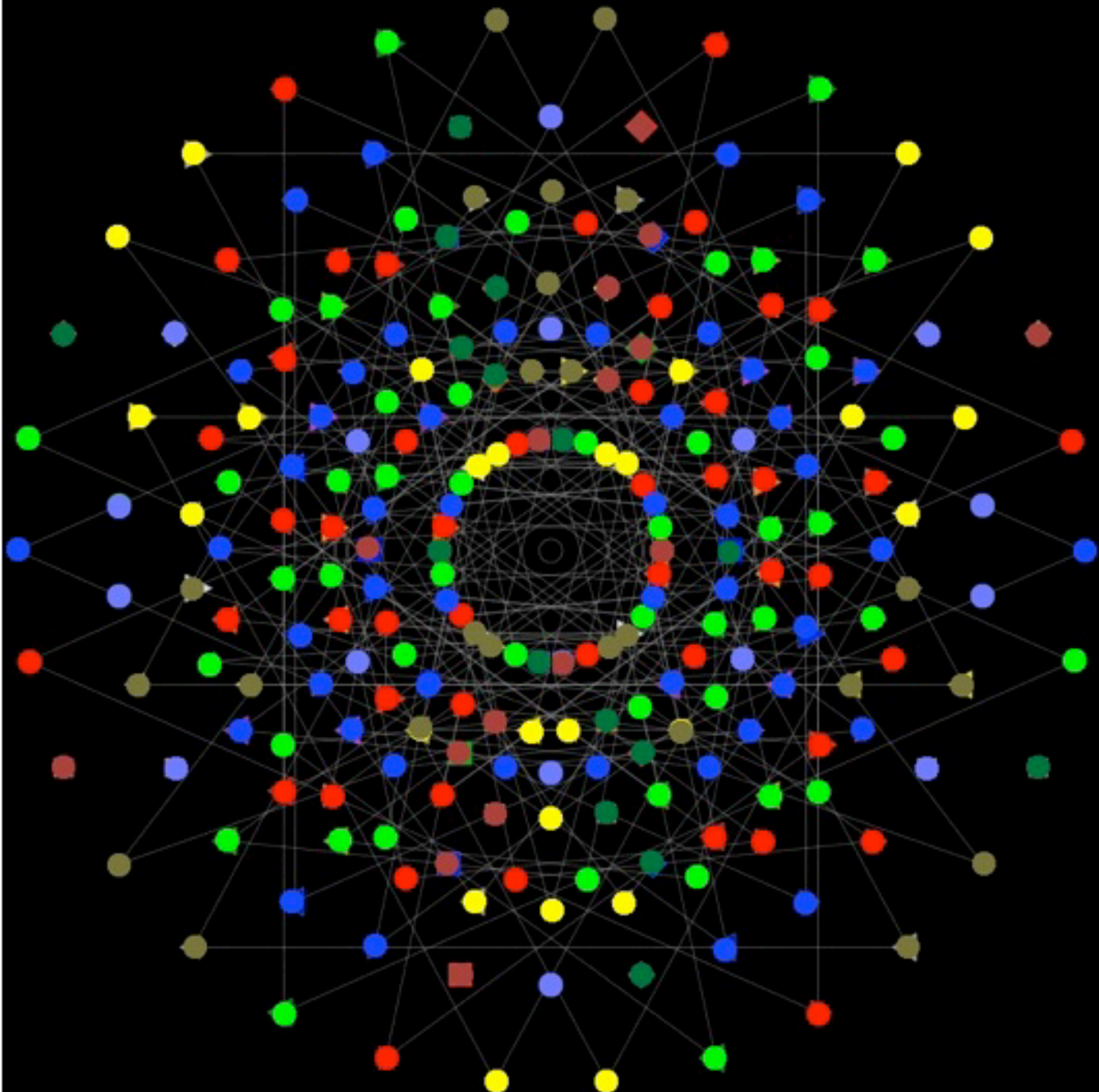
E8 and Cl(16) = Cl(8) x Cl(8)



by Frank Dodd (Tony) Smith, Jr.

Physics of E8 and Cl(16) = Cl(8) (x) Cl(8)

Frank Dodd (Tony) Smith, Jr. – Georgia – June-July 2008

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- ## Historical Introduction

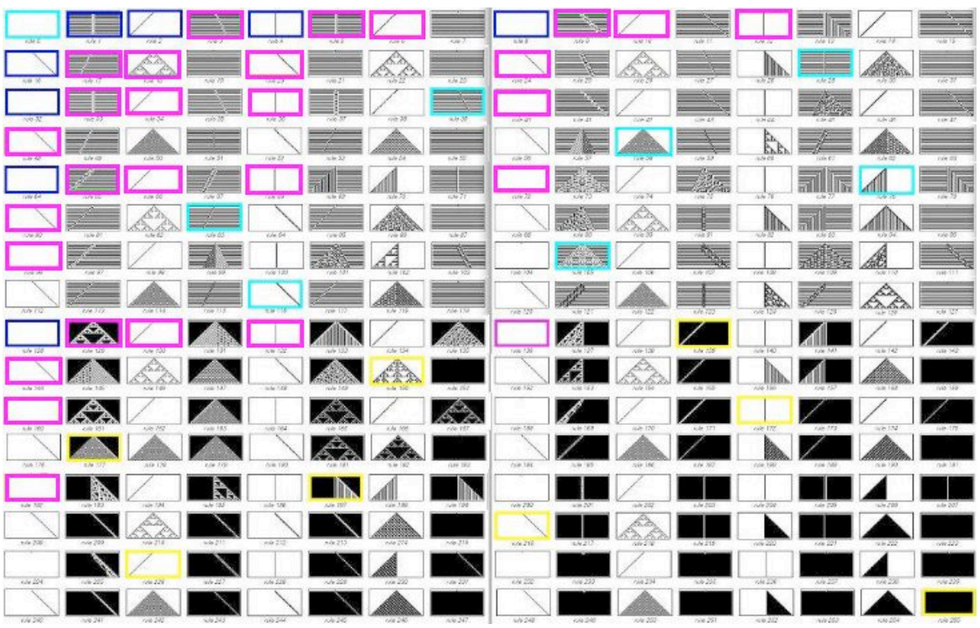
A little less than 15 billion years ago, our Universe emerged from the Void.

4 billion years ago, our Earth and Moon were orbiting our Sun.

2 billion years ago, bacteria built a nuclear fission reactor in Africa.

100,000 years ago, Humans were expanding from the African home-land to Eurasia and beyond.

12,000 years ago, Africans knew that the knowledge-patterns of 8 binary choices giving 2^8 = 256 = 16x16 possibilities could act as an Oracle. Did they realize then that those 256 possibilites corresponded to



the 256 Fundamental Cellular Automata, some of which act as Universal Computers?

From Africa, the 16x16 Oracle-patterns spread, so that by the 13th century parts of them were found in:

Judaism as the 248 positive Commandments plus the 365 negative Commandments given to Moses during the 50 days from Egypt to Sinai;

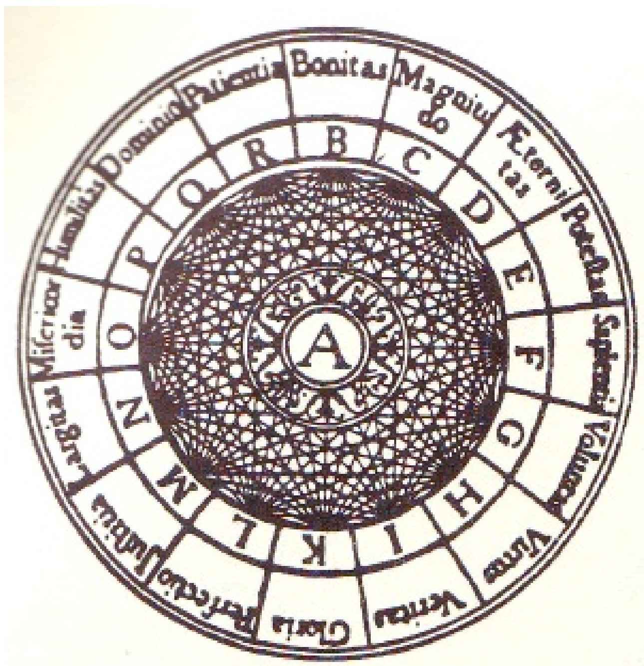
India as the 240 parts of the first sukt of the Rig Veda;

Japan as the 128 possibilities of Shinto Futomani Divination;

China as the 64 possibilities of the I Ching;

Mediterranean Africa as the 16 possibilities of the Ilm al Raml.

Near the end of the 13th century, Ramon Llull of Mallorca studied the 16 possibilities of the Ilm al Raml and realized that the 16x16 African Oracle-patterns had a Fundamental Organizational Principle that he summarized in a Wheel Diagram



wikth 16 vertices connected to each other by 120 lines. He used such structures to show the underlying unity of all human religions. However, the establishments of the various religions refused to accept Ramon Llull's revelations, and his ideas were relegated to a few obscure publications, plus an effort to preserve some aspects of the 16x16 Oracle-patterns in the form of the 78 Tarot cards and the subset of 52 cards that remains popular into the 21st century.

Did Ramon Llull understand the detailed Clifford Algebra / Lie Algebra structures implicit in his diagram? Maybe not consciously, but maybe he (like his ancient African ancestors when they developed the African Oracle) was subconsciously inspired.

Since Llull was Roman Catholic, the Islamic and Judaic bureaucracies could (and did) ignore his work as that of an irrelevant outsider.

As to the Christians, in the 14th century, Dominican Inquisitors had Ramon Llull condemned as a heretic, his works were suppressed, and his ideas were relegated to a few obscure publications, plus an effort to preserve some aspects of the 16x16 Oracle-patterns in the form of the 78 Tarot cards and the subset of 52 cards that remains popular into the 21st century.

In the 17th century the Roman Inquisition burned Giordano Bruno at the stake and sentenced Galileo to house arrest for the rest of his life, all for the sake of the Roman Inquisition's enforcement of conformity to its Consensus.

Rediscovery of the full significance of Ramon Llull's Oracle-patterns did not happen until

after 20th century science experiments progressed beyond Gravity, Electromagnetism, and early Quantum Mechanics, and

after Lise Meitner discovered the Uranium Fission Chain Reaction Process that led to the Fission Bombs that ended the Japanese part of World War II.

The Japanese defeat liberated Saul-Paul Sirag, a child of Dutch-American Baptist missionaries, from a Japanese concentration camp in Java.

During the 1950s and 1960s, David Finkelstein described Black Holes and worked on Quaternionic Physics, Jack Sarfatti studied physics (BA from Cornell and PhD from U. C. Riverside) and I learned about Lie Groups and Lie Algebras (AB in math from Princeton).

During the 1970s, Saul-Paul Sirag learned math and physics working with Arthur Young and the physics community developed the Standard Model showing how everything other than Gravity could be described, consistent with experimental results, by 3 forces of a Standard Model:

Electromagnetism, with the symmetry of a circle, denoted by $S1 = U(1)$

Weak Force with Higgs, with the symmetry of a 3-dimensional sphere, denoted by $S3 = SU(2)$

Color Force, with symmetry related to a Star of David, denoted by $SU(3)$

From the 1980s on, I learned about Clifford Algebras from David Finkelstein at Georgia Tech; about Weyl Groups and Root Vectors from the work of Saul-Paul Sirag; about Quantum Consciousness, Space-Time and Higgs as Condensates, and Bohmian Back-Reaction from the work of Jack Sarfatti; and about Compton Radius Vortices from the work of B. G. Sidharth.

In contrast to the advances in experimental results and construction of the Standard Model of physics, the social structure of the Physics Scientific Community evolved during the 20th century into a rigid Physics Consensus Community much like the Inquisitorial Consensus Community of a few hundred years ago.

For example, in the USA physics community around the middle of the 20th century, J. Robert Oppenheimer enforced his dislike of the ideas of David Bohm by declaring, as head of the Princeton Institute for Advanced Study:

"... if we cannot disprove Bohm, then we must agree to ignore him ..."

As the 20th century ended and the 21st century began, the Physics Consensus Community continued to enforce conformity to Consensus so strongly that Stanford physicist Burton Richter said:

"... scientists are imprisoned by golden bars of consensus ..."

The rigidly enforced Physics Consensus Community was so void of independent thought that the 20th century ended without anyone seeing how Ramon Llull's Oracle-patterns explained both Gravity and the Standard Model in a unified way,

but

in January 2008 the cover of the magazine of Science & Vie declared:

"Theorie du tout Enfin!

Un physicien ... chercheur hors norme ... aurait trouve la piece manquante"

The missing piece was a 248-dimensional Lie Algebra known as E8.

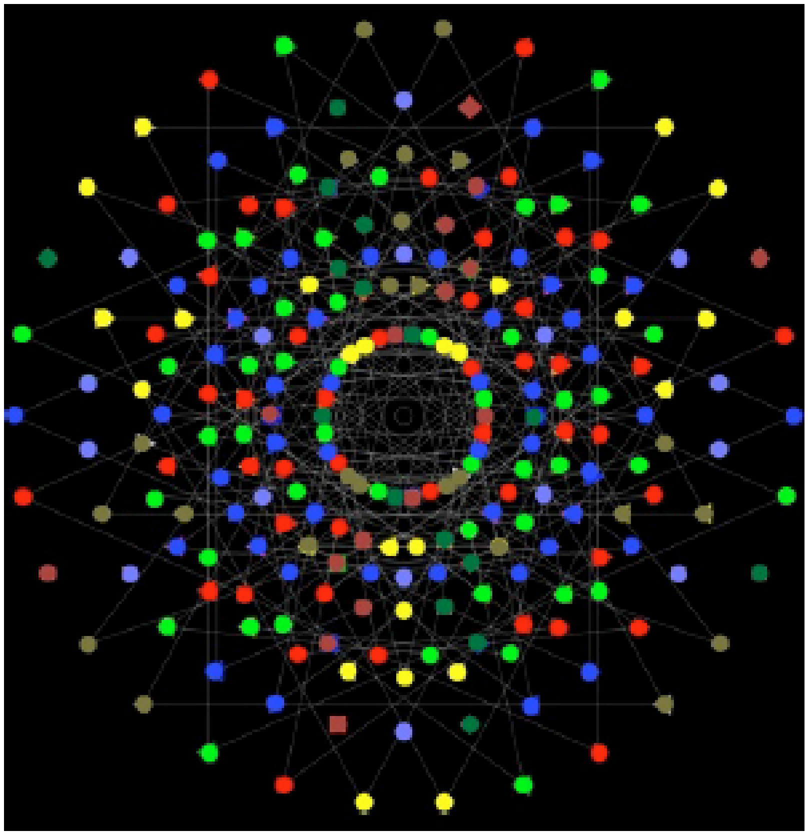
The beyond-the-norm physics researcher was a California-Hawaii Surfer Dude, Garrett Lisi, who realized that the structure of E8 could unify Gravity and the Standard Model in a way that satisfied Einstein's Criterion for

a structure "... based ... upon a faith in the simplicity ... of nature: there are no arbitrary constants ... only rationally completely determined constants ... whose ... value could ... not ... be changed without destroying the theory …"

and published his ideas on the Cornell physics arXiv as hep-th/0711.0770

When I saw Garrett Lisi's E8 ideas, I realized that the 248 dimensions of E8 only needed 8 more to give the 256 = 16x16 of Ramon Llull's Oracle-patterns, so that the African Cushite 16x16 Oracle-patterns, as interpreted by Ramon Llull, not only showed the unity of all human religions, but also showed the unity of Gravity and the Standard Model.

The 240 root vectors of E8 formed a representation of El Aleph of Jorge Luis Borges. Here is an image of the 240 root vectors and a sequence of images showing their physics meaning:



The 240 units of an E8 lattice corresponding to its integral domain represent the 8x30 = 15x15 = 240 lattice points of an E8 root vertex polytope. In terms of an E8 lattice the color-coded root vectors correspond to the similarly color-coded (with orange for the two shades of yellow) lattice points as follows. The other 6 of the 7 independent E8 lattices (and the 8th dependent one) have similar correspondences.

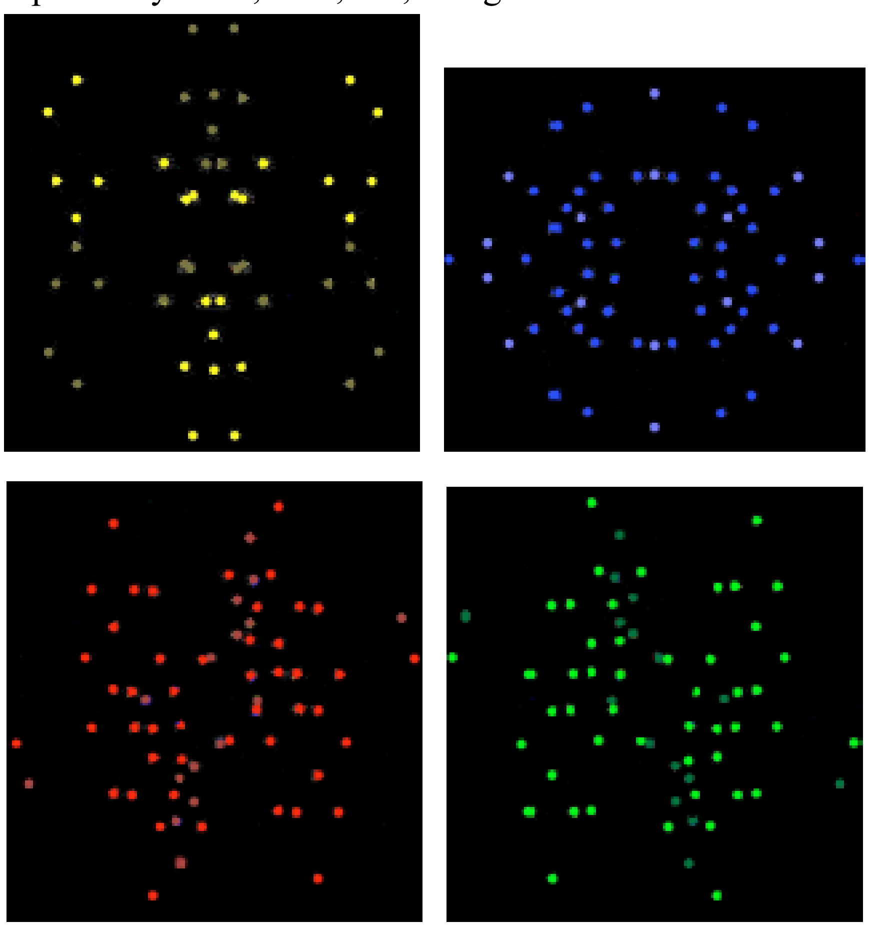
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke,

(±1 ±ie ±je ±ke)/2	**(±e ±i ±j ±k)/2**
(±1 ±ke ±e ±k)/2	**(±i ±j ±ie ±je)/2**
(±1 ±k ±i ±je)/2	**(±j ±ie ±ke ±e)/2**
(±1 ±je ±j ±e)/2	**(±ie ±ke ±k ±i)/2**
(±1 ±e ±ie ±i)/2	**(±ke ±k ±je ±j)/2**
(±1 ±i ±ke ±j)/2	**(±k ±je ±e ±ie)/2**
(±1 ±j ±k ±ie)/2	**(±je ±e ±i ±ke)/2**

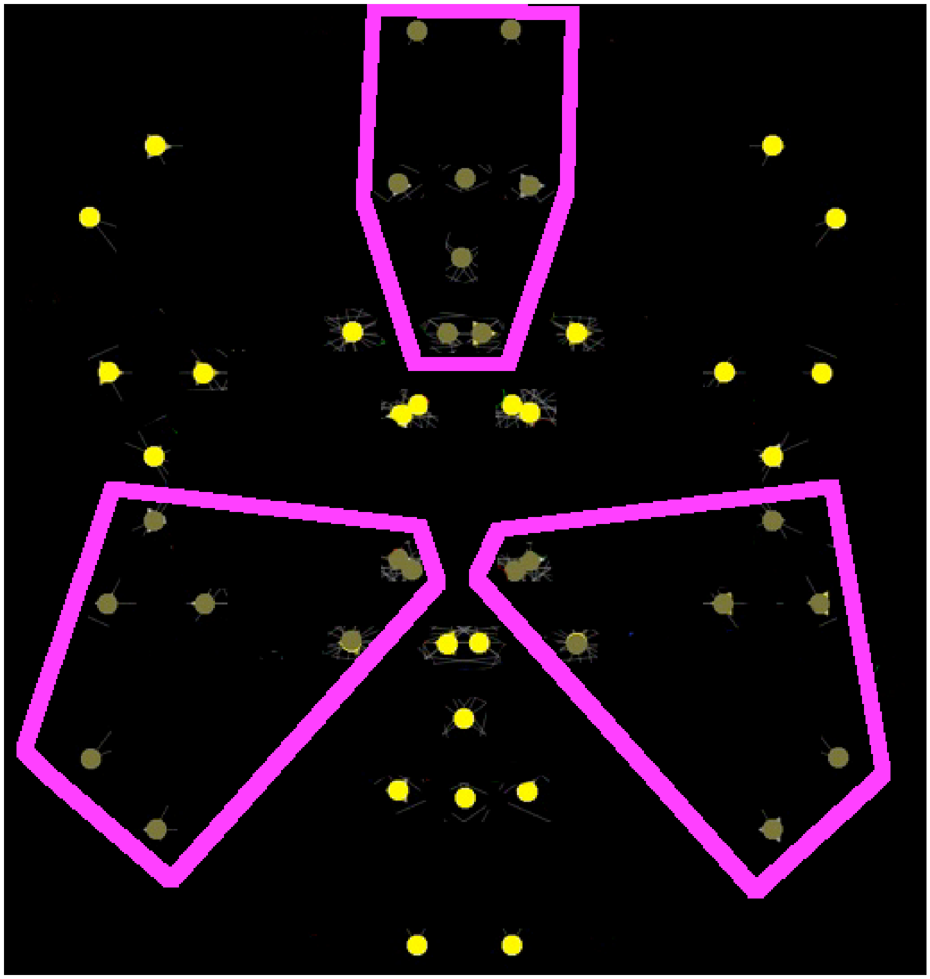
Here is another layout of those 240 lattice points:

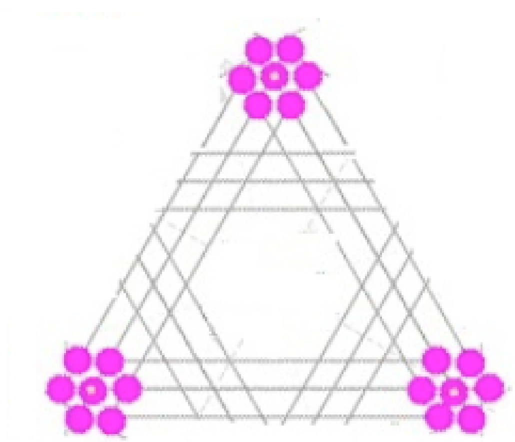
```
±1, ±i, ±j, ±k, ±e, ±ie, ±je, ±ke,
(±1                 ±ie  ±je  ±ke )/2
(   ±i  ±j  ±k  ±e                )/2
(±1         ±k  ±e            ±ke )/2
(   ±i  ±j          ±ie  ±je      )/2
(±1 ±i      ±k           ±je      )/2
(       ±j      ±e  ±ie       ±ke )/2
(±1     ±j      ±e       ±je      )/2
(   ±i      ±k      ±ie       ±ke )/2
(±1 ±i          ±e  ±ie           )/2
(       ±j  ±k           ±je  ±ke )/2
(±1 ±i  ±j                    ±ke )/2
(           ±k  ±e  ±ie  ±je      )/2
(±1     ±j  ±k      ±ie           )/2
(   ±I          ±e       ±je  ±ke )/2
```

Here are the separated yellow, blue, red, and green root vectors:



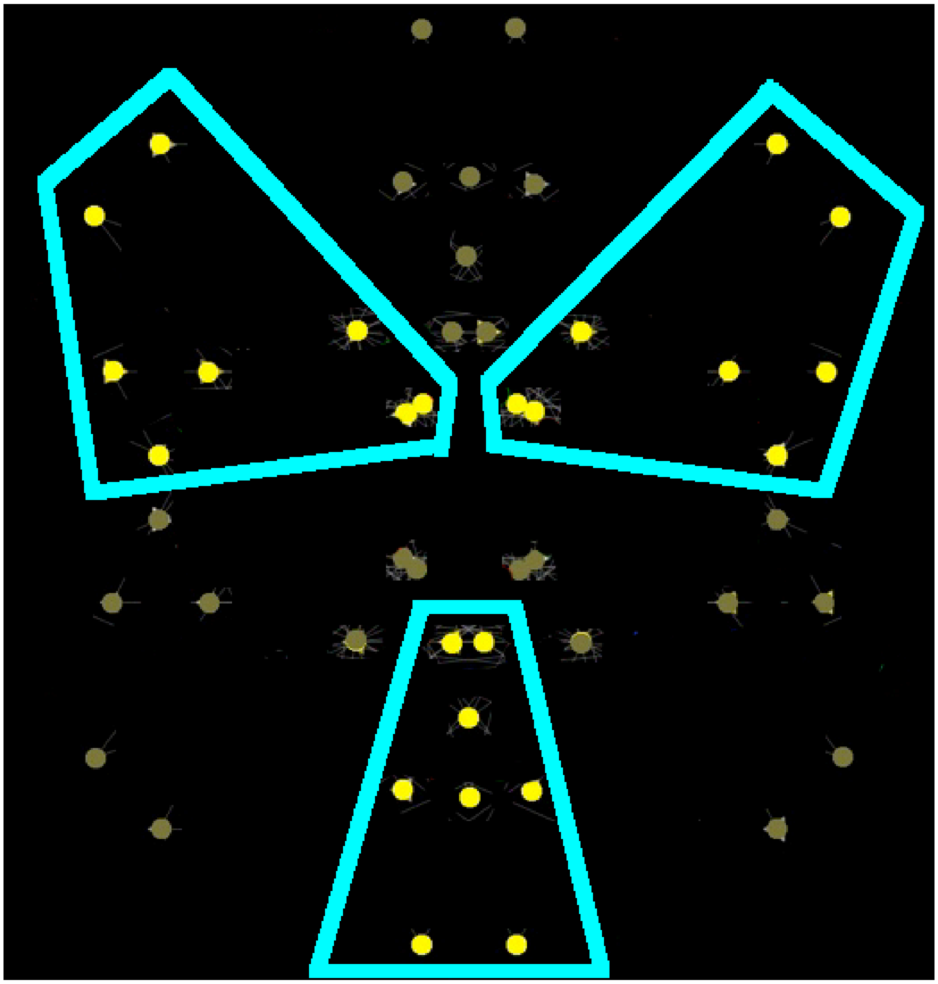
The dark yellow root vectors are grouped this way

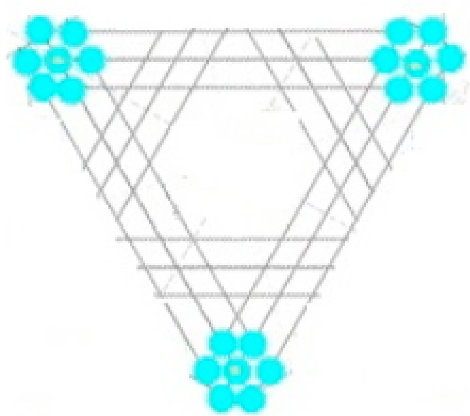




This gives Gravity similar to the Conformal Gravity of I. E. Segal, and U(1) propagator phase.

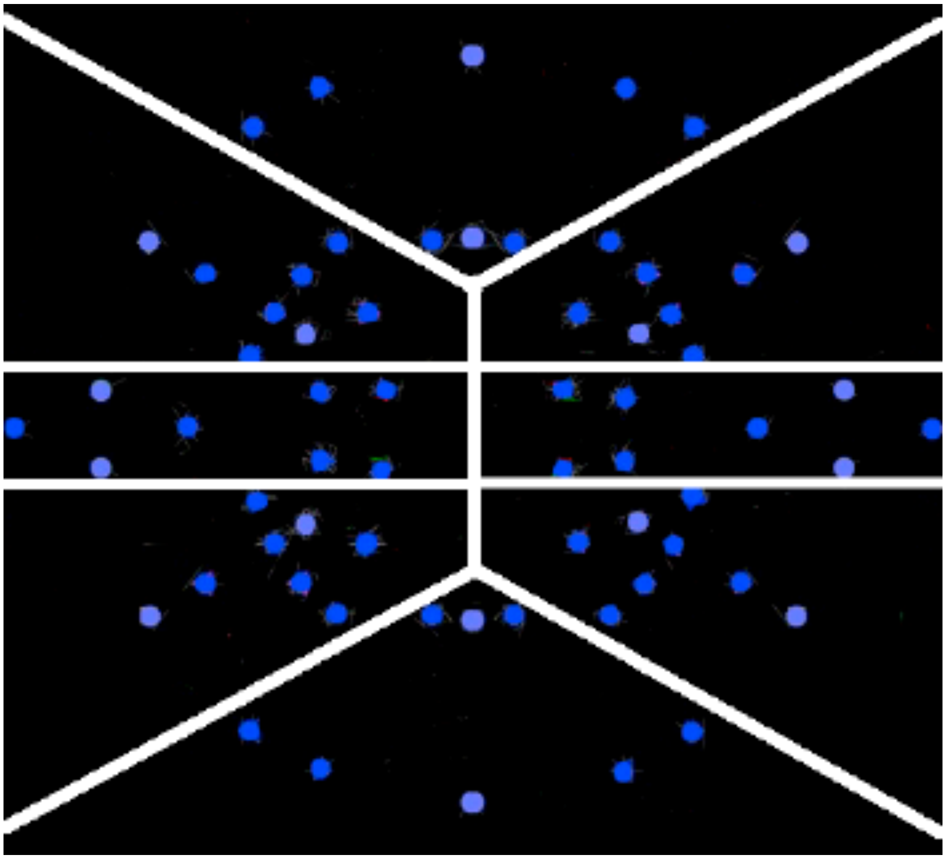
The bright yellow root vectors are grouped this way

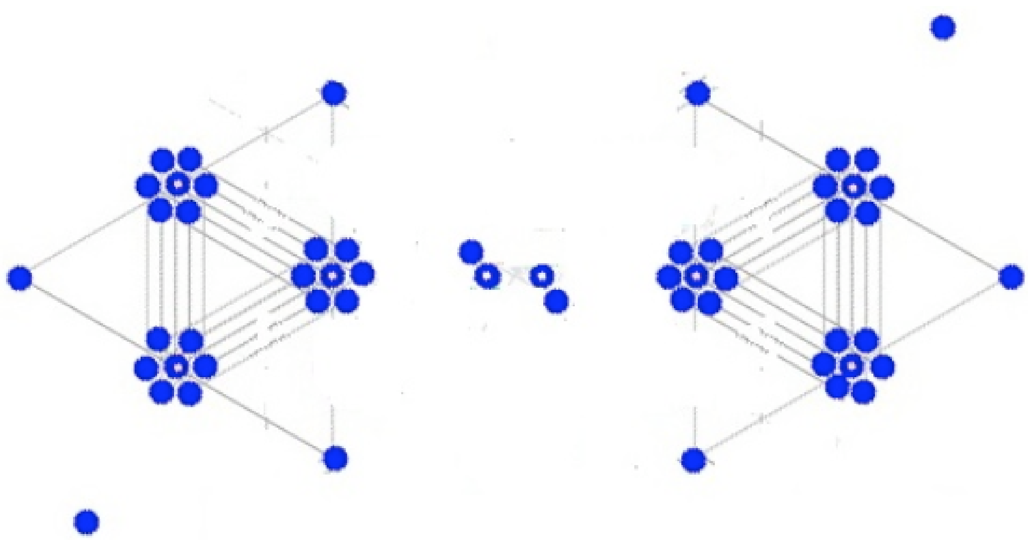




This gives SU(3) x SU(2) x U(1) of the Standard Model.

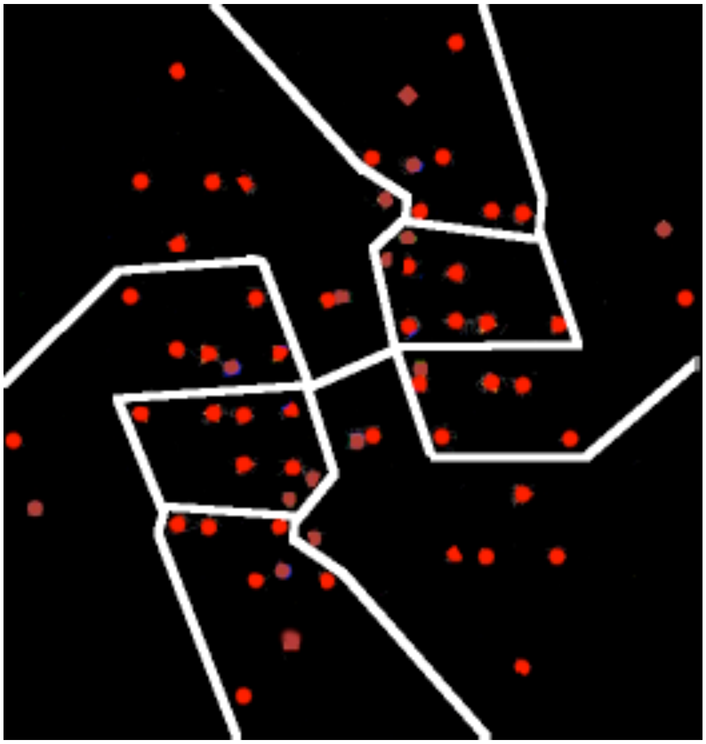
The blue root vectors are grouped this way

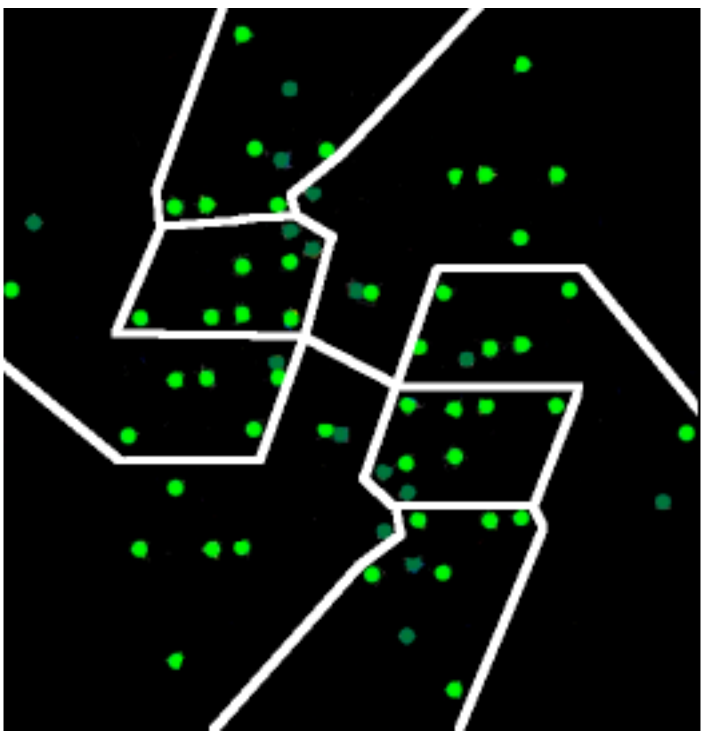


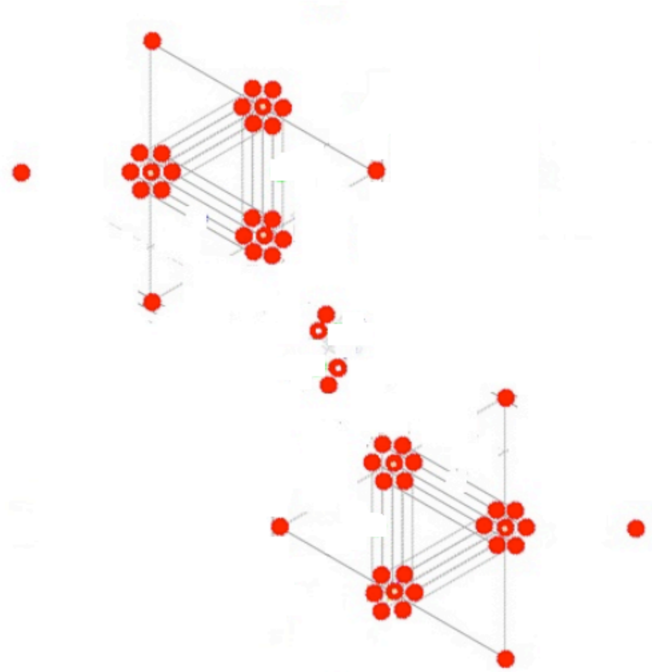


the 64 = 8_v x 8_G describes the Kaluza-Klein space and its connection to the Dirac Gammas.

