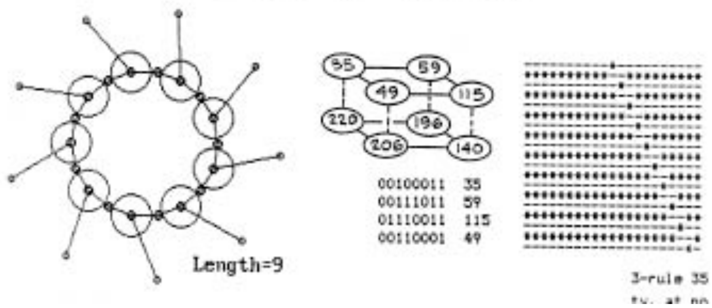

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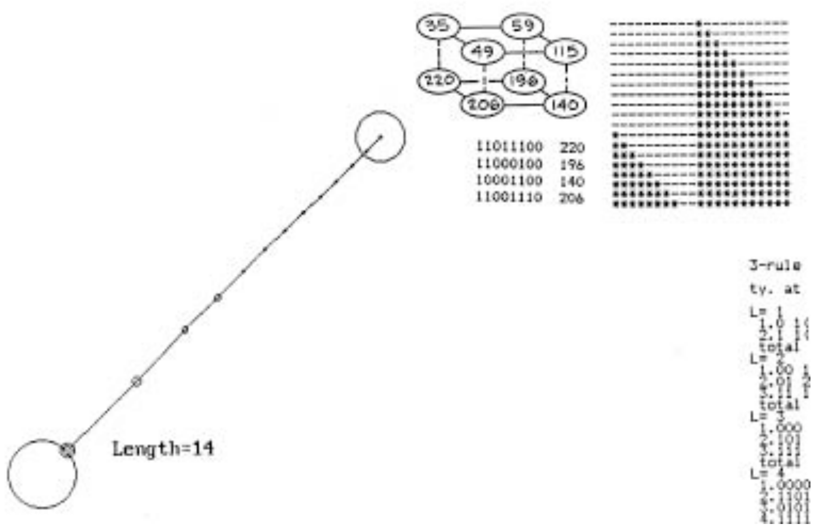

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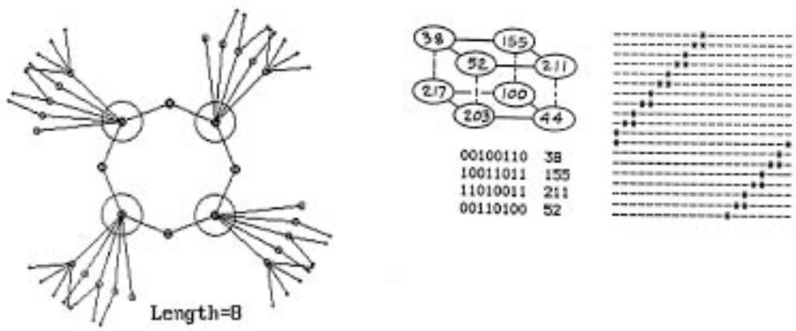

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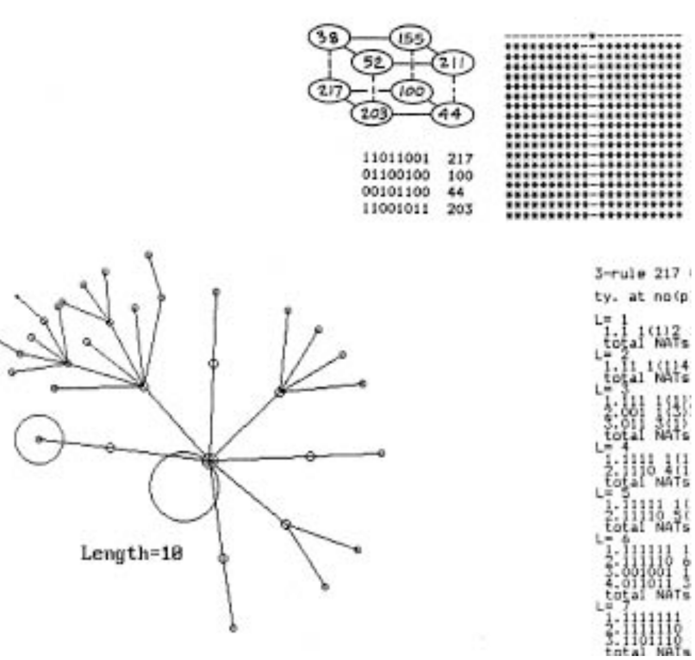

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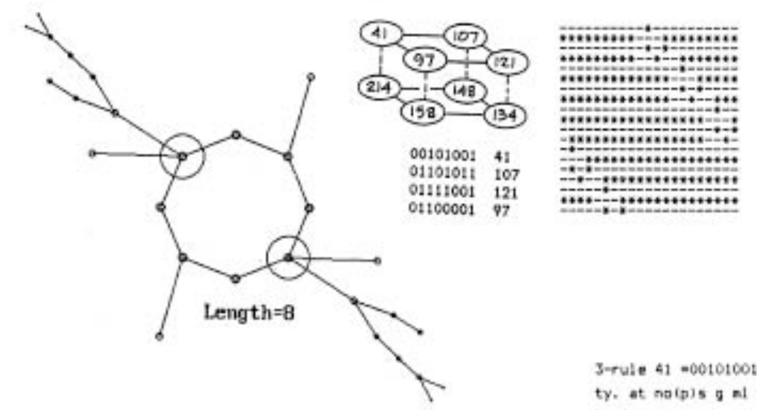
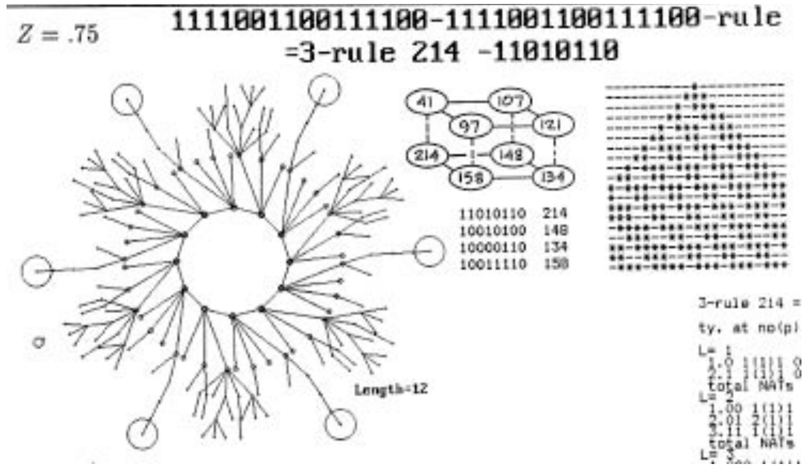
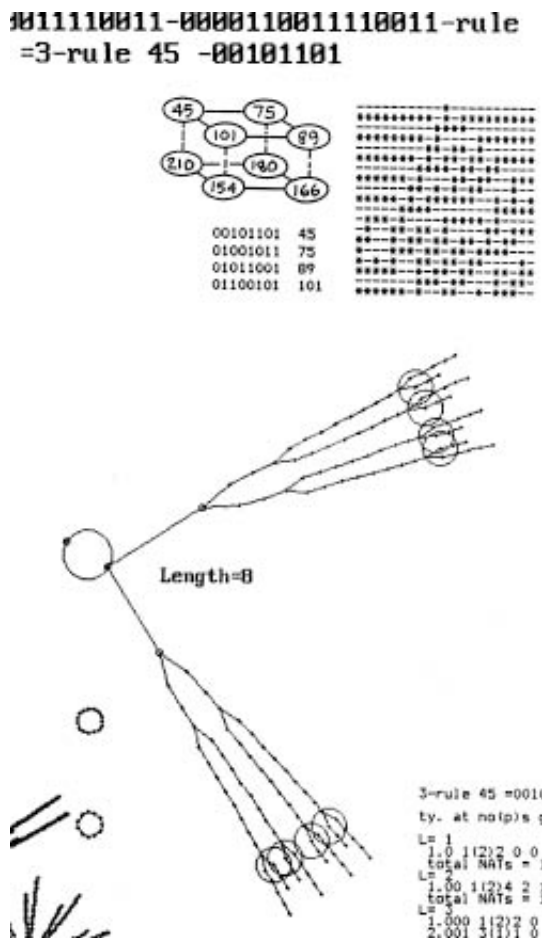



Z = .75
1111001100111100-1111001100111100-rule
=3-rule 214 -11010110
41
107
97
121
214
148
158
134
11010110 214
10010100 148
10000110 134
10011110 158
Length=12
3-rule 214 =
ty. at no(p)
total NATs
total NATs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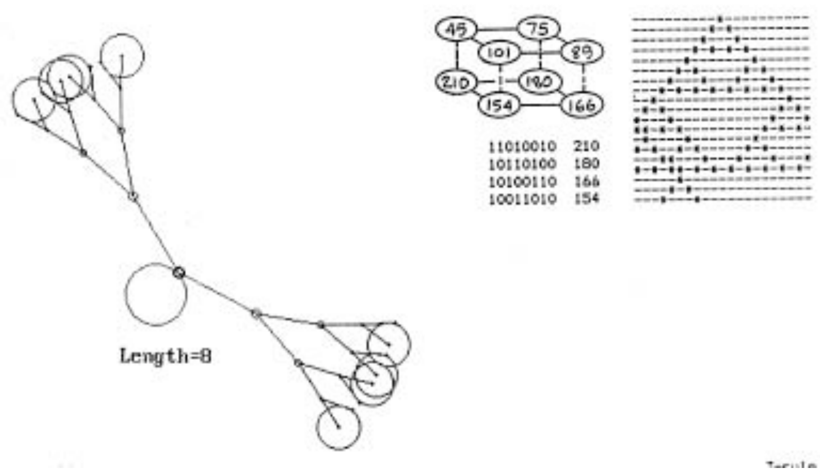
=3-rule 45 -00101101
45
75
101
89
210
180
154
166
00101101 45
01001011 75
01011001 89
01100101 101
Length=8
3-rule 45 =0010
ty. at no(p)s g
total NATs = 1
total NATs = 1

