

The Math of Hua Luogeng – 华罗庚 – Another Game in Town

by Frank D. (Tony) Smith, Jr. - June 2009

Edward Witten said (*Nature* 438 (22/29 December 2005) 1085):

“... Albert Einstein famously devoted the latter part of his life to seeking a ... ‘unified field theory’ ... String theory is the only known generalization of relativistic quantum field theory that makes sense. ... string theory ... may well be the only way to reconcile gravity and quantum mechanics ...”.

Steven Weinberg (in a 22 Feb 2009 *Telegraph* article by Graham Farmelo) is quoted as saying that “... for a truly fundamental theory of physics ... “strings are the only game in town” ...”.

Could Witten and Weinberg both be Wrong ?

Setting up Another Game in Town requires solving Three Tasks:

- 1 - Construct an EPR Physics Model that is consistent with Experimental Tests of Einstein-Podolsky-Rosen Reality;
- 2 - Use that EPR Physics Model to construct a Local Lagrangian that gives Gravity and the Standard Model with calculable Force Strengths and Particle Masses – such calculations will be seen to require the Math of Hua Luogeng;
- 3 - Quantize the Classical Local Lagrangian structure to get a Global Algebraic Quantum Field Theory (AQFT) which, as Bert Schroer said in hep-th/9608083, has “... emphasis on locality and ... insistence of separating local ... properties ... resid[ing] in ... the algebraic structure of local observables ... from global properties ... enter[ing] through ... states and ... representation spaces of local observables.

1 - An EPR Physics Model

Joy Christian in arXiv 0904.4259 “Disproofs of Bell, GHZ, and Hardy Type Theorems and the Illusion of Entanglement” says: “... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings for at least the Bell, GHZ-3, GHZ-4, and Hardy states. ... The alleged non-localities of these states ... result from misidentified [geometries] of the EPR elements of reality. ...

The correlations are ... the classical correlations among the points of a 3 or 7-sphere ... S^3 and S^7 ... are ... parallelizable ...

The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...”.

To go beyond the interesting but not completely physically realistic Bell, GHZ-3, GHZ-4, and Hardy states, we must consider more complicated spaces than S^3 and S^7 , but still require that they be parallelizable and be related to Clifford algebra structure.

As Martin Cederwall said in hep-th/9310115: “... The only simply connected compact parallelizable manifolds are the Lie groups [including $S^3 = SU(2)$] and S^7 ...”.

We know that $S^3 = SU(2) = Spin(4) / SU(2)$ so that it has global symmetry of $Spin(4)$ transformations and that 6-dimensional $Spin(4)$ is the grade-2 part of the 16-dimensional $Cl(4)$ Clifford algebra with graded structure $16 = 1 + 4 + 6 + 4 + 1$ (where grades are 0,1,2, ...).

We also know that $S^7 = Spin(8) / Spin(7)$ so that it has global symmetry of $Spin(8)$ transformations and that 28-dimensional $Spin(8)$ is the grade-2 part of the 256-dimensional $Cl(8)$ Clifford algebra with graded structure $256 = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1$.

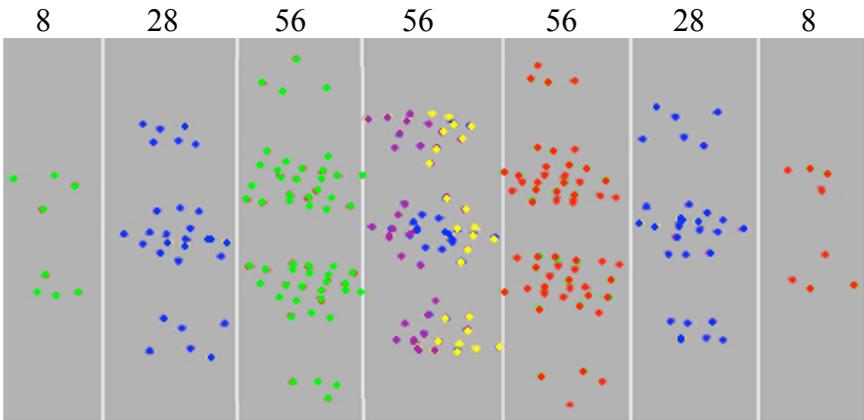
To get a Clifford algebra related parallelizable Lie group large enough to represent a realistic physics model, take the tensor product $Cl(8) \otimes Cl(8)$ which by the 8-periodicity property of Real Clifford algebras is $256 \times 256 = 65,536$ -dimensional $Cl(16)$ with graded structure $(1 \times 1) + (1 \times 8 + 8 \times 1) + (1 \times 28 + 28 \times 1 + 8 \times 8) + \dots = 1 + 16 + 120 + \dots$ whose $28 + 28 + 64 = 120$ -dimensional grade-2 part is $Spin(16)$ and whose spinor representation has $256 = 128 + 128$ dimensions.

$Spin(16)$ has $Cl(16)$ Clifford algebra structure and is a Lie group, and therefore parallelizable, but it has grade-2 bivector bosonic structure and so can only represent physical things like gauge bosons and vector spacetime, and cannot represent physical things like fermions with spinor structure.

However, if we add one of the two 128-dimensional $Cl(16)$ half-spinor representations to the bosonic adjoint 120-dimensional representation of $Spin(16)$, we get the $120 + 128 = 248$ -dimensional exceptional Lie group E_8 which Garrett Lisi in arXiv 0711.0770 used to try to describe a complete realistic physics model unifying Gravity and the Standard Model. Garrett Lisi acknowledges some flaws in his model of 0711.0770, but here we will use a variant of his approach that unifies Gravity and the Standard Model without such flaws.

248-dimensional E_8 has a 7-grading (due to Thomas Larsson)
 $8 + 28 + 56 + 64 + 56 + 28 + 8$
 (where grades are -3,-2,-1,0,1,2,3)

If 8 of the 64 central grade-0 elements are assigned to an 8-dimensional Cartan subalgebra of E_8 , the remaining $248 - 8 = 240$ elements are the 240 Root Vectors of E_8 which have a graded structure



that indicates a physical interpretation for each of them:

The 128 odd-grade Root Vectors of E8 come from the 128 half-spinors and are shown as

$$0 + 0 + 0 + 0 + 56 + 0 + 8 = 64 \text{ red}$$

The 8 grade +3 represent the basic state of 8 fermion particles.

$$8 + 0 + 56 + 0 + 0 + 0 + 0 = 64 \text{ green}$$

The 8 grade -3 represent the basic state of 8 fermion antiparticles.

The $120 - 8 = 112$ even-grade Root Vectors of E8 come from the 112 Root Vectors of Spin(16) and are shown as

$$0 + 28 + 0 + 8 + 0 + 28 + 0 = 64 \text{ blue}$$

The 8 grade 0 represent 8 basic spacetime dimensions.

$$0 + 0 + 0 + 24 + 0 + 0 + 0 = 24 \text{ purple}$$

The 24 represent the Root Vectors of a gauge group Spin(8).

$$0 + 0 + 0 + 24 + 0 + 0 + 0 = 24 \text{ gold}$$

The 24 represent the Root Vectors of another gauge group Spin(8).

2 - A Local Lagrangian that gives General Relativity and the Standard Model with Calculable Force Strengths and Particle Masses

Use the E8 physical interpretations to construct a Lagrangian
by
integration over 8-dim spacetime base manifold
of
curvature terms from the two Spin(8) gauge groups
and
a Dirac fermion particle-antiparticle term.

This differs from conventional Gravity plus Standard Model in four respects:

- 1 - 8-dimensional spacetime
- 2 - two Spin(8) gauge groups
- 3 - no Higgs
- 4 - 1 generation of fermions

These differences can be reconciled as follows:

Introduce (freezing out at lower-than-Planck energies) a preferred Quaternionic 4-dim subspace of the original (high-energy) 8-dim spacetime,
thus forming an 8-dim Kaluza-Klein spacetime $M_4 \times CP^2$
where M_4 is 4-dim physical spacetime and CP^2 is a 4-dim internal symmetry space.

Let the first Spin(8) gauge group act on the M_4 physical spacetime through the SU(3) subgroup of its U(4) subgroup.

Meinhard E. Mayer said (*Hadronic Journal* 4 (1981) 108-152):

“... each point of ... the ... fibre bundle ... E consists of a four-dimensional spacetime point x [in M_4] to which is attached

the homogeneous space $G / H [SU(3) / U(2) = CP^2] \dots$
the components of the curvature lying in the homogeneous space
 $G / H [= SU(3) / U(2)]$ could be reinterpreted as Higgs scalars
(with respect to spacetime $[M^4]$)

...

the Yang-Mills action reduces to
a Yang-Mills action for the h-components $[u(2) \text{ components }]$ of
the curvature over $M [M^4]$

and

a quartic functional for the “Higgs scalars”, which not only
reproduces the Ginzburg-Landau potential, but also gives the
correct relative sign of the constants, required for the BEHK ...
Brout-Englert-Higgs-Kibble ... mechanism to work. ...”.

So, freezing out of a Kaluza-Klein $M^4 \times CP^2$ spacetime plus internal
symmetry space produces a classical Lagrangian for the
 $SU(3) \times U(2) = SU(3) \times SU(2) \times U(1)$ Standard Model
Including a BEHK Higgs mechanism.