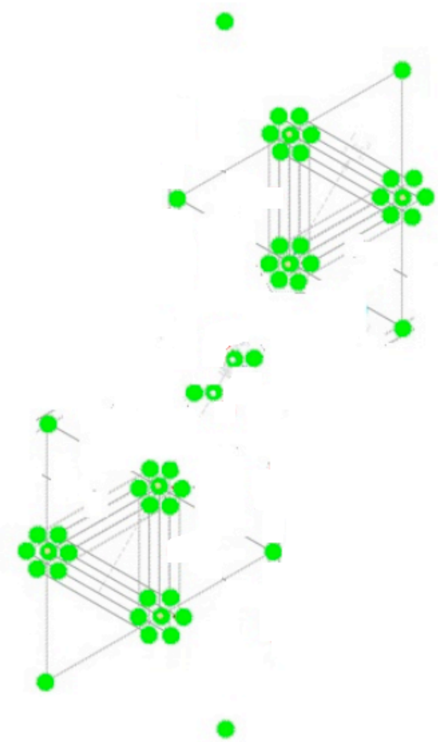
The red and green root vectors are grouped this way





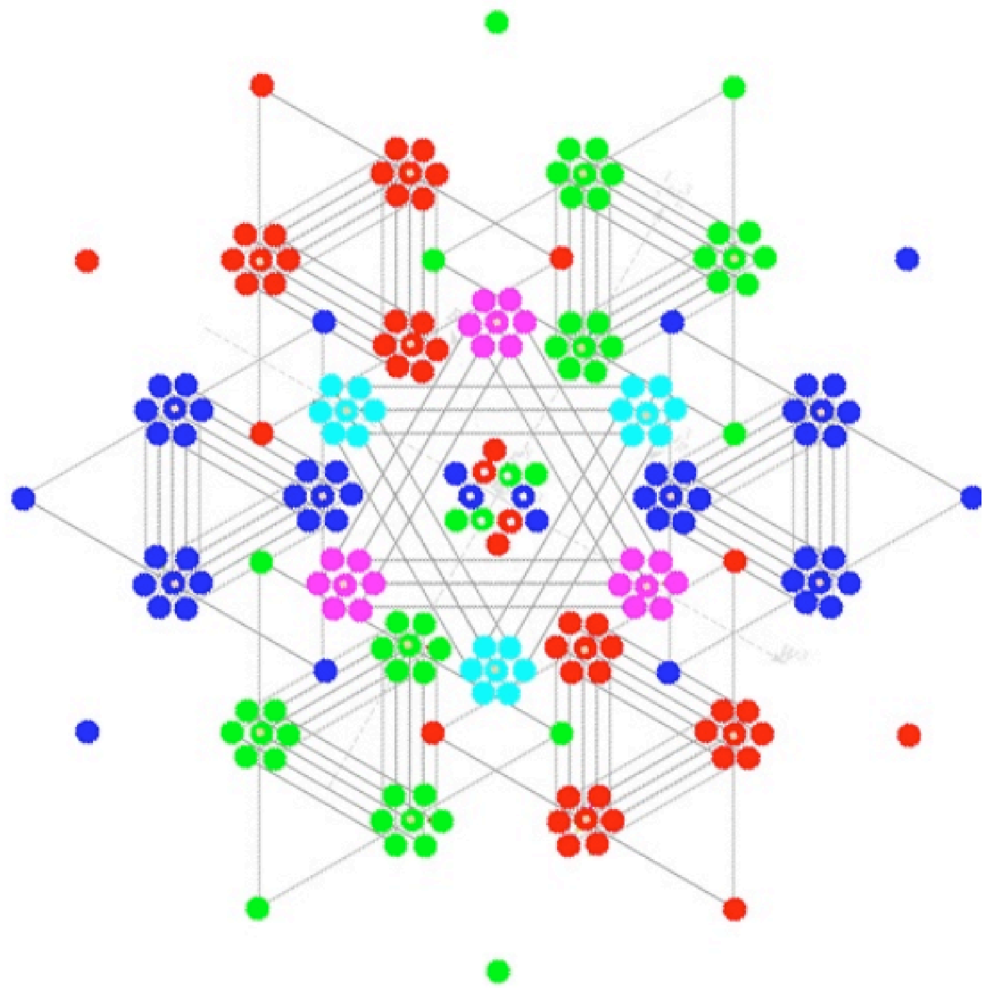


the 64 = 8_f+ x 8_G describes the 8 first-generation fermion particles (neutrino; red, blue, green up quarks; red, blue, green down quarks, electron) and their connection to the Dirac Gammas



the 64 = 8_f+ x 8_G describes the 8 first-generation fermion anti-particles and their connection to the Dirac Gammas

The interpretation point of view, for all the E8 root vectors, is



(Note that, unlike the first root vector view of 8 circles of 30 root vectors each in which all 8x30 = 240 root vectors of E8 are shown as distinct points,

from the interpretation point of view, some of the root vectors are projected onto the same point, so some of the points (white center) correspond to 3 root vectors and some (yellow center) correspond to 2 root vectors.)

Further, using the 16x16 Oracle-pattern structure along with some non-Consensus ideas of people like Irving Segal, Meinhard Mayer, Armand Wyler, and David Bohm, I had been able to do the calculations that Richard Feynman had declared to be necessary when he said:

“... The whole purpose of physics is to find a number, with decimal points, etc! Otherwise you haven’t done anything. ...”

Neither Garrett Lisi (“physicien ... chercheur hors norme” = “physics researcher beyond the norm”) nor I (lawyer working on physics in his spare time) had made their discoveries while working within the Physics Consensus Community, which was mostly hostile

to Garrett Lisi’s discovery that E8 was “la Piece Manquante” = “the Missing Piece” of the Puzzle of Unification of Gravity and the Standard Model and

to my realization that the Missing Piece E8 showed how Ramon Llull’s African Cushite 16x16 Oracle-patterns explained the fundamental Unity of Gravity and the Standard Model.

How hostile was the Physics Consensus Community? VERY!

Years ago, while I was developing his physics model, the Cornell physics e-print arXiv blacklisted me so that I could not post his work and preserve it for posterity.

The Cornell arXiv blacklisting affected not only me, but also others including but not limited to Carlos Castro (whose work, especially on Clifford Algebras and the Armand Wyler-type geometry of force strengths, is related to mine) and even Cornell graduate Jack Sarfatti.

Although Cornell made pretensions of being pro-civil-rights, under its skin Cornell was just another Roman Inquisistion.

Cornell arXiv blacklisting hurt my feelings deeply, but even it was not as bad as what happened after I had compared my calculations with published results of experiments at Fermi National Laboratory (Fermilab) near Chicago, and seen that 6 events published in 1997 were consistent with my calculations.

By 2008, Fermilab had recorded so many more new events that, if the new events were made public, they would most likely either conclusively confirm my calculations or refute them.

So, I asked a Fermilab physicist for access to data about the new events. Despite the fact that Fermilab was not a private corporation, but was a National Laboratory funded by United States taxpayers (one of which was me),

Fermilab's response was that Fermilab would keep the data about the new events secret from the public so long as Fermilab existed.

The Physics Consensus Community was so afraid of the Truth that it buried the new events deeper than the Vatican had buried records of Inquisitions.

The situation, along with some others of similar karma, depresses me.

To try to alleviate the depression, I am writing a fictional story that deals with issues some of which are similar to my real-world issues. A current draft (73 pages plus cover, pdf file less than 1 MB) is at

http://tony5m17h.net/cvr6×9Crackerville05pdftg.pdf

You can also see it by going to the front (index.html) page of

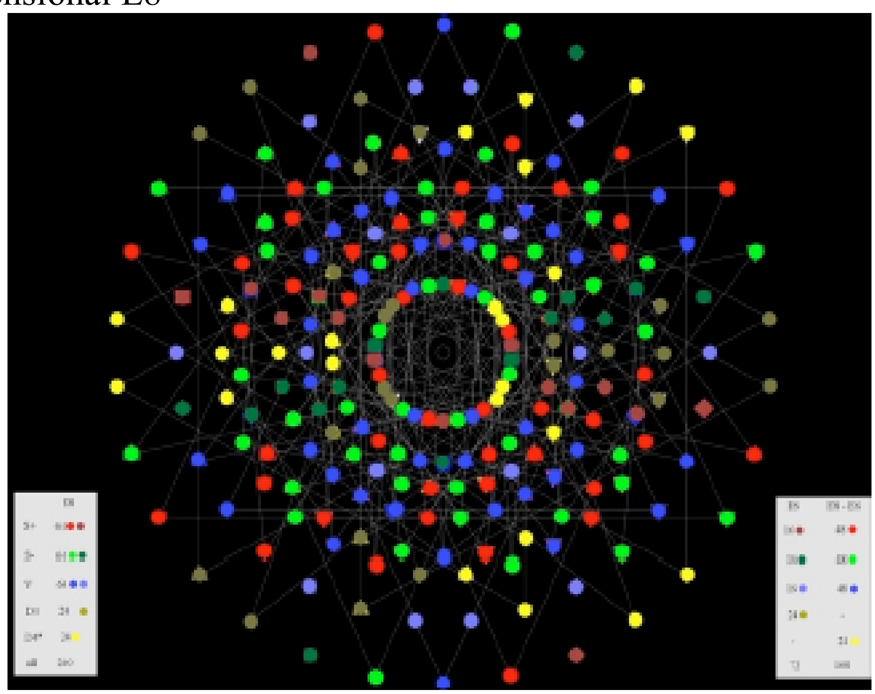
http://www.valdostamuseum.org/hamsmith/

and clicking on the link saying "Click Here For A Story.".

Technical Introduction

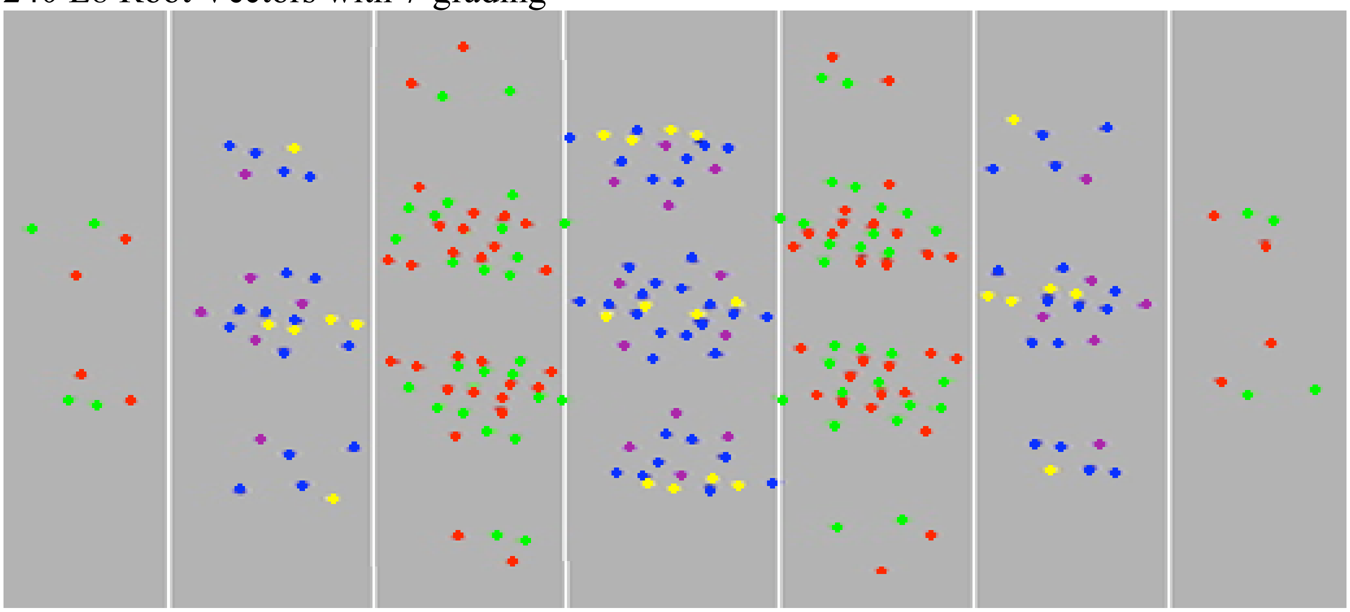
This work is intended to be an exposition of physics ideas and results, and so does not have a bibliography that cites every relevant work. No disrespect is intended to the many people whose relevant work is not explicitly mentioned here.
For ease of presentation, sometimes I will be sloppy about such things as signature, distinguishing between Pinors and Spinors, precise group structure distinctions such as between SU(3)xSU(2)xU(1) and S(U(2)xU(3)) = U(1) x SU(2) x SU(3) / I(2) x I(3), etc.
I hope that technically accurate meanings can be clearly understood from context.
This paper is based on Clifford Algebra Cl(8) physics such as CERN CDS EXT-2003-087 and CERN CDS EXT-2004-013 and CERN CDS EXT-2004-031 combined with some of Garrett Lisi's E8 ideas from hep-th/0711.0770 and related work.

Cl(16) = Cl(8) x Cl(8)
Cl(8) has 256 dimensions with 9-grading
1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 = 1 + 8 + 28 + 56 + (35+35) + 56 + 28 + 8 + 1
Octonions have graded structure 1 + 3 + 3 + 1.
Remove an Octonion 1 + 0 + 0 + 0 + (3+3) + 0 + 0 + 0 + 1
to get 248-dimensional E8



Note that removal of the scalar 1 and pseudoscalar 1 from Cl(8) to get E8 means that in E8 Physics the scalar Higgs is not an independent fundamental entity, but appears in 8-dimensional Kaluza-Klein Spacetime by the Meinhard Mayer mechanism with properties related to the Conformal Group version of the MacDowell-Mansouri mechanism and the T-quark condensate described by Yamawaki et al.

E8 has a 7-grading (similar to that of Thomas Larsson)
8 + 28 + 56 + 64 + 56 + 28 + 8
Remove 0 + 0 + 0 + 8 + 0 + 0 + 0 of the E8 Cartan subalgebra elements
to get
240 E8 Root Vectors with 7-grading



8 + 28 + 56 + 56 + 56 + 28 + 8

which includes
D4 with 24 Root Vectors at 0 + 6 + 0 + 12 + 0 + 6 + 0
D4* with 24 Root Vectors at 0 + 6 + 0 + 12 + 0 + 6 + 0
U(8) with 64 Root Vectors at 0 + 16 + 0 + 32 + 0 + 16 + 0
As to the D4 and D4*,
each gets 4 of the 8 Cartan dimensions of E8 in its middle grade,
so that in the 248-dim E8 7-grading they look like
D4 with 28 dimensions at 0 + 6 + 0 + 16 + 0 + 6 + 0
D4* with 28 dimensions at 0 + 6 + 0 + 16 + 0 + 6 + 0
and
the … 16 … looks like U(4) = D3xU(1)
and the … 6 + … + 6 … looks like 12 = real dimensionality of D4 / D3xU(1)
As to the U(8),
the … 32 … looks like U(8) / U(4)xU(4)
and the … 16 + … + 16 … looks like two copies of U(4)

Even part of E8 graded structure = D8 = Spin(16) =

= (28 = D4 = Spin(8)) + (64 = U(8)) + (28 = D4 = Spin(8))

D8 / D4 x D4 = 64 = U(8) = 8x8 =
= 8 Kaluza-Klein Spacetime dimensions x 8 Dirac Gammas

D4 / Spin(6)xSpin(2) = D4 / U(4) is a Kahler manifold with Complex Structure that allows Wick Rotation Changes of Signature so that:

D4 can be Spin(0,8) or Spin(1,7) or Quaternionic Spin(2,6)

Spin(6) can change to Spin(2,4)

U(4) can change to U(2,2)

There are two D4 in E8, so:

one D4 gives Conformal Gravity by D4 / U(2,2) = D4 / Spin(2,4)xSpin(2)

the other D4 gives the Standard Model by D4 / U(4) = D4 / Spin(6)xSpin(2)

Odd part of E8 graded structure = E8 / D8 =

= 8 + 56 + 56 + 8 = 64 + 64 = 8x8 + 8x8 =

8 Fermion Particles x 8 Dirac Gammas
+
8 Fermion AntiParticles x 8 Dirac Gammas

The Lagrangian for Gravity plus the Standard Model is based on natural structural relations among various parts of E8(8).

The second and third generations of fermions are composites of some of the 248 elements of E8 and are not directly related to triality.

Triality is useful in establishing relations among fermions, the base manifold, and gauge bosons, which relations indicate that the model satisfies Coleman-Mandula and spin-statistics.

248-dimensional E8 can be embedded in 256-dimensional Cl(8) Clifford Algebra.

Combining many copies the Cl(8) Clifford Algebra structure produces a generalized Hyperfinite II1 von Neumann Algebra factor for an Algebraic Quantum Field Theory in which a Bohm-type Quantum Potential comes from an E6 version of 26-dimensional Bosonic String Theory (16 fermionic dimensions coming from orbifolding), with strings seen as world-lines (closed strings being virtual loops) and in which Many-Worlds Quantum superposition separation plays a fundamental role in Quantum Consciousness.

The Bohm-type Quantum Potential shows how fermions, viewed as Kerr-Newman Black Hole vortices, interact in accord with Non-Relativistic Constituent Quark models.
Joy Christian in arXiv 0904.4259 "Disproofs of Bell, GHZ, and Hardy Tpe Theorems and the Illusion of Entanglement" says: "… a [geometrically] correct local-realistic framework … provides exact, deterministic, and local underpinnings for at least the Bell, GHZ-3, GHZ-4, and Hardy states. The correlations are … the classical correlations among the points of a 3 or 7-sphere … which … preserve the locality condition of Bell. The alleged non-localities of these states are thus shown to result from misidentified [geometries] of the EPR elements of reality. …". He uses Clifford algebra and Division algebra and Sphere Structure techniques.

The unit 3-sphere in R^4 with center at the origin and quaternion multiplication is the unit quaternions with Lie algebra SU(2) = Spin(3) = bivector algebra of the Cl(0,3) Clifford algebra with graded structure

0 1 2 3
1 3 3 1

with the bivectors being 3 elements the 2-graded part of Cl(0,3) which correspond to the 3 imaginary quaternions i,j,k. The even-graded part Cle(0,3) of Cl(0,3) consists of the one grade-0 scalar and the three grade-2 bivectors

0 2
1 3

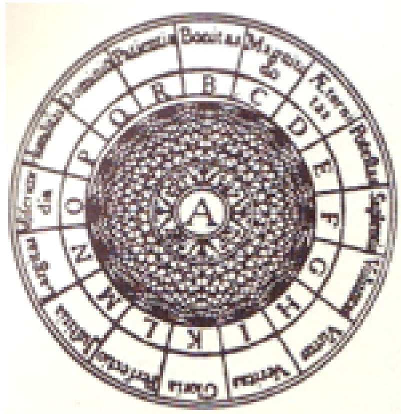
and they form the 1+3 = 4-dim Clifford algebra Cl(0,2) with graded structure
0 1 2
1 2 1
which is isomorphic to the quaternions.
Real Clifford algebras (including Cl((0,2) = quaternions) are matrix algebras (or direct sums thereof) with three types of entries:
Real, such as Cl(0,0) = R = 1x1 real matrix algebra;
Complex, such as Cl(0,1) = C = 1x1 complex matrix algebra; and
Quaternionic, such as Cl(0,2) = H = 1x1 quaternion matrix algebra
and as Cl(0,3) = H (+) H = direct sum of two quaternion 1x1 matrix algebras
and as Cl(1,3) = M(2,H) = 2x2 quaternion matrix algebra
and as Cl(1,4) = M(2,H) (+) M(2,H) = direct sum pf two quaternion 2x2 matrix algebras
and as Cl(2,4) = M(4,H) = 4x4 quaternion matrix algebra related to Spin(2,4) = SU(2,2) = Conformal group of Lie sphere light-cone geometry.

The 7-sphere is not itself a Lie algebra (it is a Malcev algebra), but its bracket product expands to form another 7-sphere and a G2, which combine with it to form a 7+7+14 = 28-dimensional Lie algebra Spin(8) of the Cl(8) Clifford algebra that is the basis for the E8 Lie algebra used in my E8 physics model, so fully physically realistic states can be represented by natural expansions of the 7-sphere structures used by Joy Christian.

The 248-dim Lie algebra E8 = 120-dim adjoint Spin(16) + 128-dim half-spinor Spin(16)

Cl(8) (x) Cl(8) = Cl(16) which has Spin(16) as its bivector Lie algebra.

As Ramon Llull showed about 700 years ago in his Wheel A, the 16 basis vectors of Cl(16) (vertices/letters) combine to form 120 bivectors (vertex pair lines) of Cl(16) which act as the 120 generators of the Lie algebra Spin(16).



The real Clifford algebra 8-periodicity tensor product factorization

$$Cl(16) = Cl(8)\ (x)\ Cl(8)$$

gives correspondences between 248-dim E8 structure and 256-dim Cl(8) structure, which has graded structure

$$Cl(8) = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1$$

Taking the tensor product Cl(8) x Cl(8) to get Cl(16) produces the following 120 Cl(16) bivectors:

- 28 Spin(8) bivectors of the first Cl(8) in the tensor product
- 28 Spin(8) bivectors of the second Cl(8) in the tensor product
- 64 = 8x8 tensor product of the two 8-dim 1-vectors of each the two Cl(8)s

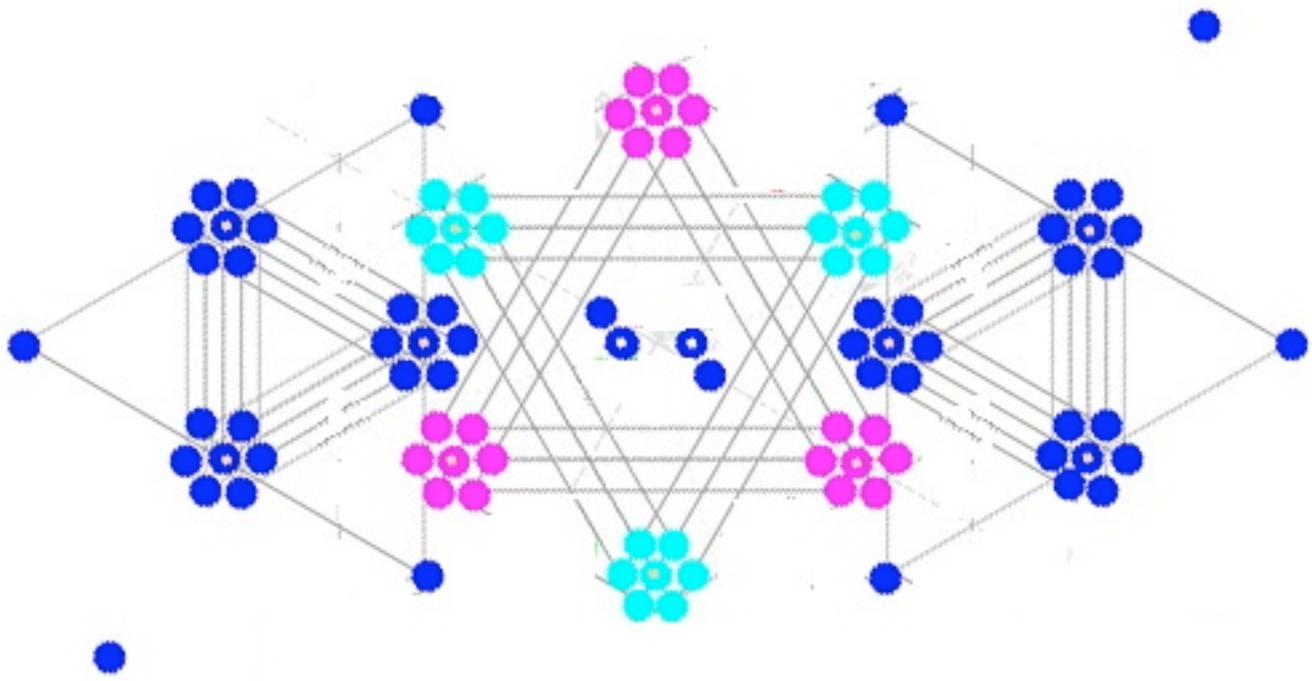
The 28+28+64 = 120-dim Cl(16) bivector algebra produces the 120-dim adjoint of the Lie algebra Spin(16).

The 248-dim Lie algebra E8 = 120-dim adjoint Spin(16) + 128-dim half-spinor Spin(16) is rank 8, and has 240 root vectors that form the vertices of an 8-dim polytope (the Witting polytope).

112 of the 240 vertices are the root vector polytope of the 120-dim rank 8 Spin(16) Lie algbra.

In terms of the 28 bivectors of the first Cl(8) factor and the 28 bivectors of the second Cl(8) factor and the 64 product-of-vectors, the 112 are:

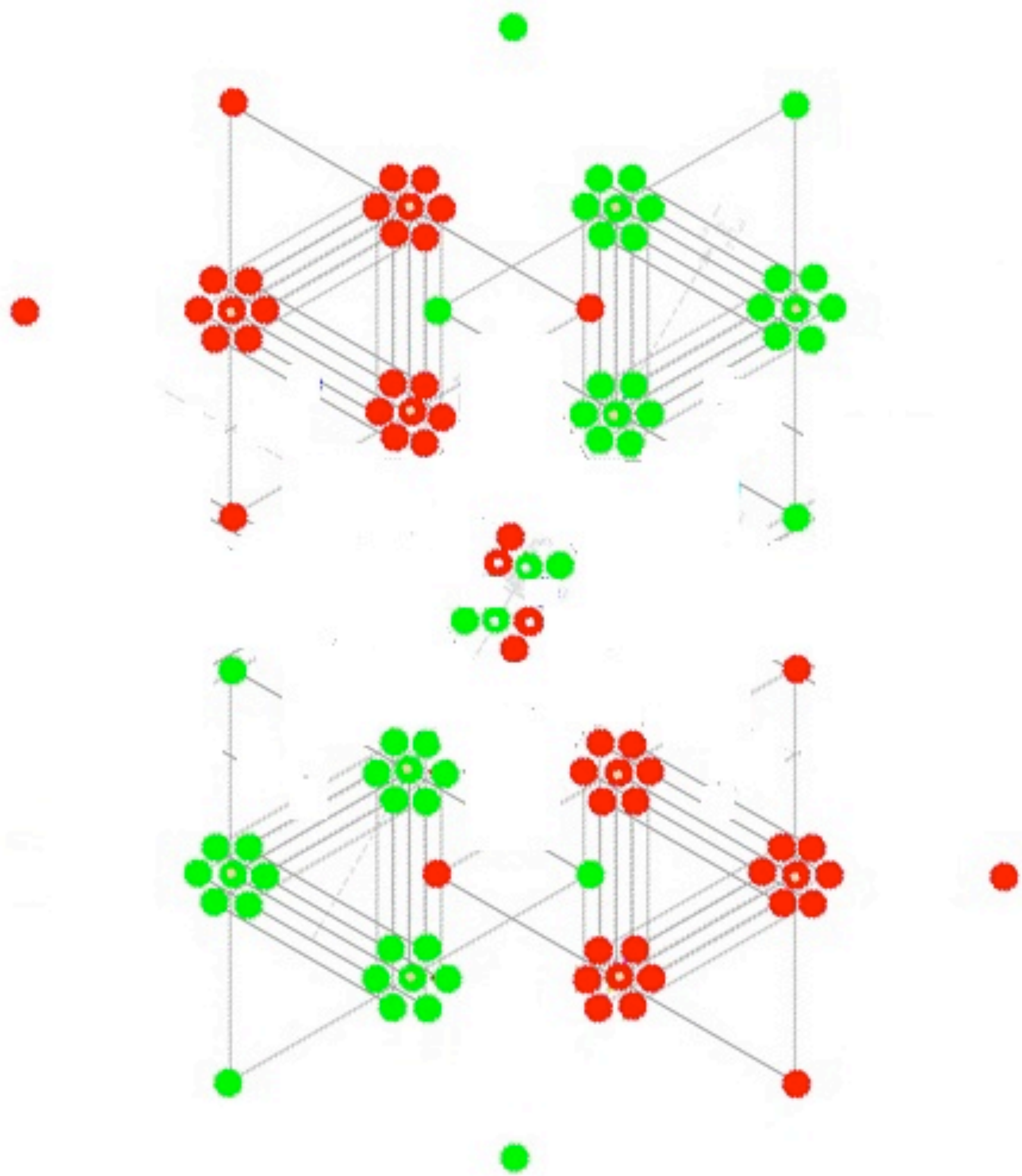
- 24 of the 24-cell root vector polytope of the rank-4 Spin(8) of the first Cl(8) (colored magenta on the following diagram)
- 24 of the 24-cell root vector polytope of the rank 4 Spin(8) of the second Cl(8) (colored cyan on the following diagram)
- 64 of the 8x8 product-of-vectors (colored blue on the following diagram)



Note that in the above image some of the 240 E8(8) vertices are projected to the same point: each of the 2 vertices in the center (with white dots) are points to which 3 vertices are projected, so that each of the 2 circles with a white dot represents 3 vertices; each of the 12 vertices surrounded by 6 same-color nearest neighbors (with yellow dots) are points to which 2 vertices are projected, so that each of the 12 circles with a yellow dot represents 2 vertices.

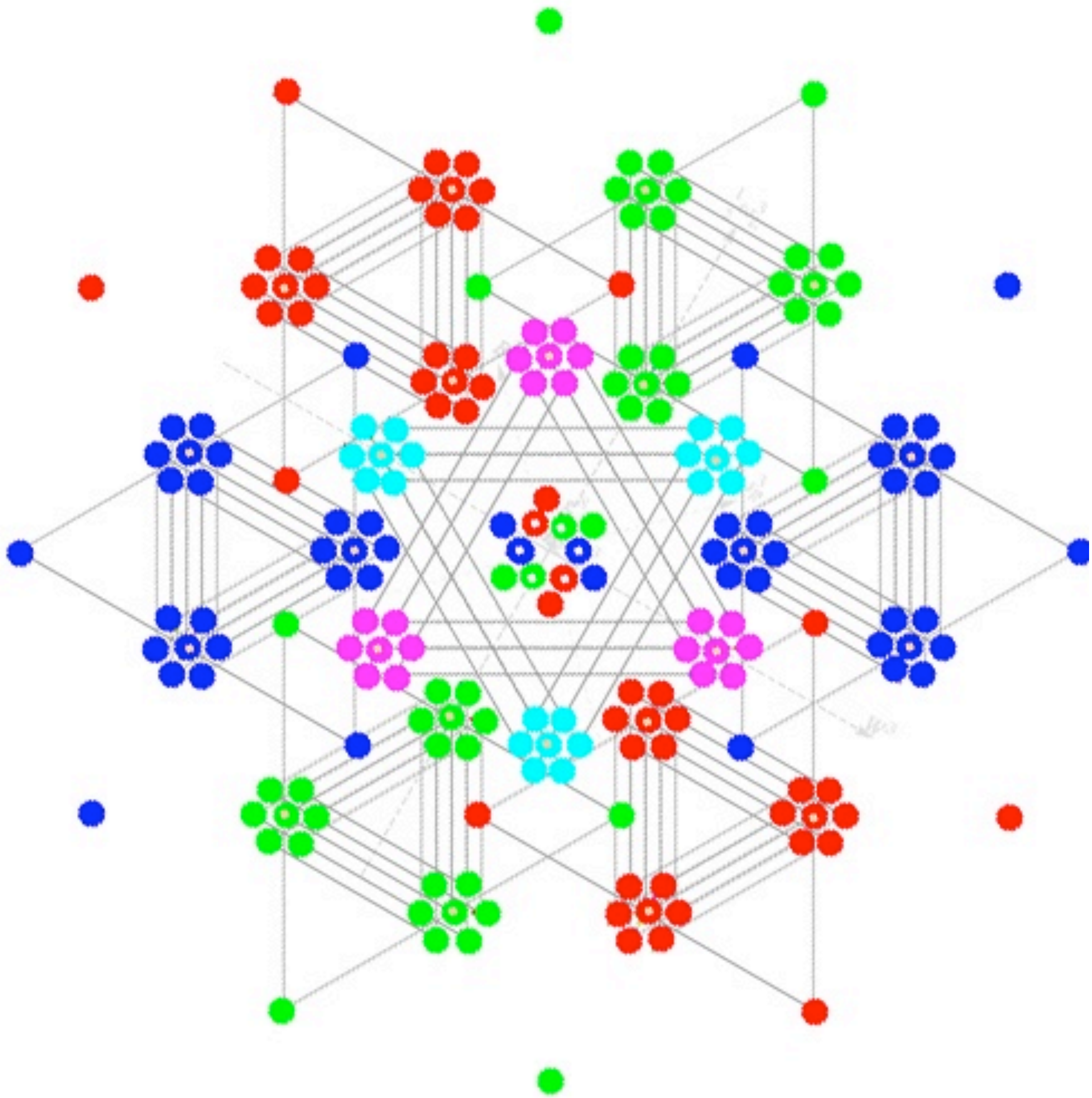
128 of the 240 vertices correspond to a half-spinor representation of the Spin(16) Lie algebra.

The 128 can be seen as the sum 64 + 64 of two 8x8 square-matrices each being 64-dim (colored red and green on the following diagram).



Note that in the above image some of the 240 E8(8) vertices are projected to the same point: each of the 4 vertices in the center (with white dots) are points to which 3 vertices are projected, so that each circle with a white dot represents 3 vertices; each of the 12 vertices surrounded by 6 same-color nearest neighbors (with yellow dots) are points to which 2 vertices are projected, so that each of the 12 circles with a yellow dot represents 2 vertices.

Putting the 112 and 128 together gives the 240 vertices of the E8 root vector polytope:



Note that in the above image some of the 240 E8(8) vertices are projected to the same point: each of the 6 vertices in the center (with white dots) are points to which 3 vertices are projected, so that each of the 6 circles with a white dot represents 3 vertices; each of the 24 vertices surrounded by 6 same-color nearest neighbors (with yellow dots) are points to which 2 vertices are projected, so that each of the 24 circles with a yellow dot represents 2 vertices.

Using the color-coding, the 240 root vector vertices of E8 correspond to the graded structure of the 256-dim Cl(8) Clifford algebra as follows:

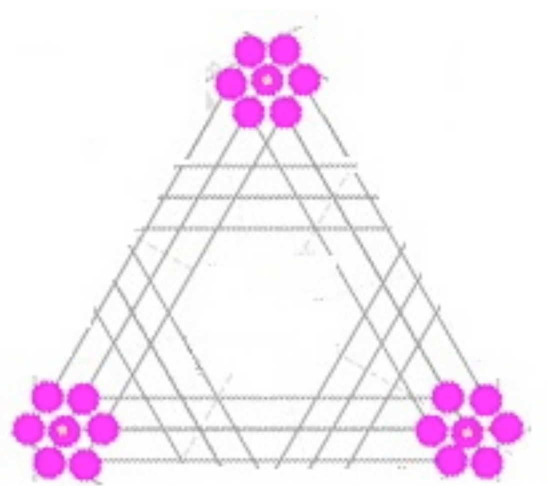
$$Cl(8) = 1 + 8 + 28 + 56 + 70 + 56 + 28 + 8 + 1 =$$

$$= 1 + 8 + (24+\underline{4}) + (24+4+28) + (32+3+3+32) + (28+4+24) + (24+\underline{4}) + 8 + 1$$

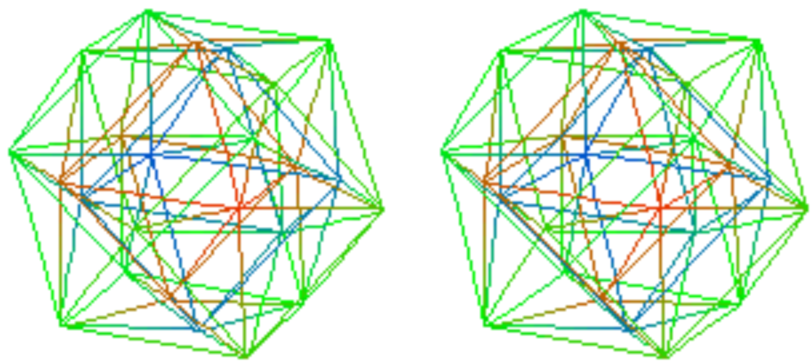
In the above, the black underlined $\underline{4}+\underline{4} = 8$ correspond to the 8 E8 Cartan subalgebra elements that are not represented by root vectors, and the black non-underlined 1+3+3+1 = 8 correspond to the 8 elements of 256-dim Cl(8) that do not directly correspond elements of 248-dim E8.