1 $Z = 1$ 1111001100001100-1111001100001100-rule
=3-rule 210 -11010010



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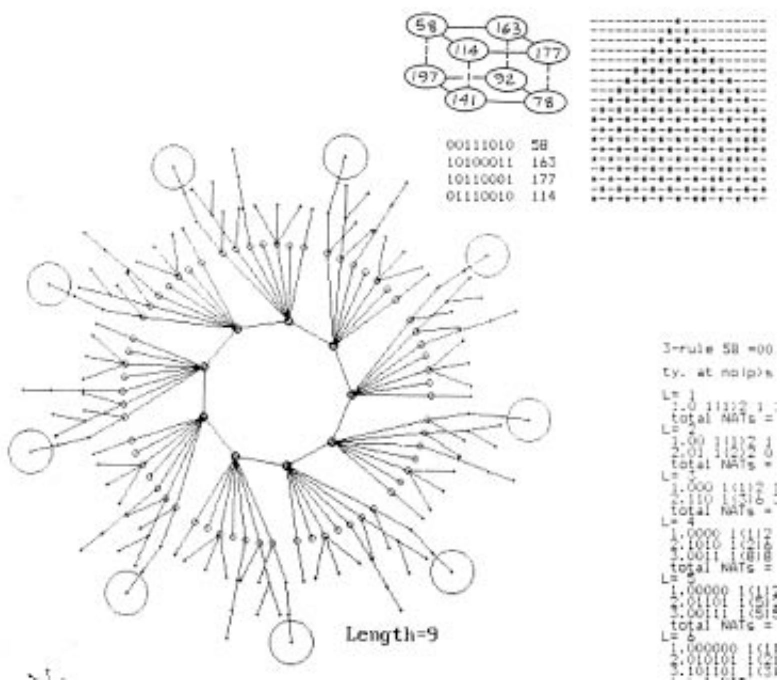
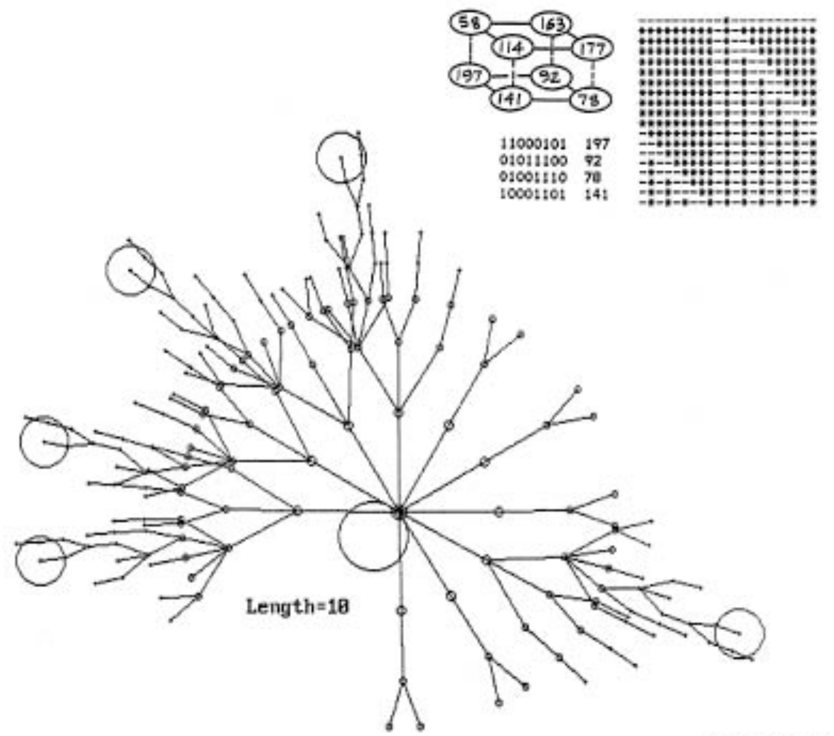
=3-rule 58 -00111010



$Z = .75$

1111000000110011-1111000000110011-rule
=3-rule 197 -11000101



3-rule 197 =11(

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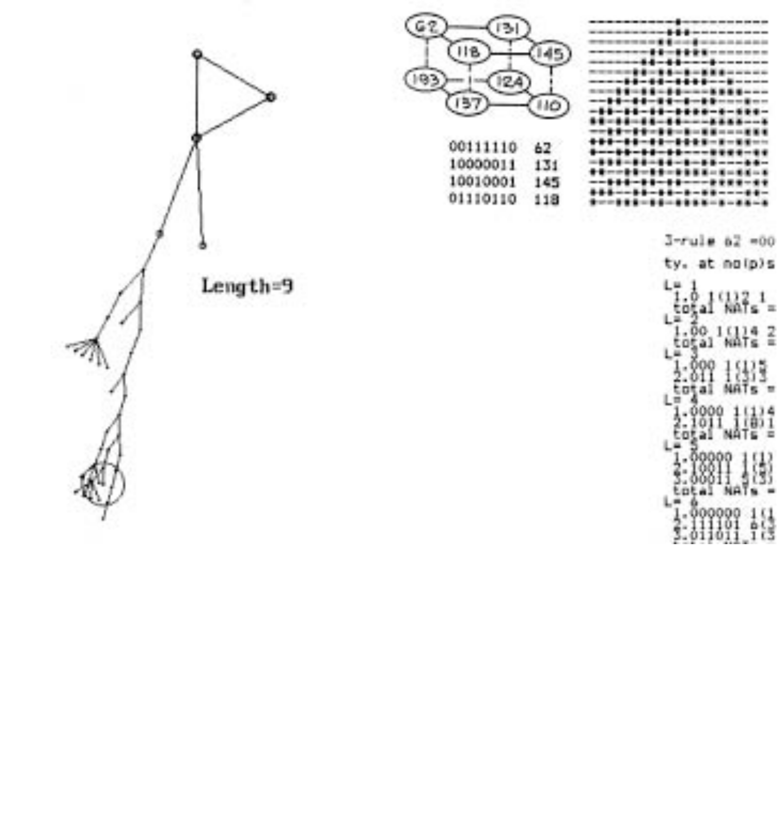
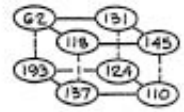


=3-rule 193 -11000001 Lengt



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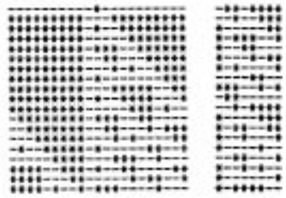
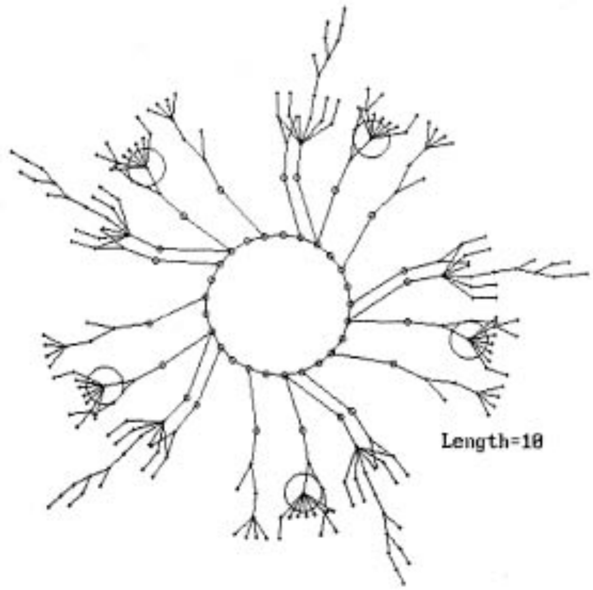
01111100	124
01101110	110
10001001	137





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.5 0000000011001100-0000000011001100-rule
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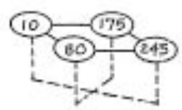

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10101111	175
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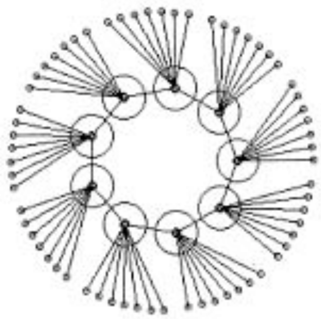
01010000	80



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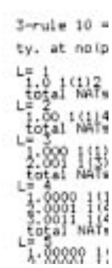
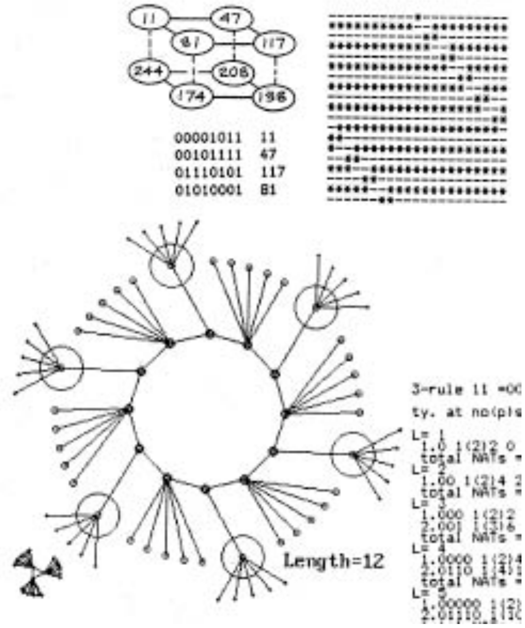


0011001111-0000000011001111-rule
=3-rule 11 -00001011



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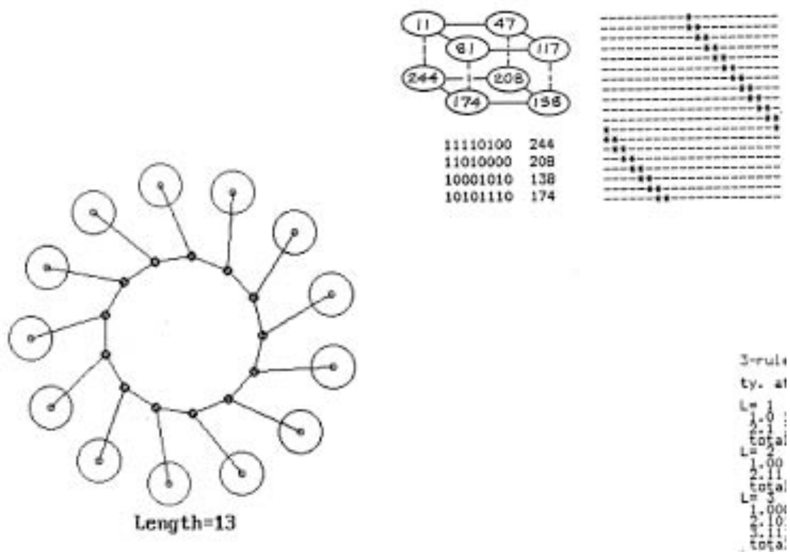
=3-rule 244 -11110100



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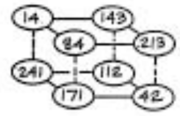