Let the second Spin(8) gauge group act on the M^4 physical
spacetime through its Conformal Subgroup $U(2,2) = Spin(2,4)$.

Rabindra Mohapatra said (section 14.6 of Unification and Supersymmetry, 2
nd edition, Springer-Verlag 1992):

“... gravitational theory can emerge from the gauging of conformal
symmetry ... we start with a Lagrangian invariant under full local
conformal symmetry and fix conformal and scale gauge to obtain
the usual action for gravity. ...”.

See also MacDowell and Mansouri (Phys. Rev. Lett. 38 (1977) 739) and
Chamseddine and West (Nucl. Phys. B 129 (1977) 39).

At this stage, we have reconciled the first 3 of the 4 differences
between our E8 Physics Model and conventional Gravity plus the
Standard Model. As to the fourth, the existence of 3 generations of

fermions, note that the 8 first generation fermion particles and the 8 first generation antiparticles can each be represented by the 8 basis elements of the Octonions O ,

and that the second and third generations can be represented by Pairs of Octonions OxO and Triples of Octonions $OxOxO$, respectively.

When the unitary Octonionic 8-dim spacetime is reduced to the Kaluza-Klein $M4 \times CP2$, there are 3 possibilities for a fermion propagator from point A to point B:

- 1 – A and B are both in $M4$, so its path can be represented by the single O ;
- 2 – Either A or B, but not both, is in $CP2$, so its path must be augmented by one projection from $CP2$ to $M4$, which projection can be represented by a second O , giving a second generation OxO ;
- 3 – Both A and B are in $CP2$, so its path must be augmented by two projections from $CP2$ to $M4$, which projections can be represented by a second O and a third O , giving a third generation $OxOxO$.

Therefore, all four differences have been reconciled, and our classical Lagrangian $E8$ Physics Model describes Gravity as well as the Standard Model with a BEHK Higgs mechanism.

However, for our classical Lagrangian $E8$ Physics Model to be said to be complete and realistic, it must allow us to calculate such things as Force Strengths and Particle Masses that are consistent with experimental and observational results.

This requires the use of the Math of Hua Luogeng, about whom Wang Yuan ("Hua Loo-Keng", translated by Peter Shiu, Springer 1999) said: "... A mathematician has to be judged by his research accomplishments and not by the number of university degrees earned. In Hua's case there are many of the former and none of the latter ... Teacher Hua only had a junior middle-school education ...

Hua visited the Soviet Union ...[in]... 1946 ... [Later]... in ... 1946 ... Hua Loo-Keng went to the Institute for Advanced Study at Princeton ... In the spring of 1948, Hua Loo-Keng was appointed full professor at the University of Illinois in Urbana. ... Hua's decision to return to China ... in 1950 ... was based on his belief ... that the Chinese Communist Party and the Chinese Government ... would ... support ... his wish ... for ... mathematics in China ... to arrive at international level ... Besides this, he saw the racial prejudice in the United States ...[and]... isolationist policy being implemented against the Chinese Communist Party and all its work ...

In 1935, E. Cartan proved that, under analytic mapping, there are precisely six types of irreducible homogeneous symmetric bounded domains. Of these there are two types of exceptional domains with dimensions 16 and ...[27]... and the remaining four types are called classical domains ... $R_I = \dots m \times n$ matrix ... $R_{II} = \dots$ symmetric matrices ... $R_{III} = \dots$ skew-symmetric matrix ... $R_{IV} [= \dots$ a homogeneous space of $2 \times n$ real matrices] ... Classical domains can thus be regarded as the generalisation of the unit disc in the plane to higher dimensions [They are the subject of Hua's book]... Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains [Institute of Mathematics, Academica Sinica, No. 4 in the A-series, Russian translation 1959, English translation 1963] ... Now let R be a bounded and simply connected domain whose points ...[are]... made up of n complex variables, so that the corresponding Euclidean space has real dimension $2n$. Suppose that L is a part of the boundary for R satisfying the following condition: Every analytic function in R takes its maximum modulus on L and that, for each point x in L , there is an analytic function $f(z)$ in R which takes its maximum modulus at x . We then call L the characteristic manifold of R [also known as the Shilov Boundary of R], and it is a uniquely determined compact manifold. Generally speaking, L is only a part of the boundary for R ... The following three kernels [and volumes of related geometric structures] can be computed: ... Bergman kernel ... Cauchy kernel ... Poisson kernel ...

Hua also generalized the notion of a bilinear (fractional linear) transformation in one complex variable to that for several complex variables ...

Hua ... seemed to have been the only person working on the subject ... but the work ... influenced ... the work on the theory of functions of several complex variables by the Russian mathematician I. I. Pyateskij-Shapiro ...”.

Hua’s calculated volumes related to the kernels and Shilov boundaries are the key to calculation of Force Strengths and Particle Masses. For example, the Lagrangian term for each of the Forces is integrated over the M4 physical spacetime base manifold, but each of the Four Forces sees M4 in terms of its own symmetry, consequently with its own measure which measure is proportional to Hua-calculated volumes. Since M4 was formed by a freezing out of a Quaternionic structure, M4 is a 4-dimensional manifold with Quaternionic structure and therefore can be seen as one of Joseph Wolf’s 4 equivalence classes:

for Electromagnetism: $T4 = U(1)^4$

for Weak Force: $S2 \times S2 = SU(2) / U(1) \times SU(2) / U(1)$

for Color Force: $CP2 = SU(3) / U(2)$

for Gravity: $S4 = Spin(5) / Spin(4) = Sp(2) / Sp(1) \times Sp(1)$

When we also take into account the relevant volumes related to the curvature term in the Lagrangian for each force, and the masses involved for forces with gauge bosons related to mass, the calculations produce results that are reasonably close to experimental observation. Full calculations of Force Strengths, the Dark Energy : Dark Matter : Ordinary Matter Ratio, Particle Masses, and Kobayashi-Maskawa Parameters, and discussion of details oversimplified here (such as signature and particle/spacetime polarizations beyond the basic level, etc.) are in my free pdf book “E8 and $Cl(16) = Cl(8) \times Cl(8)$ ” which is available at <http://www.tony5m17h.net/E8physicsbook.pdf> and <http://www.valdostamuseum.org/hamsmith/E8physicsbook.pdf>

3 - A Global Algebraic Quantum Field Theory (AQFT)

Since the E8 classical Lagrangian is Local, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing E8 Global Time.

Mathematically, this is done by embedding E8 into Cl(16) and using a copy of Cl(16) to represent each Local Lagrangian Region.

A Global Structure is then formed by taking the tensor products of the copies of Cl(16). Due to Real Clifford Algebra 8-periodicity, $Cl(16) = Cl(8) \times Cl(8)$ and any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of Cl(8), and therefore of $Cl(8) \times Cl(8) = Cl(16)$.

Just as the completion of the union of all tensor products of 2×2 complex Clifford algebra matrices produces the usual Hyperfinite III von Neumann factor that describes creation and annihilation operators on the fermionic Fock space over $C^{(2n)}$ (see John Baez's Week 175),

we can take the completion of the union of all tensor products of $Cl(16) = Cl(8) \times Cl(8)$ to produce a generalized Hyperfinite III von Neumann factor that gives a natural Algebraic Quantum Field Theory structure for our E8 Physics model,

thus

making it a complete realistic theory that satisfies Einstein's criteria (quoted by Wilczek in the winter 2002 issue of *Deadalus*) :

“... a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e., intelligibility, of nature: there are no arbitrary constants ...

that is to say,

nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur ...”.

EPR (Christian) => E8 (Lisi) + Cl(16) (Llull)

(Mayer) => M4xCP2 (Batakis) + Higgs + Standard Model

(MacDowell-Mansouri) => Conformal Gravity

(Segal) => Dark Energy : DM : OM

(HUA) => Force Strengths + Particle Masses = EXPERIMENTAL TESTS

Feynman:

The whole purpose of physics is to find a number, with decimal points, etc!

Otherwise you haven't done anything.

(Segal-Connes) => Clifford Real-Periodic HyperFinite Factor AQFT

Gradings and Triality

Frank Dodd (Tony) Smith, Jr. - August 2009

256-dimensional Clifford Algebra $Cl(8)$ has graded structure with grades 0,1,2,3,4,5,6,7,8

$$1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1$$

In E_8 physics, the $Cl(8)$ graded structure has physical interpretation:

8 = Vector Spacetime

28 = Spin(8) Gauge Bosons

1+70+1 contains 16 spinors(1+14+1) as components of $Cl(16)$ Primitive Idempotent, which 16 spinors represent 8 Fermion Particles plus 8 Fermion AntiParticles

248-dimensional Lie algebra E_8 has graded structure with grades -3,-2,-1,0,+1,+2,+3

$$8 + 28 + 56 + 64 + 56 + 28 + 8$$

with even part $28 + 64 + 28 = 120$ dim Adjoint(D_8)

and odd part $8 + 56 + 56 + 8 = 128$ -dim Half-Spinor(D_8)

In E_8 physics, the E_8 graded structure has physical interpretation:

8 = Fermion Particles

64 = 8 Spacetime + 28 Spin(8) Gauge Bosons + 28 (additional) Spin(8) Gauge Bosons

8 = Fermion AntiParticles

The $Cl(8)$ and E_8 Graded Structures and their Physical Interpretations are related by Triality.