The 240 root vectors have the following physical interpretations:

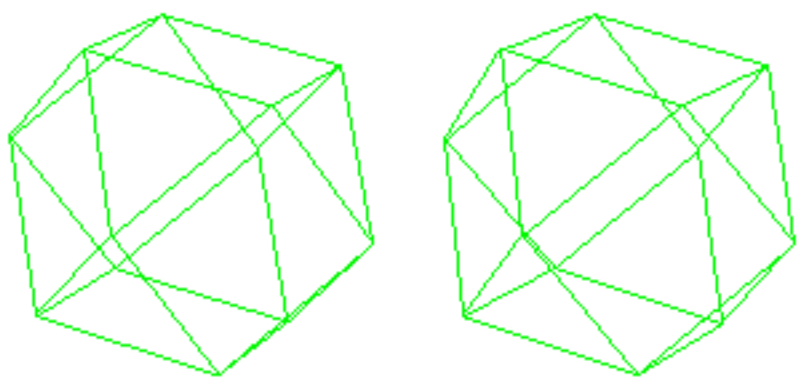
The Spin(8) whose root vector diagram is the vertices of the first 24-cell, living in the Cl(8) bivectors



A stereo view of a 24-cell (the 4th dimension color-coded red-green-blue with green in the middle)



shows that the 4-dim 24-cell has a 3-dim central polytope that is a cuboctahedron



the 12 vertices of which form the root vector polytope of the 16-dim U(2,,2) = U(1) x SU(2,2) , where 15-dim rank 3 SU(2,2) = Conformal Group Spin(2,4) produces Gravity by the MacDowell-Mansouri mechanism (see Rabindra Mohapatra, Unification and Supersymmetry (2nd edition, Springer-Verlag 1992), particularly section 14.6).

Since this group structure acts directly on the 8-dim Kaluza-Klein M4 x CP2, it acts on the associative part given by the associative 3-vector PSI of the dimensional reduction Quaternionic structure

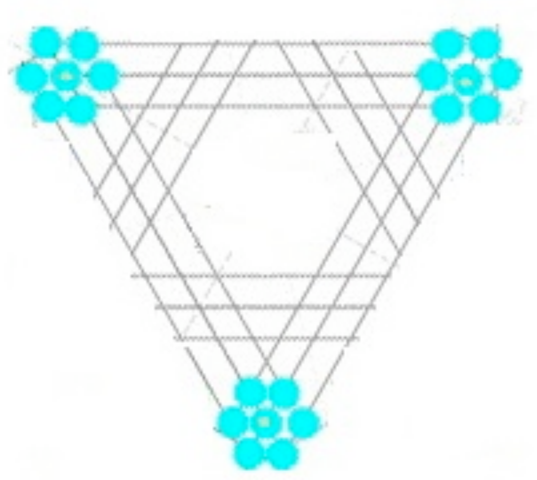
(such as occurs due to dimensional reduction of physical spacetime from 8-dim Octonionic to 4-dim Quaternionic by freezing out (at energies lower than the Planck/GUT region) a Quaternionic substructure of 8-dim Octonionic vector space)

which is the spatial part of the M4, so that the M4 on which it acts has signature -+++

The U(1) of U(2,2) provides the complex phase of propagators.

This gives Gravity similar to the Conformal Gravity of I. E. Segal, and U(1) propagator phase.

The Spin(8) whose root vector diagram is the vertices of the second 24-cell, living in the Cl(8) 6-vectors



The 28 6-vectors of Cl(8) correspond to a 28-dim rank 4 Spin(8) Lie algebra after introduction of Quaternionic structure into the E8 physics model

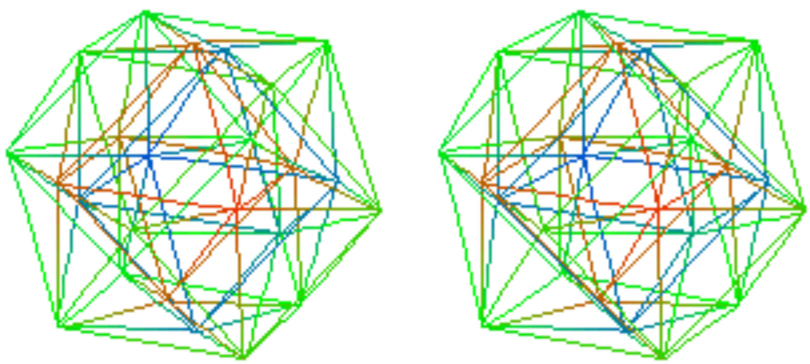
(such as occurs due to dimensional reduction of physical spacetime from 8-dim Octonionic to 4-dim Quaternionic by freezing out (at energies lower than the Planck/GUT region) a Quaternionic substructure of 8-dim Octonionic vector space)

by using the co-associative 4-vector PHI of the chosen Quaternionic structure to map any 6-vector A to a bivector A /\ PHI, and so mapping the 28 6-vectors onto 28 bivectors that form a 28-dim Lie algebra.

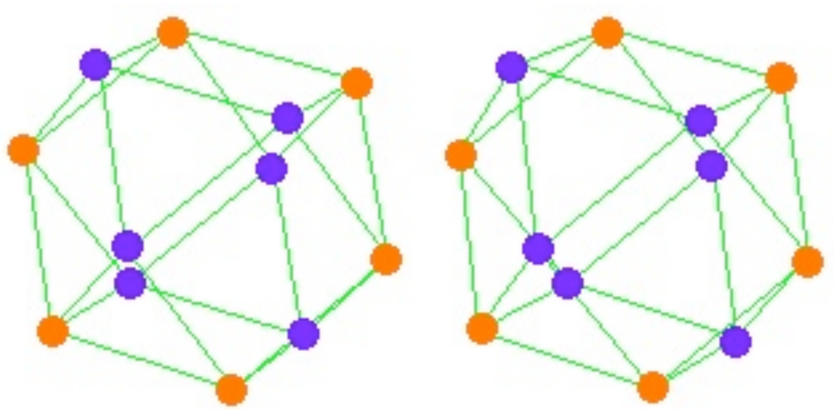
The process is somewhat analagous to using a co-associative 4-vector PHI' in Cl(7) to define a cross-product in 7-dim vector space for vectors a, b (see F. Reese Harvey, Spinors and Calibrations (Academic 1990)) by

a x b = *((a /\ b) /\ PSI)

A stereo view of a 24-cell (the 4th dimension color-coded red-green-blue with green in the middle)

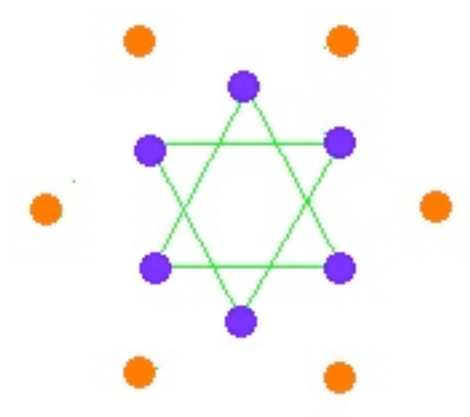


shows that the 4-dim 24-cell has a 3-dim central polytope that is a cuboctahedron



that is the root vector polytope of 15-dim rank 3 Spin(6) = SU(4) that includes 8+1 = 9-dim SU(3)xU(1) = U(3) in the Twistor construction of 6-dim CP3 = SU(4) / U(3)

Projection into a 2-dim space for the root vectors of the rank 2 group SU(3) gives



where the 6 purple vertices form the hexagonal root vector polygon of 8-dim rank 2 SU(3) and the 6 gold vertices correspond to the 6 dimensions of the CP3 Twistor space.

Introduction of a Quaternionic CP3 Twistor space "... induces a mapping of projective spaces CP3 -> QP1 ...[with]... fibres ... CP1 ..." (see R. O. Wells, Complex Geometry in Mathematical Physics (Les Presses de l'Universite de Montreal 1982), particularly section 2.6).

Since CP1 = SU(2) / U(1) an introduction of Quaternionic structure into the E8 physics model

(such as occurs due to dimensional reduction of physical spacetime from 8-dim Octonionic to 4-dim Quaternionic by freezing out (at energies lower than the Planck/GUT region) a Quaternionic substructure of 8-dim Octonionic vector space)

gives weak force SU(2) through QP1 = Sp(2)/ Sp(1)xSp(1) = Spin(5) / SU(2)xSU(2) or, equivalently, through CP3 containing CP2 = SU(3) / U(2) .

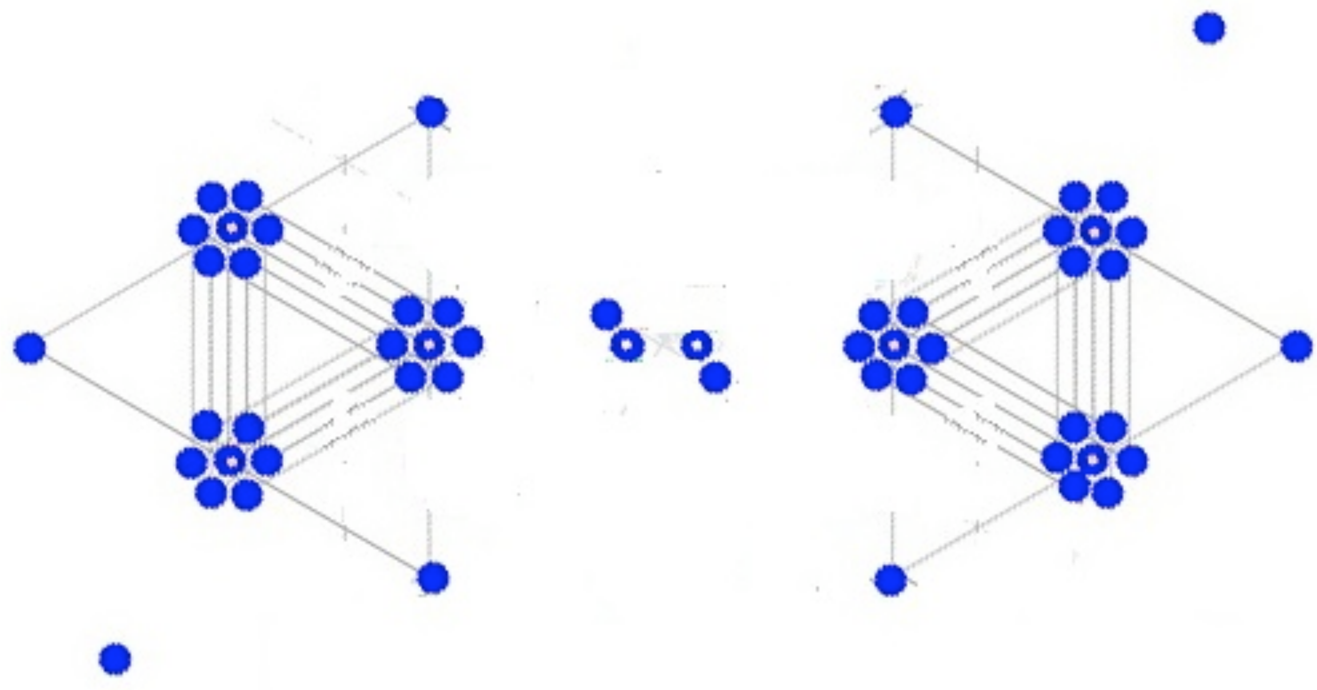
Since the U(1) of U(3) = SU(3) x U(1) is Abelian, it does not correspond to a root vector vertex and therefore does not appear in the root vector diagrams.

Since this group structure is produced by a co-associative 4-vector PHI, it acts on the co-associative part of 8-dim Kaluza-Klein M4 x CP2, which is the CP2 4-dim Internal Symmetry Space of signature ++++

As described by N. A. Batakis in Class. Quantum Grav. 3 (1986) L99-L105, the U(2) = SU(2) x U(1) acts on the CP2 as little group, or local isotropy group, while the SU(3) acts globally on the CP2 = SU(3) / U(2) = SU(3) / SU(2) x U(1)

This gives SU(3) x SU(2) x U(1) of the Standard Model.

The product-of-vectors 64 = 8 x 8



With respect to the Cl(8) grading, the first 8 of the 8x8 = 64 is the vector space, and therefore is a natural 8-dim spacetime that after introduction of a preferred Quaternionic substructure

(such as occurs due to dimensional reduction of physical spacetime from 8-dim Octonionic to 4-dim Quaternionic by freezing out (at energies lower than the Planck/GUT region) a Quaternionic substructure of 8-dim Octonionic vector space)

becomes a 4-dim plus 4-dim Kaluza-Klein space of the form M4 x CP2 as described by N. A. Batakis in Class. Quantum Grav. 3 (1986) L99-L105,

The M4 of signature -+++ contains an associative 3-dim spatial structure, while the CP2 of signature ++++ has a co-associative 4-dim structure.

So, the first 8 of the 8x8 = 64, denoted by 8_v , represents 4+4 = 8-dim M4 x CP2 Kaluza-Klein space, where the compact CP2 is small.

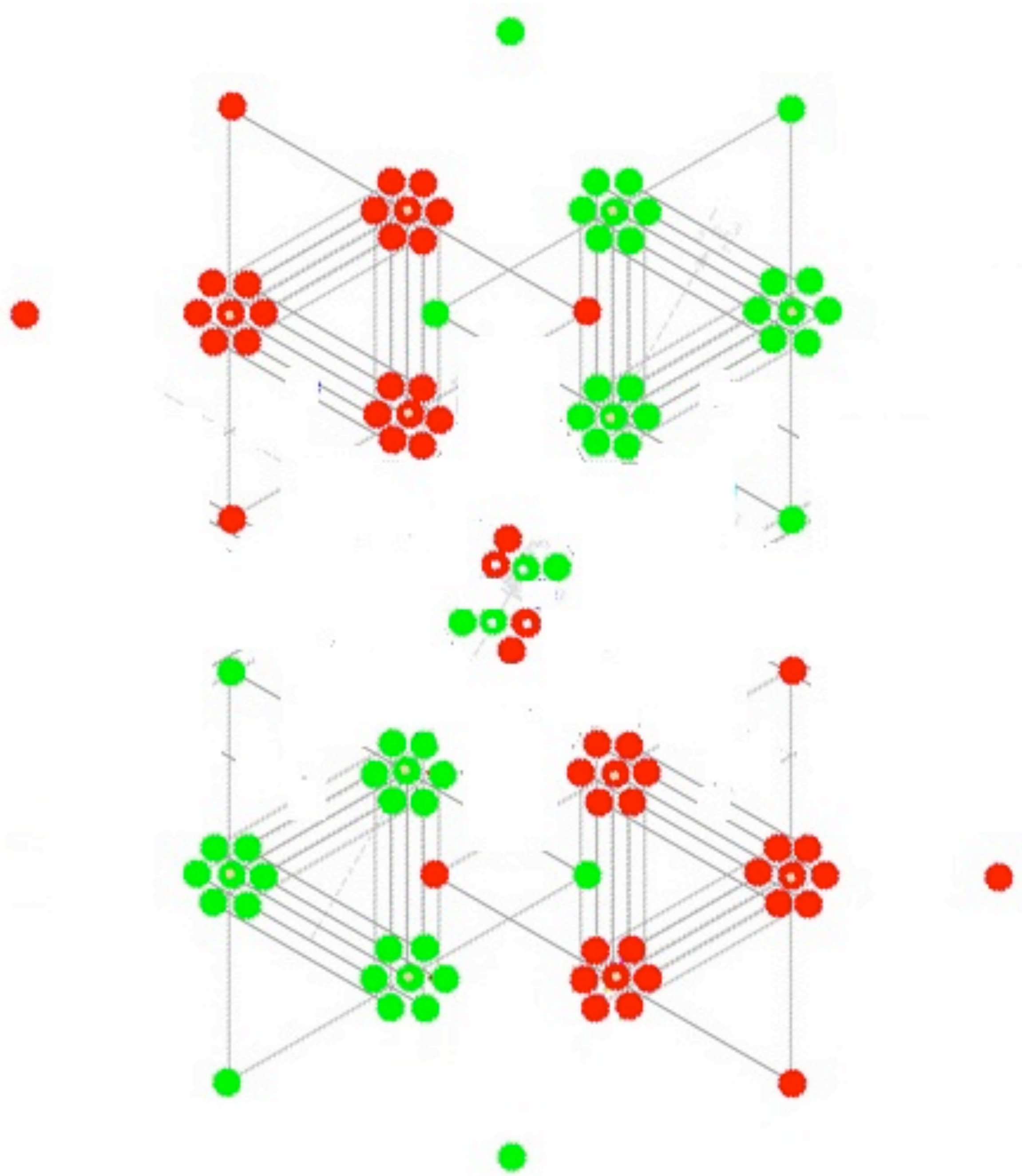
As to the second 8 of the 8_v x 8,

it lives in the 7-vectors of the Cl(8) grading,

and it should represent the 8 Dirac Gammas of the Cl(8) Clifford algebra, so denote it by 8_G so that

the 64 = 8_v x 8_G describes the Kaluza-Klein space and its connection to the Dirac Gammas.

The 128 Spin(16) half-spinors 64 + 64



The 128 is the 128-dim rank 8 symmetric space E8 / Spin(16) of type EVIII known as Rosenfeld's octo-octonionic projective plane (OxO)P2 (see Arthur

L. Besse, Einstein Manifolds (Springer 1987) and Boris Rosenfeld, Geometry of Lie Groups (Kluwer 1997)).

Since it is a plane (of 2 8x8 octo-octonionic dimensions), it has structure 128 = 64 + 64 = 8x8 + 8x8.

Since it is a half-spinor space (of Spin(16)) its elements are fundamentally fermionic, so

- one of the 8 in one of the two 8x8 = 64 should correspond to the 8 first-generation fermion particles (denote it by 8_f+)
- one of the 8 in the other of the two 8x8 = 64 should correspond to the 8 first-generation antiparticles (denote it by 8_f-)

As to the second 8 in the 8_f+ x 8 = 64 and the 8_f- x 8 = 64

it should represent the 8 Dirac Gammas of the Cl(8) Clifford algebra, so denote it by 8_G so that :

128 = 64 + 64 and

the 64 = 8_f+ x 8_G describes the 8 first-generation fermion particles (neutrino; red, blue, green up quarks; red, blue, green down quarks, electron) and their connection to the Dirac Gammas

the 64 = 8_f+ x 8_G describes the 8 first-generation fermion anti-particles and their connection to the Dirac Gammas

Note that these fermions are related to the 8-dim +half-spinor and -half-spinor representations of Spin(1,7), the Lorentz group for the 8-dim space of Cl(8), so that this physics model, based on E8 and Cl(8), satisfies the Coleman-Mandula theorem because, as Steven Weinberg says at pages 382-384 of his book The Quantum Theory of Fields, Vol. III (Cambridge 2000), the important thing about Coleman-Mandula is that fermions in a unified model must "... transform according to the fundamental spinor representations of the Lorentz group ... or, strictly speaking, of its covering group Spin(d-1,1). ..." where d is the dimension of spacetime in the model.

Note also that the fermion particles are fundamentally all left-handed, and the fermion antiparticles are fundamentally all right-handed. The other handednesses are not different fundamental states, but arise dynamically due to special relativity transformations that can switch handedness of particles that travel at less than light-speed (i.e., that have more than zero rest mass).

Chirality

Realistic Physics is Chiral (i.e., has Chiralitry) because it breaks Chiral Ivariance. A Non-Chiral model (i.e., one having Chiral Invariance) is not consistent with the parity-breaking SU(2) weak force of type V-A.

In my E8 Cl(16) physics model, E8 = Spin(16) + +half-spinor(Spin(16))

E8 has no -half-spinor(Spin(16)) and has no antigeneration of fermions.

Prior to dimensional reduction, while spacetime is fully 8-dimensional, all E8 particles are massless and the gauge group is two full copies of Spin(8).

After dimensional reduction, spacetime is M4 x CP2

CP2 has Euler number 2+1 = 3. CP2 being 4-dimensional need not have zero Hirzebruch signature. CP2 Atiyah-Singer formula gives -1/8

However, since CP2 has no spin structure, you have to give it a generalized spin structure a la Hawking and Pope, whereupon you get (for integral m) for the index n_R - n_L = (1/2)m(m+1)

For m=1, n_R - n_L = (1/2) 1 2 = 1 for 1 generation

For m=2, n_R - n_L = (1/2) 2 3 = 3 for 3 generations

so

the E8 physics model with CP2 internal symmetry space has consistent chiral fermions

for 1 generation (the case prior to dimensional reduction)

and

for 3 generations (the case after dimensional reduction).

Here are some further details and references about the construction:

- D8adj = 28 + 28 + 64 = 120-dim bosonic stuff
- D8s+ = 64 + 64 = 128-dim fermion generation
- D8s- = 64 + 64 = 128-dim fermion antigeneration

Howard Georgi says in his book "Lie Algebras in Particle Physics", 2nd ed Perseus 1999: "... The full story of the reality properties of the SO(N) spinors is ... as follows:

- Algebra Spinors
- SO(8k+3) pseudo-real
- SO(8k+4) pseudo-real
- SO(8k+5) pseudo-real
- SO(8k+6) complex
- SO(8k+7) real
- SO(8k) real
- SO(8k+1) real
- SO(8k+2) complex

The simplest example of a pseudo-real representation is the spin 1/2 representation of SU(2) generated by the Pauli matrices ...".

Since SU(2) = Spin(3) = the 3-sphere S3 in 4-dimensional quaternionic space, it is clear that what Howard Georgi characterizes as pseudo-real is quaternionic (see for example "Representation Theory" by William Fulton and Joe Harris, Springer-Verlag 1991).

In his book "Group Structure of Gauge Theories", Cambridge 1986, L. O'Raifeartaigh may not be entirely accurate about all aspects of "... reality properties ... of the simple compact Lie groups ...", because he says "... SO(4n) ... the spinor representations ... are pseudo-real ..." when, as stated by Georgi, the spinors of SO(4n) are only pseudo-real = quaternionic for odd n, and are real for even n, including the cases D4 = SO(8) and D8 = SO(16).

However,

O'Raifeartaigh does a very good job of describing physics model building, and seems to be quite accurate when he discusses "... many ... theories [for which] the fermion representations are required to be strictly complex ...[such as]... SO(4n+2) ...", including the case D5 = SO(10). In that context, he says: ".. left-handed fermions and right-handed antifermions [generations] ... are assigned to a representation f ... and their antifields [antigenerations] ... to the complex conjugate representation f* ...

SO(n) ... the defining representations are real ... all the tensor representations are real ... except for the irreducible parts F_l+/- of... the fundamental ... tensor ... representation ... F_l for SO(4n+2) [i.e., l = 4n+2]...[which are]... actually complex ...

SO(4n) ... have no anomalies, but they also have no strictly complex representations and thus a spontaneous breakdown of the real assignments must always be invoked. ...

SO(4n+2) ... the spinor representations ... D+ and D- are strictly complex, but are conjugate ...[and can]... be strictly inequivalent ... from the beginning ... with respect to ... electroweak U(2) ...[which]... violates parity ...

E8 ... has no complex representations ... all primitive representations are real ...".

As an example applying the quote of O'Raifeartaigh consider the D5 Lie algebra SO(10):

- D5adj = 28 + 16 + 1 = 45-dim bosonic stuff
- D5s+ = 16-dim fermionic stuff
- D5s- = 16-dim fermionic stuff

Since 10 = 8x1 + 2, the representations D5s+ = 16 and D5s- = 16* are strictly complex and they can be taken to be strictly inequivalent from the beginning

so D5s+ = the 16 of SO(10) can represent a generation of left-handed fermions and right-handed antifermions

and D5s- = the 16* of SO(10) can represent an antigeneration that is complex conjugate to Ds+.

To use D5 = SO(10) get k generations physically, you have to reduce the 10-dim vector spacetime to 4-dim by forming a compact 6-dim internal symmetry space whose Atiyah-Singer index is k.

A similar analysis for a D8 = SO(16) inside E8 is not as easy, but it can be done:

- D8adj = 28 + 28 + 64 = 120-dim bosonic stuff
- D8s+ = 64 + 64 = 128-dim fermionic stuff
- D8s- = 64 + 64 = 128-dim fermion stuff

Since 16 = 8x2, D8+ and D8s- are real there is no complex conjugation and a spontaneous breakdown of the real assignments must be invoked.

Let D8s+ = 128s+ = (64s+_1 + 64s+_2)

and D8s- = 128s- = (64s-_1 + 64s-_2)

One way to assign fermionic physical interpretions is:

- D8s+ containing a particle half-generation 64s+_1 and half-antigeneration 64s+_2
- D8s- containing an antiparticle half-generation 64s-_1 and half-antigeneration 64s-_2

to get

- D8s+ = generation particles + antigeneration particles
- D8s- = generation antiparticles + antigeneration antiparticles

I think that may be the assignment used by Jacques Distler in saying that Ds+ contains half a generation and half an antigeneration.

However, I prefer to assign fermionic physical interpretations differently:

- D8s+ containing a (64s+_1 + 64s+_2) generation of fermion particles and antiparticles
- D8s- containing a (64s-_1 + 64s-_2) antigeneration of fermion particles and antiparticles.

which gives:

- D8s+ = generation particles + generation antiparticles
- D8s- = antigeneration antiparticles + antigeneration antiparticles

so that 248-dim E8 = (28+28+64) =

= 120-dim D8adj + (64s+_1 + 64s+_2) = 128-dim D8s+

contains one generation of fermion particles and antiparticles (the second and third generations emerging as composites of the first),

but since E8 does not contain D8s- it does not contain any fermion antigeneration.

HERE is a corrected (there were many typos) version of my 23 July 2008 post to an ncategory cafe thread by Urs Schreiber with discussion of E8 models by Jacques Distler, Garrett Lisi, et al:

Jacques Distler said that "the (8s,8s) of D4xD4" contains "... half a generation and half an anti-generation ...".

amused asked about Garrett Lisi's E8 model "... why have not the anti-generation fermions ...[such as]... right-handed neutrinos ... been seen already in experiments along with the fermions of the SM generation to which they correspond? ...".

Here is a suggestion: The D4xD4 is part of D8. If you look at these D8 representations:

- the 120-dim adjoint - denoted by D8adj
- the 128-dim +half-spinor - denoted by D8s+
- the 128-dim -half-spinor - denoted by D8s-

and if you make the (admittedly unconventional, but it seems to me to be possibly workable) physical interpretations:

- D8adj as gauge bosons plus more bosonic stuff (possibly spacetime vectors)
- D8s+ as one generation of fermion particles and antiparticles
- D8s- as one antigeneration of fermion particles and antiparticles

THEN, if you try to form a Lie algebra from D8adj + D8s+ + D8s-

it does not work,

but if you try to form a Lie algebra from D8adj + D8s+

you succeed and get E8

with the 64+64 = 128-dim 8Ds+ representing one generation of fermion particles (one 64 of D8s+) and one generation of fermion antiparticles (the other 64 of D8s+).

So, the math structure of Lie algebras is telling you that there is no physical D8s- antigeneration of fermions, and that one generation of D8s+ fermions lives inside E8.

Given that, you have to deal with the Atiyah-Singer index giving the net number of generations, which is an issue conventionally formulated in terms of the Euler index of the compact manifold (6-dim) used to reduce 10-dim spacetime to physical 4-dim.

For an E8 model, you could see spacetime as 8-dim reduced to a Kaluza-Klein M4 x CP2 and look at the index structure of the CP2.

If you reduce 8-dim spacetime to 4-dim by using the compact 4-dim internal symmetry space CP2, you see that, although CP2 has no spin structure, you can follow Hawking and Pope (Phys. Lett. 73B (1978) 42-44) and Chakraborty and Parthasarathy (Class. Quantum Grav. 7 (1990) 1217-1224) to define a series generalized spin structures for CP2, the first two having:

- index = 1 for 1 generation (the E8 prior to dimensional reduction)
- index = 3 for 3 generations (the E8 model after dimensional reduction induces the second and third generations to emerge as effective composites of the first).

Further, if you were to insist on starting with a 10-dim spacetime, you could still reduce it using the compact CP2 with generalized spin structure, leaving a 6-dim conformal spacetime that naturally gives you 4-dim spacetime by using conformal correspondences related to quaternions, twistors, etc.

Quaternionic Structure

At energies below the Planck/GUT level, the Octonionic structure of the model changes, by freezing out of a preferred Quaternionic substructure, from Real/Octonionic 8-dim spacetime to Quaternionic -+++ associative 4-dim M4 Physical Spacetime plus Quaternionic +++ co-associative 4-dim CP2 = SU(3) / SU(2) x U(1) Internal Symmetry Space.

After Quaternionic structure freezes out,

- 64 = 8_v x 8_G x 1_Real
- 64 = 8_f+ x 8_G x 1_Real
- 64 = 8_f+ x 8_G x 1_Real

transform from 8x8 real matrices to 4x4 Quaternionic matrices

- 64 = 4_v x 4_G x 4_Quaternion
- 64 = 4_f+ x 4_G x 4_Quaternion
- 64 = 4_f+ x 4_G x 4_Quaternion

As can be seen in this chart (from F. Reese Harvey, Spinors and Calibrations (Academic 1990))

$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$	$M_{64}(\mathbf{C})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{R}) \oplus M_{128}(\mathbf{R})$	$M_{256}(\mathbf{R})$
$M_8(\mathbf{C})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H}) \oplus M_8(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{R}) \oplus M_{64}(\mathbf{R})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{C})$
$M_4(\mathbf{H})$	$M_4(\mathbf{H}) \oplus M_4(\mathbf{H})$	$M_8(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{R}) \oplus M_{32}(\mathbf{R})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{C})$	$M_{64}(\mathbf{H})$
$M_2(\mathbf{H}) \oplus M_2(\mathbf{H})$	$M_4(\mathbf{H})$	$M_8(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{C})$	$M_{32}(\mathbf{H})$	$M_{32}(\mathbf{H}) \oplus M_{32}(\mathbf{H})$
$M_2(\mathbf{H})$	$M_4(\mathbf{C})$	$M_8(\mathbf{R})$	$M_8(\mathbf{R}) \oplus M_8(\mathbf{R})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$
$M_2(\mathbf{C})$	$M_4(\mathbf{R})$	$M_4(\mathbf{R}) \oplus M_4(\mathbf{R})$	$M_8(\mathbf{R})$	$M_8(\mathbf{C})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H}) \oplus M_8(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$
$M_2(\mathbf{R})$	$M_2(\mathbf{R}) \oplus M_2(\mathbf{R})$	$M_4(\mathbf{R})$	$M_4(\mathbf{C})$	$M_4(\mathbf{H})$	$M_4(\mathbf{H}) \oplus M_4(\mathbf{H})$	$M_8(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$
$\mathbf{R} \oplus \mathbf{R}$	$M_2(\mathbf{R})$	$M_2(\mathbf{C})$	$M_2(\mathbf{H})$	$M_2(\mathbf{H}) \oplus M_2(\mathbf{H})$	$M_4(\mathbf{H})$	$M_8(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$
$\mathbf{R}$	$\mathbf{C}$	$\mathbf{H}$	$\mathbf{H} \oplus \mathbf{H}$	$M_2(\mathbf{H})$	$M_4(\mathbf{C})$	$M_8(\mathbf{R})$	$M_8(\mathbf{R}) \oplus M_8(\mathbf{R})$	$M_{16}(\mathbf{R})$

The 16x16 = 256-dim Cl(8) = Cl(1,7) = M(16,R) = 16x16 Real Matrix Algebra is transformed into the 8x8x4 = 256-dim Cl(2,6) = M(8,Q) = 8x8 Quaternionic Matrix Algebra

and the 8x8 = 64-dim Cl(6) = M(8,R) = 8x8 Real Matrix Algebra is transformed into the 4x4x4 = 64-dim Cl(2,4) = M(4,Q) = 4x4 Quaternionic Matrix Algebra

and the 8-dim Real column vectors 8_v , 8_f+ , 8_f- become the 2-Quaternionic-dim (8-Real-dim) column vectors 2_Q_v , 2_Q_f+ , 2_Q_f-

and the 8-dim Real row vectors 8_G become the 2-Quaternionic-dim (8-Real-dim) row vectors 2_Q_G

so that the relationships among the 64 , 64 , 64 , and Gravity and the Standard Model coming from the D3 Lie algebras of Spin(2,4) = SU(2,2) and Spin(6) = SU(4) are maintained after introduction of Quaternionic structure.

Triality

There is a Spin(8)-type Triality among the three 64 things

- 64 = 8_v x 8_G = 2_Q_v x 2_Q_G of Kaluza-Klein space
- 64 = 8_f+ x 8_G = 2_Q_f+ x 2_Q_G of first-generation fermion particles
- 64 = 8+f- x 8_G = 2_Q_f- x 2_Q_G of first-generation antiparticles

The model has:

- 16 gauge bosons for MacDowell-Mansouri Gravity plus a complex propagator phase and 12 Standard Model gauge bosons, for a total of 28 gauge bosons (which is also 28 = 8 /\ 8 the number of gauge bosons to be expected from 8-dim vector space)
- 8 types of fermions (the second and third generations being combinatorial combinations of first-generation fermions.

From the point of view of high-energy 8-dim space, in which gauge boson terms have dimension 1 in the Lagrangian and fermion terms have dimension 7/2 in the Lagrangian, the Triality gives a Subtle Supersymmetry

Total Boson Dimensionality = 28 x 1 = 28 = 8 x 7 / 2 = Total Fermion Lagrangian Dimensionality

The Triality Subtle Supersymmetry shows UltraViolet Finiteness of the E8 model and gives a natural physical interpretation of Quantum Path-Integral Ghosts:

As van Holten indicates in hep-th/0201124, there is a 1-1 correspondence between Y-M gauge bosons and ghosts. In the case of the 28-dimensional Spin(8) gauge group of the E8 _Physics model, the corresponding 28 ghosts can be regarded as antisymmetric pairs of 8 pre-ghosts, with one of the pre-ghosts in the pair being a particle and the other being an antiparticle. The 8 pre-ghosts are like the 8 gauge potentials that form 28 gauge field bosons by antisymmetric wedge product. From that viewpoint, you could say that the role of ghosts is played by first-generation fermion particle-antiparticle pairs, and that when you do path-integral sum-over-histories quantization you don't need to add ghosts in by hand, because virtual spinor fermion particle-antiparticle pairs will do what is needed.

Since Triality identifies 8 half-spinor fermion particles with 8 half-spinor fermion anti-particles and with 8 vectors corresponding to 8 pre-ghosts, making it unnecessary to throw in ad-hoc ghosts when you quantize, Cl(8) with triality is uniquely useful for modelling a quantum theory of gauge group physics with ghosts.

This is consistent with Garrett Lisi's view of Ghosts and BRST as he says in 0711.0770 : "… Relying on the algebraic structure of the exceptional Lie groups, the fermions may also be recast as Lie algebra elements and included naturally as parts of a BRST extended connection. … the fermionic fields … may be considered ghosts of former gauge fields …".

From a more geometric point of view, if the BRST transformation acts like the nilpotent cohomology operator on the cohomology of Spin(8), then, consider that Spin(8) cohomology looks like

S3 u S7 u S11 u S7

S7 is a fibre bundle made up of S3 and S4 = QP1 (quaternionic projective space) and S11 is a fibre bundle made up of S3 and QP2 (quaternionic projective plane).

The 28 ghosts for the 28 Spin(8) gauge bosons can be seen as:

S3
S3 u QP1
S3 u QP2
S3 u QP1
Note that S3 u S3 u QP2 = S3 u S11 is the structure of G2, the automorphism group of the octonions, and that the two S3 u QP1 = S7 7-spheres are each unit spheres in octonion space.

Since S3 looks like the quaternion unit sphere, and is generated by an associative triple of octonions, and QP1 and QP2 are obviously quaternionic, it seems obvious to ask what happens to the E8 Physics octonionic Spin(8) or Clifford(8) model when a particular quaternionic subspace is frozen out, and that is how in E8 Physics an M4xCP2 Kaluza-Klein space emerges at low temperatures (of our present world) from the full high-temperature octonion 8-dim spacetime.

Lagrangian

The natural Lagrangian for the model is

Integration over 8-dim base manifold from 64

of

MacDowell-Mansouri term from U(2,2)

and

Gauge Boson term from SU(3)xSU(2)xU(1)

and

Fermion Particle-Antiparticle term from 64 + 64

This differs from conventional Gravity plus Standard Model in three respects:

- 1 - 8-dim base manifold
- 2 - no Higgs
- 3 - 1 generation of fermions

These differences can be reconciled as follows:

Reduction to 4-dim base manifold and Higgs:

The objective is to reduce the integral over the 8-dim Kaluza-Klein M4 x CP2 to an integral over the 4-dim M4.