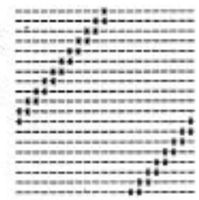

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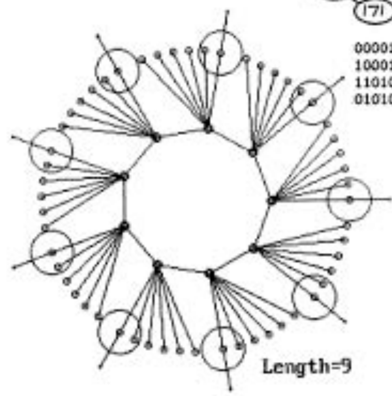
01010100	84

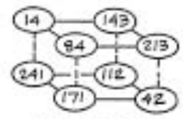


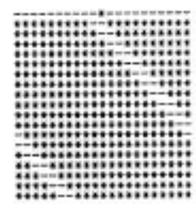
Length=9

3-rule 14 =00
ty. at no(p)s

5

11111111000000011-11111111000000011-rule
=3-rule 241 -11110001





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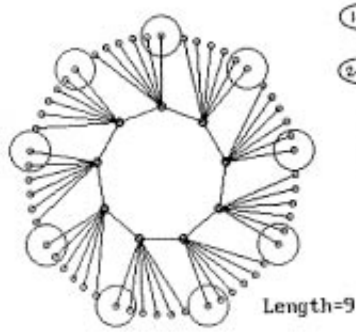
00101010	42
10101011	171



Length=9

3-rule 241
ty. at no

00000000011111111-00000000011111111-rule
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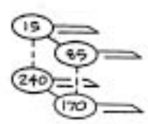
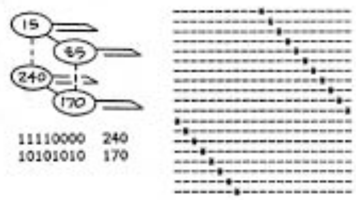


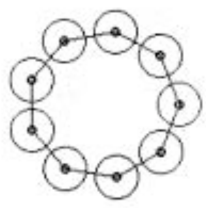
00001111	15
01010101	85

Length=9

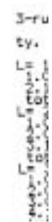
3-rule
ty. at

1111111100000000-1111111100000000-rule
=3-rule 240 -11110000





Length=9



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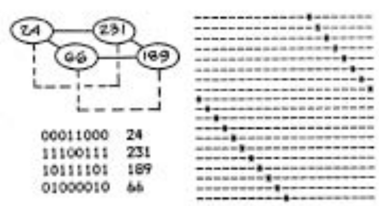
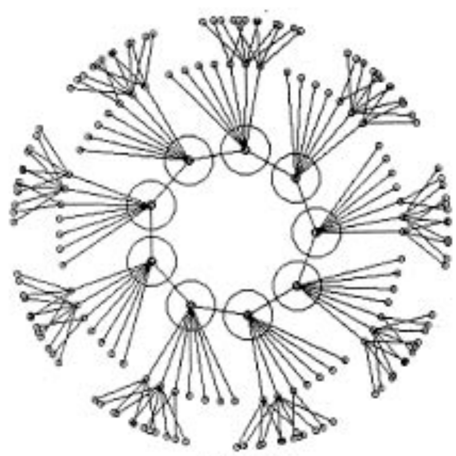
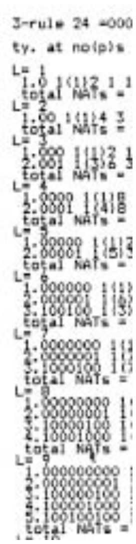
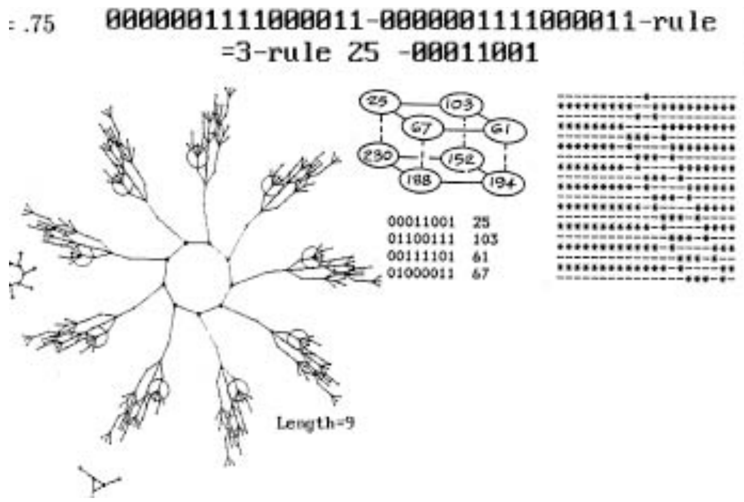
=3-rule 24 -00011000

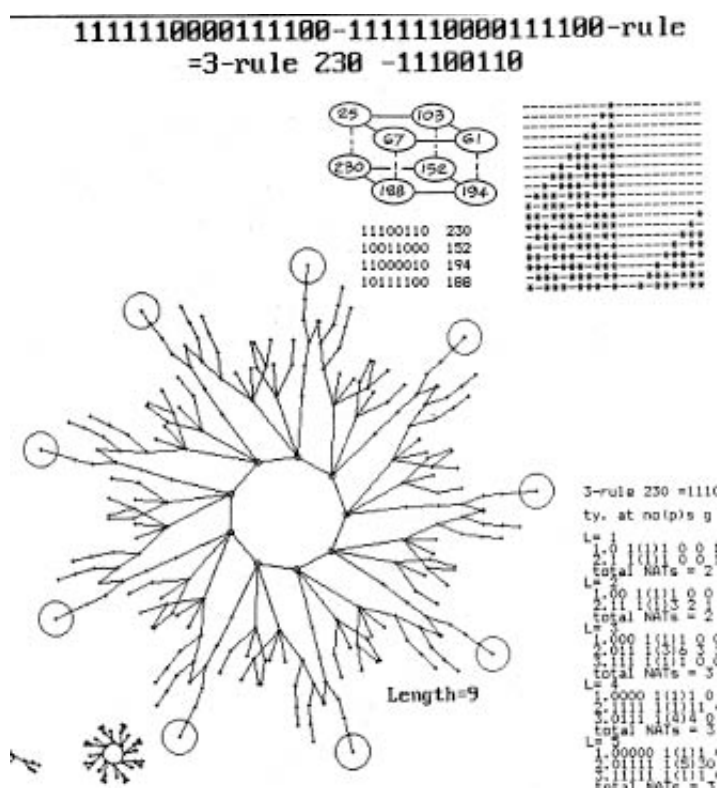




Length=9




.75
00000001111000011-00000001111000011-rule
=3-rule 25 -00011001
25
103
67
61
230
152
188
194
00011001 25
01100111 103
00111101 61
01000011 67
Length=9


11111100000111100-11111100000111100-rule
=3-rule 230 -11100110
25
103
67
61
230
152
188
194
11100110 230
10011000 152
11000010 194
10111100 188
Length=9
3-rule 230 =111(
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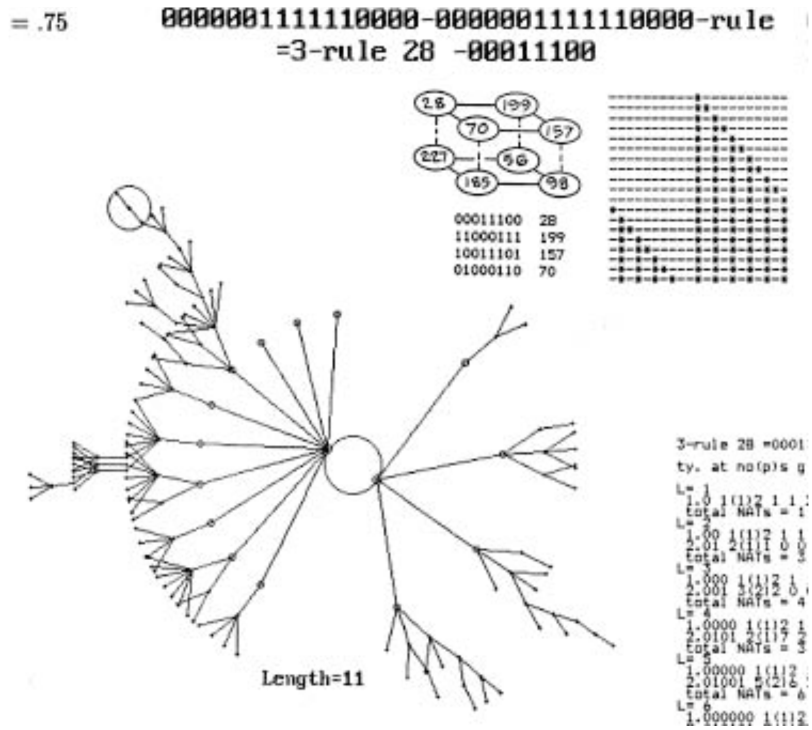
= .75
00000001111110000-00000001111110000-rule
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199
70
157
227
56
185
98
00011100 28
11000111 199
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01000110 70
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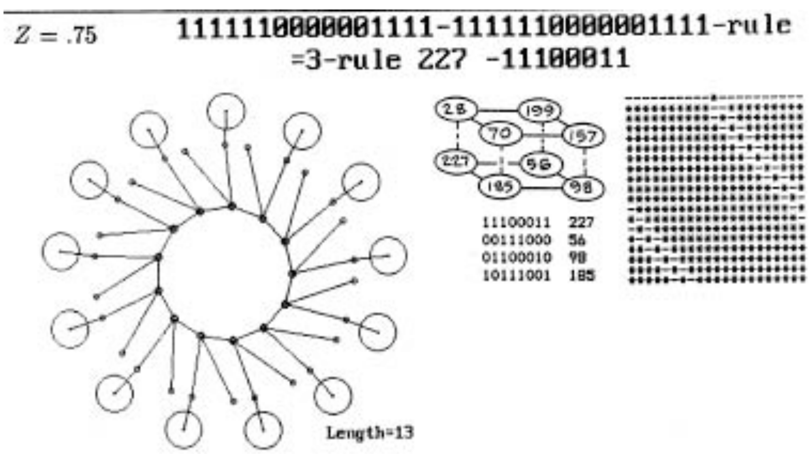
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10111001 185
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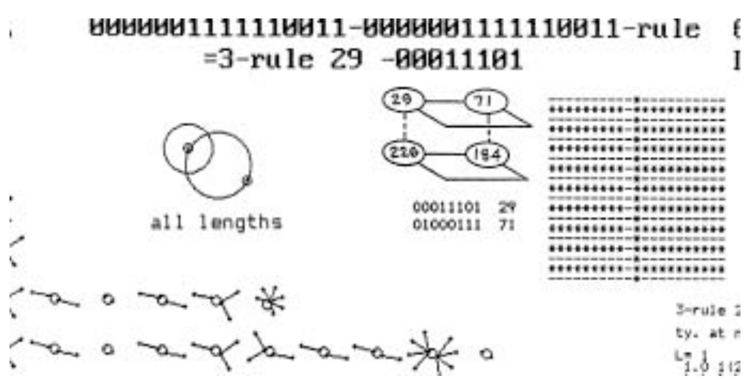
00000001111110011-00000001111110011-rule
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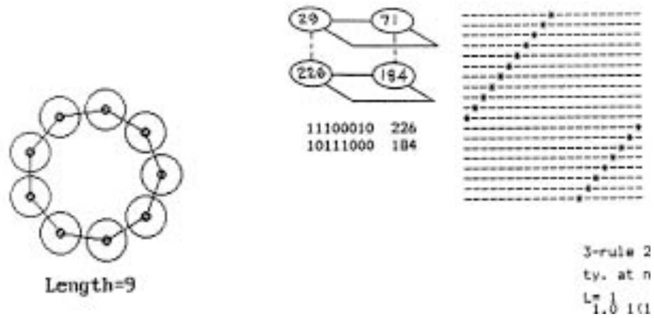
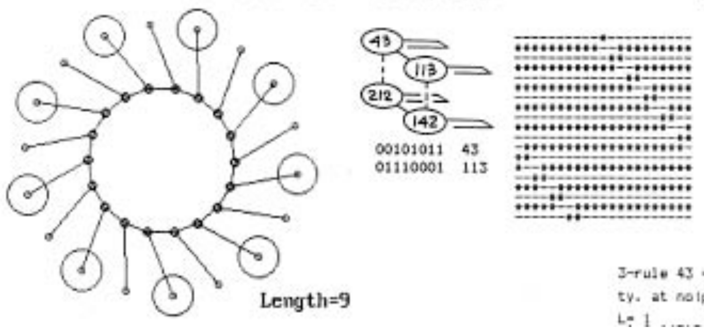
71
226
184
00011101 29
01000111 71
all lengths