First: $Cl(8)$ Triality gives isomorphisms among

- **$Cl(8)$ grade 1 Vector Spacetime**
- **$Cl(8)$ grades 0 and 4 Fermion Particles**
- **$Cl(8)$ grades 0 and 4 Fermion AntiParticles**

Second: Triality gives isomorphisms among

- Cl(8) grade 1 Vector Spacetime
- E8 grade -3 Fermion Particles
- E8 grade +3 Fermion AntiParticles

Third: Triality applied to Fermion Particle-AntiParticle Pairs and to BiVectors = Vector Pairs, along with Hodge Duality and taking into account Dirac Gamma - Spacetime Structures, gives isomorphisms among

- 28 of the E8 grade 0 Gauge Bosons
- the other 28 E8 grade 0 Gauge Bosons
- 28 Cl(8) grade 2 Gauge Bosons
- 28 Cl(8) grade 6 Dual-Gauge-Bosons
- 28 E8 grade -2 points representing 7 Dirac Gamma components of 4 Physical Spacetime dimensions of 8-dimensional Kaluza-Klein Vector Spacetime
- 28 E8 grade +2 points representing 7 Dirac Gamma components of 4 CP2 Internal Symmetry Space dimensions of 8-dimensional Kaluza-Klein Spacetime

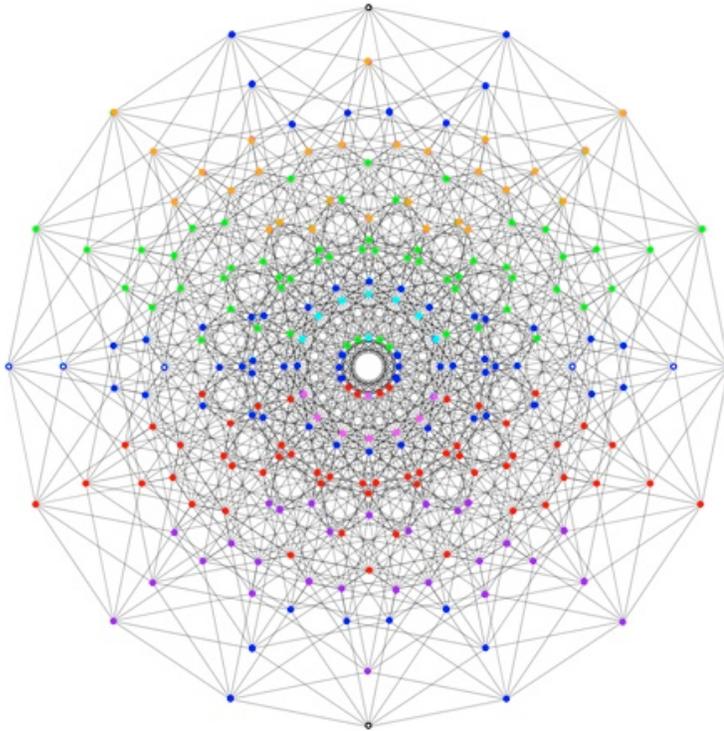
To see how all this works, start by looking at the Cl(8) Graded Structure

and a representation of $16 \times 16 = 256$ -dimensional Cl(8) as the $2^8 = 256$ vertices of an 8-dimensional HyperCube (images here are constructed from Mathematica and some graphics computer programs such as AppleWorks). This HyperCube image has 12 rings of points, with rings having 16,16,16,32,16,16,32,16,32,32,16,16 points respectively, for a total of $8 \times 16 + 4 \times 32 = 256 = 16 \times 16 = 2^8$ points.

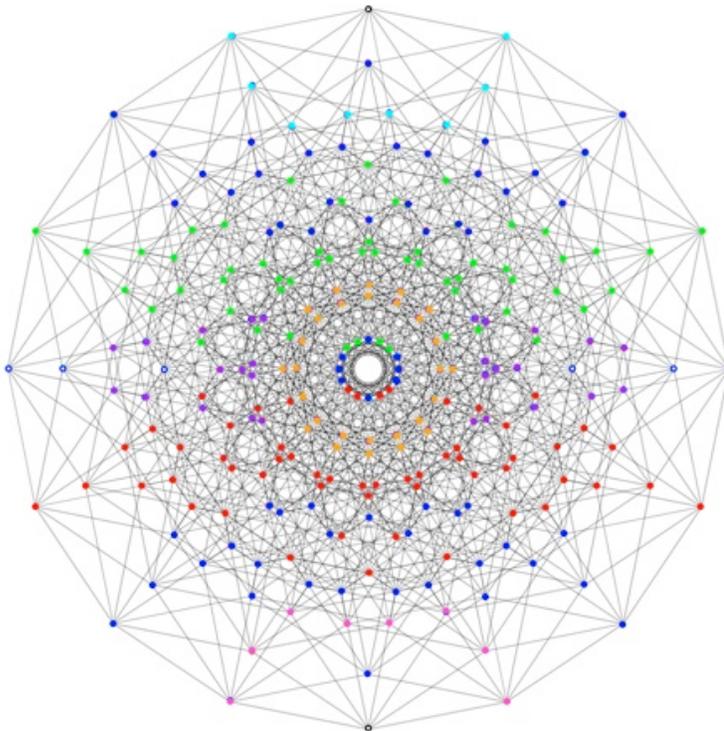
In the image the 256 basis elements of Cl(8) are represented as follows:

- the black point (with central white dot) at the top represents the grade 0 scalar Dirac Gamma
- the 8 blue points near the top represent grade 1 Vector Spacetime
- the 28 gold points near the top represent grade 2 Spin(8) Gauge Bosons
- the 56 green points above the center represent 7 polarization states of 8 Fermion Particles of grade 3
- the 70 points of grade 4 in the center are
 - 7 of the 8 cyan points represent the 7 Quark-Electron-type Fermion Particle Spinor elements which along with the scalar form half of a Cl(8) Primitive Idempotent
 - the 8th cyan point represents the Neutrino-type Fermion Particle Spinor element
 - 7 of the 8 magenta points represent the 7 Quark-Electron-type Fermion AntiParticle Spinor elements which along with the pseudo-scalar form the other half of a Cl(8) Primitive Idempotent
 - the 8th magenta point represents the Neutrino-type Fermion AntiParticle Spinor element
 - 48 blue points represent 6 Dirac Gamma components of 8-dimensional Vector Spacetime
 - 6 blue points (with central white dot) represent the Dirac Gammas beyond the scalar and pseudoscalar
- the 56 red points below the center represent 7 polarization states of 8 Fermion AntiParticles of grade 5
- the 28 purple points near the bottom represent grade 6 dual (position-momentum duality) Spin(8) Gauge Bosons
- the 8 blue points near the bottom represent grade 7 dual (position-momentum duality) Vector Spacetime
- the black point (with central white dot) at the bottom represents the grade 8 pseudo-scalar Dirac Gamma

The $1+6+1 = 8$ black and blue points with central white dot, of grades 0,4,8, are the elements of 256-dimensional Cl(8) that are not found in 248-dimensional E8.



Applying the Triality relations to the $Cl(8)$ Graded Structure gives the following picture of E_8 Graded Structure (keep in mind that the $1+6+1$ central white dot points are not present in $8+28+56+64+56+28+8 = 248$ -dimensional E_8):



In the image the 248 basis elements of E_8 are represented as follows

- the 8 cyan grade -3 points near the top represent Fermion Particles
- the 28 blue grade -2 points near the top represent 7 Dirac Gamma components of 4 Physical

Spacetime dimensions of 8-dimensional Kaluza-Klein Vector Spacetime

- the 56 green grade -1 points above the center represent 7 polarization states of 8 Fermion Particles
- the 64 points of grade 0 in the center are
 - 8 blue points represent 8-dimensional Kaluza-Klein Spacetime
 - 28 gold points represent Spin(8) Gauge Bosons
 - 28 purple points represent a second set of Spin(8) Gauge Bosons
- the 56 red grade +1 points below the center represent 7 polarization states of 8 Fermion AntiParticles
- the 28 blue grade +2 points near the bottom represent 7 Dirac Gamma components of 4 CP2 Internal Symmetry Space dimensions of 8-dimensional Kaluza-Klein Spacetime
- the 8 magenta grade + 3 points near the bottom represent Fermion AntiParticles

Using the works of Conway and Sloane (Sphere Packings, Lattices and Groups) and Coxeter (Regular Complex Polytopes, Twelve Essays, Kaleidoscopes), we can see that: The basic figure of $C1(8)$ is the 8-real-dimensional HyperCube (8-cube) with 256 vertices at each of which are 8 edges for a total of $256 \times 8 / 2 = 128 \times 8 = 1024$ edges. At each vertex are 28 squares, 56 cubes, 70 tesseracts, 56 5-cubes, 28 6-cubes, 8 7-cubes, and 1 8-cube for totals of $64 \times 28 = 1792$ squares, $32 \times 56 = 1792$ cubes, $16 \times 70 = 1120$ tesseracts, $8 \times 56 = 448$ 5-cubes, $4 \times 28 = 112$ 6-cubes, $2 \times 8 = 16$ 7-cubes, and 1 8-cube (the 8-dimensional HyperCube itself) as described by Coxeter in his works (such as Regular Complex Polytopes)

The 8-cube forms a $Z8$ lattice, the alternate checkerboard vertices of which form a $D8$ lattice that is the root vector lattice of the 120-dimensional $D8$ Lie algebra whose root vector polytope has 112 vertices. There are 128 alternate checkerboard vertices in each 8-cube of the $Z8$ lattice, and they correspond to a 128-dimensional half-spinor representation of $D8$.