Since the U(2,2) acts on the M4, there is no problem with it.

Since the CP2 = SU(3) / U(2) has global SU(3) action, the SU(3) can be considered as a local gauge group acting on the M4, so there is no problem with it.

However, the U(2) acts on the CP2 = SU(3) / U(2) as little group, and so has local action on CP2 and then on M4, so the local action of U(2) on CP2 must be integrated out to get the desired U(2) local action directly on M4.

Since the U(1) part of U(2) = U(1) x SU(2) is Abelian, its local action on CP2 and then M4 can be composed to produce a single U(1) local action on M4, so there is no problem with it.

That leaves non-Abelian SU(2) with local action on CP2 and then on M4, and the necessity to integrate out the local CP2 action to get something acting locally directly on M4. This is done by a mechanism due to Meinhard Mayer, The Geometry of Symmetry Breaking in Gauge Theories, Acta Physica Austriaca, Suppl. XXIII (1981) 477-490 where he says:

> "... We start out from ... four-dimensional M [M4] ...[and]... R ...[that is]... obtained from ... G/H [CP2 = SU(3) / U(2)] ... the physical surviving components of A and F, which we will denote by A and F, respectively, are a one-form and two form on M [M4] with values in H [SU(2)] ...the remaining components will be subjected to symmetry and gauge transformations, thus reducing the Yang-Mills action ...[on M4 x CP2]... to a Yang-Mills-Ginzburg-Landau action on M [M4] ... Consider the Yang-Mills action on R ...
>
> S_YM = Integral Tr (F /\ *F)
>
> ... We can ... split the curvature F into components along M [M4] (spacetime) and those along directions tangent to G/H [CP2] .
>
> We denote the former components by F_!! and the latter by F_?? , whereas the mixed components (one along M, the other along G/H) will be denoted by F_!? ... Then the integrand ... becomes
>
> Tr(F_!! F^!! + 2 F_!? F^!? + F_?? F^??)
>
> ... The first term .. becomes the [SU(2)] Yang-Mills action for the reduced [SU(2)] Yang-Mills theory ...

the middle term .. becomes, symbolically, Tr Sum D_! PHI(?) D^! PHI(?) where PHI(?) is the Lie-algebra-valued 0-form corresponding to the invariance of A with respect tothe vector field ? , in the G/H [CP2] direction ...

the third term ... involves the contraction F_?? of F with two vector fields lying along G/H [CP2] ... we make use of the equation [from Mayer-Trautman, Acta Physica Austriaca, Suppl. XXIII (1981) 433-476, equation 6.18]

2 F_?? = [PHI(?) , PHI(?)] - PHI([?,?])

... Thus, the third term ... reduces to what is essentially a Ginzburg-Landau potential in the components of PHI:

Tr F_?? F^?? = (1/4) Tr ([PHI , PHI] - PHI)^2

... special cases which were considered show that ...[the equation immediately above]... has indeed the properties required of a Ginzburg_Landau-Higgs potential, and moreover the relative signs of the quartic and quadratic terms are correct, and only one overall normalization constant ... is needed. ...".

(see also S. Kobayashi and K. Nomizu, Foundations of Differential Geometry, Volume I, Wiley (1963), especially section II.11)

So,

due to the work of Meinhard Mayer,

dimensional reduction to 4-dim M4 Physcial Spacetime, with respect to the SU(2) gauge group, gives the Higgs mechanism.

As to

3 Generations of Fermions:

At low (where we do experiments) energies a Quaternionic structure freezes out, splitting the 8-dim spacetime into a 4-dim physical spacetime M4 and a 4-dim internal symmetry space CP2.

First generation fermion particles are represented by octonions as follows:

Octonion Basis Element	Fermion Particle
1	e-neutrino
i	red up quark
j	green up quark
k	blue up quark
e	electron
ie	red down quark
je	green down quark
ke	blue down quark

First generation fermion antiparticles are represented by octonions in a similiar way.

Second generation fermion particles and antiparticles are represented by pairs of octonions.

Third generation fermion particles and antiparticles are represented by triples of octonions.

There are no higher generations of fermions than the Third. This can be seen geometrically as a consequence of the fact that if you reduce the original 8-dimensional spacetime into associative 4-dime M4 physical spacetime and coassociative 4-dim CP2 Internal Symmetry Space then if you look in the original 8-dimensional spacetime at a fermion (First-generation represented by a single octonion) propagating from one vertex to another there are only 4 possibilities for the same propagation after dimensional reduction:

1 - the origin o and target x vertices are both in the associative 4-dimensional physical spacetime

```
4-dim Internal Symmetry Space   -------------


4-dim Physical SpaceTime        ---o------x---
```

in which case the propagation is unchanged, and the fermion remains a FIRST generation fermion represented by a single octonion o

2 - the origin vertex o is in the associative spacetime and the target vertex * in in the Internal Symmetry Space

```
4-dim Internal Symmetry Space   ----------*---


4-dim Physical SpaceTime        ---o----------
```

in which case there must be a new link from the original target vertex * in the Internal Symmetry Space to a new target vertex x in the associative spacetime

```
4-dim Internal Symmetry Space   ----------*---


4-dim Physical SpaceTime        ---o------x---
```

and a second octonion can be introduced at the original target vertex in connection with the new link so that the fermion can be regarded after dimensional reduction as a pair of octonions o and * and therefore as a SECOND generation fermion

3 - the target vertex x is in the associative spacetime and the origin vertex o in in the Internal Symmetry Space

4-dim Internal Symmetry Space ---o---------

4-dim Physical SpaceTime ----------x---

in which case there must be a new link to the original origin vertex o in the Internal Symmetry Space from a new origin vertex * in the associative spacetime

4-dim Internal Symmetry Space ---o---------

4-dim Physical SpaceTime ---O------x---

so that a second octonion can be introduced at the new origin vertex O in connection with the new link so that the fermion can be regarded after dimensional reduction as a pair of octonions o and o and therefore as a SECOND generation fermion

4 - both the origin vertex o and the target vertex * are in the Internal Symmetry Space,

4-dim Internal Symmetry Space ---o------*---

4-dim Physical SpaceTime -------------

in which case there must be a new link to the original origin vertex o in the Internal Symmetry Space from a new origin vertex O in the associative spacetime, and a second new link from the original target vertex * in the Internal Symmetry Space to a new target vertex x in the associative spacetime

4-dim Internal Symmetry Space ---o------*---

4-dim Physical SpaceTime ---O------x---

so that a second octonion can be introduced at the new origin vertex O in connection with the first new link, and a third octonion can be introduced at the original target vertex * in connection with the second new link, so that the fermion can be regarded after dimensional reduction as a triple of octonions O and o and * and therefore as a THIRD generation fermion.

As there are no more possibilities, there are no more generations, and we have:

First generation fermions correspond to octonions O

Second generation fermions correspond to pairs of octonions O x O

Third generation fermions correspond to triples of octonions O x O x O

We now have a Lagrangian for the model

Integration over 4-dim M4 Physical Spacetime

of

MacDowell-Mansouri term from U(2,2)

and

Gauge Boson term from SU(3)xSU(2)xU(1)

and

Ginzburg-Landau-Higgs term from SU(2) amd Mayer Mechanism

and

3-Generation Fermion Particle-Antiparticle term

that gives conventional Gravity plus Standard Model.

Path integrals give a Quantum theory via the classical Lagrangian set out above.

The Lagrangian set out above is only valid in a (possibly small) neighborhood of spacetime. To get a more global theory, the local Lagrangians must be patched together. To do that, look at it from a Cl(8) point of view, and consider that, using 8-periodicity of real Clifford algebras, taking tensor products of factors of Cl(8)

Cl(8) (x) ...(N times tensor product)... (x) Cl(8) = Cl(8N)

allows construction of arbitrarily large real Clifford algebras as composites of lots of local Cl(8) factors.

By taking the completion of the union of all such Cl(8)-based tensor products, you get a generalized Real Hyperfinite II1 von Neumann Algebra factor that describes physics in terms of Algebraic Quantum Field Theory.

As to how to combine local Lagrangians in terms of E8, note that there are 7 independent Root Vector Polytopes / Lattices of type E8, denoted E8_1, E8_2, E8_3, E8_4, E8_5, E8_6, E8_7. Some of them have vertices in commmon, but they are all distinct.

All of the 7 independent Root Vector Polytope Lie algebras E8_i correspond to E8 Lattices consistent with Octonion Multiplication, and the the 7 Lie algebras / Lattices / Root Vector Polytopes E8_i are related to each other as the 7 Octonion imaginaries i,j,k,e,ie,je,ke , so the copies of E8 might combine according to the rules of octonion multiplication, globally arranging themselves like integral octonions.

If the 128 Spin(16) half-spinors are put on integral octonion vertices, and the 120-dim adjoint Spin(16) generators on links between integral octonion vertices, a realistic Spin Foam model might be produced, related to the copies of the 27-dimensional exceptional Jordan algebra contained in each E8.

Such a Spin Foam model might be related to the 26-dim Bosonic String model described in CERN preprint CERN-CDS-EXT-2004-031 in which fermions come from orbifolding and the 7 independent E8_i are used in constructing D8 branes.

Summary of Some Calculation Results

Force Strengths:

Gravity = 5 x 10^-39

Electromagnetic = 1 / 137.03608

Weak = 1.05 x 10^-5

Color at 245 MeV = 0.6286 Renormalization gives Color at 91 GeV = 0.106 and including other effects gives Color at 91 Gev = 0.125

Weak Boson Masses (based on a ground state Higgs mass of 146 GeV):

M_W+ = M_W- = 80.326 GeV;

M_Z0 = 80.326 + 11.536 = 91.862 GeV;

Tree-level fermion masses (Quark masses are constituent masses due to a Bohmian version of Many-Worlds Quantum Theory applied to a confined fermion, in which the fermion is at rest because its kinetic energy is transformed into Bohmian PSI-field potential energy.):

Neutrinos: Me-neutrino = Mmu-neutrino = Mtau-neutrino = 0 at tree-level (first order corrected masses are given below)

Electron/Positron Me = 0.5110 MeV

Up and Down Quarks Md = Mu = 312.8 MeV

Muon Mmu = 104.8 MeV

Strange Quark Ms = 625 MeV

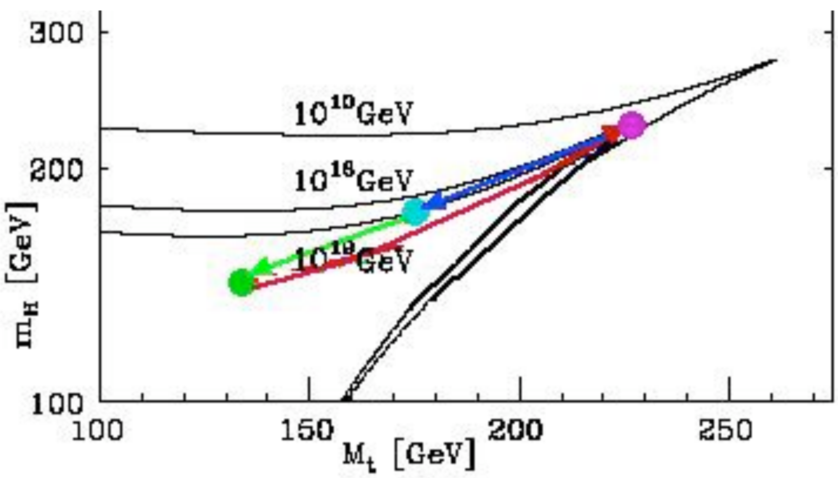
Charm Quark Mc = 2.09 GeV

Tauon Mtau = 1.88 GeV

Beauty Quark Mb = 5.63 GeV

Truth Quark Mt = 130 GeV

8-dimensional Kaluza-Klein spacetime with Truth-Quark condensate Higgs gives a 3-state system with a renormalization line connecting the 3 states:



Low ground state: Higgs = 146 GeV and T-quark = 130 GeV

Medium Triviality Bound state: Higgs = 176-188 GeV and T-quark = 172-175 GeV

High Critical Point state: Higgs = 239 +/- 3 GeV and T-quark = 218 +/- 3 GeV

Kobayashi-Maskawa parameter calculations use phase angle d13 = 1 radian (unit length on a phase circumference) to get the K-M matrix:

	d	s	b
u	0.975	0.222	0.00249-0.00388i
c	-0.222-0.000161i	0.974-0.0000365i	0.0423
t	0.00698-0.00378i	-0.0418-0.00086i	0.999

Corrections to the tree-level neutrino calculations give neutrino masses

nu_1 = 0

nu_2 = 9 x 10^(-3)eV

nu_3 = 5.4 x 10^(-2) eV

and the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.87	0.50	0
nu_m	-0.35	0.61	0.71
nu_t	0.35	-0.61	0.71

The mass of the charged pion is calculated to be 139 MeV

The Neutron-Proton mass difference is calculated to be 1.1 Mev,

The ratio

Dark Energy : Dark Matter : Ordinary Matter

for our Universe at the present time is calculated to be:

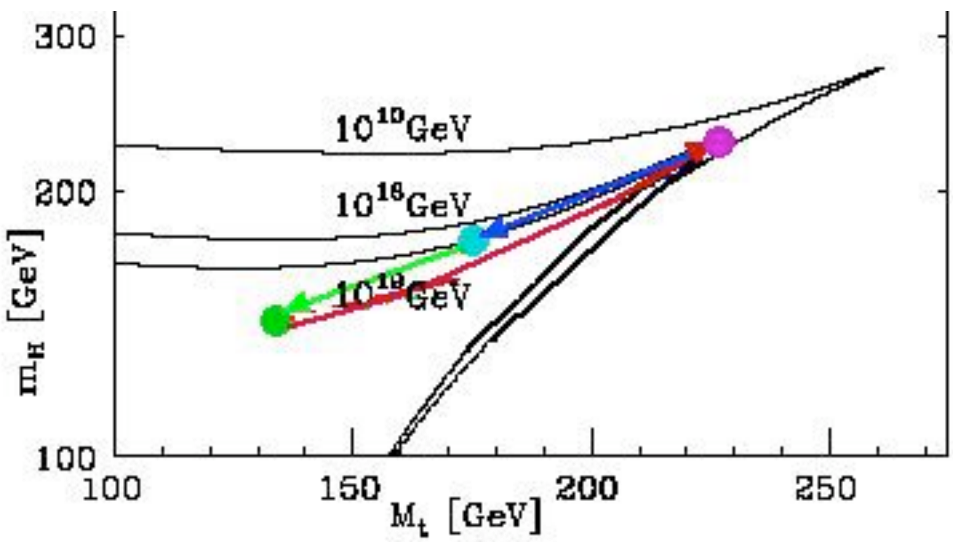
$$0.75 : 0.21 : 0.04$$

Tquark = 172-175 GeV and Higgs = 176-188 GeV

Michio Hashimoto, Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165 describe models with T-quark condensate for Higgs in 8-dimensional Kaluza-Klein spacetime with 4 compact dimensions, like M4 x CP2 of the E8 model, and calculate that

- Tquark = 172-175 GeV which is consistent with accepted experimental values
- Higgs = 176-188 GeV which is a prediction that might be tested by the LHC

Renormalization running up and down from that point on a plot of Higgs mass v. Tquark mass

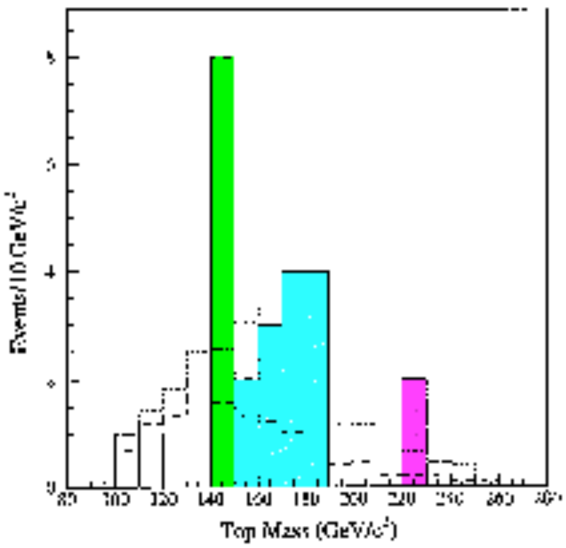


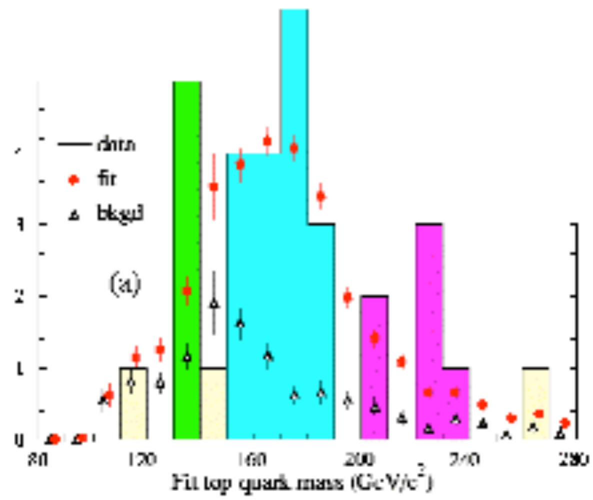
shows that the point (M_H = 176-188 , M_T = 172-175) is right on the Triviality Bound curve for as Standard Model with high-energy cut-off at the Planck energy 10^19 GeV (see hep-ph/0307138) and

- renormalization runs up to a critical point where the Triviality Bound curve intersects the Vacuum Stability curve around (M_H = 239 , M_T = 220) and
- renormalization runs down to a point in the stable region around ((M_H = 143-160 , M_T = 130-145)

There is not much data for a T-quark-Higgs state around (M_H = 239 , M_T = 220), but perhaps the LHC might shed light on that.

As to a T-quark-Higgs state around (M_H = 143-160 , M_T = 130-145) , it is not conventionally accepted that there is any evidence for such a state, but my opinion about data analysis is that there is such evidence. For example, the initial CDF and D0 histograms for semileptonic events





both independently show a tall narrow peak (green) in the 130-145 GeV range for the Tquark mass. Since mass calculations used in this E8 model had been done prior to those histograms, and had predicted a tree-level (about 10% or so accuracy) value of the Tquark mass of about 130 GeV, those independent CDF and D0 results indicate a probability around 4 sigma for M_T = 130-145 (see an entry on Tommaso Dorigo's blog around 5 September 2007).

In my opinion, recent results from CDF and D0 are still consistent with the existence of a Tquark-Higgs state around (M_H = 143-160 , M_T = 130-145), but the consensus view is otherwise. However, I disagree with that consensus, based on how I see exeperimental data, such as:

Dilepton data described by Erich Ward Varnes in Chapter 8 of his 1997 UC Berkeley PhD thesis about D0 data at Fermilab:

"… there are six t-tbar candidate events in the dilepton final states … Three of the events contain three jets, and in these cases the results of the fits using only the leading two jets and using all combinations of three jets are given …".

There being only 6 dilepton events in Figure 8.1 of Varnes's PhD thesis

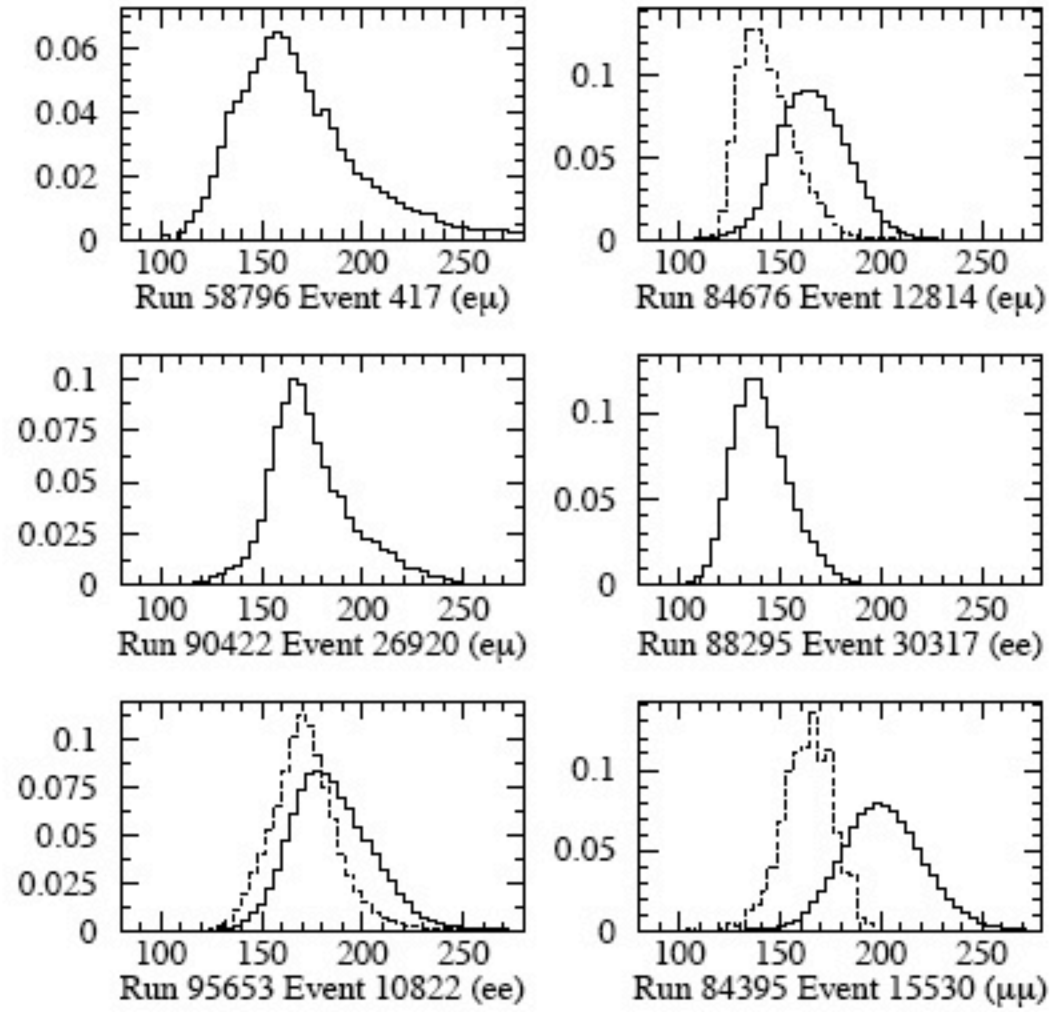


Figure 8.1: $\mathcal{W}(m_t)$ distributions for the dilepton candidates. For events with more than two jets, the dashed curves show the results of considering only the two highest E_T jets in the reconstruction.

it is reasonable to discuss each of them, so (mass is roughly estimated by me looking at the histograms) here they are:

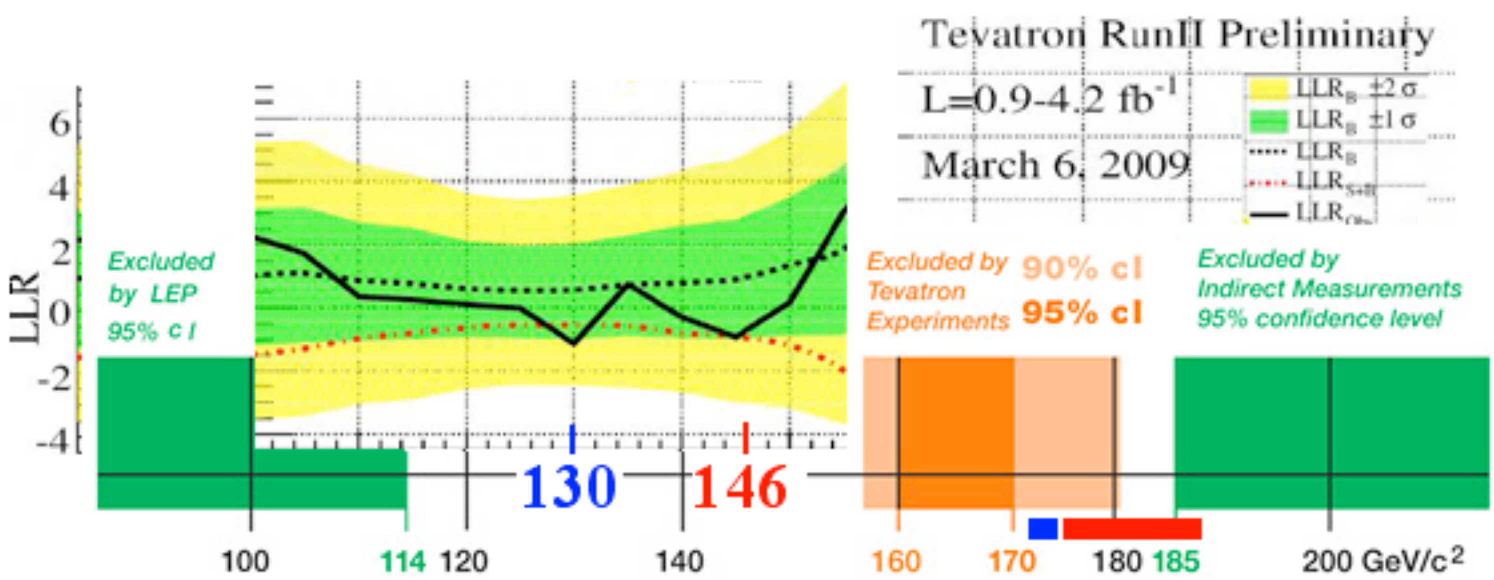
- Run 58796 Event 417 (e mu) - 2 jets - 160 GeV
- Run 90422 Event 26920 (e mu) - 2 jets - 170 GeV
- Run 88295 Event 30317 (e e) - 2 jets - 135 GeV
- Run 84676 Event 12814 (e mu) - more than 2 jets - 165 GeV - highest 2 jets - 135 GeV

- Run 95653 Event 10822 (e e) - more than 2 jets - 180 GeV - highest 2 jets - 170 GeV
- Run 84395 Event 15530 (mu mu) - more than 2 jets - 200 GeV - highest 2 jets - 165 GeV

In terms of 3 Truth Quark mass states - high around 220 GeV or so - medium around 170 GeV or so - low around 130-145 GeV or so - those look like:

- Run 58796 Event 417 (e mu) - direct 2-jet decay of medium
- Run 90422 Event 26920 (e mu) - direct 2-jet decay of medium
- Run 88295 Event 30317 (e e) - direct 2-jet decay of low
- Run 84676 Event 12814 (e mu) - decay of medium to low then 2-jet decay of low
- Run 95653 Event 10822 (e e) - direct 2-jet decay of medium with small background other jet
- Run 84395 Event 15530 (mu mu) - decay of high to medium then 2-jet decay of medium

This, and other more recent experimental subtleties (see for example www.tony5m17h.net/ and other pages on my web sites), support my E8 Physics model. For example, see a 13 March 2009 blog entry "Tevatron excludes chunk of Higgs masses!" by Tommaso Dorigo from which I constructed the following chart that shows how the E8 model tree-level 130 GeV T-quark and 146 GeV Higgs ground states and the Yamawaki et al 176-188 GeV Higgs are consistent with the Fermilab Search Signal, with the Yamawaki et al 172-175 GeV T-quark being accepted by Fermilab and therefore included in the Fermilab Search Background. Note the LLR Observed solid black curve whose valleys point to 130 GeV and 146 GeV.



Force Strengths

The model Lagrangian (just looking at spacetime and gauge bosons and ignoring spinor fermions etc) is the integral over spacetime of gauge boson terms, so THE FORCE STRENGTH IS MADE UP OF TWO PARTS:

- the relevant spacetime manifold of gauge group global action
- the relevant symmetric space manifold of gauge group local action.

Ignoring for this exposition details about the 4-dim internal symmetry space, and ignoring conformal stuff (Higgs etc), the 4-dim spacetime Lagrangian gauge boson term is the integral over spacetime as seen by gauge boson acting globally of the gauge force term of the gauge boson acting locally for the gauge bosons of each of the four forces:

- U(1) for electromagnetism
- SU(2) for weak force
- SU(3) for color force
- Spin(5) - compact version of antiDeSitter Spin(2,3) for gravity by the MacDowell-Mansouri mechanism.

In the conventional Lagrangian picture, for each gauge force the gauge boson force term contains the force strength, which in Feynman's picture is the probability to emit a gauge boson, in either an explicit (like g |F|^2) or an implicit (incorporated into the |F|^2) form. Either way, the conventional picture is that the force strength g is an ad hoc inclusion.

What I am doing is to construct the integral such that the force strength emerges naturally from the geometry of each gauge force.

To do that, for each gauge force:

1 - make the spacetime over which the integral is taken be spacetime AS IT IS SEEN BY THAT GAUGE BOSON, that is, in terms of the symmetric space with GLOBAL symmetry of the gauge boson:

- the U(1) photon sees 4-dim spacetime as T^4 = S1 x S1 X S1 x S1
- the SU(2) weak boson sees 4-dim spacetime as S2 x S2
- the SU(3) weak boson sees 4-dim spacetime as CP2
- the Spin(5) of gravity sees 4-dim spacetime as S4.

2 - make the gauge boson force term have the volume of the Shilov boundary corresponding to the symmetric space with LOCAL symmetry of the gauge boson. The nontrivial Shilov boundaries are:

- for SU(2) Shilov = RP^1xS^2
- for SU(3) Shilov = S^5
- for Spin(5) Shilov = RP^1xS^4

The result is (ignoring technicalities for exposition) the geometric factor for force strength calculation.

GLOBAL: Each gauge group is the global symmetry of a symmetric space

- S1 for U(1)
- S2 = SU(2)/U(1) = Spin(3)/Spin(2) for SU(2)
- CP2 = SU(3)/SU(2)xU(1) for SU(3)
- S4 = Spin(5)/Spin(4) for Spin(5)

LOCAL: Each gauge group is the local symmetry of a symmetric space

- U(1) for itself
- SU(2) for Spin(5) / SU(2)xU(1)
- SU(3) for SU(4) / SU(3)xU(1)
- Spin(5) for Spin(7) / Spin(5)xU(1)

The nontrivial local symmetry symmetric spaces correspond to bounded complex domains

- SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3
- SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball)
- Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5

The nontrivial bounded complex domains have Shilov boundaries

- SU(2) for Spin(5) / SU(2)xU(1) corresponds to IV3 Shilov = RP^1xS^2
- SU(3) for SU(4) / SU(3)xU(1) corresponds to B^6 (ball) Shilov = S^5
- Spin(5) for Spin(7) / Spin(5)xU(1) corresponds to IV5 Shilov = RP^1xS^4

GLOBAL AND LOCAL TOGETHER: Very roughly think of the force strength as

- the integral over the global symmetry space of
- the physical (ie Shilov Boundary) volume=strength of the force.

That is (again very roughly and intuitively): the geometric strength of the force is given by the product of

- the volume of a 4-dim thing with global symmetry of the force and
- the volume of the Shilov Boundary for the local symmetry of the force.

When you calculate the product volumes (using some normalizations etc that are described in more detail here below), you see that roughly:

Volume product for gravity is the largest volume

so since (as Feynman says) force strength = probability to emit a gauge boson means that the highest force strength or probability should be 1

I normalize the gravity Volume product to be 1, and get results roughly (for example, the fine structrure constant calculation gives 1/137.03608 but is rounded off here as 1/137):

- Volume product for gravity = 1
- Volume product for color = 2/3
- Volume product for weak = 1/4
- Volume product for electromagnetism = 1/137

There are two further main components of a force strength:

- 1 - for massive gauge bosons, a suppression by a factor of 1 / M^2
- 2 - renormalization running (important for color force).

CONSIDER MASSIVE GAUGE BOSONS: I consider gravity to be carried by virtual Planck-mass black holes, so that the geometric strength of gravity should be reduced by 1/Mp^2 and I consider the weak force to be carried by weak bosons, so that the geometric strength of gravity should be reduced by 1/MW^2 That gives the result:

- gravity strength = G (Newton's G)
- color strength = 2/3
- weak strength = G_F (Fermi's weak force G)
- electromagnetism = 1/137

FINALLY, CONSIDER RENORMALIZATION RUNNING FOR THE COLOR FORCE: That gives the result:

- gravity strength = G (Newton's G)
- color strength = 1/10 at weak boson mass scale
- weak strength = G_F (Fermi's weak force G)
- electromagnetism = 1/137

The use of compact volumes is itself a calculational device, because it would be more nearly correct, instead of

- the integral over the compact global symmetry space of
- the compact physical (ie Shilov Boundary) volume=strength of the force

to use

- the integral over the hyperbolic spacetime global symmetry space of
- the noncompact invariant measure of the gauge force term.

However, since the strongest (gravitation) geometric force strength is to be normalized to 1, the only thing that matters is RATIOS, and the compact volumes (finite and easy to look up in the book by Hua) have the same ratios as the noncompact invariant measures.

In fact, I should go on to say that continuous spacetime and gauge force geometric objects are themselves also calculational devices, and

that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.

Some of this material was written in connection with email discussion with Ark Jadczyk. More details can be found on my web site at www.valdostamuseum.org/hamsmith/

Carlos Castro and others have also done substantial work on similar geometric approaches (motivated at least in part by earlier work by Armand Wyler) to calculating force strengths. See references at

www.valdostamuseum.org/hamsmith/wfKaluzaKlein.html

Here are more details about the force strength calculations:
The force strength of a given force is

alphaforce = (1 / Mforce^2)
(Vol(MISforce))
(Vol(Qforce) / Vol(Dforce)^(1 / mforce))

where:

alphaforce represents the force strength;

Mforce represents the effective mass;

MISforce represents the part of the target Internal Symmetry Space that is available for the gauge boson to go to;

Vol(MISforce) stands for volume of MISforce, and is sometimes also denoted by the shorter notation Vol(M);

Qforce represents the link from the origin to the target that is available for the gauge boson to go through;

Vol(Qforce) stands for volume of Qforce;

Dforce represents the complex bounded homogeneous domain of which Qforce is the Shilov boundary;

mforce is the dimensionality of Qforce,
which is 4 for Gravity and the Color force,
2 for the Weak force (which therefore is considered to have two copies of QW for each spacetime HyperDiamond link),
and 1 for Electromagnetism (which therefore is considered to have four copies of QE for each spacetime HyperDiamond link)

Vol(Dforce)^(1 / mforce) stands for a dimensional normalization factor (to reconcile the dimensionality of the Internal Symmetry Space of the target vertex with the dimensionality of the link from the origin to the target vertex).

The Qforce, Hermitian symmetric space, and Dforce manifolds for the four forces are:

Gauge Group	Hermitian Symmetric Space	Type of Dforce	mforce	Qforce
Spin(5)	Spin(7) / Spin(5)xU(1)	IV5	4	RP^1xS^4
SU(3)	SU(4) / SU(3)xU(1)	B^6(ball)	4	S^5
SU(2)	Spin(5) / SU(2)xU(1)	IV3	2	RP^1xS^2
U(1)	-	-	1	-

The geometric volumes needed for the calculations are mostly taken from the book Harmonic Analysis of Functions of Several Complex Variables in the Classical Domains (AMS 1963, Moskva 1959, Science Press Peking 1958) by L. K. Hua [with unit radius scale].

Note (thanks to Carlos Castro for noticing this) that the volume lisrted for S5 is for a squashed S5, a Shilov boundary of the complex domain corresponding to the symmetric space SU(4) / SU(3) x U(1).

Note (thanks again to Carlos Castro for noticing this) also that the volume listed for CP2 is unconventional, but physically justified by noting that S4 and CP2 can be seen as having the same physical volume, with the only difference being structure at infinity.

Note also that

Force	M	Vol(M)
gravity	S^4	8pi^2/3 - S^4 is 4-dimensional
color	CP^2	8pi^2/3 - CP^2 is 4-dimensional
weak	S^2 x S^2	2 x 4pi - S^2 is a 2-dim boundary of 3-dim ball 4-dim S^2 x S^2 = = topological boundary of 6-dim 2-polyball Shilov Boundary of 6-dim 2-polyball = S^2 + S^2 = = 2-dim surface frame of 4-dim S^2 x S^
e-mag	T^4	4 x 2pi - S^1 is 1-dim boundary of 2-dim disk 4-dim T^4 = S^1 x S^1 x S^1 x S^1 = = topological boundary of 8-dim 4-polydisk Shilov Boundary of 8-dim 4-polydisk = = S^1 + S^1 + S^1 + S^1 = = 1-dim wire frame of 4-dim T^4

Also note that for U(1) electromagnetism, whose photon carries no charge, the factors Vol(Q) and Vol(D) do not apply and are set equal to 1, and from another point of view, the link manifold to the target vertex is trivial for the abelian neutral U(1) photons of Electromagnetism, so we take QE and DE to be equal to unity.