1111110000001100-1111110000001100-rule
=3-rule 226 -11100010



0000110011001111-0000110011001111-rule
=3-rule 43 -00101011



1110001100110000-1110001100110000-rule
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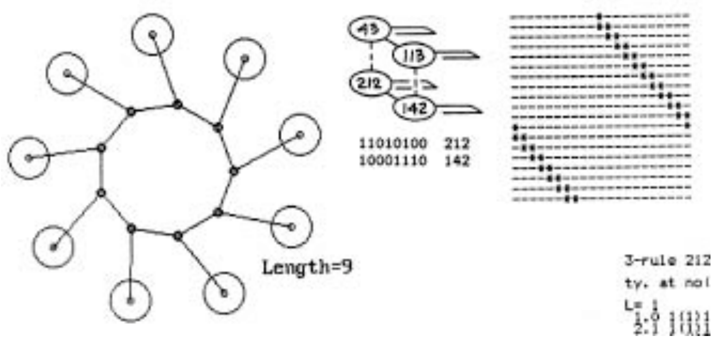
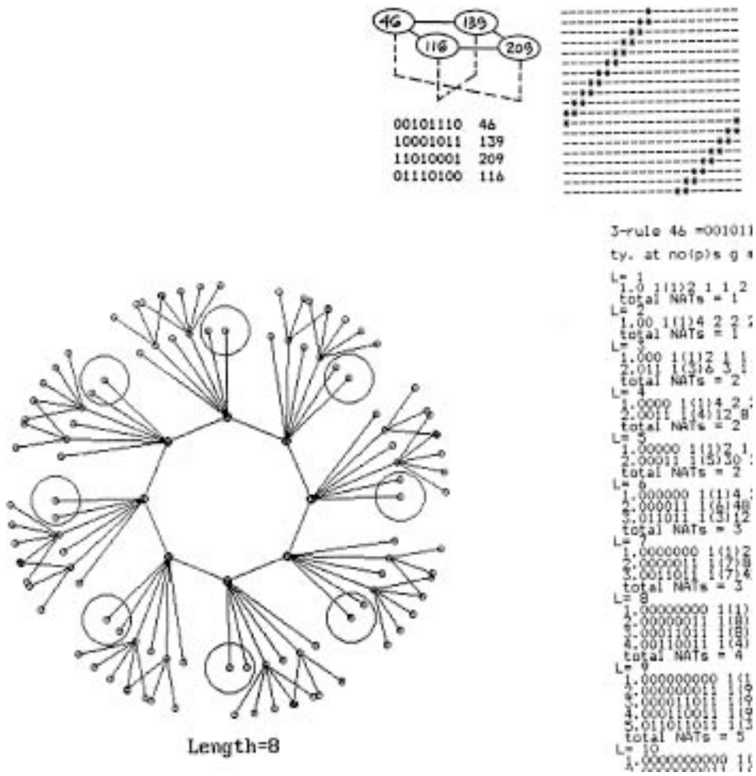


= .5

0000110011111100-0000110011111100-rule
=3-rule 46 -00101110



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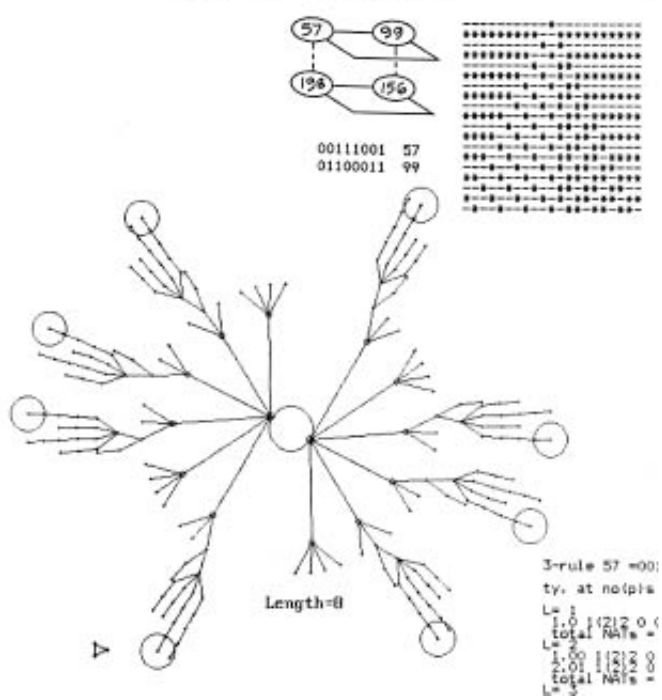
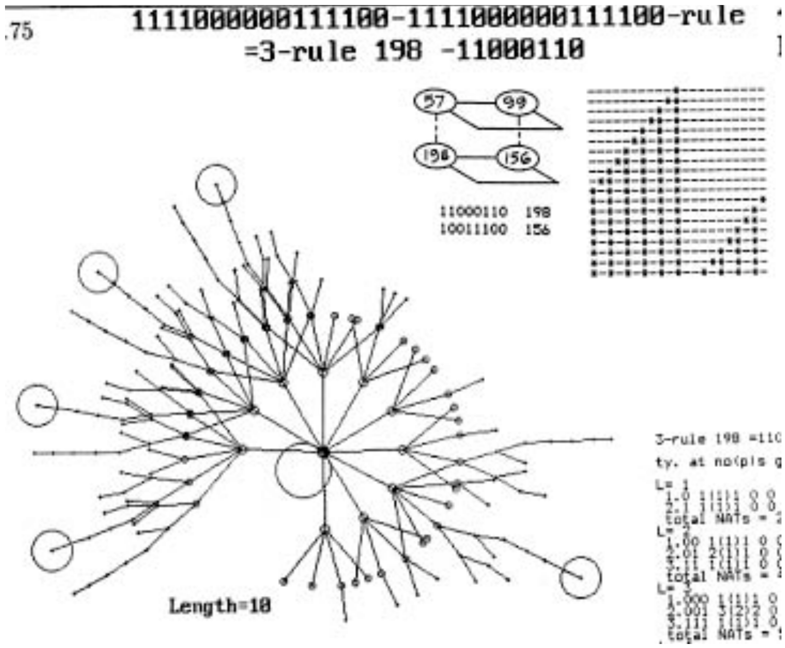
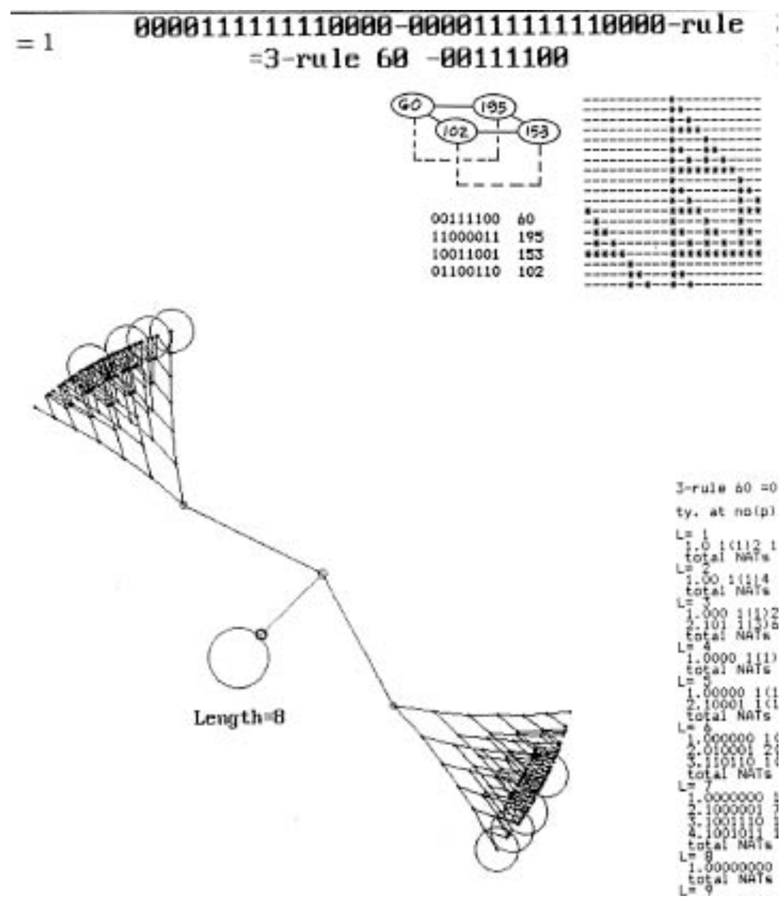