The 256 vertices of 8-cube $(+/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1)$ lie at distance $\sqrt{8} = 2\sqrt{2}$ from the origin among the 2160 vertices in that second layer out from the origin of an $E8$ lattice 5_{-21} which has $2160 = 112 + 256 + 1792$ vertices with $112 = (+/-2, +/-2, 0, 0, 0, 0, 0, 0)$ and $256 = (+/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1, +/-1)$ which correspond to 112 $D8$ root vectors + 128 $D8$ +half-spinors + 128 $D8$ -half-spinors + 1792. Note that the 128 $D8$ +half-spinors plus the 128 $D8$ -half-spinors form, as the two mirror image alternate checkerboards, the 256 vertices of an 8-cube, and also that $E8$ is made up of two $D8$ lattices, one containing the 112 $D8$ root vectors (the 112 $D8$) and the other containing one of the two 128 $D8$ half-spinors (the 128 $D8$) as described by Conway and Sloane: "... The covering radius ... the distance from a lattice point to a deep hole ... of D_n increases with n , and when $n = 8$ it is equal to the minimal distance between the lattice points. So when $n \geq 8$ we can slide another copy of $D8$ in between the points of $D8$, doubling the number of points ... without reducing the distance between them ... When $n = 8$ this construction is especially important, the lattice ... being known as $E8$...". The 128 $D8$ has by its defining 128 vertices one alternate checkerboard half of an 8-cube, and the 112 $D8$ contains, by virtue of being a $D8$ lattice, 128 vertices that can be seen as another alternate checkerboard half of the 8-cube, so that the Triality of subtle supersymmetry in $E8$ physics can be seen as automorphisms (based on duality of the alternate checkerboard halves of the 8-cube) among 112 $D8$ and each of the two 128 $D8$ lattices. To see how $E8$ is made up of 112 $D8$ plus a 128 $D8$, note that for any dimension N , the D_N root vectors can be shrunken and put around the origin to form a D_N lattice. For the 112 $D8$ root vectors, there are enough deep holes to shrink and put one of the two 128 $D8$ half-spinor sets of alternate checkerboard parts of the 8-cube in the deep holes.

The $112+128 = 240$ -vertex inner first layer lies at distance 2 from the origin of the 5_{-21} lattice and corresponds to the 240 $E8$ root vectors:

$112 = (+/-1, +/-1, 0, 0, 0, 0, 0, 0)$ with all possible permutations and signs for a total of $4 \times 8 \times 7 / 2 = 112$

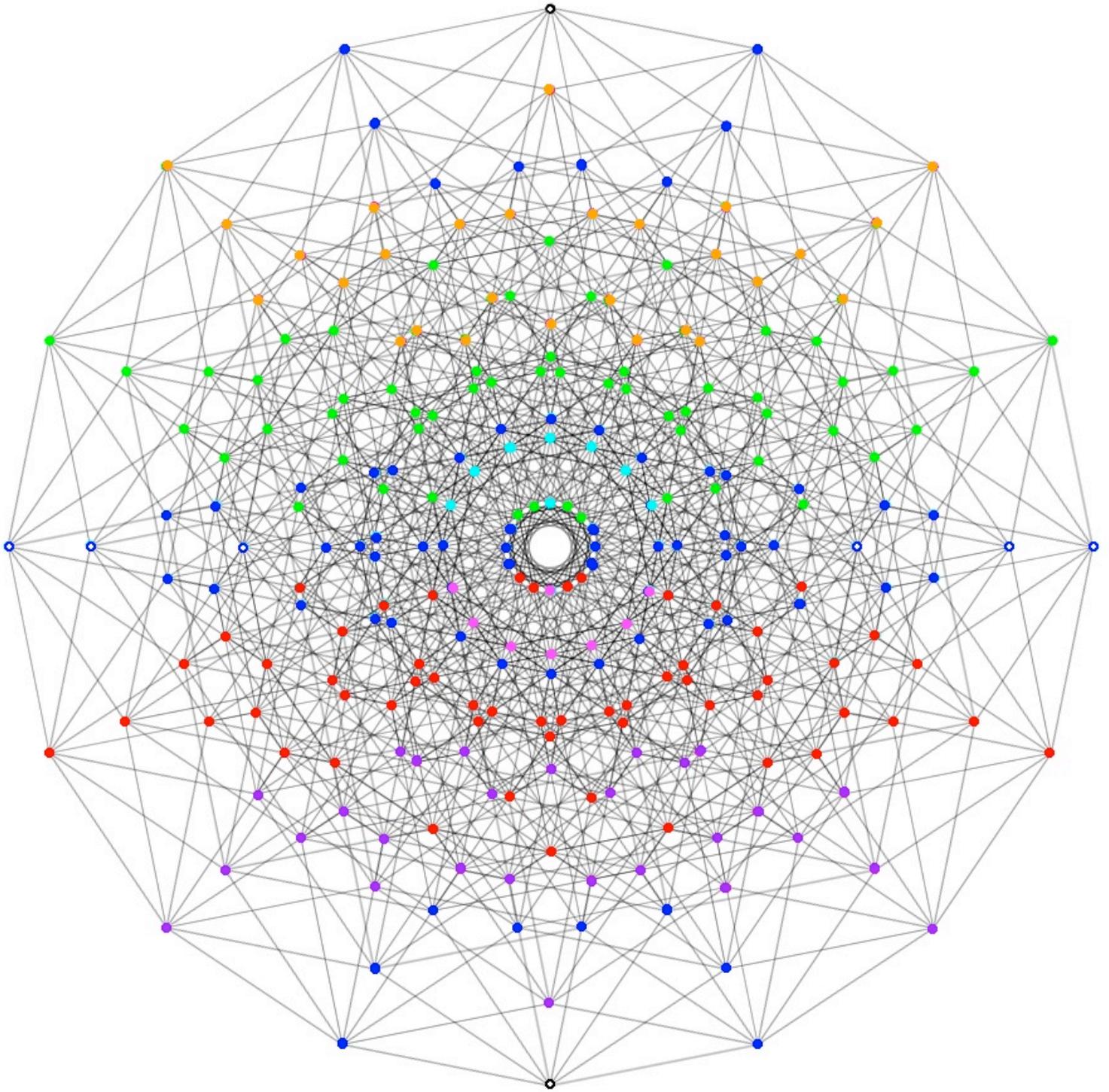
$128 = (+/-1/2, +/-1/2, +/-1/2, +/-1/2, +/-1/2, +/-1/2, +/-1/2, +/-1/2)$ with an even number of minus signs for a total of $2^8 / 2 = 128$

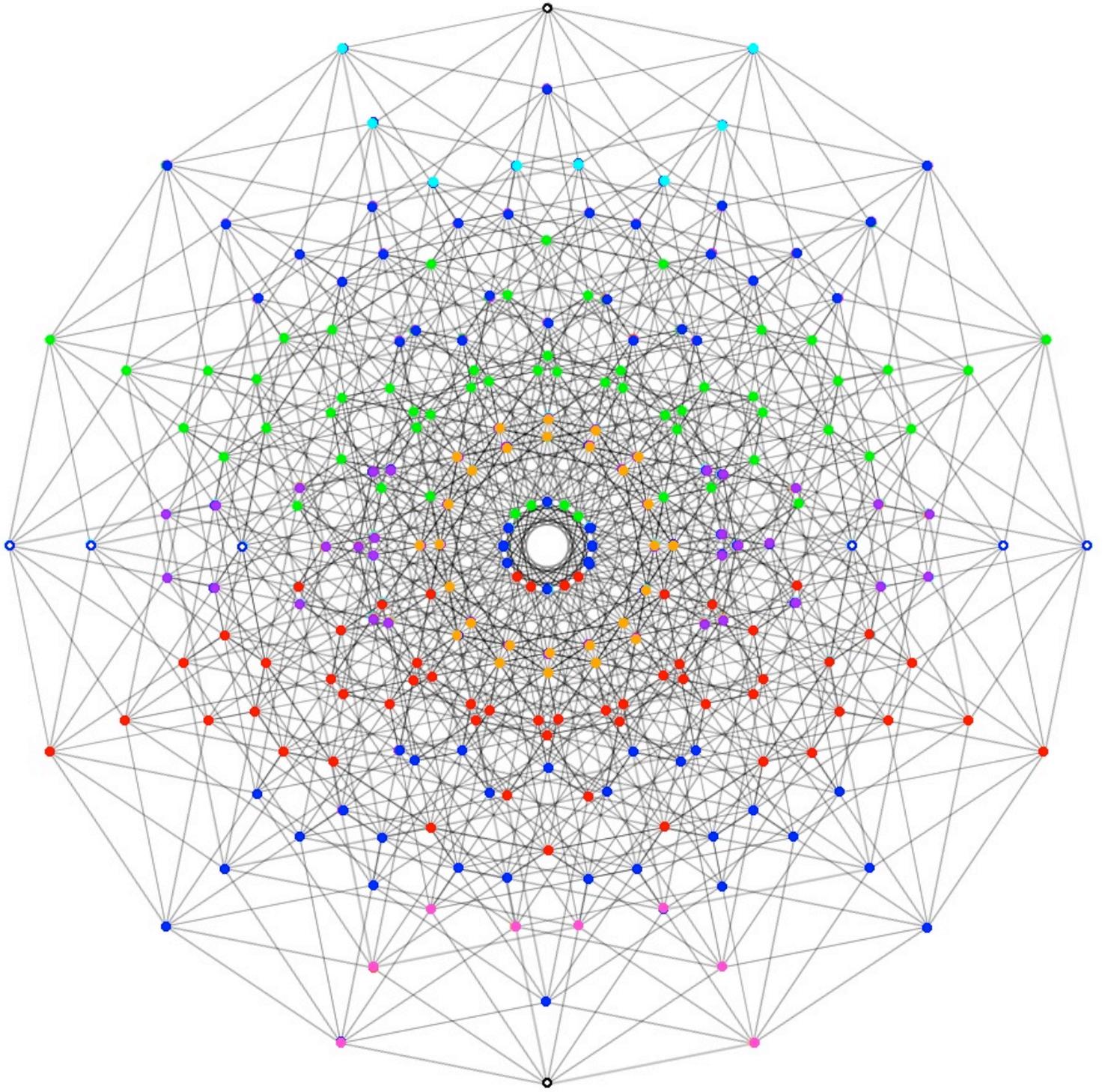
The 112 root vector polytope vertices of the original $D8$ plus the 128 alternate checkerboard 8-cube vertices of the second $D8$ form the 240 root vector vertices of the 248-dimensional Lie algebra $E8$, which can therefore be seen as the sum of the 120-dimensional Lie algebra $D8$ plus one of its 128-dimensional half-spinor representations.