Force	M	Vol(M)	Q	Vol(Q)	D	Vol(D)
gravity	S^4	8pi^2/3	RP^1xS^4	8pi^3/3	IV5	pi^5/2^4 5!
color	CP^2	8pi^2/3	S^5	4pi^3	B^6(ball)	pi^3/6
weak	S^2xS^2	2x4pi	RP^1xS^2	4pi^2	IV3	pi^3/24
e-mag	T^4	4x2pi	-	-	-	-

Using these numbers, the results of the calculations are the relative force strengths at the characteristic energy level of the generalized Bohr radius of each force:

Gauge Group	Force	Characteristic Energy	Geometric Force Strength	Total Force Strength
Spin(5)	gravity	approx 10^19 GeV	1	GGmproton^2 approx 5 x 10^-39
SU(3)	color	approx 245 MeV	0.6286	0.6286
SU(2)	weak	approx 100 GeV	0.2535	GWmproton^2 approx 1.05 x 10^-5
U(1)	e-mag	approx 4 KeV	1/137.03608	1/137.03608

The force strengths are given at the characteristic energy levels of their forces, because the force strengths run with changing energy levels.

The effect is particularly pronounced with the color force.

The color force strength was calculated using a simple perturbative QCD renormalization group equation at various energies, with the following results:

Energy Level	Color Force Strength
245 MeV	0.6286
5.3 GeV	0.166
34 GeV	0.121
91 GeV	0.106

Taking other effects, such as Nonperturbative QCD, into account, should give
a Color Force Strength of about 0.125 at about 91 GeV

Fermion Particle Masses

The E8 model Lagrangian (for this message just looking at spacetime and spinor fermions and ignoring gauge bosons etc) has

an Integral over 8-dim spacetime of a spinor fermion particle and antiparticle term,

in which first-generation fermion particles correspond to octonion basis elements

- 1 to e-neutrino
- i to red up quark
- j to green up quark
- k to blue up quark
- e to electron
- ie to red down quark
- je to green down quark
- ke to blue down quark

and first-generation fermion antiparticles correspond to octonion basis elements

- 1 to e-antineutrino
- i to red up antiquark
- j to green up antiquark
- k to blue up antiquark
- e to positron
- ie to red down antiquark
- je to green down antiquark
- ke to blue down antiquark

At low (where we do experiments) energies a specific quaternionic submanifold freezes out, splitting the 8-dim spacetime into a 4-dim M4 physical spacetime plus a 4-dim CP2 internal symmetry space and creating second and third generation fermions that live (at least in part) in the 4-dim CP2 internal symmetry space and correspond respectively to pairs and triples of octonion basis elements.

Ignoring for this exposition details about the 4-dim CP2 internal symmetry space, and ignoring conformal stuff (Higgs etc), and considering for now only first generation fermions, the 4-dim spacetime Lagrangian spinor fermion part is:

- integral over spacetime of
- spinor fermion particle and antiparticle term

In the conventional picture, the spinor fermion term is of the form m S S* where m is the fermion mass and S and S* represent the given fermion. Although the mass m is derived from the Higgs mechanism, the Higgs coupling constants are, in the conventional picture, ad hoc parameters, so that effectively the mass term is, in the conventional picuture, an ad hoc inclusion.

What I am doing is to NOT put in the mass m as an ad hoc Higgs coupling value,

but to construct the integral such that the mass m emerges naturally from the geometry of the spinor fermions.

To do that, make the spinor fermion mass term have the volume of the Shilov boundary corresponding to the symmetric space with LOCAL symmetry of the Spin(8) gauge group with respect to which the first generation spinor fermions can be seen as +half-spinor and -half-spinor spaces.

Note that due to triality, Spin(8) can act on those 8-dimensional half-spinor spaces similarly to the way it acts on 8-dimensional vector spacetime prior to dimensional reduction.

Then, take the the spinor fermion volume to be the Shilov boundary corresponding to the same symmetric space on which Spin(8) acts as a local gauge group that is used to construct 8-dimensional vector spacetime:

the symmetric space Spin(10) / Spin(8)xU(1) corresponds to a bounded domain of type IV8 whose Shilov boundary is RP^1 x S^7

Since all the first generation fermions see the spacetime over which the integral is taken in the same way (unlike what happens for the force strength calculation), the only geometric volume factor relevant for calculating first generation fermion mass ratios is in the spinor fermion volume term.

Since the physcally observed fermions in this model correspond to Kerr-Newman Black Holes, the quark mass in this model is a constituent mass.

Consider a first-generation massive lepton (or antilepton, i.e., electron or positron). For definiteness, consider an electron E (a similar line of reasoning applies to the positron).

- Gluon interactions do not affect the colorless electron (E)
- By weak boson interactions or decay, an electron (E) can only be taken into itself or a massless (at tree level) neutrino.
- As the lightest massive first-generation fermion, the electron cannot decay into a quark.

Since the electron cannot be related to any other massive Dirac fermion, its volume V(electron) is taken to be 1.

Consider a first-generation quark (or antiquark). For definiteness, consider a red down quark I (a similar line of reasoning applies to the others of the first generation).

- By gluon interactions, the red quark (I) can be interchanged with the blue and green down quarks (J and K).
- By weak boson interactions, it can be taken into the red, blue, and green up quarks (i, j, and k).
- Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons (E) and neutrinos (1).

Therefore first-generation quarks or antiquarks can by gluons, weak bosons, or decay occupy the entire volume of the Shilov boundary RP1 x S7, which volume is pi^5 / 3, so its volume V(quark) is taken to be pi^5 / 3.

Consider graviton interactions with first-generation fermions.

MacDowell-Mansouri gravitation comes from 10 Spin(5) gauge bosons, 8 of which are charged (carrying color or electric charge).

2 of the charged Spin(5) gravitons carry electric charge. However, even though the electron carries electric charge, the electric charge carrying Spin(5) gravitons can only change the electron into a (tree-level) massless neutrino, so the Spin(5) gravitons do not enhance the electron volume factor, which remains

electron volume (taking gravitons into account) = V(electron) = 1

6 of the charged Spin(5) gravitons carry color charge, and their action on quarks (which carry color charge) multiplies the quark volume V(quark) by 6, giving

quark gravity-enhanced volume = 6 x V(quark) = 6 pi^5 / 3 = 2 pi^5

The 2 Spin(5) gravitons carrying electric charge only cannot change quarks into leptons, so they do not enhance the quark volume factor, so we have (where md is down quark mass, mu is up quark mass, and me is electron mass)

md / me = mu / me = 2 pi^5 / 1 = 2 pi^5 = 612.03937

The proton mass is calculated as the sum of the constituent masses of its constituent quarks

mproton = mu + mu + md = 938.25 MeV

which is close to the experimental value of 938.27 MeV.

In the first generation, each quark corresponds to a single octonion basis element and the up and down quark constituent masses are the same:

First Generation - 8 singletons - mu / md = 1

- Down - corresponds to 1 singleton - constituent mass 312 MeV
- Up - corresponds to 1 singleton - constituent mass 312 MeV

Second and third generation calculations are generally more complicated (some details are given here below) with combinatorics indicating that in higher generations the up-type quarks are heavier than the down-type quarks. The third generation case, in which the fermions correspond to triples of octonions, is simple enough to be used in this expository overview as an illustration of the combinatoric effect:

Third Generation

8^3 = 512 triples

mt / mb = 483 / 21 = 161 / 7 = 23

- down-type (Beauty) - corresponds to 21 triples -tree-level constituent mass 5.65 GeV
- up-type (Truth) - corresponds to 483 triples - tree-level constituent mass 130 GeV

Here is a summary of the results of calculations of tree-level fermion masses (quark masses are constituent masses):

- Me-neutrino = Mmu-neutrino = Mtau-neutrino = 0 at tree-level (first order corrected masses are given HERE)
- Me = 0.5110 MeV
- Md = Mu = 312.8 MeV
- Mmu = 104.8 MeV
- Ms = 625 MeV
- Mc = 2.09 GeV
- Mtau = 1.88 GeV
- Mb = 5.63 GeV
- Mt = 130 GeV

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- the compact physical (ie Shilov Boundary) volume=strength of the force

to use

- the integral over the hyperbolic spacetime global symmetry space of
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that it would be even more nearly correct to do the calculations with respect to a discrete generalized hyperdiamond Feynman checkerboard.

Some of this material was written in connection with email discussion with Ark Jadczyk. More details can be found on my web site at www.valdostamuseum.org/hamsmith/

Here are more details about the fermion mass calculations:

Fermion masses are calculated as a product of four factors:

V(Qfermion) x N(Graviton) x N(octonion) x Sym

- V(Qfermion) is the volume of the part of the half-spinor fermion particle manifold S^7 x RP^1 that is related to the fermion particle by photon, weak boson, and gluon interactions.
- N(Graviton) is the number of types of Spin(0,5) graviton related to the fermion. The 10 gravitons correspond to the 10 infinitesimal generators of Spin(0,5) = Sp(2). 2 of them are in the Cartan subalgebra. 6 of them carry color charge, and may therefore be considered as corresponding to quarks. The remaining 2 carry no color charge, but may carry electric charge and so may be considered as corresponding to electrons. One graviton takes the electron into itself, and the other can only take the first-generation electron into the

massless electron neutrino. Therefore only one graviton should correspond to the mass of the first-generation electron. The graviton number ratio of the down quark to the first-generation electron is therefore 6/1 = 6.

- N(octonion) is an octonion number factor relating up-type quark masses to down-type quark masses in each generation.
- Sym is an internal symmetry factor, relating 2nd and 3rd generation massive leptons to first generation fermions. It is not used in first-generation calculations.

The ratio of the down quark constituent mass to the electron mass is then calculated as follows:

> Consider the electron, e. By photon, weak boson, and gluon interactions, e can only be taken into 1, the massless neutrino. The electron and neutrino, or their antiparticles, cannot be combined to produce any of the massive up or down quarks. The neutrino, being massless at tree level, does not add anything to the mass formula for the electron. Since the electron cannot be related to any other massive Dirac fermion, its volume V(Qelectron) is taken to be 1.
>
> Next consider a red down quark ie. By gluon interactions, ie can be taken into je and ke, the blue and green down quarks. By also using weak boson interactions, it can be taken into i, j, and k, the red, blue, and green up quarks.
>
> Given the up and down quarks, pions can be formed from quark-antiquark pairs, and the pions can decay to produce electrons and neutrinos.
>
> Therefore the red down quark (similarly, any down quark) is related to any part of S^7 x RP^1, the compact manifold corresponding to { 1, i, j, k, ie, ie, ke, e } and therefore a down quark should have a spinor manifold volume factor V(Qdown quark) of the volume of S^7 x RP^1.
>
> The ratio of the down quark spinor manifold volume factor tothe electron spinor manifold volume factor is just
>
> V(Qdown quark) / V(Qelectron) = V(S^7x RP^1)/1 = pi^5 / 3.

Since the first generation graviton factor is 6,

md/me = 6V(S^7 x RP^1) = 2 pi^5 = 612.03937

As the up quarks correspond to i, j, and k, which are the octonion transforms under e of ie, je, and ke of the down quarks, the up quarks and down quarks have the same constituent mass

mu = md.

Antiparticles have the same mass as the corresponding particles.

Since the model only gives ratios of massses, the mass scale is fixed so that the electron mass me = 0.5110 MeV.

Then, the constituent mass of the down quark is md = 312.75 MeV,

and the constituent mass for the up quark is mu = 312.75 MeV.

These results when added up give a total mass of first generation fermion particles:

Sigmaf1 = 1.877 GeV

As the proton mass is taken to be the sum of the constituent masses of its constituent quarks

mproton = mu + mu + md = 938.25 MeV

The theoretical calculation is close to the experimental value of 938.27 MeV.

The third generation fermion particles correspond to triples of octonions. There are 8^3 = 512 such triples.

The triple { 1,1,1 } corresponds to the tau-neutrino.

The other 7 triples involving only 1 and e correspond to the tauon:

- { e ,e, e } { e, e, 1} { e, 1, e } { 1, e, e } { 1, 1, e } { 1, e, 1 } { e, 1, 1 }

The symmetry of the 7 tauon triples is the same as the symmetry of the 3 down quarks, the 3 up quarks, and the electron, so the tauon mass should be the same as the sum of the masses of the first generation massive fermion particles. Therefore the tauon mass is calculated at tree level as 1.877 GeV.

The calculated Tauon mass of 1.88 GeV is a sum of first generation fermion masses, all of which are valid at the energy level of about 1 GeV.

However, as the Tauon mass is about 2 GeV, the effective Tauon mass should be renormalized from the energy level of 1 GeV (where the mass is 1.88 GeV) to the energy level of 2 GeV. Such a renormalization should reduce the mass. If the renormalization reduction were about 5 percent,

the effective Tauon mass at 2 GeV would be about 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a Tauon mass of 1.777 GeV.

Note that all triples corresponding to the tau and the tau-neutrino are colorless.

The beauty quark corresponds to 21 triples.

They are triples of the same form as the 7 tauon triples, but for 1 and ie, 1 and je, and 1 and ke, which correspond to the red, green, and blue beauty quarks, respectively.

The seven triples of the red beauty quark correspond to the seven triples of the tauon, except that the beauty quark interacts with 6 Spin(0,5) gravitons while the tauon interacts with only two.

The beauty quark constituent mass should be the tauon mass times the third generation graviton factor 6/2 = 3, so the B-quark mass is

mb = 5.63111 GeV.

The calculated Beauty Quark mass of 5.63 GeV is a consitituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Beauty Quark mass of 5.63 GeV corresponds to a conventional pole mass of 5.32 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a lattice gauge theory Beauty Quark pole mass as 5.0 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known. The conventional value of alpha_s at about 5 GeV is about 0.22. Using alpha_s (5 GeV) = 0.22, a pole mass of 5.0 GeV gives an MSbar 1-loop Beauty Quark mass of 4.6 GeV, and

an MSbar 1,2-loop Beauty Quark mass of 4.3, evaluated at about 5 GeV.

If the MSbar mass is run from 5 GeV up to 90 GeV, the MSbar mass decreases by about 1.3 GeV, giving an expected MSbar mass of about 3.0 GeV at 90 GeV.

DELPHI at LEP has observed the Beauty Quark and found a 90 GeV MSbar Beauty Quark mass of about 2.67 GeV, with error bars +/- 0.25 (stat) +/- 0.34 (frag) +/- 0.27 (theo).

Note that the theoretical model calculated mass of 5.63 GeV corresponds to a pole mass of 5.32 GeV, which is somewhat higher than the conventional value of 5.0 GeV. However, the theoretical model calculated value of the color force strength constant alpha_s at about 5 GeV is about 0.166, while the conventional value of the color force strength constant alpha_s at about 5 GeV is about 0.216, and the theoretical model calculated value of the color force strength constant alpha_s at about 90 GeV is about 0.106, while the conventional value of the color force strength constant alpha_s at about 90 GeV is about 0.118.

The theoretical model calculations gives a Beauty Quark pole mass (5.3 GeV) that is about 6 percent higher than the conventional Beauty Quark pole mass (5.0 GeV), and a color force strength alpha_s at 5 GeV (0.166) such that 1 + alpha_s = 1.166 is about 4 percent lower than the conventional value of 1 + alpha_s = 1.216 at 5 GeV.

Note particularly that triples of the type { 1, ie, je } , { ie, je, ke }, etc., do not correspond to the beauty quark, but to the truth quark.

The truth quark corresponds to the remaining 483 triples, so the constituent mass of the red truth quark is 161/7 = 23 times the red beauty quark mass, and the red T-quark mass is

mt = 129.5155 GeV

The blue and green truth quarks are defined similarly.

All other masses than the electron mass (which is the basis of the assumption of the value of the Higgs scalar field vacuum expectation value v = 252.514 GeV), including the Higgs scalar mass and Truth quark mass, are calculated (not assumed) masses in the E8 model.

These results when added up give a total mass of third generation fermion particles:

Sigmaf3 = 1,629 GeV

The second generation fermion particles correspond to pairs of octonions.

There are 8^2 = 64 such pairs. The pair { 1,1 } corresponds to the mu-neutrino. The pairs { 1, e }, { e, 1 }, and { e, e } correspond to the muon.

Compare the symmetries of the muon pairs to the symmetries of the first generation fermion particles.

The pair { e, e } should correspond to the e electron.

The other two muon pairs have a symmetry group S2, which is 1/3 the size of the color symmetry group S3 which gives the up and down quarks their mass of 312.75 MeV.

Therefore the mass of the muon should be the sum of

- the { e, e } electron mass and
- the { 1, e }, { e, 1 } symmetry mass, which is 1/3 of the up or down quark mass.

Therefore, mmu = 104.76 MeV .

According to the 1998 Review of Particle Physics of the Particle Data Group, the experimental muon mass is about 105.66 MeV.

Note that all pairs corresponding to the muon and the mu-neutrino are colorless.

There is an interesting alternative way, due to G. Sardin, to calculate the Muon mass:

Since the Second-Generation Muon can be regarded as a pair (o , *) of Octonions,
if you regard o as a point corresponding to the 0.511 MeV mass of an Electron,
and if you regard * as the space of a 1-dim harmonic oscillator about o ,
then you can calculate the Muon mass as
the $n = 1$ Schroedinger equation quantum number $E(1)$
plus the base mass $V = m c^2$ = Electron mass = 0.511 MeV represented by o .
The calculation, based on work of G. Sardin at www.terra.es/personal/gsardin/ , is:
$E(1) = (3/2) (h c m c^2 / 2 \pi q^2) = (3/2) (\hbar c / q^2) \times (m c^2) =$
$= (3/2) (1.054 \times 10^{-27} \times 3 \times 10^{10} / 4.8 \times 10^{-10} \times 4.8 \times 10^{-10}) \times m c^2 =$
$= 205.9 \times m c^2 = 205.9 \times 0.511$ MeV = 105.22 MeV
Therefore, by this interesting Sardin-type calculation method,
the Muon mass is calculated to be 105.22 MeV + 0.511 MeV = 105.73 MeV

Note that it is not so easy to apply this method directly to the Tauon,
as its Octonion triple structure is not so easy to represent
in terms of a 1-dim harmonic oscillator.

The red, blue and green strange quark each corresponds to the 3 pairs involving 1 and ie, je, or ke.

The red strange quark is defined as the three pairs 1 and i, because i is the red down quark.Its mass should be the sum of two parts:

- the { i, i } red down quark mass, 312.75 MeV, and
- the product of the symmetry part of the muon mass, 104.25 MeV, times the graviton factor.

Unlike the first generation situation, massive second and third generation leptons can be taken, by both of the colorless gravitons that may carry electric charge, into massive particles. Therefore the graviton factor for the second and third generations is 6/2 = 3.

Therefore the symmetry part of the muon mass times the graviton factor 3 is 312.75 MeV, and the red strange quark constituent mass is

$$m_s = 312.75 \text{ MeV} + 312.75 \text{ MeV} = 625.5 \text{ MeV}$$

The blue strange quarks correspond to the three pairs involving j, the green strange quarks correspond to the three pairs involving k, and their masses are determined similarly.

The charm quark corresponds to the other 51 pairs. Therefore, the mass of the red charm quark should be the sum of two parts:

- the { i, i }, red up quark mass, 312.75 MeV; and
- the product of the symmetry part of the strange quark mass, 312.75 MeV, and the charm to strange octonion number factor 51/9, which product is 1,772.25 MeV.

Therefore the red charm quark constituent mass is

mc = 312.75 MeV + 1,772.25 MeV = 2.085 GeV

The blue and green charm quarks are defined similarly, and their masses are calculated similarly.

The calculated Charm Quark mass of 2.09 GeV is a consitituent mass, that is, it corresponds to the conventional pole mass plus 312.8 MeV.

Therefore, the calculated Charm Quark mass of 2.09 GeV corresponds to a conventional pole mass of 1.78 GeV.

The 1996 Particle Data Group Review of Particle Physics gives a range for the Charm Quark pole mass from 1.2 to 1.9 GeV.

The pole mass can be converted to an MSbar mass if the color force strength constant alpha_s is known. The conventional value of alpha_s at about 2 GeV is about 0.39, which is somewhat lower than the teoretical model value. Using alpha_s (2 GeV) = 0.39, a pole mass of 1.9 GeV gives an MSbar 1-loop mass of 1.6 GeV, evaluated at about 2 GeV.

These results when added up give a total mass of second generation fermion particles:

Sigmaf2 = 32.9 GeV

Higgs and W-boson Masses

As with forces strengths, the calculations produce ratios of masses, so that only one mass need be chosen to set the mass scale.

In the E8 model, the value of the fundamental mass scale vacuum expectation value v = <PHI> of the Higgs scalar field is set to be the sum of the physical masses of the weak bosons, W+, W-, and Z0,

whose tree-level masses will then be shown by ratio calculations to be 80.326 GeV, 80.326 GeV, and 91.862 GeV, respectively,

and so that the electron mass will then be 0.5110 MeV.

The relationship between the Higgs mass and v is given by the Ginzburg-Landau term from the Mayer Mechanism as

(1/4) Tr ([PHI , PHI] - PHI)^2

or, in the notation of hep-ph/9806009 by Guang-jiong Ni

(1/4!) lambda PHI^4 - (1/2) sigma PHI^2

where the Higgs mass M_H = sqrt(2 sigma)

Ni says: "... the invariant meaning of the constant lambda in the Lagrangian is not the coupling constant, the latter will change after quantization ... The invariant meaning of lambda is nothing but the ratio of two mass scales:

lambda = 3 (M_H / PHI)^2

which remains unchanged irrespective of the order ...".

Since <PHI>^2 = v^2, and assuming at tree-level that lambda = 1 (a value consistent with the Higgs Tquark condensate model of Michio Hashimoto, Masaharu Tanabashi, and Koichi Yamawaki in their paper at hep-ph/0311165), we have, at tree-level

M_H^2 / v^2 = 1 / 3

In the E8 model, the fundamental mass scale vacuum expectation value v of the Higgs scalar field is the fundamental mass parameter that is to be set to define all other masses by the mass ratio formulas of the model and

v is set to be 252.514 GeV

so that

M_H = v /sqrt(3) = 145.789 GeV

To get W-boson masses, denote the 3 SU(2) high-energy weak bosons (massless at energies higher than the electroweak unification) by W+, W-, and W0, corresponding to the massive physical weak bosons W+, W-, and Z0.

The triplet { W+, W-, W0 } couples directly with the T - Tbar quark-antiquark pair, so that the total mass of the triplet { W+, W-, W0 } at the electroweak unification is equal to the total mass of a T - Tbar pair, 259.031 GeV.

The triplet { W+, W-, Z0 } couples directly with the Higgs scalar, which carries the Higgs mechanism by which the W0 becomes the physical Z0, so that the total mass of the triplet { W+, W-, Z0 } is equal to the vacuum expectation value v of the Higgs scalar field, v = 252.514 GeV.

What are individual masses of members of the triplet { W+, W-, Z0 } ?

First, look at the triplet { W+, W-, W0 } which can be represented by the 3-sphere S^3. The Hopf fibration of S^3 as

S^1 --> S^3 --> S^2

gives a decomposition of the W bosons into the neutral W0 corresponding to S^1 and the charged pair W+ and W- corresponding to S^2.

The mass ratio of the sum of the masses of W+ and W- to the mass of W0 should be the volume ratio of the S^2 in S^3 to the S^1 in S3.

- The unit sphere S^3 in R^4 is normalized by 1 / 2.

- The unit sphere S^2 in R^3 is normalized by 1 / sqrt(3).
- The unit sphere S^1 in R^2 is normalized by 1 / sqrt(2).

The ratio of the sum of the W+ and W- masses to the W0 mass should then be

$$(2 / \sqrt{3})\, V(S^2) / (2 / \sqrt{2})\, V(S^1) = 1.632993$$

Since the total mass of the triplet { W+, W-, W0 } is 259.031 GeV, the total mass of a T - Tbar pair, and the charged weak bosons have equal mass, we have

$$M_{W+} = M_{W-} = 80.326 \text{ GeV and } M_{W0} = 98.379 \text{ GeV.}$$

The charged W+/- neutrino-electron interchange must be symmetric with the electron-neutrino interchange, so that the absence of right-handed neutrino particles requires that the charged W+/- SU(2) weak bosons act only on left-handed electrons.

Each gauge boson must act consistently on the entire Dirac fermion particle sector, so that the charged W+/- SU(2) weak bosons act only on left-handed fermion particles of all types.

The neutral W0 weak boson does not interchange Weyl neutrinos with Dirac fermions, and so is not restricted to left-handed fermions, but also has a component that acts on both types of fermions, both left-handed and right-handed, conserving parity.

However, the neutral W0 weak bosons are related to the charged W+/- weak bosons by custodial SU(2) symmetry, so that the left-handed component of the neutral W0 must be equal to the left-handed (entire) component of the charged W+/-.

Since the mass of the W0 is greater than the mass of the W+/-, there remains for the W0 a component acting on both types of fermions.

Therefore the full W0 neutral weak boson interaction is proportional to $(M_{W+/-}^2 / M_{W0}^2)$ acting on left-handed fermions and

$(1 - (M_{W+/-}^2 / M_{W0}^2))$ acting on both types of fermions.

If (1 - (M_W+/-2 / M_W0^2)) is defined to be sin(theta_w)^2 and denoted by K,

and if the strength of the W+/- charged weak force (and of the custodial SU(2) symmetry) is denoted by T,

then the W0 neutral weak interaction can be written as W0L = T + K and W0LR = K.

Since the W0 acts as W0L with respect to the parity violating SU(2) weak force

and as W0LR with respect to the parity conserving U(1) electromagnetic force of the U(1) subgroup of SU(2), the W0 mass mW0 has two components:

the parity violating SU(2) part mW0L that is equal to M_W+/-

the parity conserving part M_W0LR that acts like a heavy photon.

As M_W0 = 98.379 GeV = M_W0L + M_W0LR, and as M_W0L = M_W+/- = 80.326 GeV, we have M_W0LR = 18.053 GeV.

Denote by *alphaE = *e^2 the force strength of the weak parity conserving U(1) electromagnetic type force that acts through the U(1) subgroup of SU(2).

The electromagnetic force strength alphaE = e^2 = 1 / 137.03608 was calculated above using the volume V(S^1) of an S^1 in R^2, normalized by 1 / sqrt(2).

The *alphaE force is part of the SU(2) weak force whose strength alphaW = w^2 was calculated above using the volume V(S^2) of an S^2 \subset R^3, normalized by 1 / sqrt(3).

Also, the electromagnetic force strength alphaE = e^2 was calculated above using a 4-dimensional spacetime with global structure of the 4-torus T^4 made up of four S^1 1-spheres, while the SU(2) weak force strength alphaW = w^2 was calculated above using two 2-spheres S^2 x S^2, each of which contains one 1-sphere of the *alphaE force.

Therefore

- *alphaE = alphaE (sqrt(2) / sqrt(3))(2 / 4) = alphaE / sqrt(6),
- *e = e / (4th root of 6) = e / 1.565 ,

and the mass mW0LR must be reduced to an effective value M_W0LReff = M_W0LR / 1.565 = 18.053/1.565 = 11.536 GeV for the *alphaE force to act like an electromagnetic force in the E8 model:

*e M_W0LR = e (1/5.65) M_W0LR = e M_Z0,

where the physical effective neutral weak boson is denoted by Z0.

Therefore, the correct E8 model values for weak boson masses and the Weinberg angle theta_w are:

M_W+ = M_W- = 80.326 GeV;

M_Z0 = 80.326 + 11.536 = 91.862 GeV;

Sin(theta_w)^2 = 1 - (M_W+/- / M_Z0)^2 = 1 - (6452.2663 / 8438.6270) = 0.235.

Radiative corrections are not taken into account here, and may change these tree-level values somewhat.

Kobayashi-Maskawa Parameters

The Kobayashi-Maskawa parameters are determined in terms of the sum of the masses of the 30 first-generation fermion particles and antiparticles, denoted by Smf1 = 7.508 GeV,

and the similar sums for second-generation and third-generation fermions, denoted by Smf2 = 32.94504 GeV and Smf3 = 1,629.2675 GeV.

The reason for using sums of all fermion masses (rather than sums of quark masses only) is that all fermions are in the same spinor representation of Spin(8), and the Spin(8) representations are considered to be fundamental.

The following formulas use the above masses to calculate Kobayashi-Maskawa parameters:

- phase angle d13 = 1 radian (unit length on a phase circumference)
- sin(alpha) = s12 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+[mmu^2+3ms^2+3mc^2]) = 0.222198
- sin(beta) = s13 = [me+3md+3mu]/sqrt([me^2+3md^2+3mu^2]+[mtau^2+3mb^2+3mt^2]) = 0.004608
- sin(*gamma) = [mmu+3ms+3mc]/sqrt([mtau^2+3mb^2+3mt^2]+[mmu^2+3ms^2+3mc^2])
- sin(gamma) = s23 = sin(*gamma) sqrt(Sigmaf2 / Sigmaf1) = 0.04234886

The factor sqrt(Smf2 /Smf1) appears in s23 because an s23 transition is to the second generation and not all the way to the first generation, so that the end product of an s23 transition has a greater available energy than s12 or s13 transitions by a factor of Smf2 / Smf1 .

Since the width of a transition is proportional to the square of the modulus of the relevant KM entry and the width of an s23 transition has greater available energy than the s12 or s13 transitions by a factor of Smf2 / Smf1

the effective magnitude of the s23 terms in the KM entries is increased by the factor sqrt(Smf2 /Smf1) .

The Chau-Keung parameterization is used, as it allows the K-M matrix to be represented as the product of the following three 3x3 matrices:

1	0	0
0	cos(gamma)	sin(gamma)
0	-sin(gamma)	cos(gamma)

---

cos(beta)	0	sin(beta)exp(-i d13)
0	1	0
-sin(beta)exp(i d13)	0	cos(beta)

---

cos(alpha)	sin(alpha)	0
-sin(alpha)	cos(alpha)	0
0	0	1

The resulting Kobayashi-Maskawa parameters for W+ and W- charged weak boson processes, are:

	d	s	b
u	0.975	0.222	0.00249 -0.00388i
c	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
t	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

The matrix is labelled by either (u c t) input and (d s b) output, or, as above, (d s b) input and (u c t) output.

For Z0 neutral weak boson processes, which are suppressed by the GIM mechanism of cancellation of virtual subprocesses, the matrix is labelled by either (u c t) input and (u'c't') output, or, as below, (d s b) input and (d's'b') output:

	d	s	b
d'	0.975	0.222	0.00249 -0.00388i
s'	-0.222 -0.000161i	0.974 -0.0000365i	0.0423
b'	0.00698 -0.00378i	-0.0418 -0.00086i	0.999

Since neutrinos of all three generations are massless at tree level, the lepton sector has no tree-level K-M mixing.

According to a Review on the KM mixing matrix by Gilman, Kleinknecht, and Renk in the 2002 Review of Particle Physics:

"... Using the eight tree-level constraints discussed below together with unitarity, and assuming only three generations, the 90% confidence limits on the magnitude of the elements of the complete matrix are

	d	s	b
u	0.9741 to 0.9756	0.219 to 0.226	0.00425 to 0.0048
c	0.219 to 0.226	0.9732 to 0.9748	0.038 to 0.044
t	0.004 to 0.014	0.037 to 0.044	0.9990 to 0.9993

... The constraints of unitarity connect different elements, so choosing a specific value for one element restricts the range of others. ... The phase d13 lies in the range 0 < d13 < 2 pi, with non-zero values generally breaking CP invariance for the weak interactions. ... Using tree-level processes as constraints only, the matrix elements ...[of the 90% confidence limit shown above]... correspond to values of the sines of the angles of s12 = 0.2229 +/- 0.0022, s23 = 0.0412 +/- 0.0020, and s13 = 0.0036 +/- 0.0007. If we use the loop-level processes discussed below as additional constraints, the sines of the angles remain unaffected, and the CKM phase, sometimes referred to as the angle gamma = phi3 of the unitarity triangle ... is restricted to d13 = (1.02 +/- 0.22) radians = 59 +/- 13 degrees. ... CP-violating amplitudes or differences of rates are all proportional to the product of CKM factors ... s12 s13 s23 c12 c13^2 c23 sind13. This is just twice the area of the unitarity triangle. ... All processes can be quantitatively understood by one value of the CKM phase d13 = 59 +/- 13 degrees. The value of beta = 24 +/- 4 degrees from the overall fit is consistent with the value from the CP-asymmetry measurements of 26 +/- 4 degrees. The invariant measure of CP violation is J = (3.0 +/- 0.3) x 10^(-5). ... From a combined fit using the direct measurements, B mixing, epsilon, and sin2beta, we obtain: Re Vtd = 0.0071 +/- 0.0008 , Im Vtd = -0.0032 +/- 0.0004 ... Constraints... on the position of the apex of the unitarity triangle following from | Vub | , B mixing, epsilon, and sin2beta. ...".

In hep-ph/0208080, Yosef Nir says: "... Within the Standard Model, the only source of CP violation is the Kobayashi-Maskawa (KM) phase ... The study of CP violation is, at last, experiment driven. ... The CKM matrix provides a consistent picture of all the measured flavor and CP violating processes. ... There is no signal of new flavor physics. ... Very likely, the KM mechanism is the dominant source of CP violation in flavor changing processes. ... The result is consistent with the SM predictions. ...".

Neutrino Masses

Consider the three generations of neutrinos:

- nu_e (electron neutrino);
- nu_m (muon neutrino);
- nu_t (tauon neutrino)

and three neutrino mass states: nu_1 ; nu_2 : nu_3

and the division of 8-dimensional spacetime into

- 4-dimensional physical M4 Minkowski spacetime
- plus 4-dimensional CP2 internal symmetry space.

The lightest mass state nu_1 corresponds to a neutrino whose propagation begins and ends in physical Minkowski spacetime, lying entirely therein. According to the E8 model, the mass of nu_1 is zero at tree-level and it picks up no first-order correction while propagating entirely through physical Minkowski spacetime, so the first-order corrected mass of nu_1 is zero.

Since only two of the three neutrinos have first-order mass, and since in the E8 model theneutrinos are not Majorana particles, there is no neutrino CP-violation or phase at first order.

Consider the neutrino mixing matrix

	nu_1	nu_2	nu_3
nu_e	Ue1	Ue2	Ue3
nu_m	Um1	Um2	Um3
nu_t	Ut1	Ut2	Ut3

Assume the simplest mixing scheme with a massless nu_1 andnu_3 with no nu_e component so that Ue3 = 0

or, in conventional notation, mixing angle theta_13 = 0 = sin(theta_13) and cos(theta_13) = 1.

Then we have (as described in the 2004 Particle Data Book):

	nu_1	nu_2	nu_3
nu_e	cos(theta_12)	sin(theta_12)	0
nu_m	-sin(theta_12)cos(theta_23)	cos(theta_12)cos(theta_23)	sin(theta_23)
nu_t	sin(theta_12)sin(theta_23)	-cos(theta_12)sin(theta_23)	cos(theta_23)

Assume that nu_3 has equal components of nu_m and nu_t so that Um3 = Ut3 = 1/sqrt(2)

or, in conventional notation, mixing angle theta_23 = pi/4.

Then we have:

	nu_1	nu_2	nu_3
nu_e	cos(theta_12)	sin(theta_12)	0
nu_m	-sin(theta_12)/sqrt(2)	cos(theta_12)/sqrt(2)	1/sqrt(2)
nu_t	sin(theta_12)/sqrt(2)	-cos(theta_12)/sqrt(2)	1/sqrt(2)

The heaviest mass state nu_3 corresponds to a neutrino whose propagation begins and ends in CP2 internal symmetry space, lying entirely therein.

According to the E8 model the mass of nu_3 is zero at tree-level but it picks up a first-order correction propagating entirely through internal symmetry space by merging with an electron through the weak and electromagnetic forces, effectively acting not merely as a point but as a point plus an electron loop at both beginning and ending points

so the first-order corrected mass of nu_3 is given by

M_nu_3 x (1/sqrt(2)) = M_e x GW(mproton^2) x alpha_E

where the factor (1/sqrt(2)) comes from the Ut3 component of the neutrino mixing matrix so that

M_nu_3 = sqrt(2) x M_e x GW(mproton^2) x alpha_E =

= 1.4 x 5 x 10^5 x 1.05 x 10^(-5) x (1/137) eV =

= 7.35 / 137 = 5.4 x 10^(-2) eV.