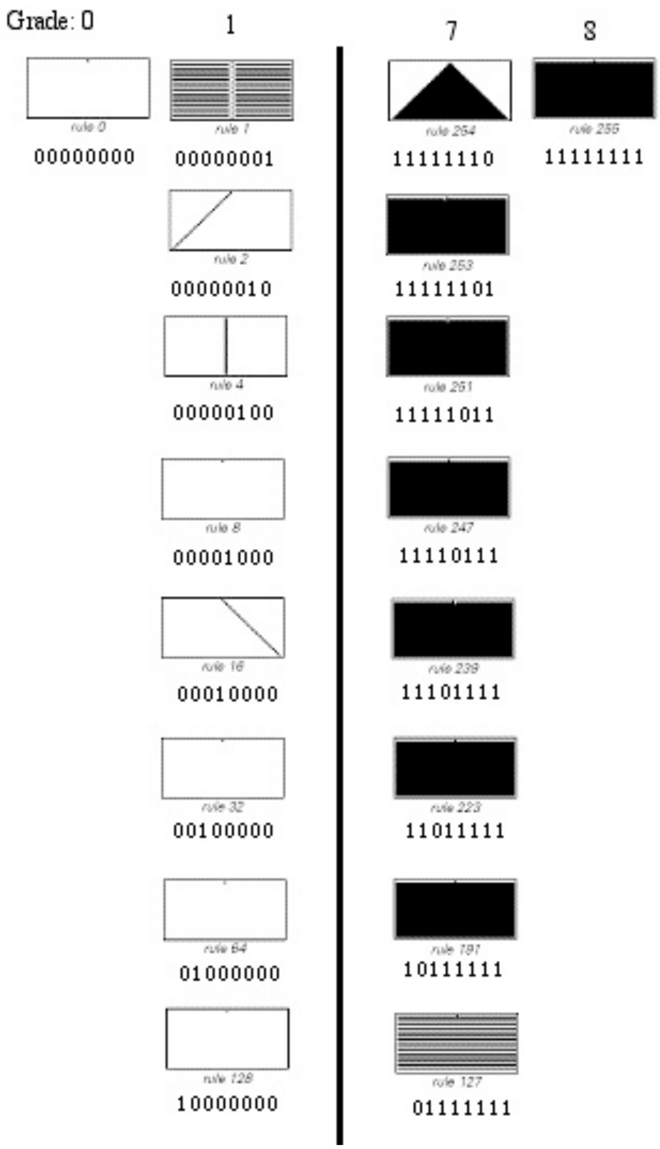


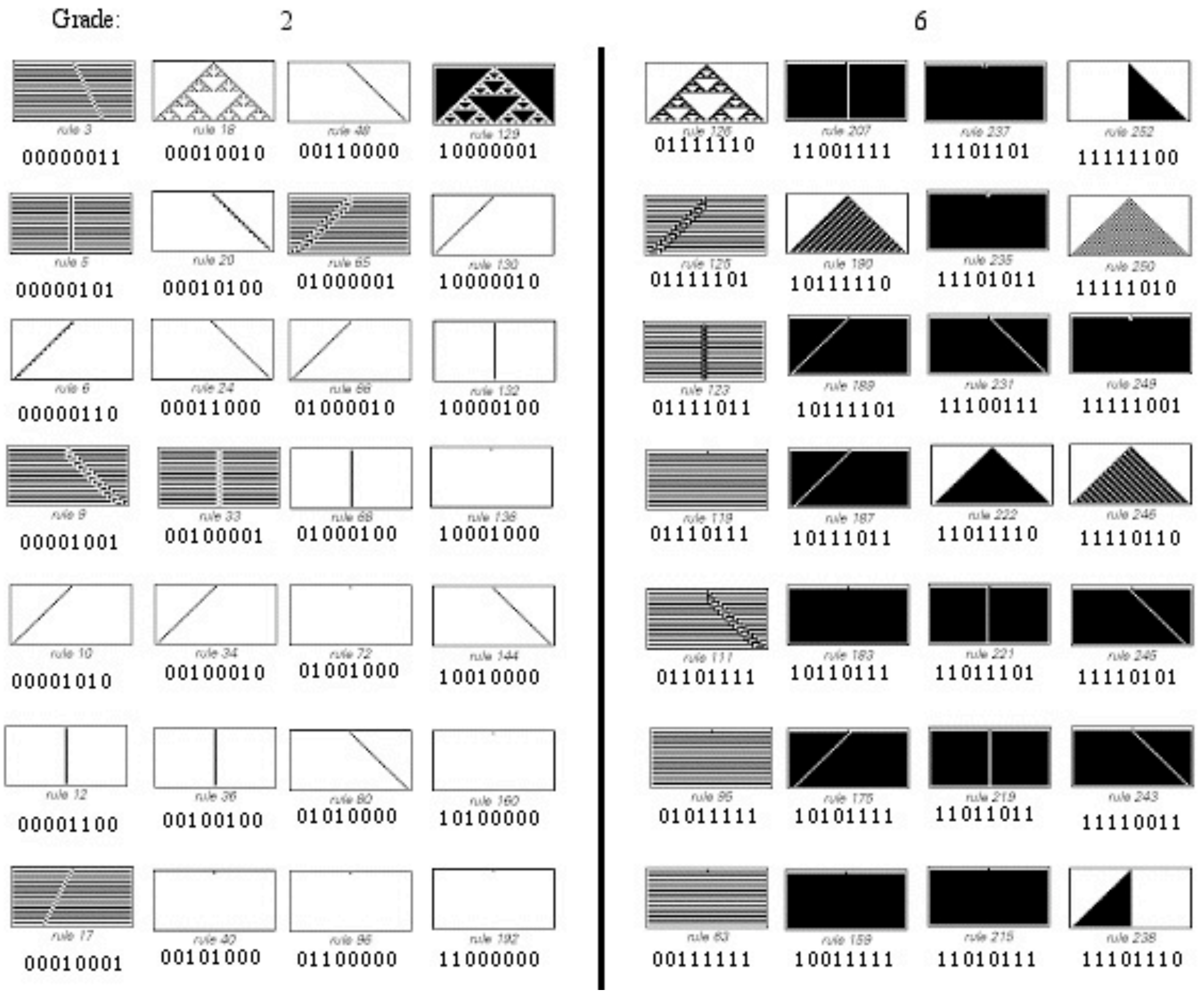
256 Elementary Rule Patterns

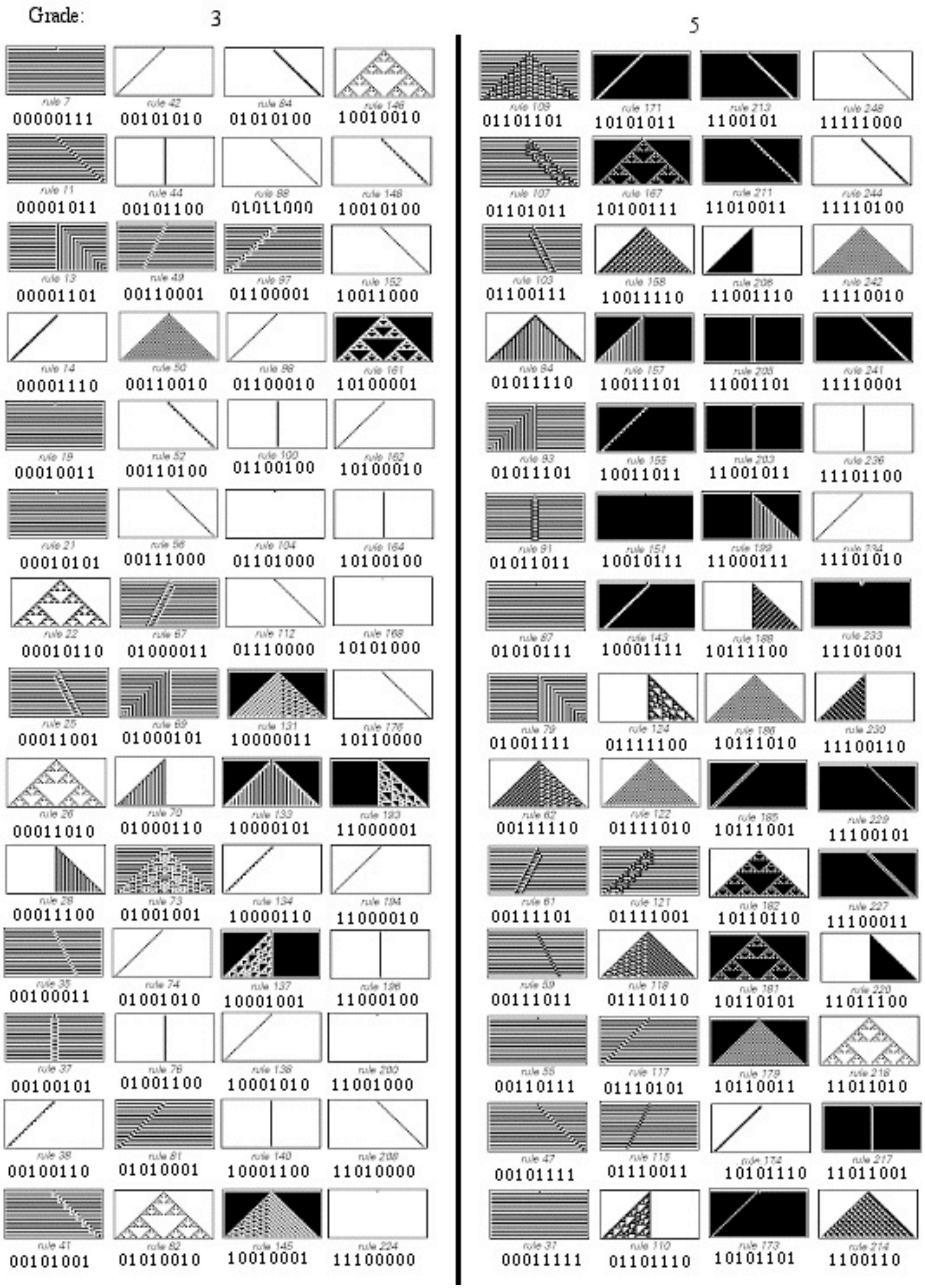
with Graded Structure

1 8 28 56 70 56 28 8 1

(images from "A New Kind of Science" by Stephen Wolfram (Wolfram 2002))




Grade:
2
6
rule 3
00000011
rule 18
00010010
rule 48
00110000
rule 129
10000001
rule 5
00000101
rule 20
00010100
rule 65
01000001
rule 130
10000010
rule 6
00000110
rule 24
00011000
rule 66
01000010
rule 132
10000100
rule 9
00001001
rule 33
00100001
rule 68
01000100
rule 136
10001000
rule 10
00001010
rule 34
00100010
rule 72
01001000
rule 144
10010000
rule 12
00001100
rule 36
00100100
rule 80
01010000
rule 160
10100000
rule 17
00010001
rule 40
00101000
rule 96
01100000
rule 192
11000000
rule 126
01111110
rule 207
11001111
rule 237
11101101
rule 252
11111100
rule 125
01111101
rule 190
10111110
rule 235
11101011
rule 250
11111010
rule 123
01111011
rule 189
10111101
rule 231
11100111
rule 249
11111001
rule 119
01110111
rule 187
10111011
rule 222
11011110
rule 246
11110110
rule 111
01101111
rule 183
10110111
rule 221
11011101
rule 245
11110101
rule 95
01011111
rule 175
10101111
rule 219
11011011
rule 243
11110011
rule 63
00111111
rule 159
10011111
rule 215
11010111
rule 238
11101110