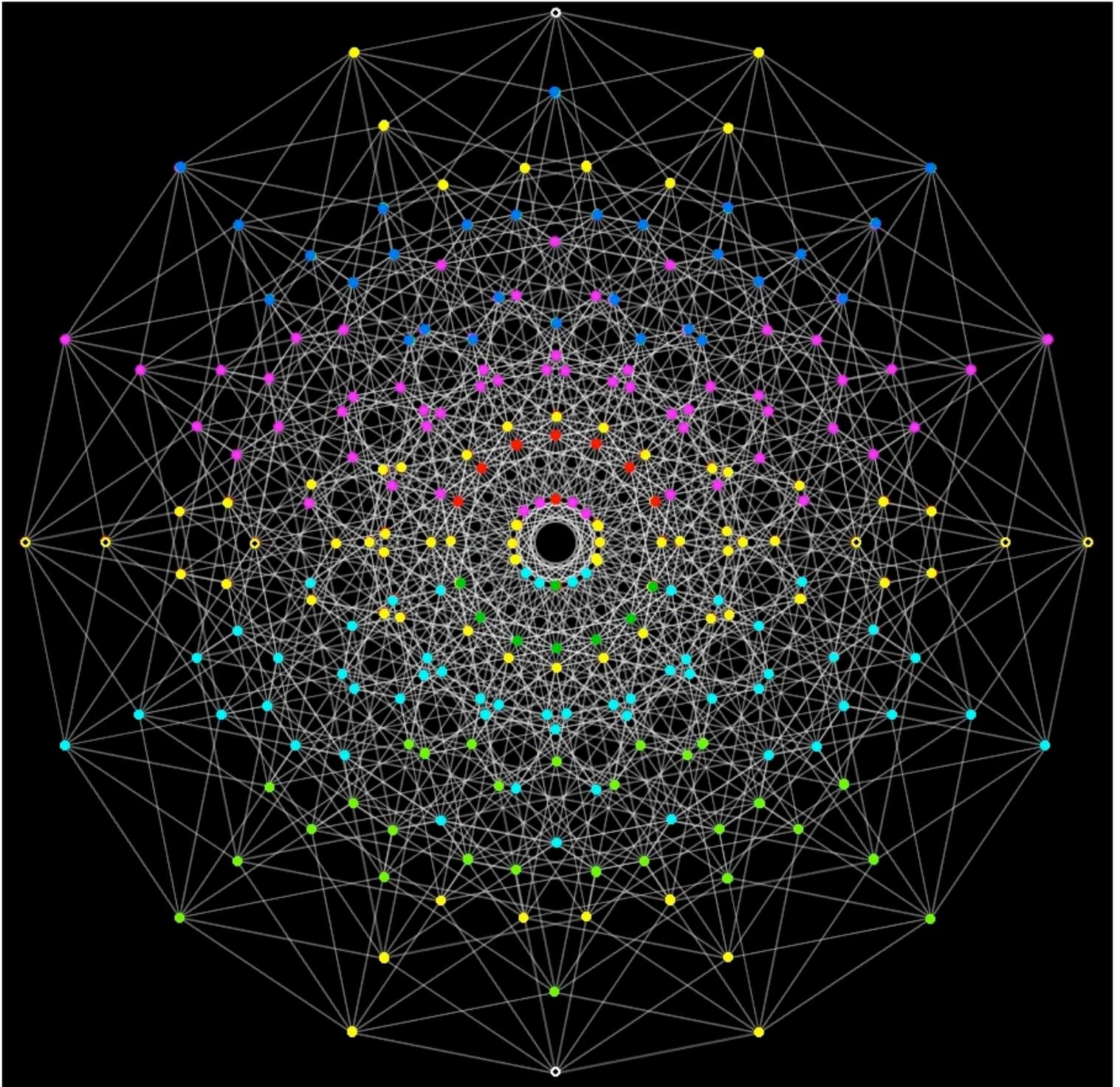
The basic real figure of $E8$ is the 8-real-dimensional Witting Polytope 4_{-21} with 240 vertices at each of which are 56 edges for a total of $240 \times 56 / 2 = 6720$ edges. The 56 real edges correspond to the 56 vertices of the 7-real-dimensional polytope 3_{-21} and to the 56-dimensional representation of $E7$ and to the 56-dimensional Freudenthal algebra $Fr(3,O)$ of which $E6$ is its automorphism group.

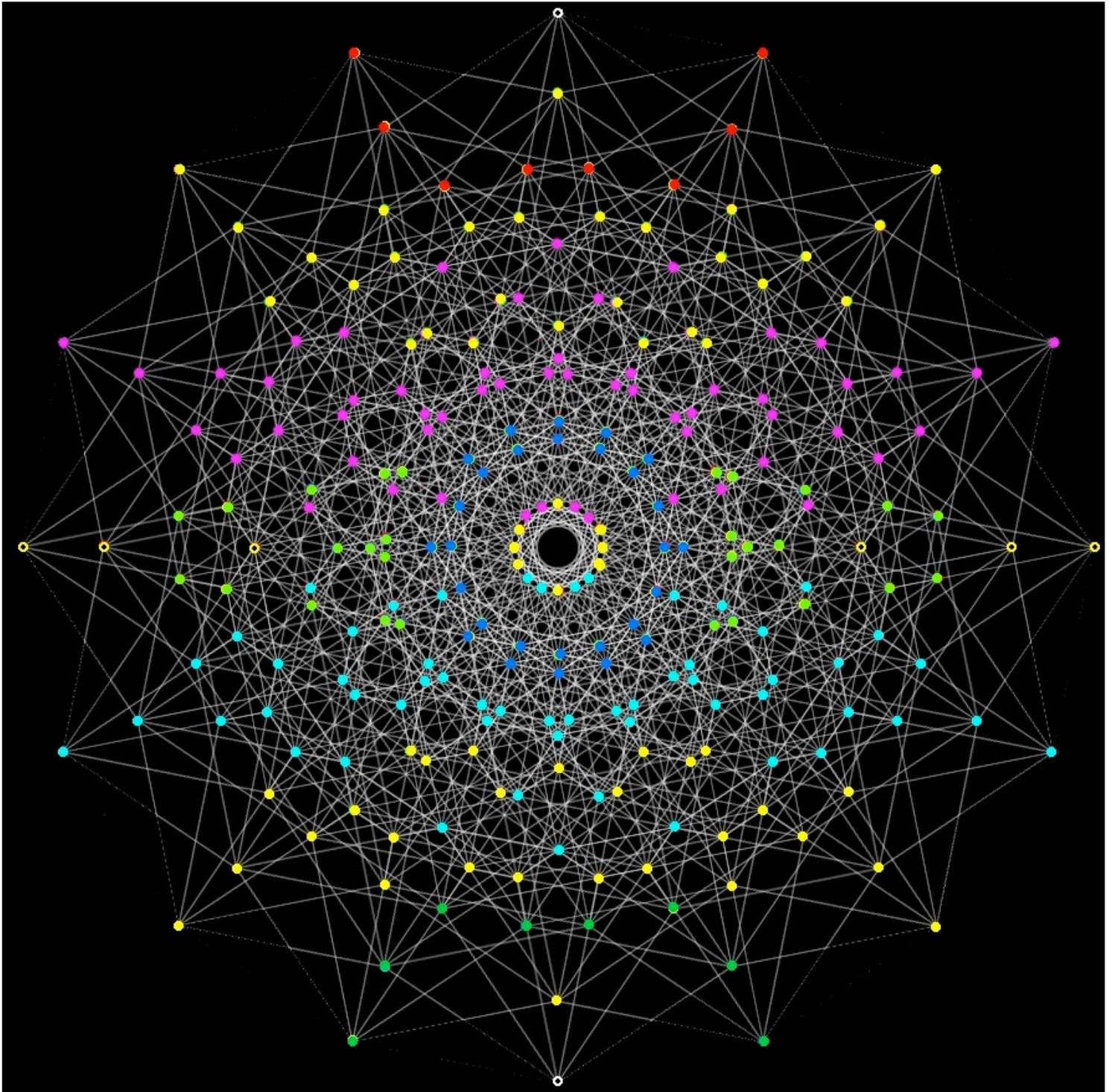
The basic complex figure of $E8$ is the 4-complex-dimensional Witting Polytope with 240 vertices at each of which are 27 complex (2-real-dimensional triangular) edges for a total of $240 \times 27 / 3 = 2160$ complex edges. The 27 complex edges correspond to the 27 lines on a general cubic surface and the 27 vertices of the 6-real-dimensional polytope 2_{-21} and to the 27-dimensional representation of $E6$ and to the 27-dimensional Jordan algebra $J(3,O)$ of which $F4$ is its automorphism group.