Note that the neutrino-plus-electron loop can be anchored by weak force action through any of the 6 first-generation quarks at each of the beginning and ending points, and that the anchor quark at the beginning point can be different from the anchor quark at the ending point, so that there are 6x6 = 36 different possible anchorings.

The intermediate mass state nu_2 corresponds to a neutrino whose propagation begins or ends in CP2 internal symmetry space and ends or begins in physical Minkowski spacetime, thus having only one point (either beginning or ending) lying in CP2 internal symmetry space where it can act not merely as a point but as a point plus an electron loop.

According to the E8 model the mass of nu_2 is zero at tree-level but it picks up a first-order correction at only one (but not both) of the beginning or ending points

so that so that there are 6 different possible anchorings for nu_2 first-order corrections, as opposed to the 36 different possible anchorings for nu_3 first-order corrections,

so that the first-order corrected mass of nu_2 is less than the first-order corrected mass of nu_3 by a factor of 6, so the first-order corrected mass of nu_2 is

M_nu_2 = M_nu_3 / Vol(CP2) = 5.4 x 10^(-2) / 6

= 9 x 10^(-3)eV.

Therefore: the mass-squared difference D(M23^2) is

D(M23^2) = M_nu_3^2 - M_nu_2^2 =

= (2916 - 81) x 10^(-6) eV^2 =

= 2.8 x 10^(-3) eV^2

and

the mass-squared difference D(M12^2) is

D(M12^2) = M_nu_2^2 - M_nu_1^2 =

= (81 - 0) x 10^(-6) eV^2 =

= 8.1 x 10^(-5) eV^2

Set theta_12 = pi/6= 0.866 so that cos(theta_12) = 0.866 = sqrt(3)/2 and sin(theta_12) = 0.5 = 1/2 = Ue2 = fraction of nu_2 begin/end points that are in the physical spacetime where massless nu_e lives. Then we have for the neutrino mixing matrix:

	nu_1	nu_2	nu_3
nu_e	0.87	0.50	0
nu_m	-0.35	0.61	0.71
nu_t	0.35	-0.61	0.71

The E8 model calculations are substantially consistent with experimental results as described in the 2004 Particle Data Book and in the presentation by deGouvea at the 2004 APS DPF meeting at UC Riverside.

Dark Energy : Dark Matter : Ordinary Matter

Gravity and the Cosmological Constant come from the MacDowell-Mansouri Mechanism and the 15-dimensional Spin(2,4) = SU(2,2) Conformal Group, which is made up of:

- 3 Rotations;
- 3 Boosts;
- 4 Translations;
- 4 Special Conformal transformations; and
- 1 Dilatation.

According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:

> "... If the fundamental spacetime symmetry of the laws of Physics is that given by the de Sitter instead of the Poincare group, the P-symmetry of the weak cosmological-constant limit and the Q-symmetry of the strong cosmological-constant limit can be considered as limiting cases of the fundamental symmetry. ...
>
> ... N ...[is the space]... whose geometry is gravitationally related to an infinite cosmological constant ...[and]... is a 4-dimensional cone-space in which ds = 0, and whose group of motion is Q. Analogously to the Minkowski case, N is also a homogeneous space, but now under the kinematical group Q, that is, N = Q/L [where L is the Lorentz Group of Rotations and Boosts]. In other words, the point-set of N is the point-set of the special conformal transformations.
>
> Furthermore, the manifold of Q is a principal bundle P(Q/L,L), with Q/L = N as base space and L as the typical fiber. The kinematical group Q, like the Poincare group, has the Lorentz group L as the subgroup accounting for both the isotropy and the equivalence of inertial frames in this space. However, the special conformal transformations introduce a new kind of homogeneity. Instead of ordinary translations, all the points of N are equivalent through special conformal transformations. ...

... Minkowski and the cone-space can be considered as dual to each other, in the sense that their geometries are determined respectively by a vanishing and an infinite cosmological constants. The same can be said of their kinematical group of motions: P is associated to a vanishing cosmological constant and Q to an infinite cosmological constant.

The dual transformation connecting these two geometries is the spacetime inversion x^u -> x^u / sigma^2 . Under such a transformation, the Poincare group P is transformed into the group Q, and the Minkowski space M becomes the cone-space N. The points at infinity of M are concentrated in the vertex of the cone-space N, and those on the light-cone of M becomes the infinity of N. It is interesting to notice that, despite presenting an infinite scalar curvature, the concepts of space isotropy and equivalence between inertial frames in the cone-space N are those of special relativity. The difference lies in the concept of uniformity as it is the special conformal transformations, and not ordinary translations, which act transitively on N. ..."

- Since the Cosmological Constant comes from the 10 Rotation, Boost, and Special Conformal generators of the Conformal Group Spin(2,4) = SU(2,2), the fractional part of our Universe of the Cosmological Constant should be about 10 / 15 = 67%.
- Since Black Holes, including Dark Matter Primordial Black Holes, are curvature singularities in our 4-dimensional physical spacetime, and since Einstein-Hilbert curvature comes from the 4 Translations of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2) through the MacDowell-Mansouri Mechanism (in which the generators corresponding to the 3 Rotations and 3 Boosts do not propagate), the fractional part of our Universe of Dark Matter Primordial Black Holes should be about 4 / 15 = 27%.
- Since Ordinary Matter gets mass from the Higgs mechanism which is related to the 1 Scale Dilatation of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2), the fractional part of our universe of Ordinary Matter should be about 1 / 15 = 6%.

Therefore, our Flat Expanding Universe should, according to the cosmology of the model, have (without taking into account any evolutionary changes with time) roughly:

- 67% Cosmological Constant
- 27% Dark Matter - possilbly primordial stable Planck mass black holes
- 6% Ordinary Matter

As Dennnis Marks pointed out to me, since density rho is proportional to (1+z)^3(1+w) for red-shift factor z and a constant equation of state w:

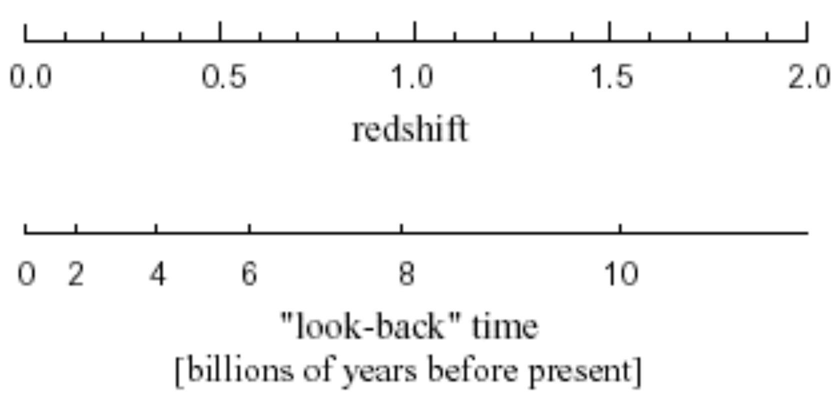
- w = -1 for /\ and the average overall density of /\ Dark Energy remains constant with time and the expansion of our Universe; and
- w = 0 for nonrelativistic matter so that the overall average density of Ordinary Matter declines as 1 / R^3 as our Universe expands; and
- w = 0 for primordial black hole dark matter - stable Planck mass black holes - so that Dark Matter also has density that declines as 1 / R^3 as our Universe expands;

so that the ratio of their overall average densities must vary with time, or scale factor R of our Universe, as it expands.

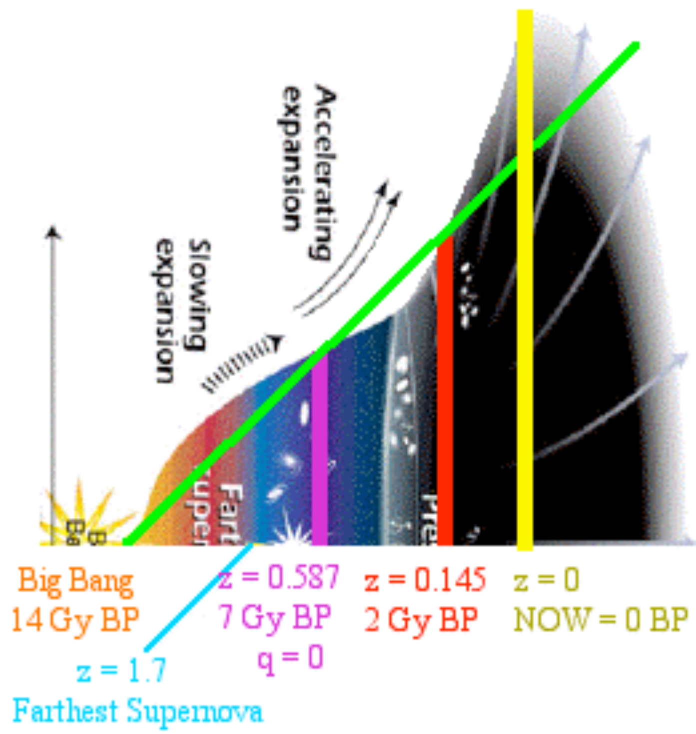
Therefore, the above calculated ratio 0.67 : 0.27 : 0.06 is valid only for a particular time, or scale factor, of our Universe.

When is that time ? Further, what is the value of the ratio NOW ?

Since WMAP observes Ordinary Matter at 4% NOW, the time WHEN Ordinary Matter was 6% would be at redshift z such that 1 / (1+z)^3 = 0.04 / 0.06 = 2/3 , or (1+z)^3 = 1.5 , or 1+z = 1.145 , or z = 0.145. To translate redshift into time, in billions of years before present, or Gy BP, use this chart



from a www.supernova.lbl.gov file SNAPoverview.pdf. to see that the time WHEN Ordinary Matter was 6% would have been a bit over 2 billion years ago, or 2 Gy BP.



In the diagram, there are four Special Times in the history of our Universe:

- the Big Bang Beginning of Inflation (about 13.7 Gy BP);
- the End of Inflation = Beginning of Decelerating Expansion (beginning of green line also about 13.7 Gy BP);
- the End of Deceleration (q=0) = Inflection Point = Beginning of Accelerating Expansion (purple vertical line at about z = 0.587 and about 7 Gy BP). According to a hubblesite web page credited to Ann Feild, the above diagram "... reveals changes in the rate of expansion since the universe's birth 15 billion years ago. The more shalow the curve, the faster the rate of expansion. The curve changes noticeably about 7.5 billion years ago, when objects in the universe began flying apart as a faster rate. ...". According to a CERN Courier web page: "... Saul Perlmutter, who is head of the Supernova Cosmology Project ... and his team have studied altogether some 80 high red-shift type Ia supernovae. Their results imply that the universe was decelerating for the first half of its existence, and then began accelerating approximately 7 billion years ago. ...". According to astro-ph/0106051

by Michael S. Turner and Adam G. Riess: "... current supernova data ... favor deceleration at $z > 0.5$ ... SN 1997ff at $z = 1.7$ provides direct evidence for an early phase of slowing expansion if the dark energy is a cosmological constant ...".

- the Last Intersection of the Accelerating Expansion of our Universe with Linear Expansion (green line) from End of Inflation (first interesection) through Inflection Point (second intersection, at purple vertical line at about $z = 0.587$ and about 7 Gy BP) to the Third Intersection (at red vertical line at $z = 0.145$ and about 2 Gy BP), which is also around the times of the beginning of the Proterozoic Era and Eukaryotic Life, Fe_2O_3 Hematite ferric iron Red Bed formations, a Snowball Earth, and the start of the Oklo fission reactor. 2 Gy is also about 10 Galactic Years for our Milky Way Galaxy and is on the order of the time for the process of a collision of galaxies.

Those four Special Times define four Special Epochs:

- The Inflation Epoch, beginning with the Big Bang and ending with the End of Inflation. The Inflation Epoch is described by Zizzi Quantum Inflation ending with Self-Decoherence of our Universe (see gr-qc/0007006).
- The Decelerating Expansion Epoch, beginning with the Self-Decoherence of our Universe at the End of Inflation. During the Decelerating Expansion Epoch, the Radiation Era is succeeded by the Matter Era, and the Matter Components (Dark and Ordinary) remain more prominent than they would be under the "standard norm" conditions of Linear Expansion.
- The Early Accelerating Expansion Epoch, beginning with the End of Deceleration and ending with the Last Intersection of Accelerating Expansion with Linear Expansion. During Accelerating Expansion, the prominence of Matter Components (Dark and Ordinary) declines, reaching the "standard norm" condition of Linear Expansion at the end of the Early Accelerating Expansion Epoch at the Last Intersection with the Line of Linear Expansion.
- The Late Accelerating Expansion Epoch, beginning with the Last Intersection of Accelerating Expansion and continuing forever, with New Universe creation happening many times at Many Times. During the Late Accelerating Expansion Epoch, the Cosmological Constant /\ is more prominent than it would be under the "standard norm" conditions of Linear Expansion.

NOW happens to be about 2 billion years into the Late Accelerating Expansion Epoch.
What about Dark Energy : Dark Matter : Ordinary Matter NOW ?

As to how the Dark Energy /\ and Cold Dark Matter terms have evolved during the past 2 Gy, a rough estimate analysis would be:

- /\ and CDM would be effectively created during expansion in their natural ratio 67 : 27 = 2.48 = 5 / 2, each having proportionate fraction 5 / 7 and 2 / 7, respectively;
- CDM Black Hole decay would be ignored; and
- pre-existing CDM Black Hole density would decline by the same 1 / R^3 factor as Ordinary Matter, from 0.27 to 0.27 / 1.5 = 0.18.

The Ordinary Matter excess 0.06 - 0.04 = 0.02 plus the first-order CDM excess 0.27 - 0.18 = 0.09 should be summed to get a total first-order excess of 0.11, which in turn should be distributed to the /\ and CDM factors in their natural ratio 67 : 27, producing, for NOW after 2 Gy of expansion:

CDM Black Hole factor = 0.18 + 0.11 x 2/7 = 0.18 + 0.03 = 0.21

for a total calculated Dark Energy : Dark Matter : Ordinary Matter ratio for NOW of

0.75 : 0.21 : 0.04

so that the present ratio of 0.73 : 0.23 : 0.04 observed by WMAP seems to me to be substantially consistent with the cosmology of the E8 model.

Pion Mass

The quark content of a charged pion is a quark - antiquark pair: either Up plus antiDown or Down plus antiUp. Experimentally, its mass is about 139.57 MeV.

The quark is a Naked Singularity Kerr-Newman Black Hole, with electromagnetic charge e and spin angular momentum J and constituent mass M 312 MeV, such that $e^2 + a^2$ is greater than M^2 (where $a = J / M$).

The antiquark is a also Naked Singularity Kerr-Newman Black Hole, with electromagnetic charge e and spin angular momentum J and constituent mass M 312 MeV, such that $e^2 + a^2$ is greater than M^2 (where $a = J / M$).

According to General Relativity, by Robert M. Wald (Chicago 1984) page 338 [Problems] ... 4. ...:

> "... Suppose two widely separated Kerr black holes with parameters (M1 , J1) and (M2 , J2) initially are at rest in an axisymmetric configuration, i.e., their rotation axes are aligned along the direction of their separation.
>
> Assume that these black holes fall together and coalesce into a single black hole.
>
> Since angular momentum cannot be radiated away in an axisymmetric spacetime, the final black hole will have momentum J = J1 + J2. ...".

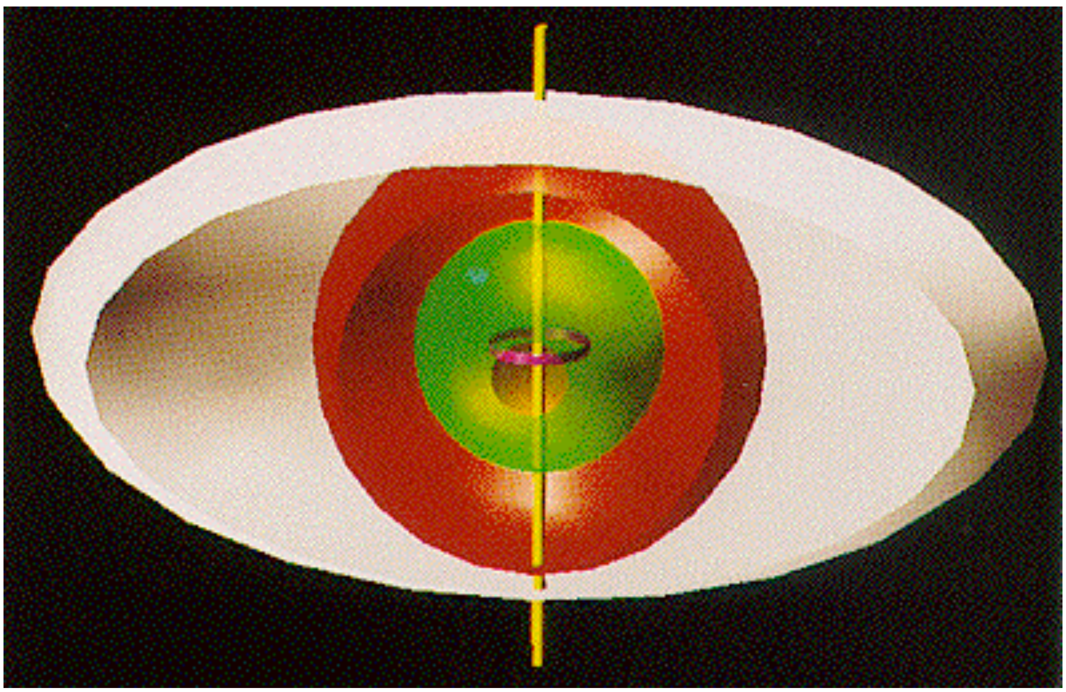
The neutral pion produced by the quark - antiquark pair would have zero angular momentum, thus reducing the value of $e^2 + a^2$ to e^2 .

For fermion electrons with spin 1/2, $1 / 2 = e / M$ (see for example Misner, Thorne, and Wheeler, Gravitation (Freeman 1972), page 883) so that $M^2 = 4 e^2$ is greater than e^2 for the electron. In other words, the angular momentum term a^2 is necessary to make $e^2 + a^2$ greater than M^2 so that the electron can be seen as a Kerr-Newman naked singularity.

Since the magnitude of electromagnetic charge of each quarks or antiquarks less than that of an electron, and since the mass of each quark or antiquark (as well as the pion mass) is greater than that of an electron, and since the quark - antiquark pair (as well as the pion) has angular momentum zero, the quark - antiquark pion has M^2 greater than e^2 + a^2 = e^2.

(Note that color charge, which is nonzero for the quark and the antiquark and is involved in the relation M^2 less than sum of spin-squared and charges-squared by which quarks and antiquarks can be see as Kerr-Newman naked singularities, is not relevant for the color-neutral pion.)

Therefore, the pion itself is a normal Kerr-Newman Black Hole with Outer Event Horizon = Ergosphere at r = 2M (the Inner Event Horizon is only the origin at r = 0) as shown in this image

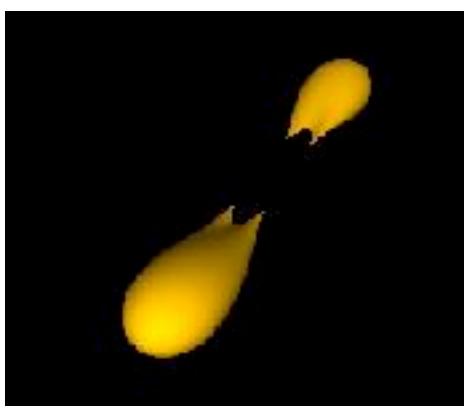


from Black Holes - A Traveller's Guide, by Clifford Pickover (Wiley 1996) in which the Ergosphere is white, the Outer Event Horizon is red, the Inner Event Horizon is green, and the Ring Singularity is purple. In the case of the pion, the white and red surfaces coincide, and the green surface is only a point at the origin.

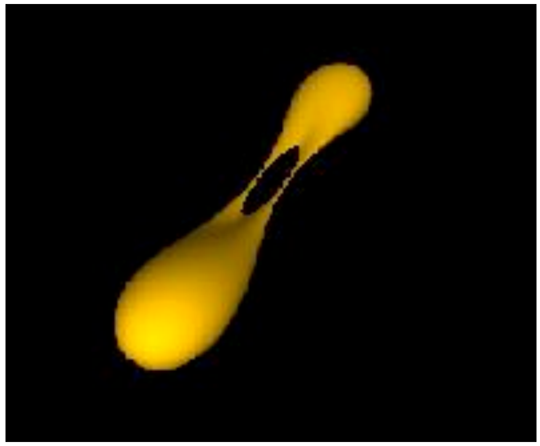
According to section 3.6 of Jeffrey Winicour's 2001 Living Review of the Development of Numerical Evolution Codes for General Relativity (see also a 2005 update):

> "... The black hole event horizon associated with ... slightly broken ... degeneracy [of the axisymmetric configuration]... reveals new features not seen in the degenerate case of the head-on collision ... If the degeneracy is slightly broken, the individual black holes form with spherical topology but as they approach, tidal distortion produces two sharp pincers on each black hole just prior to merger.
>
> ... Tidal distortion of approaching black holes ...

> ... Formation of sharp pincers just prior to merger ..



... toroidal stage just after merger ...



At merger, the two pincers join to form a single ... toroidal black hole.

The inner hole of the torus subsequently [begins to] close... up (superluminally) ... [If the closing proceeds to completion, it]... produce[s] first a peanut shaped black hole and finally a spherical black hole. ...".

In the physical case of quark and antiquark forming a pion, the toroidal black hole remains a torus. The torus is an event horizon and therefore is not a 2-spacelike dimensional torus, but is a (1+1)-dimensional torus with a timelike dimension.

The effect is described in detail in Robert Wald's book General Relativity (Chicago 1984). It can be said to be due to extreme frame dragging, or to timelike translations becoming spacelike as though they had been Wick rotated in Complex SpaceTime.

As Hawking and Ellis say in The LargeScale Structure of Space-Time (Cambridge 1973):

"... The surface r = r+ is ... the event horizon ... and is a null surface ...

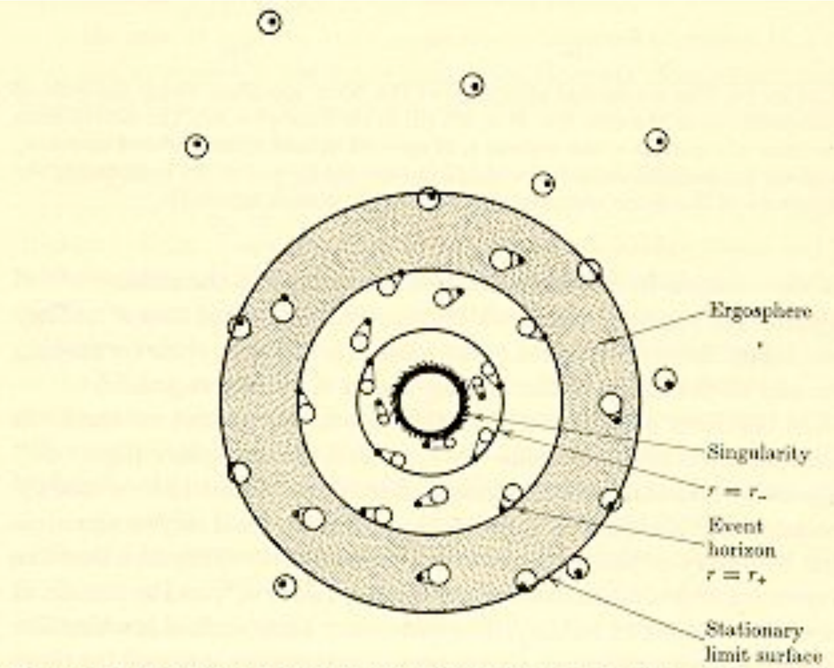


FIGURE 30. The equatorial plane of a Kerr solution with $m^2 > a^2$. The circles represent the position a short time later of flashes of light emitted by the points represented by heavy dots.

> ... On the surface r = r+ .... the wavefront corresponding to a point on this surface lies entirely within the surface. ...".

A (1+1)-dimensional torus with a timelike dimension can carry a Sine-Gordon Breather, and the soliton and antisoliton of a Sine-Gordon Breather correspond to the quark and antiquark that make up the pion.

Sine-Gordon Breathers are described by Sidney Coleman in his Erica lecture paper Classical Lumps and their Quantum Descendants (1975), reprinted in his book Aspects of Symmetry (Cambridge 1985), where Coleman writes the Lagrangian for the Sine-Gordon equation as (Coleman's eq. 4.3):

L = (1 / B^2) ((1/2) (df)^2 + A (cos(f) - 1))

and Coleman says:

> "... We see that, in classical physics, B is an irrelevant parameter: if we can solve the sine-Gordon equation for any non-zero B, we can solve it for any other B. The only effect of

changing B is the trivial one of changing the energy and momentum assigned to a given soluition of the equation. This is not true in quantum physics, becasue the relevant object for quantum physics is not L but [eq. 4.4]

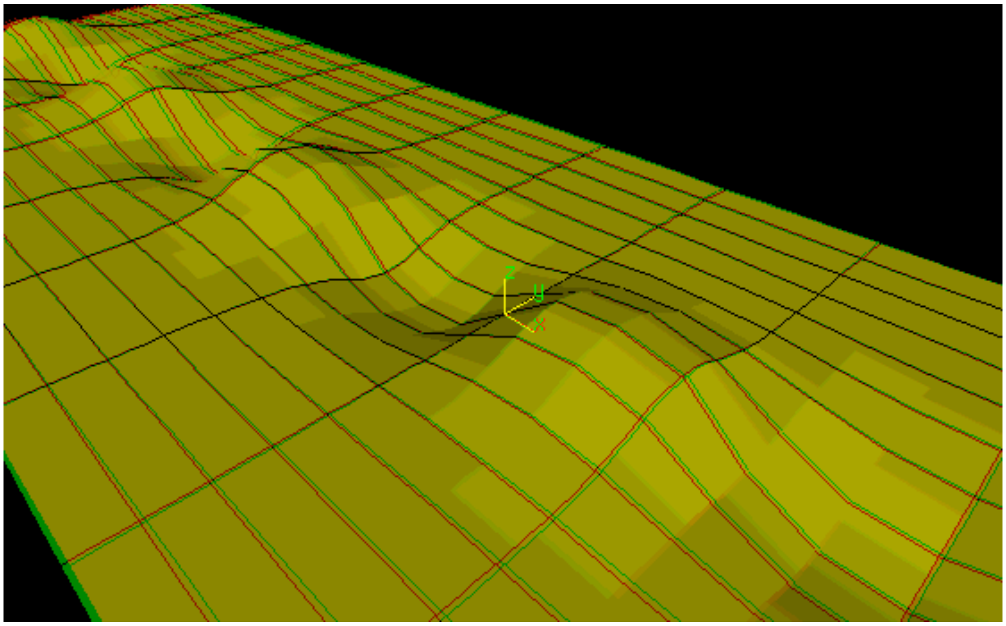
L / hbar = (1 / (B^2 hbar)) ((1/2) (df)^2 + A (cos(f) - 1))

An other way of saying the same thing is to say that in quantum physics we have one more dimensional constant of nature, Planck's constant, than in classical physics. ... the classical limit, vanishingf hbar, is exactly the same as the small-coupling limit, vanishing B ... from now on I will ... set hbar equal to one. ...

... the sine-Gordon equation ...[has]... an exact periodic solution ...[eq. 4.59]...

f(x, t) = (4 / B) arctan((n sin(w t) / cosh(n w x))

where [eq. 4.60] n = sqrt(A - w^2) / w and w ranges from 0 to A. This solution has a simple physical interpretation ... a soliton far to the left ...[and]... an antisoliton far to the right. As sin(w t) increases, the soliton and antisoliton mover farther apart from each other. When sin(w t) passes thrpough one, they turn around and begin to approach one another. As sin(w t) comes down to zero ... the soliton and antisoliton are on top of each other ... when sin(w t) becomes negative .. the soliton and antisoliton have passed each other. ...[



This stereo image of a Sine-Gordon Breather was generated by the program 3D-Filmstrip for Macintosh by Richard Palais. You can see the stereo with red-green or red-cyan 3D glasses. The program is on the WWW at http://rsp.math.brandeis.edu/3D-Filmstrip. The Sine-Gordon Breather is confined in space (y-axis) but periodic in time (x-axis), and therefore naturally lives on the (1+1)-dimensional torus with a timelike dimension of the Event Horizon of the pion. ...]

... Thus, Eq. (4.59) can be thought of as a soliton and an antisoliton oscillation about their common center-of-mass. For this reason, it is called 'the doublet [or Breather] solution'. ... the energy of the doublet ...[eq. 4.64]

E = 2 M sqrt(1 - (w^2 / A))

where [eq. 4.65] M = 8 sqrt(A) / B^2 is the soliton mass. Note that the mass of the doublet is always less than twice the soliton mass, as we would expect from a soltion-antisoliton pair. ... Dashen, Hasslacher, and Neveu ... Phys. Rev. D10, 4114; 4130; 4138 (1974). A pedagogical review of these methods has been written by R. Rajaraman (Phys. Reports 21, 227 (1975 ... Phys. Rev. D11, 3424 (1975) ...[Dashen, Hasslacher, and Neveu found that]... there is only a single series of bound states, labeled by the integer N ... The energies ... are ... [eq. 4.82]

E_N = 2 M sin(B'^2 N / 16)

where N = 0, 1, 2 ... < 8 pi / B'^2 , [eq. 4.83]

B'^2 = B^2 / (1 - (B^2 / 8 pi))

and M is the soliton mass. M is not given by Eq. (4.675), but is the soliton mass corrected by the DHN formula, or, equivalently, by the first-order weak coupling expansion. ... I have written the equation in this form .. to eliminate A, and thus avoid worries about renormalization conventions. Note that the DHN formula is identical to the Bohr-Sommerfeld formula, except that B is replaced by B'. ... Bohr and Sommerfeld['s] ... quantization formula says that if we have a one-parameter family of periodic motions, labeled by the period, T, then an energy eigenstate occurs whenever [eq. 4.66]

[Integral from 0 to T](dt p qdot = 2 pi N,

where N is an integer. ... Eq.(4.66) is cruder than the WKB formula, but it is much more general; it is always the leading approximation for any dynamical system ... Dashen et al speculate that Eq. (4.82) is exact. ...

the sine-Gordon equation is equivalent ... to the massive Thirring model. This is surprising, because the massive Thirring model is a canonical field theory whose Hamiltonian is expressedin terms of fundamental Fermi fields only. Even more surprising, when B^2 = 4 pi , that sine-Gordon equation is equivalent to a free massive Dirac theory, in one spatial dimension. ... Furthermore, we can identify the mass term in the Thirring model with the sine-Gordon interaction, [eq. 5.13]

M = - (A / B^2) N_m cos(B f)

.. to do this consistently ... we must say [eq. 5.14]

B^2 / (4 pi) = 1 / (1 + g / pi)

....[where]... g is a free parameter, the coupling constant [for the Thirring model]... Note that if B^2 = 4 pi , g = 0 , and the sine-Gordon equation is the theory of a free massive Dirac field. ... It is a bit surprising to see a fermion appearing as a coherent state of a Bose field. Certainly this could not happen in three dimensions, where it would be forbidden by the spin-statistics theorem. However, there is no spin-statistics theorem in one dimension, for the excellent reason that there is no spin. ... the lowest fermion-antifermion bound state of the massive Thirring model is an obvious candidate for the fundamental meson of sine-Gordon theory. ... equation (4.82) predicts that all the doublet bound states disappear when B^2 exceeds 4 pi . This is precisely the point where the Thirring model interaction switches from attractive to repulsive. ... these two theories ... the massive Thirring model .. and ... the sine-Gordon equation ... define identical physics. ... I have computed the predictions of ...[various]... approximation methods for the ration of the soliton mass to the meson mass for three values of B^2 : 4 pi (where the qualitative picture of the soliton as a lump totally breaks down), 2 pi, and pi . At 4 pi we know the exact answer

... I happen to know the exact answer for 2 pi, so I have included this in the table. ...

Method	B^2 = pi	B^2 = 2 pi	B^2 = 4 pi
Zeroth-order weak coupling expansion eq2.13b	2.55	1.27	0.64
Coherent-state variation	2.55	1.27	0.64
First-order weak coupling expansion	2.23	0.95	0.32
Bohr-Sommerfeld eq4.64	2.56	1.31	0.71
DHN formula eq4.82	2.25	1.00	0.50
Exact	?	1.00	0.50

...[eq. 2.13b] E = 8 sqrt(A) / B^2 ...[is the]... energy of the lump ... of sine-Gordon theory ... frequently called 'soliton...' in the literature ... [Zeroth-order is the classical case, or classical limit.] ...

... Coherent-state variation always gives the same result as the ... Zeroth-order weak coupling expansion ... .

The ... First-order weak-coupling expansion ... explicit formula ... is (8 / B^2) - (1 / pi). ...".

Note that, using the VoDou Physics constituent mass of the Up and Down quarks and antiquarks, about 312.75 MeV, as the soliton and antisoliton masses, and setting B^2 = pi and using the DHN formula, the mass of the charged pion is calculated to be

(312.75 / 2.25) MeV = 139 MeV

which is in pretty good agreement with the experimental value of about 139.57 MeV.

Why is the value B^2 = pi (or, using Coleman's eq. (5.14), the Thirring coupling constant g = 3 pi) the special value that gives the pion mass ?

Because B^2 = pi is where the First-order weak coupling expansion substantially coincides with the (probably exact) DHN formula.

In other words, the physical quark - antiquark pion lives where the first-order weak coupling expansion is exact.

Near the end of his article, Coleman expressed "Some opinions":

> "... This has been a long series of physics lectures with no reference whatsoever to experiment. This is embarrassing.
>
> ... Is there any chance that the lump will be more than a theoretical toy in our field? I can think of two possiblities.
>
> One is that there will appear a theory of strong-interaction dynamics in which hadrons are thought of as lumps, or, ... as systems of quarks bound into lumps. ... I am pessimistic about the success of such a theory. ... However, I stand ready to be converted in a moment by a convincing computation.
>
> The other possibility is that a lump will appear in a realistic theory ... of weak and electromagnetic interactions ... the theory would have to imbed the U(1)xSU(2) group ... in a larger group without U(1) factors ... it would be a magnetic monopole. ...".

This description of the hadronic pion as a quark - antiquark system governed by the sine-Gordon - massive Thirring model should dispel Coleman's pessimism about his first stated possibility and relieve his embarrassment about lack of contact with experiment.

As to his second stated possibility, very massive monopoles related to SU(5) GUT are still within the realm of possible future experimental discoveries.

Further material about the sine-Gordon doublet Breather and the massive Thirring equation can be found in the book Solitons and Instantons (North-Holland 1982,1987) by R. Rajaraman, who writes:

> "... the doublet or breather solutions ... can be used as input into the WKB method. ... the system is ... equivalent to the massive Thirring model, with the SG soliton state identifiable as a fermion. ... Mass of the quantum soliton ... will consist of a classical term followed by quantum corrections. The energy of the classical soliton ... is ... [eq. 7.3]
>
> $$E_{cl}[f_{sol}] = 8 m^3 / L$$
>
> The quantum corrections ... to the 'soliton mass' ... is finite as the momentum cut-off goes to infinity and equals ($- m / \pi$). Hence the quantum soliton's mass is [eq. 7.10]
>
> $$M_{sol} = (8 m^3 / L) - (m / \pi) + O(L).$$
>
> The mass of the quantum antisoliton will be, by ... symmetry, the same as M_{sol}. ...
>
> The doublet solutions ... may be quantised by the WKB method. ... we see that the coupling constant (L / m^2) has been replaced by a 'renormalised' coupling constant G ... [eq. 7.24]
>
> $$G = (L / m^2) / (1 - (L / 8 \pi m^2))$$
>
> ... as a result of quantum corrections. ... the same thing had happened to the soliton mass in eq. (7.10). To leading order, we can write [eq. 7.25]
>
> $$M_{sol} = (8 m^3 / L) - (m / \pi) = 8 m / G$$
>
> ... The doublet masses ... bound-state energy levels ... $E = M_N$, where ... [eq. 7.28]
>
> $$M_N = (16 m / G) \sin(N G / 16) ; N = 1, 2, ... < 8 \pi / G$$

> Formally, the quantisation condition permits all integers N from 1 to oo , but we run out of classical doublet solutions on which these bound states are based when N > 8 pi / G . ... The classical solutions ... bear the same relation to the bound-state wavefunctionals ... that Bohr orbits bear to hydrogen atom wavefunctions. ...
>
> Coleman ... show[ed] explicitly ... the SG theory equivalent to the charge-zero sector of the MT model, provided ... L / 4 pi m^2 = 1 / (1 + g / pi)
>
> ...[where in Coleman's work set out above such as his eq. (5.14) , B^2 = L / m^2]...
>
> Coleman ... resurrected Skyrme's conjecture that the quantum soliton of the SG model may be identified with the fermion of the MT model. ... ".