Grade:
3
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00110010
01100010
10100001
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00110100
01100100
10100010
00010101
00111000
01101000
10100100
00010110
01000011
01110000
10101000
00011001
01000101
10000011
10110000
00011010
01000110
10000101
11000001
00011100
01001001
10000110
11000010
00100011
01001010
10001001
11000100
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01001100
10001010
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01010001
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11010000
00101001
01010010
10010001
11100000
01101101
10101011
1100101
11111000
01101011
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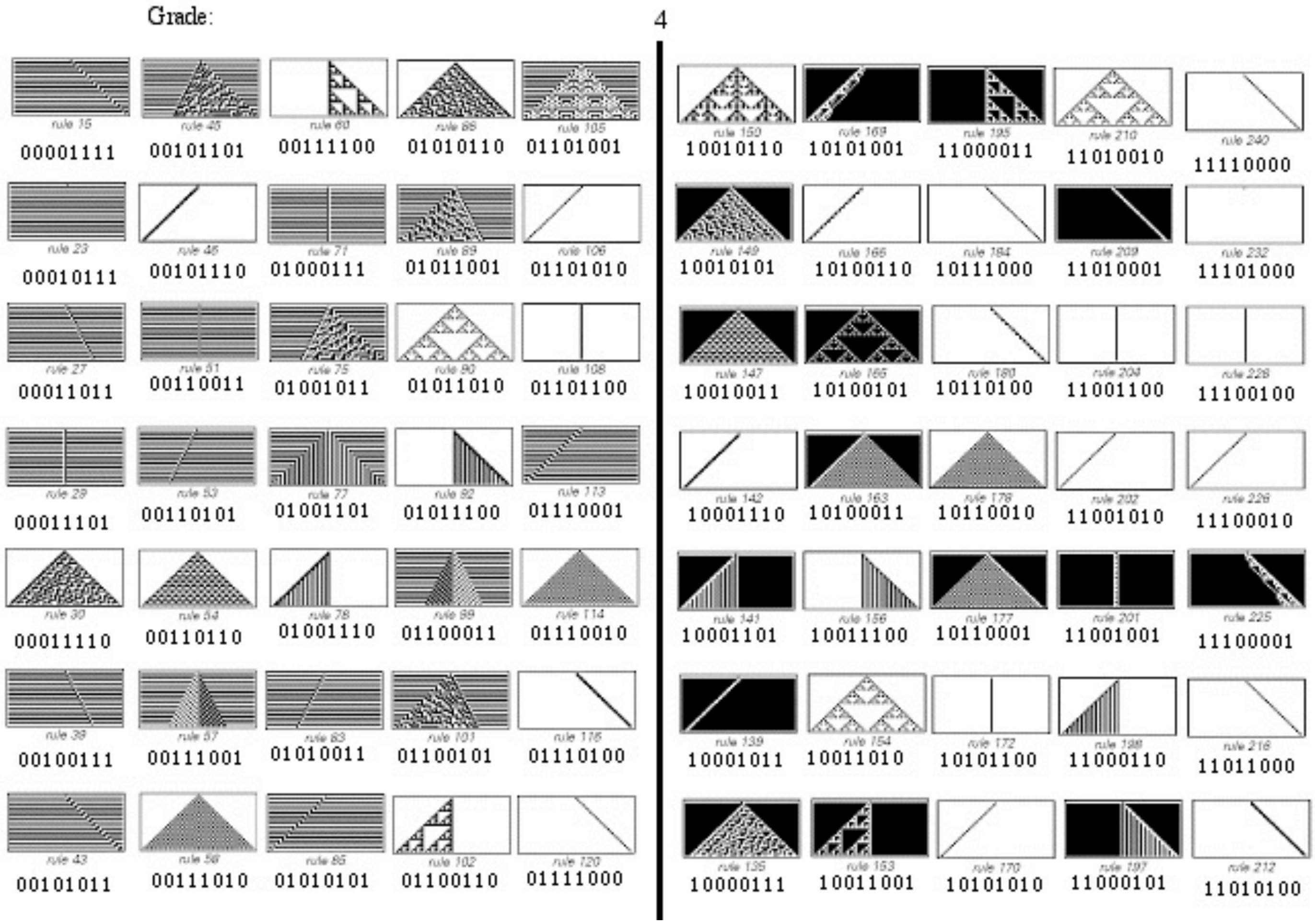
1100110


Grade:
4
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00101101
00111100
01010110
01101001
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00101110
01000111
01011001
01101010
00011011
00110011
01001011
01011010
01101100
00011101
00110101
01001101
01011100
01110001
00011110
00110110
01001110
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01110010
00100111
00111001
01010011
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10001101
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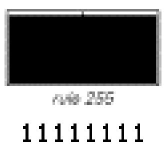
10110001
11001001
11100001
10001011
10011010
10101100
11000110
11011000
10000111
10011001
10101010
11000101
11010100

the 16 terms in the Cl(8) primitive idempotent

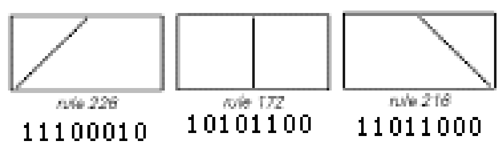
f = (1/2)(1 + e_1248) (1/2)(1 + e_2358) (1/2)(1 + e_3468) (1/2)(1 + e_4578) =

=(1/16)(1 + e_1248 + e_2358 + e_3468 + e_4578 + e_5618 + e_6728 + e_7138 - e_3567 - e_4671 - e_5712 - e_6123 - e_7234 - e_1345 - e_2456 + e_J)

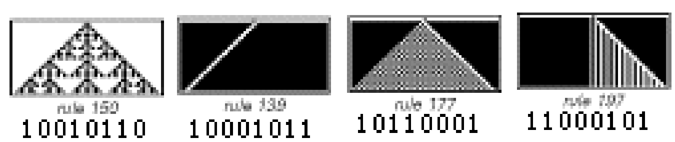
correspond to 16 of the 256 Cellular Automata



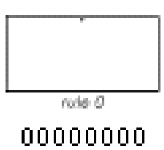
- + e_12345678



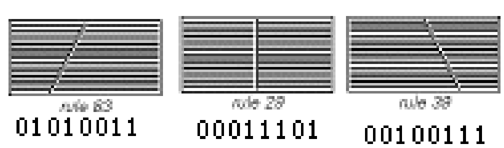
- + e_6728 + e_3468 + e_4578 to



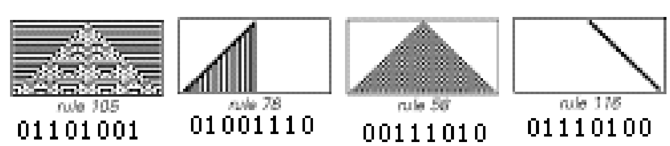
- + e_2358 + e_1248 + e_5618 + e_7138



- + 1 to



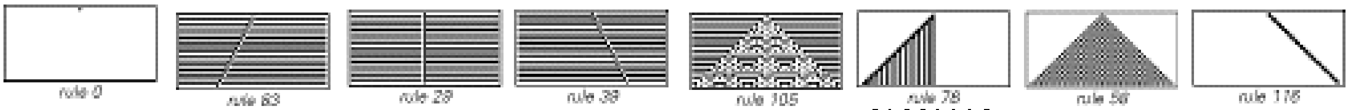
- - e_5712 - e_1345 - e_6123



- - e_4671 - e_7234 - e_2456 - e_3567

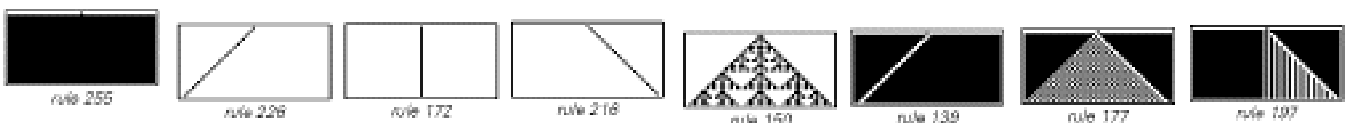
Note the Cl(0,8) = Cl(1,7) triality correspondences among:

- the 8 +half-spinors



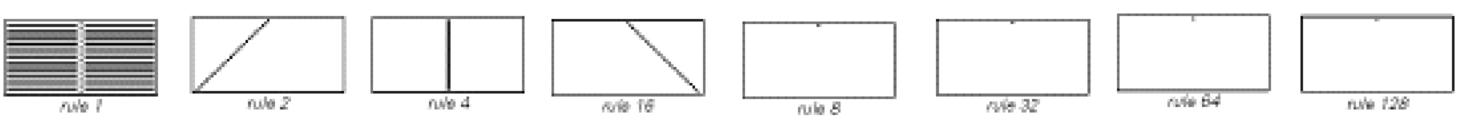
00000000 01010011 00011101 00100111 01101001 01001110 00111010 01110100

- the 8 -half-spinors



11111111 11100010 10101100 11011000 10010110 10001011 10110001 11000101

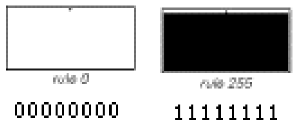
- the 8 vectors



00000001 00000010 00000100 00010000 00001000 00100000 01000000 10000000

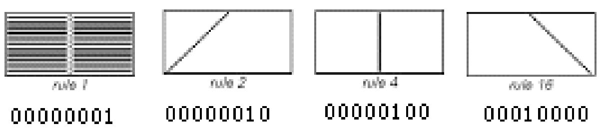
Note that:

the grade-0 scalars



are related to the Spinors and Primitive Idempotents of Cl(0,8);

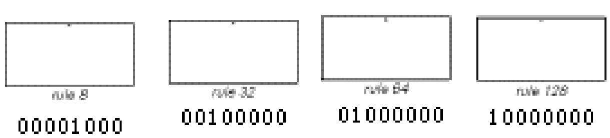
the grade-1 vectors 1, 2, 4, 16 (the subset sequence 2^0 = 1, 2^1 = 2, 2^2 = 4, 2^4 = 16 related to Fermat primes)



correspond to the 4 dimensions of physical spacetime;

- 1 gives a succession of bands, the procession of time;
- 2 gives a slope to the left, one of three space dimensions;
- 4 gives a vertical slope, a second of three space dimensions;
- 16 gives a slope to the right, the third of three space dimensions;