Here are higher-resolution versions of the above two images, first in the above colors and white background, and then in black background and inverted colors (black->white, cyan->red, blue->yellow, green->magenta, gold->blue, purple->pale green, red->cyan, magenta->green):



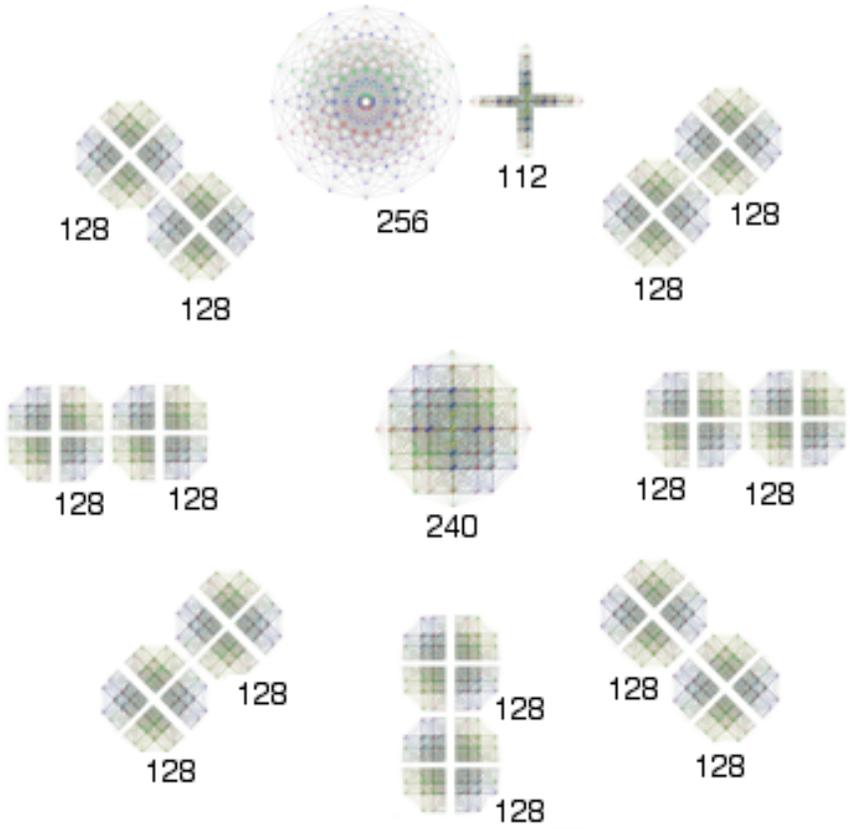






E8 Physics

by Frank D. (Tony) Smith, Jr. – September 2009



The First and Second shells of an E8 lattice have 240 and $2160 = 112 + 256 + 7(128+128)$ vertices.

The 256 is an 8-HyperCube with vertices

$(\pm 1, \pm 1)$ of which

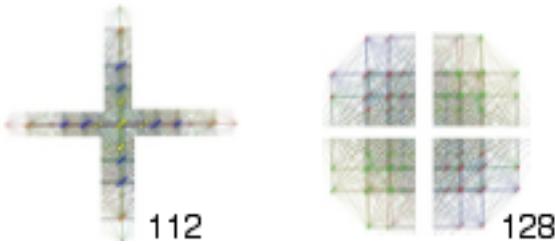
one checkerboard half represents the 128 +half-spinors of D8 and the other mirror image checkerboard half represents the 128 -half-spinors of

D8.

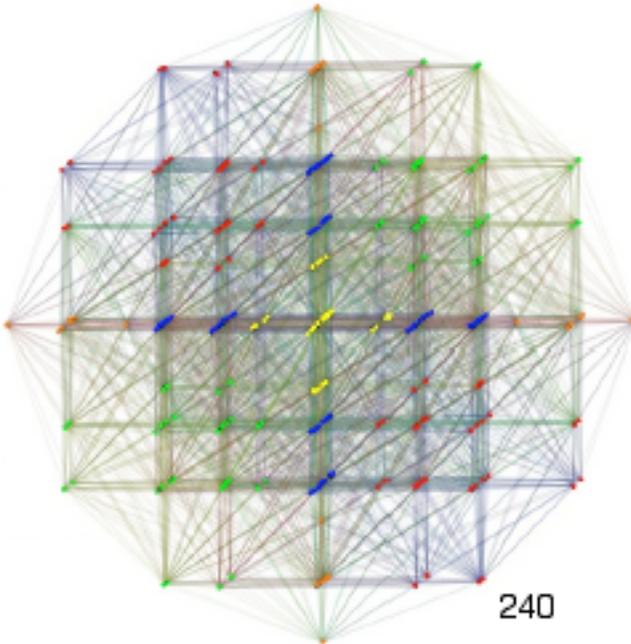
The 112 represents the 112 root vectors of 120-dim D8.

Each of the 7 pairs of 128 are also representations of the 128 +half-spinors and 128 -half-spinors of D8.

The 112 can be combined with any of the 128



to form the 240 of the First Shell of an E8 lattice



that represents the $112 + 128 = 240$ root vectors of E8.

There are 7 pairs of 128 in the Second Shell. Each choice of a pair from which to get a 128 to combine with the 112 produces one of the 7 independent E8 lattices.

You can also choose half of the 256 to combine with the 112 to form an 8th E8 lattice. Although the 8th E8 lattice is not independent of the 7, it is useful in constructing a physics model based on 8-Brane spacetime that in the continuum limit at low (compared to Planck) energies has M4 x CP2 Kaluza-Klein structure. Denote the 7 independent E8 lattices by 1E8, 2E8, 3E8, 4E8, 5E8, 6E8, and 7E8 and the 8th E8 lattice by 8E8.

Note that each of the 8 E8 lattices uses only one of the 128 of a pair (or one half of the 256), and that it corresponds to one of the D8 half-spinor spaces.

Physically, the chosen 128 represents Fermion Particles and AntiParticles of one generation, so the E8 contains one generation of Fermion Particles and AntiParticles (the second and third generations emerge at low energies).

The 128 not chosen represents one antigeration of Fermion Particles and AntiParticles, so the E8 does not contain a Fermion antigeration. Therefore, the E8 model has realistic chirality properties.

My goal in this paper is to explain how this E8 model is realistic and overcomes the acknowledged shortcomings of Garrett Lisi's E8 model of arXiv 0711.0770 which model was the motivation for me to work on this E8 model. I think that Garrett Lisi should get full credit for doing the basic ground-work for the E8 model.

I hope that this paper shows to its readers that the E8 model and its AQFT constitute a complete realistic theory that satisfies Einstein's criteria (quoted by Wilczek in the winter 2002 issue of Deadalus) :

“... a theorem which at present can not be based upon anything more than upon a faith in the simplicity, i.e., intelligibility, of nature: there are no arbitrary constants ... that is to say, nature is so constituted that it is possible logically to lay down such strongly determined laws that within these laws only rationally completely determined constants occur ...”.

The remainder of this paper consists of the following sections:

10 spacetime Dimensions of 26-dim Bosonic Strings

16 Fermionic Dimensions of 26-dim Bosonic Strings

Closed Bosonic String World-Lines

Quaternionic $M_4 \times CP^2$ Kaluza-Klein

Calculations of Masses, Force Strengths, etc

-for detailed results of the calculations etc, see web book at

<http://www.tony5m17h.net/E8physicsbook.pdf> and

www.valdostamuseum.org/hamsmith/E8physicsbook.pdf

AQFT

EPR Entanglement

10 spacetime Dimensions of 26-dim Bosonic Strings:

An 8-Brane is constructed as a superposition of all of the 8 E8 lattices.

Each 8-Brane represents a local neighborhood of spacetime.

Global spacetime is a collection of 8-Branes parameterized by two real variables a, b that are analagous to the conformal dimensions (1,1) that extend (1,3) Minkowski physical spacetime of Spin(1,3) to the (2,4)

Conformal spacetime of Spin(2,4) = SU(2,2).