WHAT ABOUT THE NEUTRAL PION?

The quark content of the charged pion is u_d or d_u , both of which are consistent with the sine-Gordon picture. Experimentally, its mass is 139.57 Mev.

The neutral pion has quark content (u_u + d_d)/sqrt(2) with two components, somewhat different from the sine-Gordon picture, and a mass of 134.96 Mev.

The effective constituent mass of a down valence quark increases (by swapping places with a strange sea quark) by about DcMdquark = (Ms - Md) (Md/Ms)2 aw V12 = 312x0.25x0.253x0.22 Mev = 4.3 Mev.

Similarly, the up quark color force mass increase is about

DcMuquark = (Mc - Mu) (Mu/Mc)2 aw V12 = 1777x0.022x0.253x0.22 Mev = 2.2 Mev.

The color force increase for the charged pion DcMpion± = 6.5 Mev.

Since the mass Mpion± = 139.57 Mev is calculated from a color force sine-Gordon soliton state, the mass 139.57 Mev already takes DcMpion± into account.

For pion0 = (u_u + d_d)/ sqrt 2 , the d and _d of the the d_d pair do not swap places with strange sea quarks very often because it is energetically preferential for them both to become a u_u pair.

Therefore, from the point of view of calculating DcMpion0, the pion0 should be considered to be only u_u , and DcMpion0 = 2.2+2.2 = 4.4 Mev.

If, as in the nucleon, DeM(pion0-pion±) = -1 Mev, the theoretical estimate is

DM(pion0-pion±) = DcM(pion0-pion±) + DeM(pion0-pion±) = 4.4 - 6.5 -1 = -3.1 Mev,

roughly consistent with the experimental value of -4.6 Mev.

Proton-Neutron Mass Difference

According to the 1986 CODATA Bulletin No. 63, the experimental value of the neutron mass is 939.56563(28) Mev, and the experimental value of the proton is 938.27231(28) Mev.

The neutron-proton mass difference 1.3 Mev is due to the fact that the proton consists of two up quarks and one down quark, while the neutron consists of one up quark and two down quarks.

The magnitude of the electromagnetic energy difference mN - mP is about 1 Mev, but the sign is wrong: mN - mP = -1 Mev, and the proton's electromagnetic mass is greater than the neutron's.

The difference in energy between the bound states, neutron and proton, is not due to a difference between the Pre-Quantum constituent masses of the up quark and the down quark, which are calculated in the E8 model to be equal.

It is due to the difference between the Quantum color force interactions of the up and down constituent valence quarks with the gluons and virtual sea quarks in the neutron and the proton.

An up valence quark, constituent mass 313 Mev, does not often swap places with a 2.09 Gev charm sea quark, but a 313 Mev down valence quark can more often swap places with a 625 Mev strange sea quark.

Therefore the Quantum color force constituent mass of the down valence quark is heavier by about

(ms - md) (md/ms)^2 a(w) |Vds| = 312 x 0.25 x 0.253 x 0.22 Mev = 4.3 Mev,

(where a(w) = 0.253 is the geometric part of the weak force strength and |Vds| = 0.22 is the magnitude of the K-M parameter mixing first generation down and second generation strange)

so that the Quantum color force constituent mass Qmd of the down quark is

$$Qmd = 312.75 + 4.3 = 317.05 \text{ MeV}.$$

Similarly,

the up quark Quantum color force mass increase is about

(mc - mu) (mu/mc)^2 a(w) |V(uc)| = 1777 x 0.022 x 0.253 x 0.22 Mev = 2.2 Mev,

(where |Vuc| = 0.22 is the magnitude of the K-M parameter mixing first generation up and second generation charm)

so that the Quantum color force constituent mass Qmu of the up quark is

$$Qmu = 312.75 + 2.2 = 314.95 \text{ MeV}.$$

Therefore, the Quantum color force Neutron-Proton mass difference is

$$mN - mP = Qmd - Qmu = 317.05 \text{ Mev} - 314.95 \text{ Mev} = 2.1 \text{ Mev}.$$

Since the electromagnetic Neutron-Proton mass difference is roughly mN - mP = -1 MeV

the total theoretical Neutron-Proton mass difference is

$$mN - mP = 2.1 \text{ Mev} - 1 \text{ Mev} = 1.1 \text{ Mev},$$

an estimate that is fairly close to the experimental value of 1.3 Mev.

Note that in the equation (ms - md) (md/ms)^2 a(w) |Vds| = 4.3 Mev , Vds is a mixing of down and strange by a neutral Z0, compared to the more conventional Vus mixing by charged W. Although real neutral Z0 processes are suppressed by the GIM mechanism, which is a cancellation of virtual processes, the process of the equation is strictly a virtual process.

Note also that the K-M mixing parameter |Vds| is linear. Mixing (such as between a down quark and a strange quark) is a two-step process, that goes approximately as the square of |Vds|:

- First the down quark changes to a virtual strange quark, producing one factor of |Vds|.
- Then, second, the virtual strange quark changes back to a down quark, producing a second factor of |Vsd|, which is approximately equal to |Vds|.

Only the first step (one factor of |Vds|) appears in the Quantum mass formula used to determine the neutron mass.

If you measure the mass of a neutron, that measurement includes a sum over a lot of histories of the valence quarks inside the neutron. In some of those histories, in my view, you will "see" some of the two valence down quarks in a virtual transition state that is at a time after the first action, or change from down to strange, and before the second action, or change back. Therefore, you should take into account those histories in the sum in which you see a strange valence quark, and you get the linear factor |Vds| in the above equation.

Planck Mass

In the E8 model, a Planck-mass black hole is not a tree-level classical particle such as an electron or a quark, but a quantum entity resulting from the Many-Worlds quantum sum over histories at a single point in spacetime.

Consider an isolated single point, or vertex in the lattice picture of spacetime. In the E8 model, fermions live on vertices, and only first-generation fermions can live on a single vertex. (The second-generation fermions live on two vertices that act at our energy levels very much like one, and the third-generation fermions live on three vertices that act at our energy levels very much like one.)

At a single spacetime vertex, a Planck-mass black hole is the Many-Worlds quantum sum of all possible virtual first-generation particle-antiparticle fermion pairs permitted by the Pauli exclusion principle to live on that vertex.

Once a Planck-mass black hole is formed, it is stable in the E8 model. Less mass would not be gravitationally bound at the vertex. More mass at the vertex would decay by Hawking radiation.

In the E8 model, a Planck-mass black hole can be formed: as the end product of Hawking radiation decay of a larger black hole; by vacuum fluctuation; or perhaps by using a pion laser.

Since Dirac fermions in 4-dimensional spacetime can be massive (and are massive at low enough energies for the Higgs mechanism to act), the Planck mass in 4-dimensional spacetime is the sum of masses of all possible virtual first-generation particle-antiparticle fermion pairs permitted by the Pauli exclusion principle.

There are 8 fermion particles and 8 fermion antiparticles for a total of 64 particle-antiparticle pairs. A typical combination should have several quarks, several antiquarks, a few colorless quark-antiquark pairs that would be equivalent to pions, and some leptons and antileptons.

Due to the Pauli exclusion principle, no fermion lepton or quark could be present at the vertex more than twice unless they are in the form of boson pions, colorless first-generation quark-antiquark pairs not subject to the Pauli exclusion principle. Of the 64 particle-antiparticle pairs, 12 are pions.

A typical combination should have about 6 pions.

If all the pions are independent, the typical combination should have a mass of about .14x6 GeV = 0.84 GeV. However, just as the pion mass of .14 GeV is less than the sum of the masses of a quark and an antiquark, pairs of oppositely charged pions may form a bound state of less mass than the sum of two pion masses. If such a bound state of oppositely charged pions has a mass as small as .1 GeV, and if the typical combination has one such pair and 4 other pions, then the typical combination could have a mass in the range of 0.66 GeV.

Summing over all 2^64 combinations, the total mass of a one-vertex universe should give a Planck mass roughly around 0.66 x 2^64 = 1.217 x 10^19 GeV.

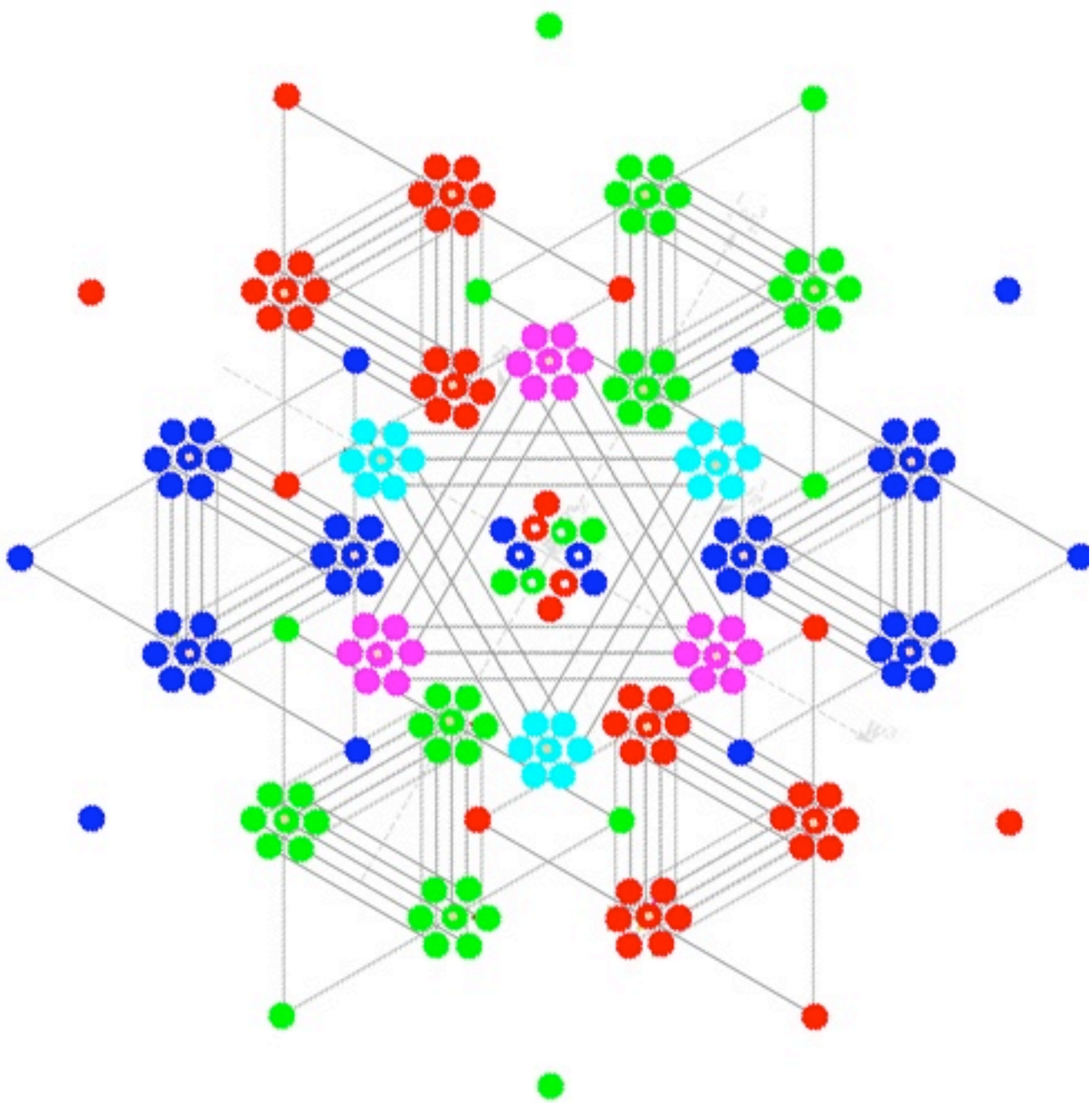
Since each fermion particle has a corresponding antiparticle, a Planck-mass Black Hole is neutral with respect to electric and color charges.

The value for the Planck mass given in the Particle Data Group's 1998 review is 1.221 x 10^19 GeV.

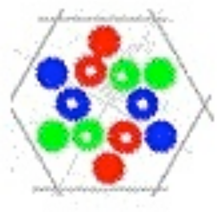
Monster Symmetry of Local Neighborhood Physics

Each E8 or Cl(8) only describes physics in a Local Neighborhood (it takes the Algebraic Quantum Field Theory of the Generalized Hyperfinite II1 von Neumann Factor to describe a more global theory).

Consider the E8(8) root vector polytope



and particularly the central 24 vertices:



made up of 8+8+8 = 24 central vertices.

If consider the 24-dim space generated by those 24 elements,

and consider the 24-dimensional Leech Lattice as a lattice in that 24-dim space,

and then compactify the 24-dim space by taking its quotient modulo the Leech Lattice,

you get a representation of a single E8 alone, the simplest building block element of the full E8 model.

According to James Lepowsky in math.QA/0706.4072:

> "... the Fischer-Griess Monster M ... was constructed by Griess as a symmetry group (of order about 10^54) of a remarkable new commutative but very, very highly nonassociative, seemingly ad-hoc, algebra B of dimension 196,883. ... One takes the torus that is the quotient of 24-dimensional Euclidean space modulo the Leech lattice ... The Monster is the automorphism group of the smallest nontrival string theory that nature allows ... Bosonic 26-dimensional space-time ... "compactified" on 24 dimensions ...".

It is a conjecture that the Monster is also the automorphism group of the smallest nontrivial part of the E8 model, and that the common relationship to the Monster might show an equivalence of the E8 model and the 26-dim Bosonic String Model (with fermions from orbifolding) described at CERN-CDS-EXT-2004-031. It might even be that both the E8 model and such String models are substantially equivalent to a Spin Foam model with E8(8) structures organized according to the 27-dim exceptional Jordan algebra J3(O).

As to possible physical meaning of such Monster symmetry of elemental E8 model structures, consider that the order of the Monster Group is

8080, 17424, 79451, 28758, 86459, 90496, 17107, 57005, 75436, 80000, 00000

=

2^46 .3^20 .5^9 .7^6 .11^2 .13^3 .17.19.23.29.31.41.47.59.71

or about 8 x 10^53.

If you use positronium (electron-positron bound state of the two lowest-nonzero-mass Dirac fermions) as a unit of mass Mep = 1 MeV, then it is interesting that the product of the squares of the Planck mass Mpl = 1.2 x 10^22 MeV and W-boson mass Mw = 80,000 MeV gives ((Mpl/Mep)(Mw/Mep))^2 = 9 x 10^53 which is roughly the Monster order.

- The Mpl part of M may be related to Aut(Leech Lattice) = double cover of Co1.
- The order of Co1 is 2^21.3^9.5^4.7^2.11.13.23 or about 4 x 10^18.
- The Mw part of M may be related to Aut(Golay Code) = M24.
- The order of M24 is 2^10.3^3.5.7.11.23 or about 2.4 x 10^8.

If you look at the physically realistic superposition of 8 such Cells, you get 8 copies of the Monster of total order about 6.4 x 10^54, which is roughly the product of the Planck mass and Higgs VEV squared:

(1.22 x 10^22)^2 x (2.5 x 10^5)^2 = 9 x 10^54

The full 26-dimensional Lattice Bosonic String Theory, and the full E8 model, and the full J3(O) Spin Foam model, might in that view all be regarded as an infinite-dimensional Affinization of the Theory of that Single Cell.

Inflation, Octonion Non-Unitarity, and Entropy and Bohm

In his book Quaternionic Quantum Mechanics and Quantum Fields ((Oxford 1995), Stephen L. Adler says at pages 50-52, 561:

> "... If the multiplication is associative, as in the complex and quaternionic cases, we can remove parentheses in ... Schroedinger equation dynamics ... to conclude that ... the inner product < f(t) | g(t) > ... is invariant ... this proof fails in the octonionic case, and hence one cannot follow the standard procedure to get a unitary dynamics. ...[so
>
> there is a]... failure of unitarity in octonionic quantum mechanics...".

Conventionally, creation of the particles in our universe occurred during inflation with unitarity and energy conservation being due to an inflaton field that is addition to the fields we now observe in the Standard Model plus Gravity.

In the E8 model, our present 4-dimensional physical spacetime freezes out from a high-energy 8-dimensional octonionic spacetime due to selection of a preferred quaternionic subspacetime. A question is whether the dimensional reduction occurs at the initial Big Bang beginning of inflation or continues through inflation to its end.

If our spacetime remains octonionic 8-dimensional throughout inflation, then the non-associativity and non-unitarity of octonions might account for particle creation without the need for tapping the energy of an inflaton field.

The non-associative structure of octonions manifests itself in interesting ways, such as the expansion of the 7-dim 7-sphere S7 under the Lie algebra bracket operation to the 28-dim Lie algebra Spin(8) that is made up of two S7 spheres and a 14-dim G2 Lie algebra.

Consider that the initial Big Bang produced a particle-antiparticle pair of the 7 charged fermions, plus the 8th fermion (neutrino) corresponding to the real number 1.

In gr-qc/0007006 Paola Zizzi says:

> "... during inflation, the universe can be described as a superposed state of quantum ... [qubits]. The self-reduction of the superposed quantum state is ... reached at the end of inflation ...[at]... the decoherence time ... [Tdecoh = 10^9 Tplanck = 10^(-34) sec] ... and corresponds to a superposed state of ... [10^19 = 2^64 qubits]. ... ... This is also the number of superposed tubulins-qubits in our brain ... leading to a conscious event. ...".

The number of doublings (also known as e-foldings) is also estimated by in astro-ph/0307459, by Banks and Fischler, who say:

> "... If the present acceleration of the universe is due to an asymptotically deSitter universe with small cosmological constant, then the number of e-foldings during inflation is bounded. ... The essential ingredient is that because of the UV-IR connection, entropy requires storage space. The existence of a small cosmological constant restricts the available storage space. ... We obtain the upper bound ... N_e = 85 ... where we took [the cosmological constant] /\ to be of O(10^(-3) eV). For the sake of comparison, the case k = 1/3 [corresponding to the equation of state for a radiation-dominated fluid, such as the cosmic microwave background] yields ... N_e= 65 ... This value for the maximum number of e-foldings is close to the value necessary to solve the "horizon problem".

If at each of the 64 doubling stages of Zizzi inflation the 2 particles of such a pair produced 8+8 = 16 fermions,

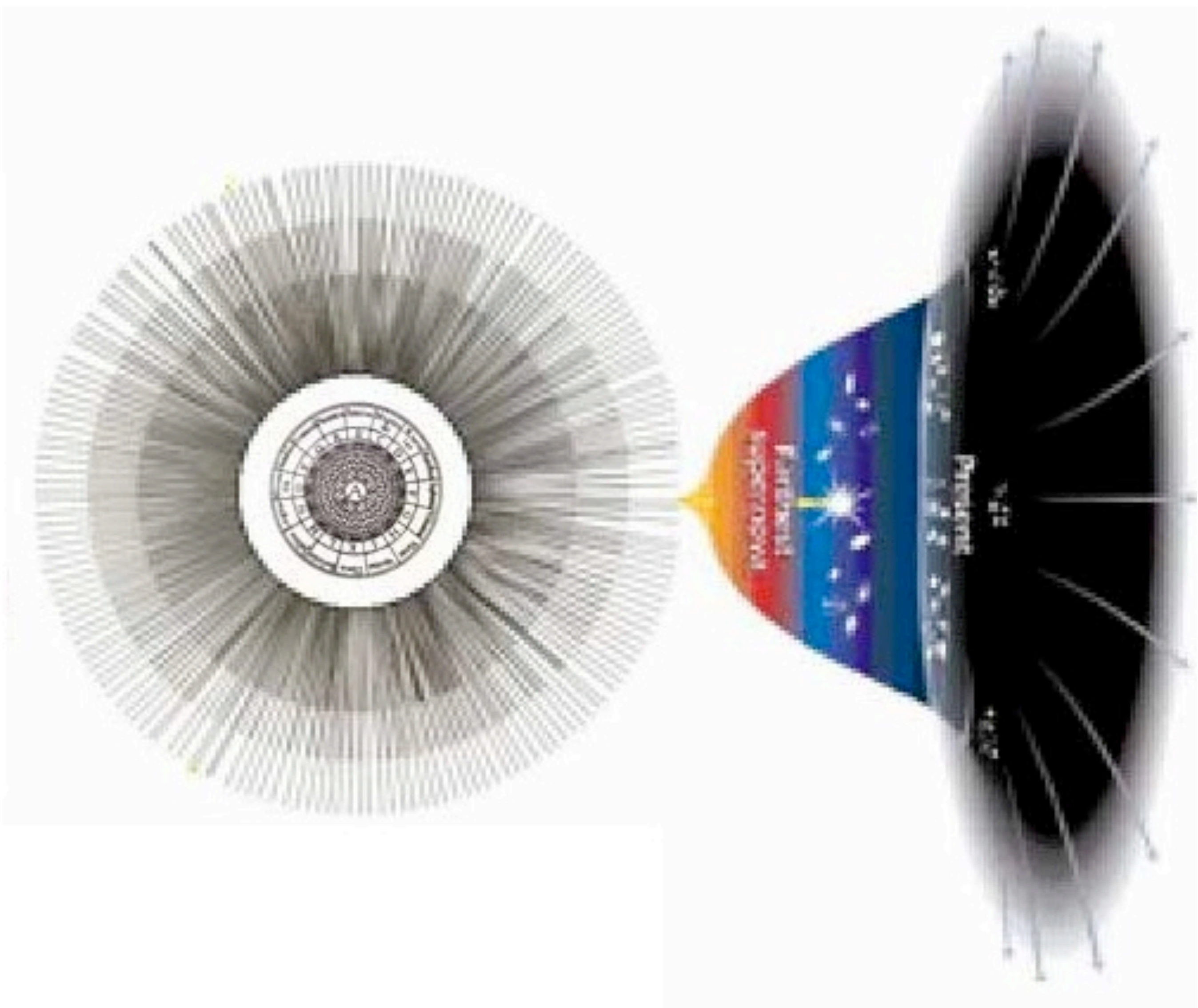
then at the end of inflation such a non-unitary octonionic process would have produced about 2 x 16^64 = 4 x (2^4)^64 = 4 x 2^256 = 4 x 10^77 fermion particles.

The figure of 4 x 10^77 is similar number of particles estimated by considering the initial fluctuation to be a Planck mass Black Hole and the 64 doublings to act on such Black Holes (which process can also be considered due to octonionic non-associativity non-unitarity).

Roger Penrose, in his book The Emperor's New Mind (Oxford 1989, pages 316-317) said:

> "... in our universe ... Entropy ... increases ... Something forced the entropy to be low in the past. ... the low-entropy states in the past are a puzzle. ...".

The Zizzi Inflation phase of our universe ends with decoherence "collapse" of the 2^64 Superposition Inflated Universe into Many Worlds of the Many-Worlds Quantum Theory, only one of which Worlds is our World.



In this image:

- the central white circle containing Llull's A-wheel is the Inflation Era in which everything is in Superposition;
- the boundary of the central circle marks the decoherence/collapse at the End of Inflation; and
- each line radiating from the central circle corrresponds to one decohered/collapsed Universe World (of course, there are many more lines than actually shown), only three of which are explicitly indicated in the image, and only one of which is Our Universe World.

Since our World is only a tiny fraction of all the Worlds, it carries only a tiny fraction of the entropy of the 2^64 Superposition Inflated Universe, thus solving Penrose's Puzzle.

Penrose (in his book The Emperor's New Mind (Oxford 1989, page 339)) proposed that the solution of his Puzzle might be related to Weyl Curvature, saying

> "... For some reason, the universe was created in a very special (low entropy) state, with something like the WEYL = 0 constraint of the FRW-models imposed upon it ...".

In the book The Dawning of Gauge Theory (Princeton 1997, pages 45,77-81,86,120,144) Lochlainn O'Raifeartaigh said:

> "... Weyl's ... 1918 paper ... showed how a geometrical significance could be ascribed to the electromagnetic field ... in 1922 ... Shroedinger ... suggested ... the flaw in the original Weyl theory might be removed by quantum mechanics ... the exponent of the non-integrable Weyl factor became quantized ...
>
> ... London in his 1927 paper ... establish[ed] the relation between Weyl's non-integrable scale factor and the gauge principle as it ocurs in the Hamilton-Jacobi, de Broglie and Schroedinger equations ... it is the complex amplitude of the de Broglie wave ... The fault in Weyl's original theory lay not in the presence of Weyl's non-integrable scale-factor but in the fact that it was real and applied to the metric. It should be converted to a phase-factor and applied to the wave-function. ...

> Weyl's reaction ... was ... enthusiasm ... in ... 1929 ... electromagnetism is an accompanying phenomenon of the material wave-field and not of gravitation ...
>
> ... Pauli proceeded to incorporate many of Weyl's ideas into his Handbuch article and by 1953 he had become an ardent proponent of the gauge principle ...".

In the early 1950s, Bohm developed his theory, an elaboration of de Broglie-Schroedinger quantum theory.

In physics/0211012 B. G. Sidharth said:

> "... Santamato ... Phys.Rev.D. 29 (2), 216ff, 1984 ... J. Math. Phys. 26 (8), 2477ff, 1984 ... Phys.Rev.D 32 (10), 2615ff, 1985 ... further developed the deBroglie-Bohm formulation by relating the ... Quantum potential to ... Weyl's geometry ...".

In The Anthropic Cosmological Principle (Oxford 1986, pages 446-447) Barrow and Tipler said:

> "... Penrose ... suggested that the Weyl curvature could be intimately related to the gravitational entropy of space-time ... Unfortunately, as yet there is no obvous candidate to use as a gravitational entropy Sg ...".

As Penrose said in his book The Emperor's New Mind (Oxford 1989, pages 210-211):

> "... REIMANN = WEYL + RICCI ... Einstein's equations become ... RICCI = ENERGY ...
>
> The Weyl tensor WEYL measures a tidal distortion of our sphere of freely falling particles (i.e., an initial change in shape, rather than in size), and the Ricci tensor RICCI measures its initial change in volume. ... the Weyl tensor ... is an important quantity. The tidal effect that is experienced in empty space is entirely due to WEYL. ... there are differential equations connecting WEYL with ENERGY, rather like the Maxwell equations ... a fruitful point of view is to regard WEYL as a

kind of gravitational analogue of the electromagnetic field quantity ...".

These remarks of Penrose seem to me to justify seeing the Weyl curvature as a Weyl gauge quantum phase for a Bohm-type Quantum Potential, especially in view of my model in which the Bohm-type Quantum Potential comes from what is commonly viewed as a gravitational part of Bosonic String Theory and in which Many-Worlds gravitational superposition separation plays a fundamental role in Quantum Consciousness.

Since, from the Many-Worlds point of view, the branching of the Worlds of our Universe as time moves forward towards the future might give a realistic definition of gravitational entropy Sg and since Deutsch has indicated that the Bohm potential can be seen to be equivalent to Many-Worlds Quantum theory, it seems to me that the Weyl-Schroedinger-London-Santamato description of the Quantum potential in terms of Weyl curvature could be seen as Penrose's Weyl curvature entropy.

Moreover, the fact that the Weyl curvature WEYL is the conformal part of the RIEMANN tensor is interesting, and the unification of RICCI for gravity and WEYL for quantum potential indicate to me that Jack Sarfatti's idea that BOTH should have back-reaction may be correct.

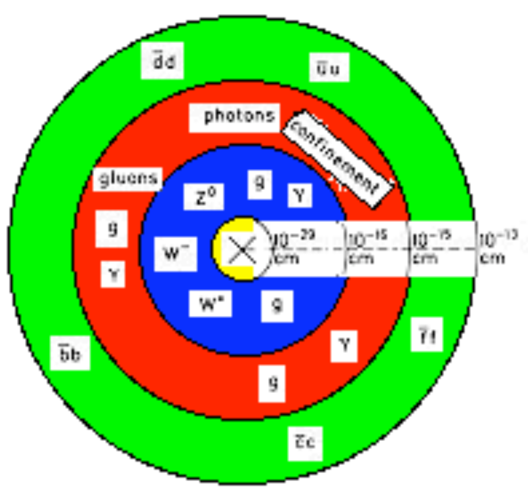
Another useful aspect of Bohm’s Quantum Potential is the effectiveness of the NonRelativistic Quark Model of hadrons, which can be explained by Bohm’s quantum theory applied to a fermion confined in a box, in which the fermion is at rest because its kinetic energy is transformed into PSI-field potential energy (see quant-ph/9806023).

Angular Momentum, Mass, Magnetic Dipole Moment

At T = 10^19 GeV, the Planck Mass/Energy, the Inflation Era begins.

At T = 10^16 GeV, the SU(5) Monopole Mass/Energy ... [According to The Early Universe, by G. Borner (Springer-Verlag 1988), from which book's Fig. 6.21 the SU(5) GUT illustration below is taken,

> "... For GUT physics monopoles are extremely interesting objects: they have an onion-like structure ... which contains the whole world of grand unified theories.
>
> Near the center (about 10^(-29) cm) there is a GUT symmetric vacuum.
>
> At about 10^(-16) cm, out to the Yukawa tail ... exp(- Mw r), the field is the electroweak colour field of the (3,2,1) standard model, and
>
> at ...[10^(-15) cm]... it is made up of photons and gluons, while
>
> at the edge [10^(-13) cm] there are fermion-antifermion pairs.
>
> Far beyond nuclear distances it behaves as a magnetically-charged pole of the Dirac type.



This view of the GUT monopole raises the possibility that it may catalyze the decay of the proton ...".]...

SU(5) GUT Monopole formation ends and the Inflationary X-Boson Higgs mechanism eliminates the relic Monopoles.

According to The Early Universe, by Kolb and Turner (1994 paperback edition, Adddison-Wesley, page 526):

"... the full symmetry of the GUT cannot be manifest; if it were the proton would decay in 10^(-24) sec. The gauge group ... must be spontaneously broken to [SU(3) x SU(2) x U(1)]. For SU(5), this is accomplished by ... masses of the order of the unification scale for the twelve X ... gauge bosons. ...[

	X color charges	X electric charges
3 3 3 X X	red red	-4/3 -1/3
3 3 3 X X	green green	-4/3 -1/3
3 3 3 X X	blue blue	-4/3 -1/3
X X X 2 2	antired antigreen antiblue	+4/3 +4/3 +4/3
X X X 2 2	antired antigreen antiblue	+1/3 +1/3 +1/3

]... Thus, ... at energies below 10^14 GeV or so the processes mediated by X ... boson exchange can be treated as a four-fermion interaction with strength ... [proportional to 1 / M^2] ... where M = 3 x 10^14 GeV is the unification scale. ... these new ... interactions are extremely weak at energies below 10^14 GeV. ... the proton lifetime must be ...[about]... 10^31 yr. ...".

In The Early Universe (paperback edition Addison-Wesley 1994) Kolb and Turner say (at p. 526):

"... SU(5) GUT ...[has]... at the very least one complex 5-dimensional Higgs. The 5-dimensional Higgs contains

the usual doublet Higgs required for W-Boson SSB ...[which]... must acquire a mass of order of a few 100 GeV and

a color triplet Higgs ... which can also mediate B,L [baryon,lepton] violation. The triplet component must acquire a mass comparable to ... M = 3 x 10^14 GeV ... to guarantee the proton's longevity, ...".

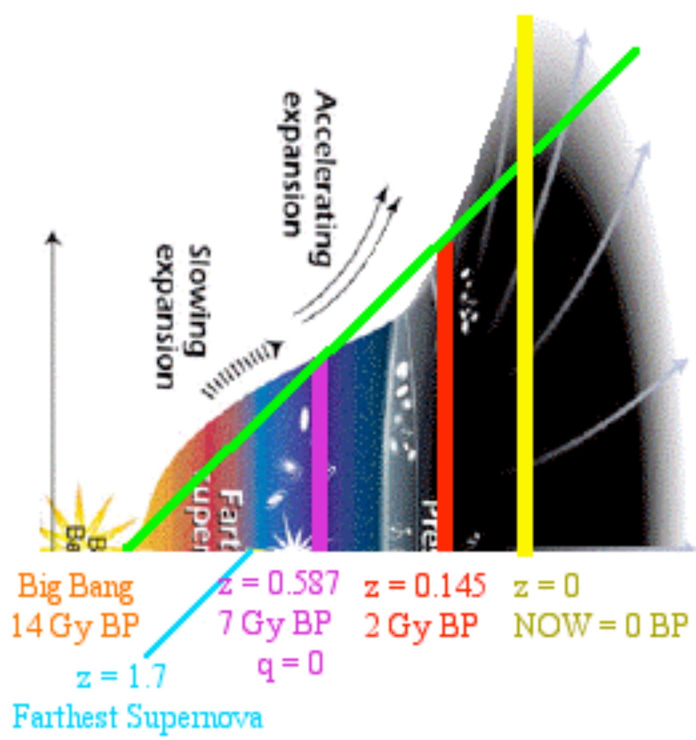
At T = 10^15 GeV or about 10^(-34) sec the size of Our Universe is about 10 cm, and the Inflation Era ends.

At T = 10^14 GeV, the SU(5) X-Boson Mass/Energy, Zizzi Reheating occurs and SU(5) Unification ends. At the phase transition at 10^14 GeV the GUT SU(5) is broken to U(3)xU(2)

3 3 3
3 3 3
3 3 3
2 2
2 2

and then to the Standard Model SU(3) x SU(2) x U(1) with the usual Higgs doublet with VEV around 250 GeV.

After the Inflation Era, Our Universe begins its current phase of expansion



controlled by Gravity according to a MacDowell-Mansouri Mechanism based on the Conformal Group Spin(2,4) = SU(2,2) with 15 generators:

- 6 Lorentz Rotation and Boost Generators;
- 4 Special Conformal Generators;
- 4 Translation Generators; and
- 1 Scalar Dilation Generator.

According to gr-qc/9809061 by R. Aldrovandi and J. G. Peireira:

> "... By the process of Inonu-Wigner group contraction taking the limit R -> 0, ...[where R is the de Sitter pseudo-radius, the] ... de Sitter group... [whether of metric ... (-1,+1,+1,+1,-1) or (-1,+1,+1,+1,+1) , is]... contracted to the group Q, formed by a semi-direct product between Lorentz and special conformal transformation groups, and ... de Sitter space...[is]... reduced to the cone-space N, which is a space with vanishing Riemann and Ricci curvature tensors. As the scalar curvature of the de Sitter space goes to infinity in this limit, we can say that N is a spacetime gravitationally related to an infinite cosmological constant.".

If the 2+4 = 6-dimensional spacetime on which the full Conformal Group Spin(2,4) acts linearly is viewed in terms of an elastic Aether, its rigidity would correspond to the VEV of the X-Boson Higgs Condensate on the order of 10^14 GeV. Since the action of the Conformal Group Spin(2,4) = SU(2,2) is nonlinear on 4-dimensional physical spacetime, the 4-dimensional elastic Aether can, within the Conformal Expanding Domain of Our Universe, be deformed by Special Conformal and Dilation transformations without the restrictions of X-Higgs VEV rigidity on the order of 10^14 GeV.

The Aldrovandi-Peireira paper shows that the 10 Generators (4 Special Conformal and 6 Lorentz) describe Our Universe expanding due to Dark Energy (also known, somewhat inaccurately as it is variable, a cosmological constant).

What about the other Generators?

- The 4 Translation Generators describe spacetime, singularities of which are black holes, and Primordial Black Holes after the End of the Inflation Era make up the Dark Matter of Our Universe that organizes the Large Scale Structure of Galaxy Formation.

- The 1 Scalar Dilation Generator corresponds to the Scalar Higgs of the W-Bosons, with VEV 250 GeV, that gives mass to Ordinary Matter in Our Universe.