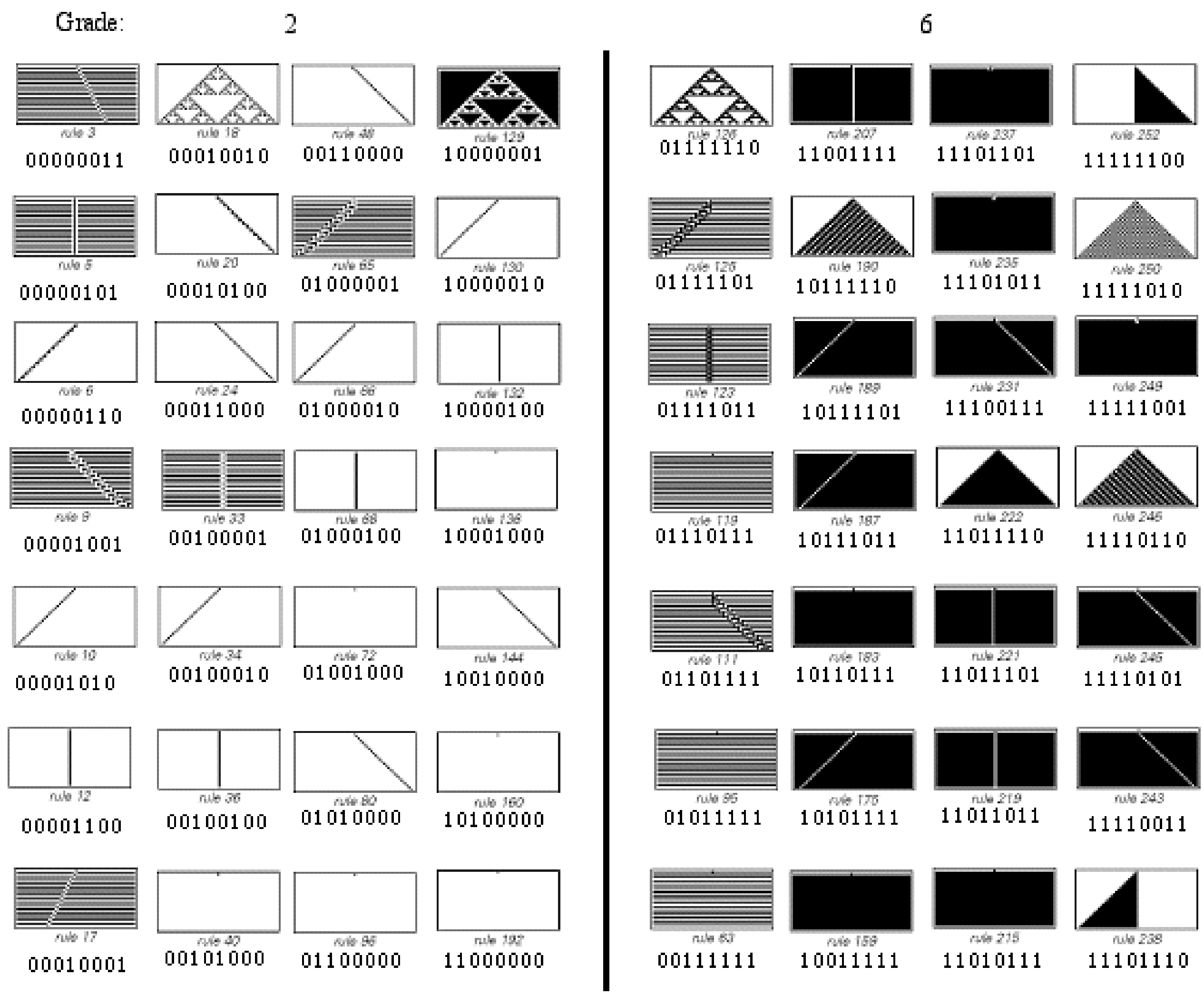
the grade-1 vectors 8, 32, 64, 128 (all giving all white)



correspond to the 4 dimensions of internal symmetry space;

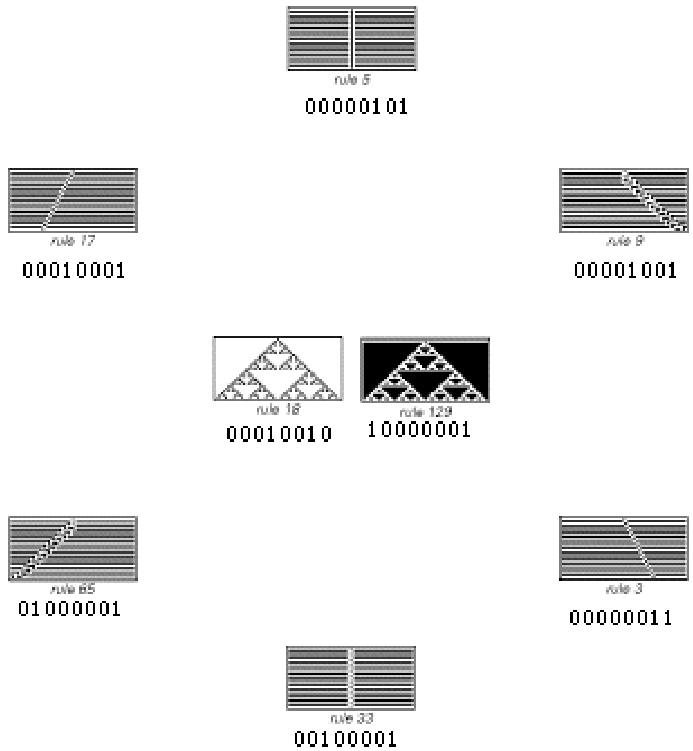
- rule 18 = 00010010 is the first rule to include both 16 = 00010000 with right slope and 2 = 00000010 with left slope and is the first rule with traingular self-similar fractal structure;
- rule 30 = 00011110 is the first rule to include 16, 8, 4, and 2 and is in the self-dual grade-4 and is the first rule with triangular chaotic behavior.

Here are all 28 rules for each of grades 2 and 6.



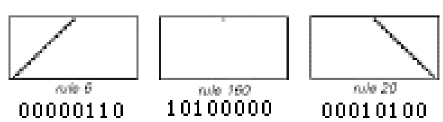
all 28 grade-2 bivectors correspond to the 28 generators of the Spin(8) Lie algebra;

8 of the grade-2 bivectors,



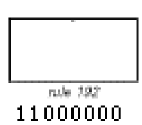
after dimensional reduction to 4-dimensional physical spacetime, correspond to the 8 generators of color force SU(3), whose root vector diagram is illustrated above;

3 of the grade-2 bivectors,



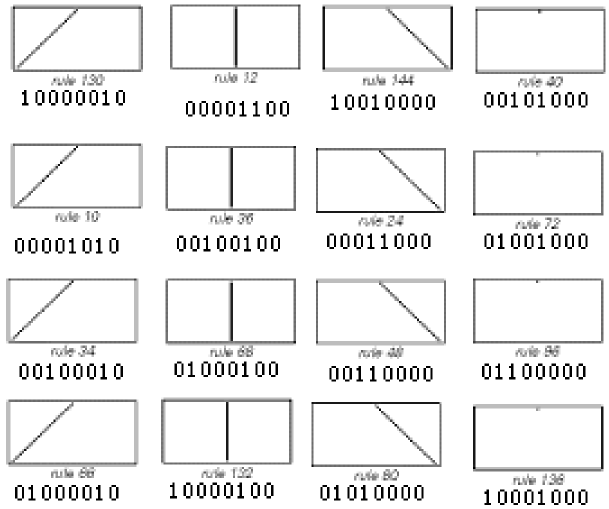
after dimensional reduction to 4-dimensional physical spacetime, correspond to the 3 generators of weak force SU(2);

1 of the grade-2 bivectors,



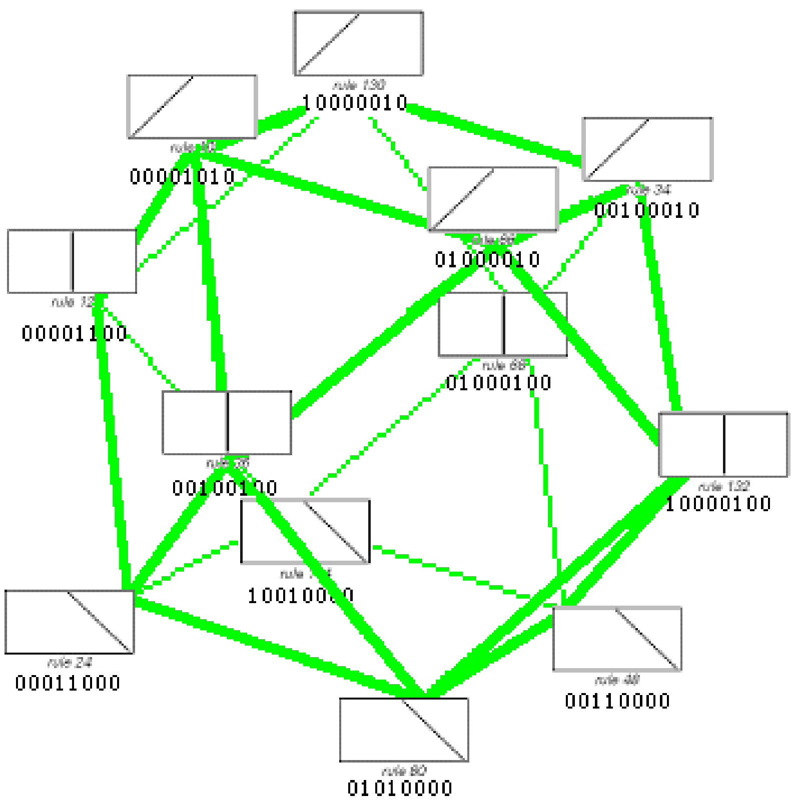
after dimensional reduction to 4-dimensional physical spacetime, correspond to the 1 generator of electromagnetic U(1);

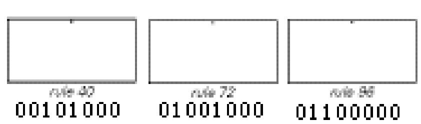
16 of the grade-2 bivectors,



after [dimensional reduction to 4-dimensional physical spacetime](), correspond to [the 16 generators of Gravity/Higgs/phase U(2,2)](). One of them

corresponds to the propagator phase U(1) while the other 15 correspond to the [Conformal]() Group SU(2,2) = Spin(2,4) [whose root vector diagram]()



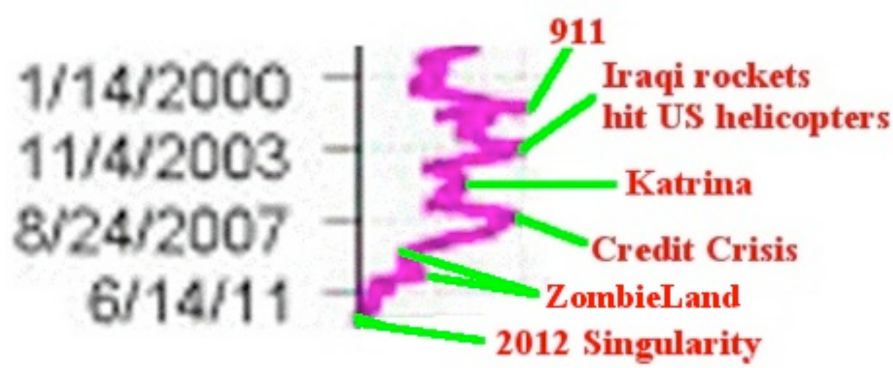


is a 12-vertex cuboctahedron (the other 3 bivectors corresponding to the 3 generators of the Cartan Subalgebra).