Physical Gauge Bosons link an 8-Brane to a successor 8-Brane along the World Line of that Gauge Boson as follows:

A Gauge Boson emanating from only the 8E8 lattice in the 8-Brane is a U(1) Electromagnetic Photon;

A Gauge Boson emanating from only the 8E8 and the 4E8 lattice in the 8-Brane is a U(2) Weak Boson (note that their common 8E8 unifies the Electromagnetic Photon with the Weak Bosons);

A Gauge Boson emanating from only the 5E8, 6E8, and 7E8 lattices in the 8-Brane is a U(3) Gluon;

A Gauge Boson emanating from only only the 8E8 lattice and the 1E8, 2E8, and 3E8 lattices in the 8-Brane is a $U(2,2) = U(1) \times SU(2,2) = U(1) \times Spin(2,4)$ Conformal Gauge Boson that gives Gravity by the MacDowell-Mansouri mechanism.

16 Fermionic Dimensions of 26-dim Bosonic Strings:

We now have constructed the 10 dimensions of the base manifold of 26-dim Closed Unoriented Bosonic String Theory, as well as the Gauge Bosons of the Standard Model plus Gravity, in which Strings are physically interpreted as World-Lines, with relatively large Closed Strings corresponding to World-Lines of particles that locally appear to be free and relatively small Closed Strings corresponding to paths of virtual particles in the Path Integral Sum-Over-Histories picture.

To describe the one fundamental generation of Fermion Particles and AntiParticles of the E8 model add, to the 10 dimensions we already have, a 16-dimensional space that is discretized by Orbifolding it with respect to the 16-element discrete Octonionic multiplicative group $\{+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K\}$ to reduce the 16-dim Fermionic representation space to 16 points $\{-1,-i,-j,-k,-E,-I,-J,-K;+1,+i,+j,+k,+E,+I,+J,+K\}$ for which Fermion Particles (nu, ru, gu, bu, e, rd, gd, bu) are represented by $\{-1,-i,-j,-k,-E,-I,-J,-K\}$ and the corresponding Fermion AntiParticles are represented by $\{+1,+i,+j,+k,+E,+I,+J,+K\}$.

Now our E8 model has realistic first-generation Fermions as well as a base manifold with the Standard Model plus Gravity (M4 x CP2 Kaluza-Klein spacetime, with its 4-dim physical spacetime, and the second and third generations of Fermions, emerge at low temperatures when a preferred Quaternionic substructure freezes out from the high-temperature Octonionic structure).

Closed Bosonic String World-Lines:

Interaction of Closed Bosonic Strings as World-Lines looks like Andrew Gray's idea in quant-ph/9712037

"... probabilities are ... assigned to entire fine-grained histories ... this new formulation makes the same experimental predictions as quantum

field theory ..."

so it seems that physical results of Bosonic String Theory can be interpreted as:

String Tachyons can be physically interpreted as describing the virtual particle-antiparticle clouds that dress the orbifold Fermion particles (As Lubos Motl said in his on 13 July 2005: "... closed string tachyons ... can be localized if they appear in a twisted sector of an orbifold ... tachyons condense near the tip which smears out the tip of the cone which makes the tip nice and round. ..." and as Bert Schroer said in hep-th/9908021: "... any compactly localized operator applied to the vacuum generates clouds of pairs of particle/antiparticles ...").

String spin-2 Gravitons can be physically interpreted as describing a Bohm-like Quantum Potential and what Penrose (in "Shadows of the Mind" (Oxford 1994) with respect to Quantum Consciousness) describes as "... the gravitational self-energy of that mass distribution which is the difference between the mass distributions of ... states that are to be considered in quantum linear superposition ...".

The 128 in the 240 of the E8 model breaks up into two 64-element things. One $64 = 8 \times 8$ represents the 8 Dirac gamma covariant components (with respect to high-energy 8-dim spacetime) of each of the 8 fundamental first-generation Fermion Particles; the other $64 = 8 \times 8$ represents the 8 Dirac gamma covariant components (with respect to high-energy 8-dim spacetime) of each of the 8 fundamental first-generation Fermion AntiParticles.

The 112 in the 240 of the E8 model breaks up into three parts: a 64 plus a 24 plus a dual 24.

The $64 = 8 \times 8$ in the 112 represents 8 Dirac gammas for the 8 dimensions of high-energy spacetime; the 24 represents the 24 root vectors of a 28-dim D4 Lie algebra whose generators include those of the Standard Model Gauge Bosons; the dual 24 represents the 24 root vectors of a second 28-dim D4 Lie algebra whose generators include those of the conformal U(2,2) that

produces Gravity.

Quaternionic M4 x CP2 Kaluza-Klein:

At this stage, the E8 model differs from conventional Gravity plus Standard Model in four respects:

- 1 - 8-dimensional spacetime
- 2 – two Spin(8) gauge groups from the two D4 in 112
- 3 - no Higgs
- 4 - 1 generation of fermions

These differences can be reconciled as follows:

Introduce (freezing out at lower-than-Planck energies) a preferred Quaternionic 4-dim subspace of the original (high-energy) 8-dim spacetime,
thus forming an 8-dim Kaluza-Klein spacetime M4xCP2
where M4 is 4-dim physical spacetime and CP2 is a 4-dim internal symmetry space.

Let the first Spin(8) gauge group act on the M4 physical spacetime through the SU(3) subgroup of its U(4) subgroup. As Meinhard E. Mayer said (Hadronic Journal 4 (1981) 108-152): "... each point of ... the ... fibre bundle ... E consists of a four-dimensional spacetime point x [in M4] to which is attached the homogeneous space G / H [$SU(3) / U(2) = CP2$] ... the components of the curvature lying in the homogeneous space G / H [$= SU(3) / U(2)$] could be reinterpreted as Higgs scalars (with respect to spacetime [M4])

...

the Yang-Mills action reduces to a Yang-Mills action for the h-components [U(2) components] of the curvature over M [M4] and

a quartic functional for the "Higgs scalars", which not only reproduces the Ginzburg-Landau potential, but also gives the correct relative sign of the constants, required for the BEHK ... Brout-Englert-Higgs-Kibble ... mechanism to work. ...".

So, freezing out of a Kaluza-Klein $M4 \times CP2$ spacetime plus internal symmetry space produces a classical Lagrangian for the $SU(3) \times U(2) = SU(3) \times SU(2) \times U(1)$ Standard Model including a BEHK Higgs mechanism.

Let the second Spin(8) gauge group act on the $M4$ physical spacetime through its Conformal Subgroup $U(2,2) = Spin(2,4)$. As Rabindra Mohapatra said (section 14.6 of Unification and Supersymmetry, 2nd edition, Springer-Verlag 1992): "... gravitational theory can emerge from the gauging of conformal symmetry ... we start with a Lagrangian invariant under full local conformal symmetry and fix conformal and scale gauge to obtain the usual action for gravity. ...".

At this stage, we have reconciled the first 3 of the 4 differences between our E8 Physics Model and conventional Gravity plus the Standard Model. As to the fourth, the existence of 3 generations of fermions, note that the 8 first generation fermion particles and the 8 first generation antiparticles can each be represented by the 8 basis elements of the Octonions O , and that the second and third generations can be represented by Pairs of Octonions $O \times O$ and

Triples of Octonions $O \times O \times O$, respectively.

When the unitary Octonionic 8-dim spacetime is reduced to the Kaluza-Klein $M4 \times CP2$, there are 3 possibilities for a fermion propagator from point A to point B:

- 1 – A and B are both in $M4$, so its path can be represented by the single O ;
- 2 – Either A or B, but not both, is in $CP2$, so its path must be augmented by one projection from $CP2$ to $M4$, which projection can be represented by a second O , giving a second generation $O \times O$;
- 3 – Both A and B are in $CP2$, so its path must be augmented by two projections from $CP2$ to $M4$, which projections can be represented by a second O and a third O , giving a third generation $O \times O \times O$.

Therefore, all four differences have been reconciled, and our classical Lagrangian E8 Physics Model describes Gravity as well as the Standard Model with a BEHK Higgs mechanism.

Calculations of Masses, Force Strengths, etc:

However, for our classical Lagrangian E8 Physics Model to be said to be complete and realistic, it must allow us to calculate such things as Force Strengths and Particle Masses that are consistent with experimental and observational results. To do that, we use the results of Hua in his book “Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains”. (Similar use of the work of Hua was made years ago by Armand Wyler, and recently by a few others, such as Carlos Castro.)

Hua’s calculated volumes related to kernels and Shilov boundaries are the key to calculation of Force Strengths and Particle Masses. For example, the Lagrangian term for each of the Forces is integrated over the M4 physical spacetime base manifold, but each of the Four Forces sees M4 in terms of its own symmetry, consequently with its own measure which measure is proportional to Hua-calculated volumes. Since M4 was formed by a freezing out of a Quaternionic structure, M4 is a 4-dimensional manifold with Quaternionic structure and therefore can be seen as one of Joseph Wolf’s 4 equivalence classes:

for Electromagnetism: $T4 = U(1)^4$

for Weak Force: $S2 \times S2 = SU(2) / U(1) \times SU(2) / U(1)$

for Color Force: $CP2 = SU(3) / U(2)$

for Gravity: $S4 = Spin(5) / Spin(4) = Sp(2) / Sp(1) \times Sp(1)$

When we also take into account the relevant volumes related to the curvature term in the Lagrangian for each force,

and the masses involved for forces with gauge bosons related to mass, the calculations produce results that are reasonably close to experimental observation:

Force Strengths:

Gravity = 5×10^{-39}

Electromagnetic = $1 / 137.03608$

Weak = 1.05×10^{-5}

Color at 245 MeV = 0.6286

Renormalization gives Color at 91 GeV = 0.106

and including other effects gives Color at 91 GeV = 0.125

Tree-level fermion masses (Quark masses are constituent masses due to a Bohmian version of Many-Worlds Quantum Theory applied to a confined fermion, in which the fermion is at rest because its kinetic energy is transformed into Bohmian PSI-field potential energy.):

Neutrinos: $m_e\text{-neutrino} = m_{\mu}\text{-neutrino} = m_{\tau}\text{-neutrino} = 0$ at tree-level
(first order corrected masses are given below)

Electron/Positron $m_e = 0.5110$ MeV

Up and Down Quarks $m_d = m_u = 312.8$ MeV

Muon $m_{\mu} = 104.8$ MeV

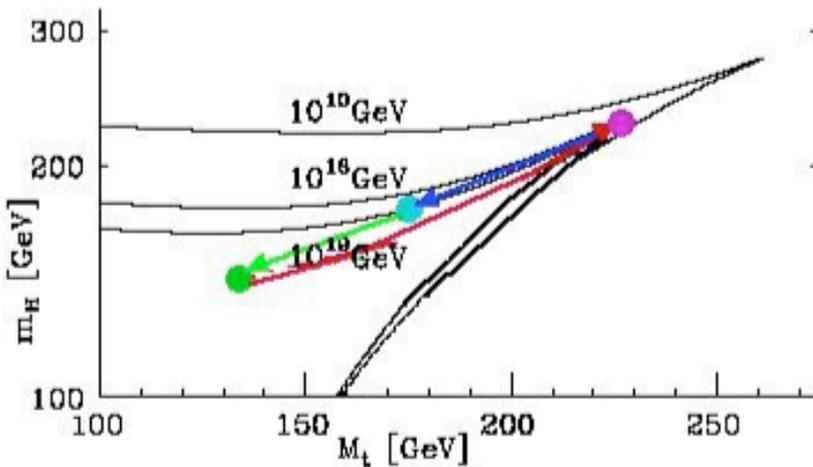
Strange Quark $m_s = 625$ MeV

Charm Quark $m_c = 2.09$ GeV

Tauon $m_{\tau} = 1.88$ GeV

Beauty Quark $m_b = 5.63$ GeV

Truth Quark $m_t = 130$ GeV ground state - 8-dimensional Kaluza-Klein spacetime with Truth-Quark condensate Higgs gives a 3-state system with a renormalization line connecting the 3 states:



(see hep-ph/0307138 for background for chart immediately above)

Low ground state:

Higgs = 146 GeV and T-quark = 130 GeV

Medium Triviality Bound state:

Higgs = 176-188 GeV and T-quark = 172- 175 GeV

High Critical Point state:

Higgs = 239 +/- 3 GeV and T-quark = 218 +/- 3 GeV

Weak Boson Masses (based on a ground state Higgs mass of 146 GeV):
 $M_{W^+} = M_{W^-} = 80.326 \text{ GeV}$;
 $M_{Z^0} = 80.326 + 11.536 = 91.862 \text{ GeV}$

Kobayashi-Maskawa parameter calculations use phase angle $\delta_{13} = 1$ radian (unit length on a phase circumference) to get the K-M matrix:

	d	s	b
u	0.975	0.222	0.00249-0.00388i
c	-0.222-0.000161i	0.974-0.0000365i	0.0423
t	0.00698-0.00378i	-0.0418-0.00086i	0.999

Corrections to the tree-level neutrino calculations give neutrino masses
 $\nu_1 = 0$
 $\nu_2 = 9 \times 10^{-3} \text{ eV}$
 $\nu_3 = 5.4 \times 10^{-2} \text{ eV}$
 and
 the neutrino mixing matrix:

	ν_1	ν_2	ν_3
ν_e	0.87	0.50	0
ν_μ	-0.35	0.61	0.71
ν_τ	0.35	-0.61	0.71

The mass of the charged pion is calculated to be 139 MeV based on a Kerr-Newman Black Hole model of the pion and its constituent quark-antiquark pair. The pair of Black Holes form a Toroidal Black Hole for which the Torus is an Event Horizon that is (1+1)-dimensional with a timelike dimension which carries a Sine-Gordon Breather whose soliton and antisoliton are the quark and antiquark. The physically relevant Sine-Gordon solution for which the first-order weak coupling expansion is exact gives the ratio of quark constituent mass to the pion mass.

The Neutron-Proton mass difference is calculated to be 1.1 MeV based on the down quark having virtual states related to the strange quark and the up quark having virtual states related to the charm quark, and the higher probability of strange quark states emerging from the nucleon sea.

The ratio Dark Energy : Dark Matter : Ordinary Matter for our Universe at the present time is calculated to be:

$$0.75 : 0.21 : 0.04$$

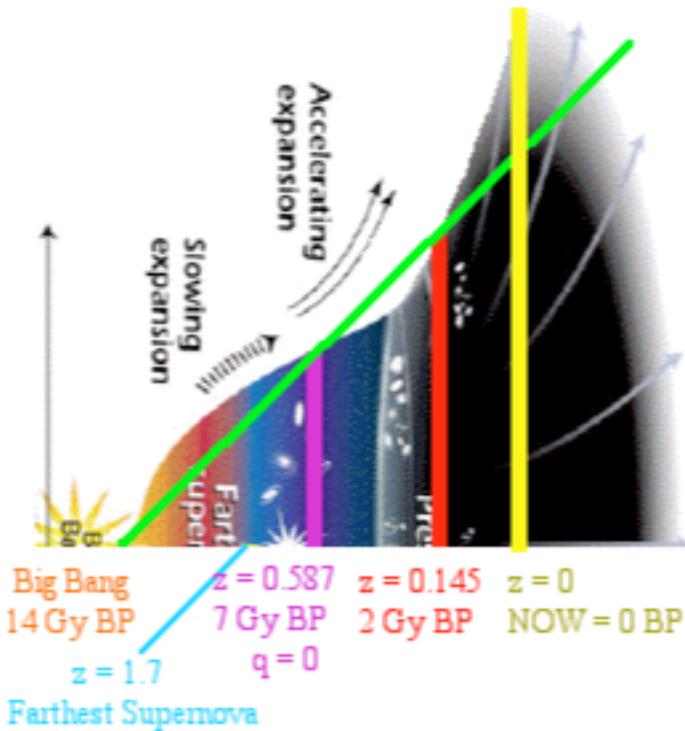
based on the Conformal Gravity model of Irving Ezra Segal and the 15 generators of the Conformal Group $\text{Spin}(2,4) = \text{SU}(2,2)$

10 = 6 Lorentz plus 4 Special Conformal = Dark Energy

4 Translations = Dark Matter Primordial Black Holes

1 Dilation = Ordinary Matter mass from Higgs

and the evolution of that basic ratio 10 : 4 : 1 = 0.67 : 0.27 : 0.06 as our universe has expanded



Details of calculations and discussion of some things that here are oversimplified can be found in my free pdf book “E8 and $\text{Cl}(16) = \text{Cl}(8) \times \text{Cl}(8)$ ” which is available at

<http://www.tony5m17h.net/E8physicsbook.pdf> and

<http://www.valdostamuseum.org/hamsmith/E8physicsbook.pdf>

AQFT:

Since the E8 classical Lagrangian is Local, it is necessary to patch together Local Lagrangian Regions to form a Global Structure describing a Global E8 Algebraic Quantum Field Theory (AQFT).

Mathematically, this is done by using Clifford Algebras (others now using Clifford algebras in related ways include Carlos Castro and David Finkelstein) to embed E8 into Cl(16) and using a copy of Cl(16) to represent each Local Lagrangian Region. A Global Structure is then formed by taking the tensor products of the copies of Cl(16). Due to Real Clifford Algebra 8-periodicity, $Cl(16) = Cl(8) \times Cl(8)$ and any Real Clifford Algebra, no matter how large, can be embedded in a tensor product of factors of Cl(8), and therefore of $Cl(8) \times Cl(8) = Cl(16)$. Just as the completion of the union of all tensor products of 2x2 complex Clifford algebra matrices produces the usual Hyperfinite III von Neumann factor that describes creation and annihilation operators on the fermionic Fock space over $C^{(2n)}$ (see John Baez's Week 175), we can take the completion of the union of all tensor products of $Cl(16) = Cl(8) \times Cl(8)$ to produce a generalized Hyperfinite III von Neumann factor that gives a natural Algebraic Quantum Field Theory structure to the E8 model.

EPR Entanglement:

For the E8 model AQFT to be realistic, it must be consistent with EPR entanglement relations. Joy Christian in arXiv 0904.4259 "Disproofs of Bell, GHZ, and Hardy Type Theorems and the Illusion of Entanglement" said: "... a [geometrically] correct local-realistic framework ... provides exact, deterministic, and local underpinnings for at least the Bell, GHZ-3, GHZ-4, and Hardy states. ... The alleged non-localities of these states ... result from misidentified [geometries] of the EPR elements of reality. ... The correlations are ... the classical correlations among the points of a 3 or 7-sphere ... S^3 and S^7 ... are ... parallelizable ... The correlations ... can be seen most transparently in the elegant language of Clifford algebra ...". The E8 model AQFT is based on the parallelizable Lie group E8 and related Clifford algebras, so the E8 model seems consistent with EPR.