Can the African IFA tell us about Past and Future History ?

A few thousand years ago in Shang China, King Wen took the 64-element I Ching subset of IFA and modified its binary number Earlier Heaven sequence to describe the History of that Time and Place. In the 20th century, Terence McKenna used a similar technique to describe History of the Earth through 2012.
His Predictions for 2000 to 2012 seem to be quite relevant to the State of Our Earth as of now (2009):

Terence McKenna died in April 2000, but the Predictions of his



I Ching Resonance TimeWave of History live on to 2012.

Now (April 2009) we know that his 2000 to 2009 TimeWave History peaks:

- 9/11
- Iraqi War escalation (Rocket Attacks on USA helicopters)
- Hurricane Katrina
- the Credit Crisis of the USA/UK Global Financial System

all did happen coincident with TimeWave Peaks.

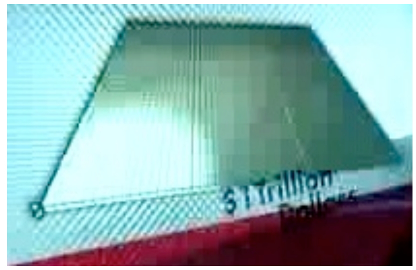
What does the TimeWave tell us about next 3 years, 2009 to 2012 ?

From now (April 2009) into 2010 is a nearly flat (or seemingly slightly rising) plateau that I call **ZombieLand**. In ZombieLand, Trillions of USA Dollars are fed to keep insolvent (but politically influential) financial institutions in an undead state - that is, not a living useful part of the Global Economy, but not officially recognized as being dead. To understand ZombieLand, you need to understand what is a **Trillion USA Dollars**. Glenn Beck (on his TV show) made a very good effort to show what is a Trillion USA Dollars. He said (I am paraphrasing): :

"... Here are $100 USA Dollars and $1 Million USA Dollars (It would fit in a suitcase):

Here are $1 Billion USA Dollars (It is about the size of a car) and **$1 Trillion USA Dollars (It is about the**

size of a Thousand Cars in a big parking lot.)



...".

Another way to visualize the size of $1 Trillion USA Dollars is to realize that the population of the USA (around 300 Million people) is about 100 Million Families.

$1 Billion is about $10 for every Family in the USA.

$1 Trillion is about $10,000 for every Family in the USA.

Another thing to keep in mind is that the amount of Bailouts so far is on the order of a few Trilllion USA Dollars, but **the total amount of possibly-worthless derivatives is on the order of $500 Trillion USA Dollars**. **That would be 500 x 10,000 = $5 Million for every Family in the USA**.

Since the population of the Earth is around 6 Billion, that would be almost $100,000 for every Person on Earth.

According to a 15 March 2009 huffingtonpost.com article by Arianna Huffington:

> "... The battle lines over **how to deal with the banking crisis** have been drawn. **On the one side are those who know what needs to be done. On the other are those who know what needs to be done -- but won't admit it**. Because it is against their self-interest. Unlike the conflict over the stimulus package, this is not an ideological fight. **This is a battle between the status quo and the future, between the interests of the financial/lobbying establishment and the public interest**. **What needs to be done is hard but straightforward**. As Martin Wolf of the Financial Times sums it up: "**Admit reality, restructure banks and, above all, slay zombie institutions at once.**" This tough love for bankers is being promoted by everyone from Nouriel Roubini, Paul Krugman, and Ann Pettifor to Niall Ferguson, the Wall Street Journal, and Milton Friedman's old partner, Anna Schwartz ... "They should not be recapitalizing firms that should be shut down," says Schwartz. "Firms that made wrong decisions should fail."... **Tim Geithner ... is on the wrong side of the issue, more worried about the banking industry than the American people**. Like Hank Paulson before him, Geithner appears more concerned about saving particular banks than saving the banking system. ... As Ann Pettifor puts it on HuffPost: "Much of Wall Street is effectively insolvent. **It's not that these banks lack cash or capital -- it's just that they're never going to meet all their financial liabilities -- i.e. repay their debts. Ever**." ...".

According to a 23 March 2009 democracynow.org interview of Paul Krugman:

> "... **"The Zombie Ideas Have Won" - Paul Krugman on $1 Trillion Geithner Plan to Buy Toxic Bank Assets** ... **Treasury Secretary Timothy Geithner** is preparing to unveil a plan ... to **purchase as much as $1 trillion in troubled mortgages and other assets from banks**. ... **The Obama administration has described the plan as a public-private partnership, but most of the actual money will be put up by the government**. ... Paul Krugman: A zombie

> idea is ... a bad idea, but it just keeps on coming back. ... to ... have taxpayers go in and buy ... these toxic assets ... 85 percent of the money is going to be a loan from the government, which is a non-recourse loan ... this is not Geithner. Ultimately, **the buck stops in the Oval Office.** The question is, why is President Obama going with the soft side, the hope over analysis, on this stuff? ... **the view still, apparently, dominant ... in this administration is that there's nothing really fundamentally wrong with the system. ... those people who we thought were so smart ... really are smart, and we want to keep them on the job**. ... the Obama administration is still partying like it's 2006. ...".

According to a 22 April 2009 economictimes.indiatimes.com article by Swaminathan S. Anklesaria Alyar:

> "... **the Obama administration is prolonging the recession by avoiding surgery to remove dead wood from its financial sector**. Some call this cowardice. Others, such as former IMF chief economist Simon Johnson, writing in The Atlantic, say **Wall Street has captured the White House**. ... Johnson says **the US now resembles Russia, where business oligarchs and government officials protect each others' financial interests, at the expense of the economy**. ... This ... highlights the priority given by the Obama administration to save the titans of Wall Street rather than end the recession quickly. ... **Technically, the financial sector is comprehensively bust**. It needs to recognise the losses, writing off trillions. ... **The market solution would be to force insolvent banks into bankruptcy, with shareholders and creditors taking a huge hit** ... Many titans of Wall Street will disappear ... **the Obama administration refuses to contemplate this obvious solution**. ... **Wall Street has captured the White House, so nothing will be done to imperil the politico-financial network that rules the US**. Robert Rubin and Hank Paulson, treasury secretaries of Clinton and Bush, were both from Goldman Sachs. Larry Summers, the current treasury secretary, earned millions as a hedge fund consultant. **In a market economy, well-managed companies should be rewarded with profits, while mismanaged companies should go bust. This basic rule has been suspended almost entirely for the titans of Wall Street. ... Accounting norms have been tweaked to permit zombie banks to pretend they are alive and solvent.**...".

If Terence McKenna's TimeWave is accurate, then for the next year or so we will live in a ZombieLand of accounting fiction and Empty Words of Hope,

only to see a sharp collapse of the USA/UK Global Financial System during 2010.

What will the 2012 Singularity Be Like ?

On the bad side, maybe the USA/UK leaders will be so unhappy at the prospect of losing their Global Hegemony that they might have a Temper Tantrum and Kick Over the Table of the Game of Life and have a Big War.

A more optimistic possibility is that a new Global Hegemony might form consistent with a Confucian Mandate of Heaven.

The Confucian I Ching (used by Terence McKenna in constructing the TimeWave) seems to be derived from Ancient African Mathematical Divination. Ron Eglash, in his book "African Fractals" (Rutgers 1999), said: "... fractal settlement patterns of Africa stand in sharp contrast to the Cartesian grids of Euro-American settlements. ... Euro-American cultures are organized by ... "top-down" organization. Precolonial African cultures included ... societies that are organized "bottom-up" rather than "top-down". ... most of the indigenous African societies were neither utterly anarchic, nor frozen in static order; rather they utilized an adaptive flexibility ... African traditions of decentralized decision making could ... be combined with new information technologies, creating new forms that combine democratic rule with collective information sharing ...".

How does such a New Collective Democracy fit with the most likely successor entity to the USA/UK Global Hegemon - China ?

Although some regard China as a top-down dictatorship, the true situation is quite different, as is exemplified by the Chinese Computer Hacker Community. According to a 23 April 2009 popsci.com article by Mara Hvistendahl: "... In the past two years, Chinese hackers have intercepted critical NASA files, breached the computer system in a sensitive Commerce Department bureau, and launched assaults on the Save Darfur Coalition, pro-Tibet groups and CNN. And those are just the attacks that have been publicly acknowledged. Were these initiated by the Chinese government? Who is doing this? ... **It's hundreds of thousands of everyday civilians**. ... This ... Hacker ... culture thrives on a viral, Internet-driven nationalism. ... **China's Internet patriots, who call themselves "red hackers," may not be acting on direct behalf of their government, but the effect is much the same**. ... The Red Hacker Alliance, often described in the Western press as a monolithic group, is in fact a loose association allowing disparate cells to coordinate their efforts. ... **the largest unifying characteristic is nationalism**. In a 2005 Hong Kong Sunday Morning Post article, a man identified as "the Godfather of hackers" explains, "Unlike our Western [hacker] counterparts, most of whom are individualists or anarchists, Chinese hackers tend to get more involved with politics because most of them are young, passionate, and patriotic." Nationalism is hip, and hackers -- who spearhead nationalist campaigns with just a laptop and an Internet connection -- are figures to revere. Henderson ... emphasizes that **the relationship between citizen and state is fluid in China**, and that the Chinese government tends not to prosecute hackers unless they attack within China. To Henderson, that lack of supervision is tacit approval, and it constitutes a de facto partnership between civilian hackers and the Chinese government. Jack Linchuan Qiu, a communications professor at the Chinese University of Hong Kong ... agrees. "**Chinese hackerism** is not the American 'hacktivism' that wants social change," he says. "**It's actually very close to the state. The Chinese distinction between the private and public domains is very small**." ...".

Perhaps the Chinese Government/People System, is in fact in accord with "African traditions of decentralized decision making",

and so might give us a Better World than the USA/UK system that is now collapsing under the weight of its own greedy profiteering.