Those 15 Conformal Group Spin(2,4) = SU(2,2) Generators indicate that the basic tree-level ratio Dark Energy : Dark Matter : Ordinary Matter is 10 : 4 : 1 = 67 : 27 : 6 . After taking into account the history of Our Universe to the Present Time, that ratio is calculated in the E8 model to be, as of Now, consistent with observations including WMAP:

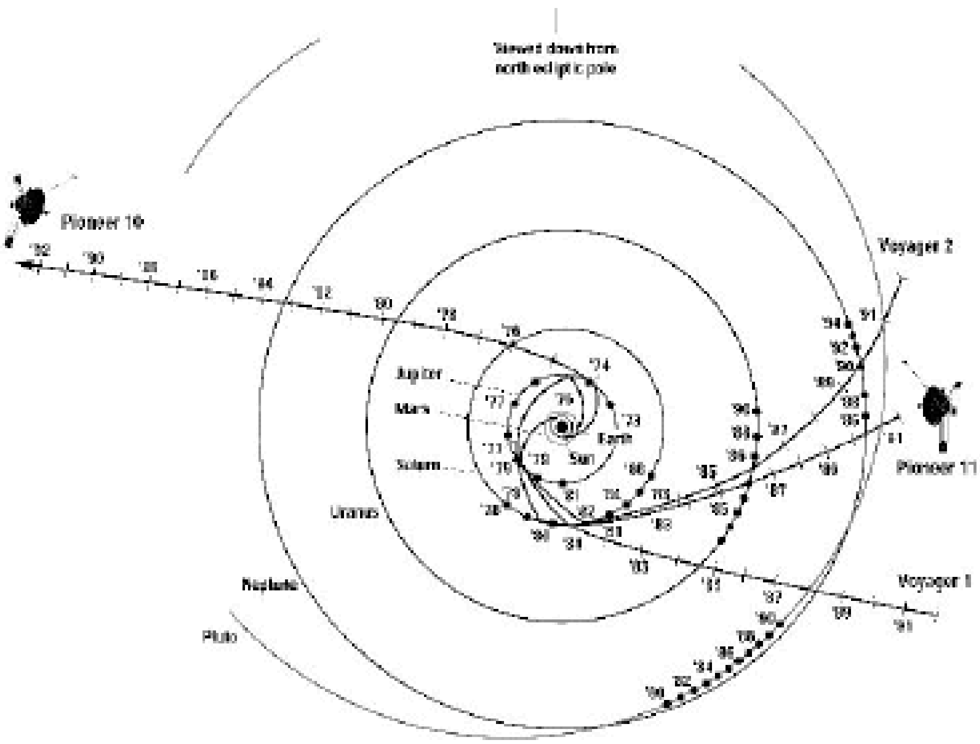
Dark Energy : Dark Matter : Ordinary Matter = 75.3 : 20.2 : 4.5

After conventional expansion of our universe begins, some regions of our Universe become Gravitationally Bound Domains (such as, for example, Galaxies) in which the 4 Conformal GraviPhoton generators are frozen out, forming domains within our Universe like IceBergs in an Ocean of Water. Within each Gravitationally Bound Domain, spacetime (regarded as Aether) is incompressible with a rigidity on the order of the W-Boson Higgs VEV = 250 GeV.

On the scale of our Earth-Sun Solar System, the region of our Earth, where we do our local experiments, is in a Gravitationally Bound Domain.

Since the Pioneer spacecraft are not bound to our Solar System, the Pioneer Spacecraft are experiments beyond the Gravitationally Bound Domain of our Earth-Sun Solar System.

In their Study of the anomalous acceleration of Pioneer 10 and 11, gr-qc/0104064, John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev say: "... The latest successful precession maneuver to point ...[Pioneer 10]... to Earth was accomplished on 11 February 2000, when Pioneer 10 was at a distance from the Sun of 75 AU. [The distance from the Earth was [about] 76 AU with a corresponding round-trip light time of about 21 hour.] ... The next attempt at a maneuver, on 8 July 2000, was unsuccessful ... conditions will again be favorable for an attempt around July, 2001. ... At a now nearly constant velocity relative to the Sun of 12.24 km/s, Pioneer 10 will continue its motion into interstellar space, heading generally for the red star Aldebaran ... about 68 light years away ... it should take Pioneer 10 over 2 million years to reach its neighborhood....



Ecliptic pole view of Pioneer 10, Pioneer 11, and Voyager trajectories.
Digital artwork by T. Esposito. NASA ARC Image # AC97-0036-3.

... on 1 October 1990 ... Pioneer 11 ... was [about] 30 AU away from the Sun ... The last communication from Pioneer 11 was received in November 1995, when the spacecraft was at distance of [about] 40 AU from the Sun. ... Pioneer 11 should pass close to the nearest star in the constellation Aquila in about 4 million years ...

... Calculations of the motion of a spacecraft are made on the basis of the range time-delay and/or the Doppler shift in the signals. This type of data was used to determine the positions, the velocities, and the magnitudes of the orientation maneuvers for the Pioneer, Galileo, and Ulysses spacecraft considered in this study. ... The Pioneer spacecraft only have two- and three-way S-band Doppler. ... analyses of radio Doppler ... data ... indicated that an apparent anomalous acceleration is acting on Pioneer 10 and 11 ... The data implied an anomalous, constant acceleration with a magnitude a_P = 8 x

10^{-8} cm / s^2 , directed towards the Sun ...

... the size of the anomalous acceleration is of the order c H, where H is the Hubble constant ...

... Without using the apparent acceleration, CHASMP shows a steady frequency drift of about -6×10^{-9} Hz / s, or 1.5 Hz over 8 years (one-way only). ... This equates to a clock acceleration, $-a_t$, of -2.8×10^{-18} s / s^2 . The identity with the apparent Pioneer acceleration is $a_P = a_t c$. ...

... Having noted the relationships

$$a_P = c\, a_t$$

and that of ...

$$a_H = c H \rightarrow 8 \times 10^{-8} \text{ cm / s}^2$$

if H = 82 km / s / Mpc ...

we were motivated to try to think of any ... "time" distortions that might ... fit the CHASMP Pioneer results ... In other words ...

Is there any evidence that some kind of "time acceleration" is being seen? ... In particular we considered ... Quadratic Time Augmentation. This model adds a quadratic-in-time augmentation to the TAI-ET (International Atomic Time - Ephemeris Time) time transformation, as follows

$$ET \rightarrow ET + (1/2)\, a_{ET}\, ET^2$$

The model fits Doppler fairly well ...

... There was one [other] model of the ...[time acceleration]... type that was especially fascinating. This model adds a quadratic in time term to the light time as seen by the DSN station:

$$\Delta_{TAI} = TAI_{received} - TAI_{sent} \rightarrow$$
$$\rightarrow \Delta_{TAI} + (1/2)\, a_{quad} (TAI_{received}^2 - TAI_{sent}^2)$$

It mimics a line of sight acceleration of the spacecraft, and could be thought of as an expanding space model.

Note that a_quad affects only the data. This is in contrast to the a_t ... that affects both the data and the trajectory. ... This model fit both Doppler and range very well. Pioneers 10 and 11 ... the numerical relationship between the Hubble constant and a_P ... remains an interesting conjecture. ...".

In his book Mathematical Cosmology and Extragalactic Astronomy (Academic Press 1976) (pages 61-62 and 72), Segal says:

"... Temporal evolution in ... Minkowski space ... is

$$H \to H + s I$$

... unispace temporal evolution ... is ...

$$H \to (H + 2 \tan(a/2)) / (1 - (1/2) H \tan(a/2)) = H + a I + (1/4) a H^2 + O(s^2)$$

...".

Therefore,

the Pioneer Doppler anomalous acceleration is an experimental observation of a system that is not gravitationally bound in the Earth-Sun Solar System, and its results are consistent with Segal's Conformal Theory.

Rosales and Sanchez-Gomez say, at gr-qc/9810085 :

"... the recently reported anomalous acceleration acting on the Pioneers spacecrafts should be a consequence of the existence of some local curvature in light geodesics when using the coordinate speed of light in an expanding spacetime. This suggests that the Pioneer effect is nothing else but the detection of cosmological expansion in the solar system. ... the ... problem of the detected misfit between the calculated and the measured position in the spacecrafts ... this quantity differs from the expected ... just in a systematic "bias" consisting on an effective residual acceleration directed toward the center of coordinates; its constant value is ... H c ... This is the acceleration observed in Pioneer 10/11 spacecrafts. ... a periodic orbit does not experience the systematic bias but only a very small correction ... which is not detectable ... in the old Foucault pendulum experiment ... the motion of the pendulum experiences the effect of the Earth based reference system being not an inertial frame relatively to the "distant stars". ... Pioneer effect

is a kind of a new cosmological Foucault experiment, the solar system based coordinates, being not the true inertial frame with respect to the expansion of the universe, mimics the role that the rotating Earth plays in Foucault's experiment ...".
The Rosales and Sanchez-Gomez idea of a 2-phase system in which objects bound to the solar system (in a "periodic orbit") are in one phase (non-expanding pennies-on-a-balloon) while unbound (escape velocity) objects are in another phase (expanding balloon) that "feels" expansion of our universe is very similar to my view of such things as described on this page.

The Rosales and Sanchez-Gomez paper very nicely unites:

- the physical 2-phase (bounded and unbounded orbits) view;
- the Foucault pendulum idea; and
- the cosmological value H c.

My view, which is consistent with that of Rosales and Sanchez-Gomez, can be summarized as a 2-phase model based on Segal's work which has two phases with different metrics:

a metric for outside the inner solar system, a dark energy phase in which gravity is described in which all 15 generators of the conformal group are effective, some of which are related to the dark energy by which our universe expands; and

a metric for where we are, in regions dominated by ordinary matter, in which the 4 special conformal and 1 dilation degrees of freedom of the conformal group are suppressed and the remaining 10 generators (antideSitter or Poincare, etc) are effective, thus describing ordinary matter phenomena.
If you look closely at the difference between the metrics in those two regions, you see that the full conformal dark energy region gives an "extra acceleration" that acts as a "quadratic in time term" that has been considered as an explanation of the Pioneer effect by John D. Anderson, Philip A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Martin Nieto, and Slava G. Turyshev in their paper at gr-qc/0104064 .

Jack Sarfatti has a 2-phase dark energy / dark matter model that can give a similar anomalous acceleration in regions where c^2 /\ dark energy / dark matter is effectively present. If there is a phase transition (around Uranus at 20 AU) whereby ordinary matter dominates inside that distance from the sun and exotic dark energy / dark matter appears at greater distances, then Jack's model could also explain the Pioneer anomaly and it may be that Jack's model with ordinary and exotic phases and my model with deSitter/Poincare and Conformal phases may be two ways of looking at the same thing. As to what might be the physical mechanism of the phase transition, Jack says

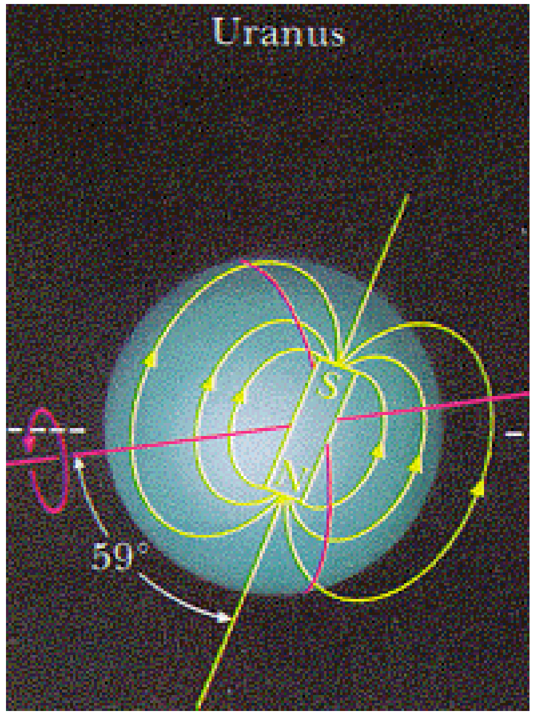
"... Rest masses of [ordinary matter] particles ... require the smooth non-random Higgs Ocean ... which soaks up the choppy random troublesome zero point energy ...".
In other words in a region in which ordinary matter is dominant, such as the Sun and our solar system, the mass-giving action of the Higgs mechanism "soaks up" the Dark Energy zero point conformal degrees of freedom that are dominant in low-ordinary mass regions of our universe (which are roughly the intergalactic voids that occupy most of the volume of our universe). That physical interpretation is consistent with my view.

Transition at Orbit of Uranus:

It may be that the observation of the Pioneer phase transition at Uranus from ordinary to anomalous acceleration is an experimental result that gives us a first look at dark energy / dark matter phenomena that could lead to energy sources that could be even more important than the nuclear energy discovered during the past century.

In gr-qc/0104064, Anderson et al say:

"... Beginning in 1980 ... at a distance of 20 astronomical units (AU) from the Sun ... we found that the largest systematic error in the acceleration residuals was a constant bias, aP, directed toward the Sun. Such anomalous data have been continuously received ever since. ...",
so that the transition from inner solar system Minkowski acceleration to outer Segal Conformal acceleration occurs at about 20 AU, which is about the radius of the orbit of Uranus. That phase transition may account for the unique rotational axis of Uranus,



which lies almost in its orbital plane.

The most stable state of Uranus may be with its rotational axis pointed toward the Sun, so that the Solar hemisphere would be entirely in the inner solar system Minkowski acceleration phase and the anti-Solar hemisphere would be in entirely in the outer Segal Conformal acceleration phase.

Then the rotation of Uranus would not take any material from one phase to the other, and there would be no drag on the rotation due to material going from phase to phase.

Of course, as Uranus orbits the Sun, it will only be in that most stable configuration twice in each orbit, but an orbit in the ecliptic containing that most stable configuration twice (such as its present orbit) would be in the set of the most stable ground states, although such an effect would be very small now.

However, such an effect may have been been more significant on the large gas/dust cloud that was condensing into Uranus and therefore it may have caused Uranus to form initially with its rotational axis pointed toward the Sun.

In the pre-Uranus gas/dust cloud, any component of rotation that carried material from one phase to another would be suppressed by the drag of undergoing phase transition, so that, after Uranus condensed out of the gas/dust cloud, the only remaining component of Uranus rotation would be on an axis pointing close to the Sun, which is what we now observe.

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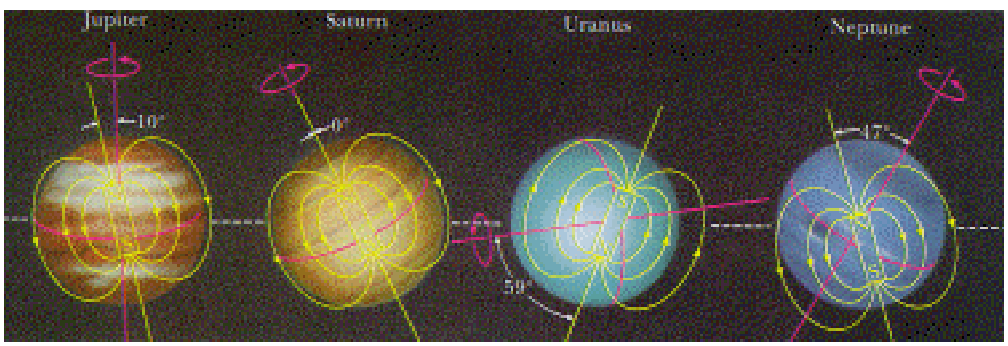
Much of the perpendicular (to Uranus orbital plane) angular momentum from the original gas/dust cloud may have been transferred (via particles "bouncing" off the phase boundary) to the clouds forming Saturn (inside the phase boundary) or Neptune (outside the phase boundary, thus accounting for the substantial (relative to Jupiter) deviation of their rotation axes from exact perpendicularity (see images above and below from Universe, 4th ed, by William Kaufmann, Freeman 1994).

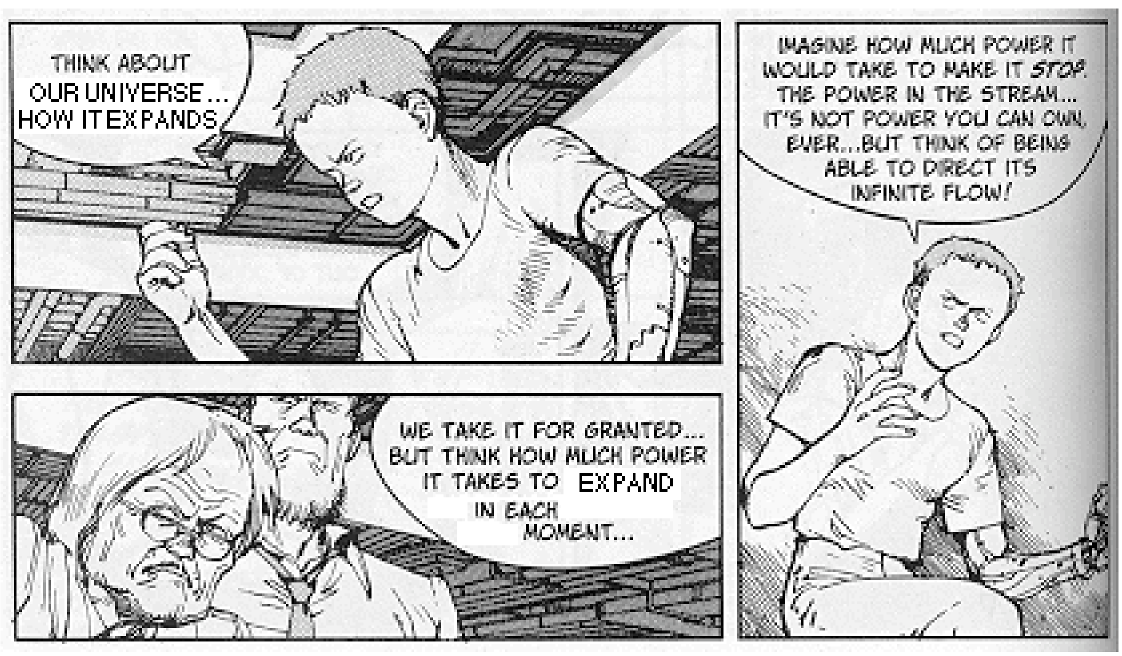


According to Utilizing Minor Planets to Assess the Gravitational Field in the Outer Solar System, astro-ph/0504367, by Gary L. Page, David S. Dixon, and John F. Wallin:

"... the great distances of the outer planets from the Sun and the nearly circular orbits of Uranus and Neptune makes it very difficult to use them to detect the Pioneer Effect. ... The ratio of the Pioneer acceleration to that produced by the Sun at a distance equal to the semimajor axis of the planets is 0.005, 0.013, and 0.023 percent for Uranus, Neptune, and Pluto, respectively. ... Uranus' period shortens by 5.8 days and Neptune's by 24.1, while Pluto's period drops by 79.7 days. ... an equivalent change in aphelion distance of 3.8 x 10^10, 1.2 x 10^11, and 4.3 x 10^11 cm for Uranus, Neptune, and Pluto. In the first two cases, this is less than the accepted uncertainty in range of 2 x 10^6 km [or 2 x 10^11 cm] (Seidelmann 1992). ... Pluto['s] ... orbit is even less well-determined ... than the other outer planets. ... . [C]omets ... suffer ... from outgassing ... [and their nuclei are hard to locate precisely] ...".

According to a google cache of an Independent UK 23 September 2002 article by Marcus Chown:
"... The Pioneers are "spin-stabilised", making them a particularly simple platform to understand. Later probes ... such as the Voyagers and the Cassini probe ... were stabilised about three axes by intermittent rocket boosts. The unpredictable accelerations caused by these are at least 10 times bigger than a small effect like the Pioneer acceleration, so they completely cloak it. ...".

Earth Laboratory Experiments (image below from Akira Manga)



Can we use Laboratory Experiments on Earth to get access to the energy of all 15 generators of Conformal Spin(2,4), including the 4 Conformal GraviPhotons ?

In astro-ph/0512327 Christian Beck says: "... if dark energy is produced by vacuum fluctuations then there is a chance to probe some of its properties by simple laboratory tests based on Josephson junctions. These electronic devices can be used to perform 'vacuum fluctuation spectroscopy', by directly measuring a noise spectrum induced by vacuum fluctuations. One would expect to see a cutoff near 1.7 THz in the measured power spectrum, provided the new physics underlying dark energy couples to electric charge.

The effect exploited by the Josephson junction is a subtile nonlinear mixing effect and has nothing to do with the Casimir effect or other effects based on van der Waals forces. A Josephson experiment of the suggested type will now be built, and we should know the result within the next 3 years. ...".

That Josephson experiment is by P A Warburton of University College London. It is EPSRC Grant Reference: EP/D029783/1, "Externally-Shunted High-Gap Josephson Junctions: Design, Fabrication and Noise Measurements", starting1 February 2006 and ending 31 January 2009 with £ Value: 242,348. Its abstract states:
"... In the late 1990's measurements of the cosmic microwave background radiation and distant supernovae confirmed that around 70% of the energy in the universe is in the form of gravitationally-repulsive dark energy. This dark energy is not only responsible for the accelerating expansion of the universe but also was the driving force for the big bang. A possible source of this dark energy is vacuum fluctuations which arise from the finite zero-point energy of a quantum mechanical oscillator, hf/2 (where f is the oscillator frequency). … dark energy may be measured in the laboratory using resistively-shunted Josephson junctions (RS-JJ's). Vacuum fluctuations in the resistive shunt at low temperatures can be measured by non-linear mixing within the Josephson junction. If vacuum fluctuations are responsible for dark energy, the finite value of the dark energy density in the universe (as measured by astronomical observations) sets an upper frequency limit on the spectrum of the quantum fluctuations in this resistive shunt. Beck and Mackey calculated an upper bound on this cut-off frequency of 1.69 THz. … We therefore propose to perform measurements of the quantum noise in RS-JJ's fabricated using superconductors with sufficiently large gap energies that the full noise spectrum up to and beyond 1.69 THz can be measured. … Nitride junctions have cut-off frequencies of around 2.5 THz, which should give sufficiently low quasiparticle current noise around 1.69 THz at accessible measurement temperatures. Cuprate superconductors have an energy gap an order of magnitude higher than the nitrides, but here there is finite quasiparticle tunnelling at voltages less than the gap voltage, due to the d-wave pairing symmetry. By performing experiments on both the

nitrides and the cuprates we will have two independent measurements of the possible cut-off frequency in two very different materials systems. This would give irrefutable confirmation (or indeed refutation) of the vacuum fluctuations hypothesis. ...".

Beck and Mackey in astro-ph/0406504 say: "... the zero-point term has proved important in explaining X-ray scattering in solids ... ; understanding of the Lamb shift ... in hydrogen ... ; predicting the Casimir effect ... ; understanding the origin of Van der Waals forces ... ; interpretation of the Aharonov-Bohm effect ... ; explaining Compton scattering ... ; and predicting the spectrum of noise in electrical circuits ... .

It is this latter effect that concerns us here. ... We predict that the measured spectrum in Josephson junction experiments must exhibit a cutoff at the critical frequency nu_c ... [corresponding to the currently observed Dark Energy density 0.73 x critical density = 0.73 x 5.3 GeV/m^3 = 3.9 GeV/m^3]... If not, the corresponding vacuum energy density would exceed the currently measured dark energy density of the universe. ... The energy associated with the computed cutoff frequency nu_c ...[about 1.7 x 10^12 Hz]...

E_c = h nu_c = (7.00 ± 0.17) x 10^(-3) eV ...

coincides with current experimental estimates of neutrino masses. .. It is likely that the Josephson junction experiment only measures the photonic part of the vacuum fluctuations, since this experiment is purely based on electromagnetic interaction. ... If the frequency cutoff is observed, it could be used to determine the fraction ... of dark energy density that is produced by electromagnetic processes ...

Finally, we conjecture that it will be interesting to re-analyze experimentally observed 1/f noise in electrical circuits under the hypothesis that it could be a possible manifestation of suppressed zero-point fluctuations. ... Our simple theoretical considerations show that 1/f noise arises naturally if bosonic vacuum fluctuations are suppressed by fermionic ones. ...".

Some points that may be relevant to the experiment are:

1 - the critical density in our universe now is about 5 GeV/m^3

2 - it is made up of Dark Energy : Dark Matter : Ordinary Matter in a ratio DE : DM : OM = 73 : 23 : 4

3 - the density of the various types of stuff in our universe now is

DE = about 4 GeV / m^3
DM = about 1 GeV / m^3
OM = about 0.2 GeV / m^3

4 - the density of vacuum fluctuations already observed in Josephson Junctions is about 0.062 GeV/m^3 which is for frequencies up to about 6 x 10^11 Hz

5 - the radiation density (for photons) varies with frequency as the 4th power of the frequency, i.e., as (pi h / c^3) nu^4

6 - if Josephson Junction frequencies were to be experimentally realized up to 2 x 10^12 Hz, then, if the photon vacuum fluctuation energy density formula were to continue to hold, the vacuum energy density would be seen to be 0.062 x (20/6)^4 = about 7 GeV/m^3 which exceeds the total critical density of our universe now

7 - to avoid such a divergence being physically realized, neutrinos should appear in the vacuum at frequencies high enough that E = h nu exceeds their mass of about 8 x 10^(-3) eV, or at frequencies over about 1.7 x 10^12 Hz

8 - if Josephson Junctions could be developed to see vacuum fluctuation frequencies up to 10^12 Hz, and if the photon equation were to hold there, then the obseved vacuum fluctuation density would be about 0.5 GeV/m^3 which is well over the 0.2 GeV / m^3 Ordinary Matter energy density which means that DE and/or DM components would be seen in vacuum fluctuations in Josephson Junctions that go up to 10^12 Hz

How do your build a Josephson Junction sensitive to terahertz fluctuations?

According to a paper by Chen, Horiguchi, Wang, Nakajima, Yamashita, and Wu at http://www.iop.org/EJ/abstract/0953-2048/15/12/309
Terahertz frequency metrology based on high-Tc Josephson junctions
Published 22 November 2002:
"... Using YBa2Cu3O7/MgO bicrystal Josephson junctions operating between 6-77 K, we have studied their responses to monochromatic electromagnetic radiation from 50 GHz to 4.25 THz. We have obtained direct detections for radiation at 70 K from 50 GHz to 760 GHz and at 40 K from 300 GHz to 3.1 THz. ...".

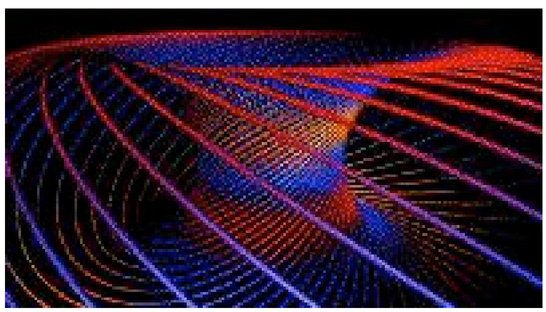
Some details of how to make such things were outlined at
http://fy.chalmers.se/~tarasov/e1109m_draft.htm
by Stepantsov, Tarasov, Kalabukhov, Lindstrooem, Ivanov, and Claeson, dated August 2001: "... Submicron YBCO bicrystal Josephson junctions and devices for high frequency applications were designed, fabricated and experimentally studied. The key elements of these devices are bicrystal sapphire substrates. ... A technological process based on deep ultraviolet photolithography using a hard carbon mask was developed for the fabrication of 0.4-0.6 mm wide Josephson junctions. ... These junctions were used as Josephson detectors and spectrometers
at frequencies up to 1.5 THz ...".

As to the possibility of using arrays of Josephson junctions, a paper entitled
Averaged Equations for Distributed Josephson Junction Arrays
At http://www.physics.gatech.edu/mbennett/dist2003.pdf
by Matthew Bennett and Kurt Wiesenfeld says:
"... The Kirchhoff limit is valid provided the size of the system is small compared to the wavelength of the electromagnetic radiation. As it happens, the twin technological goals of generating higher operating frequencies ... and larger output powers (and thus more junctions) both work against this limit. ... To take an example, an array operating at 300 GHz - not a particularly high frequency for Josephson junctions - corresponds to a wavelength of 0.4 millimeters when the index of refraction is 2.5;
for a typical spacing of 10 micrometers, this is about the same size as an array of about 40 junctions - not a particularly large number for Josephson arrays ... at higher frequencies the current in the wire is not necessarily spatially uniform, so the wire becomes a significant dynamical entity which

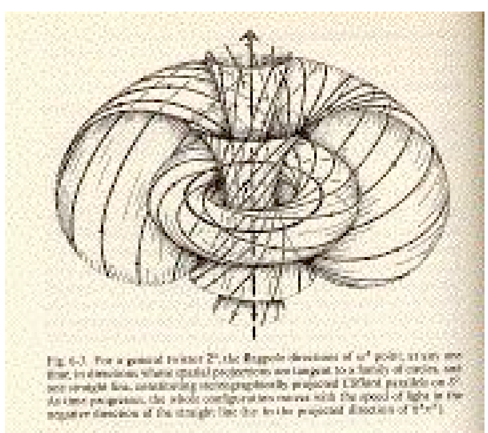
couples the junctions along its length. ... we model the wire as a lossless transmission line ... The resonant case is especially revealing, and leads to significant physical insight into achieving attracting synchronized dynamics. The tighter the clusters, the more likely it is that phase locked solutions appear. ... There are also hints that distributed arrays exhibit fundamentally different phenomena than their lumped counterparts. In one case, experiments on distributed Josephson arrays reported evidence of super-radiance ...".

Here is picture that I have in my mind for building a Josephson Junction Array device for exploring vacuum fluctuations:

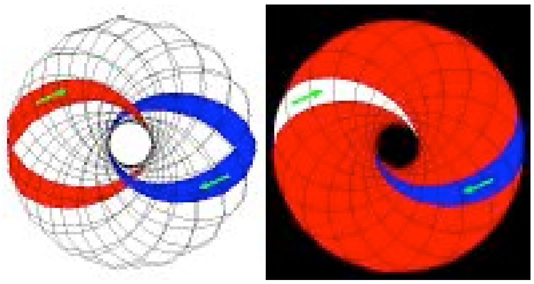
Consider the nested tori and linked circles of a Clifford-Hopf 3-sphere fibration. This picture (from a movie on a UBC web page)



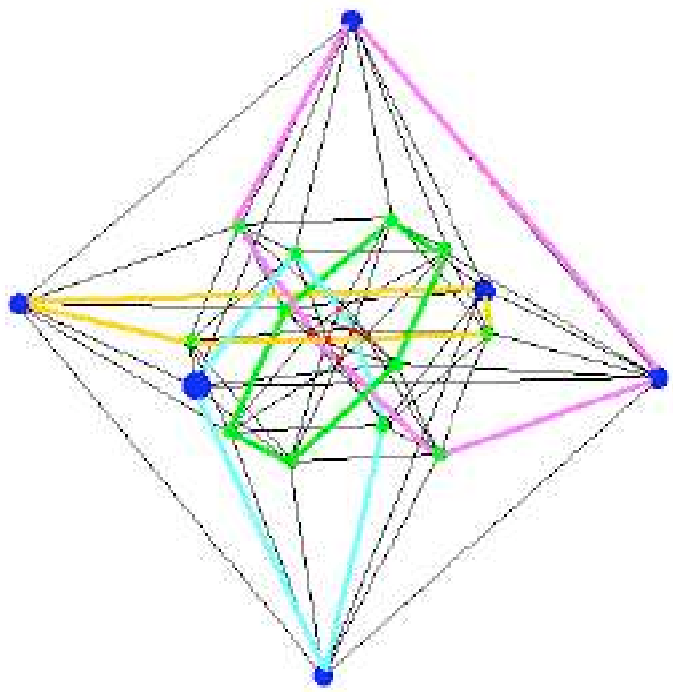
shows one torus, so imagine a lot of tori nested like a Rodin Coil
(image from Spinors and Spacetime, volume 2, by Penrose and Rindler (Cambridge 1986))



These pictures (from 3D-Filmstrip by Richard S. Palais)

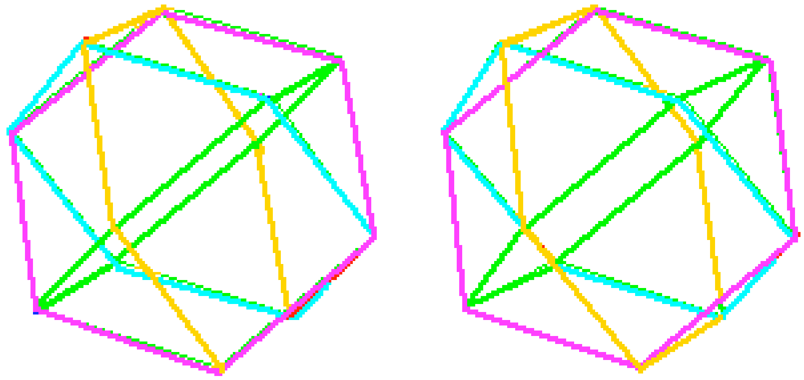


show that for any given torus in the nesting the circles are interlinked similarly to 24-cell paths (image from Fig. 172 of Geometry and the Imagination (Anschauliche Geometrie) by David Hilbert and S. Cohn-Vossen (Chelsea 1952))



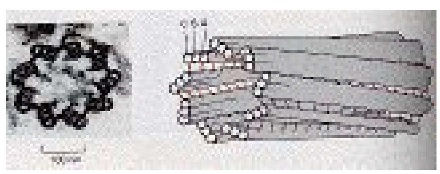
Let each circle be a superconducting wire carrying some current, and let all the circles be embedded in an insulator so that the whole thing has characteristics of a lot of Josephson Junctions.

Then assemble 4 coils, one for each of the 4 physical dimensions of SpaceTime, configured as 4 axes that are 3-dim projections of the 4-dim coordinate axes of the 4-dim 24-cell, i.e., as 4 axes of Fuller's Vector Equilibrium, the cuboctahedron (3-dimensional projection of the 24-cell),



Then play with various magnetic field configurations and then watch what happens.

In order to get up to the terahertz energy level you might have to fabricate the thing on sub-millmeter scales, which should be fun. When you get down to micron - nanometer scales, you get to scales of subcellular biological structures such as microtubules and centrioles (image of Centriole illustration from Molecular Biology of the Cell, 2nd ed, by Alberts, Bray, Lewis, Raff, Roberts, and Watson (Garland 1989))



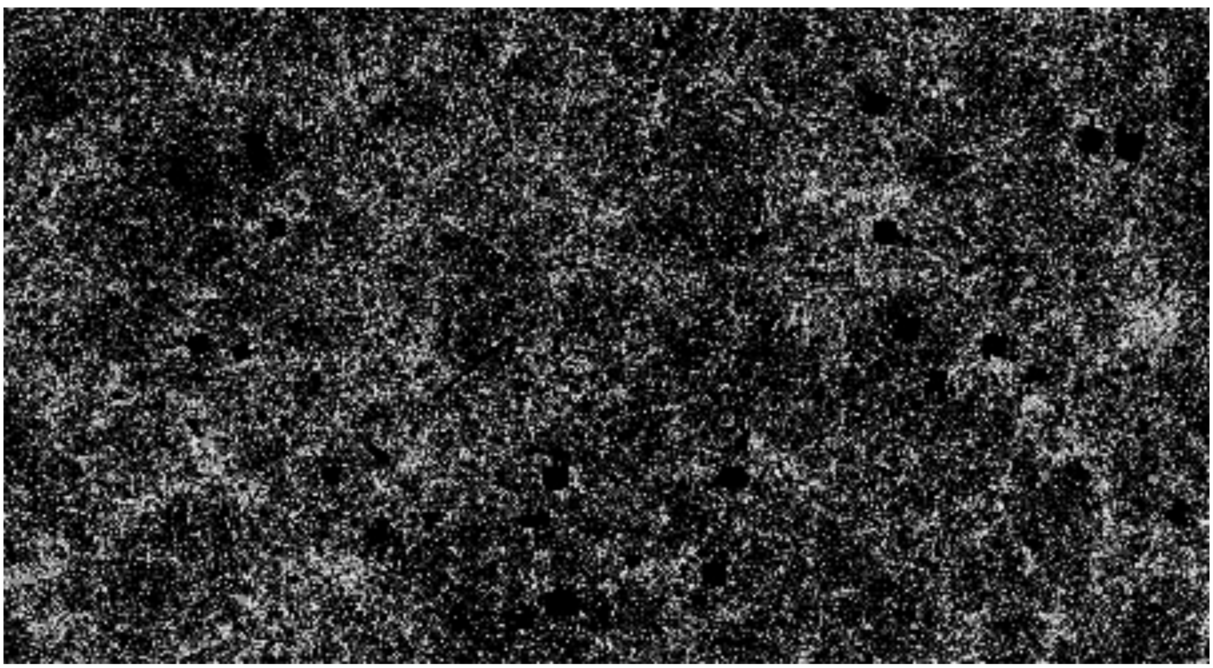
so maybe evolution has already built some related stuff into our cells, and maybe this stuff is on the borderline between conventional semiconductor/superconductor fabrication and biological growth of structures.

Such Rodin Coil Josephson Junction Array experiments may be useful in building StarGate Ring Ships and in construction of star-gate worm-holes, whose stability might be interpretable in terms of ghosts, in addition to the utilization on Earth of $\Lambda > 0$ Zero-Point Dark Energy,

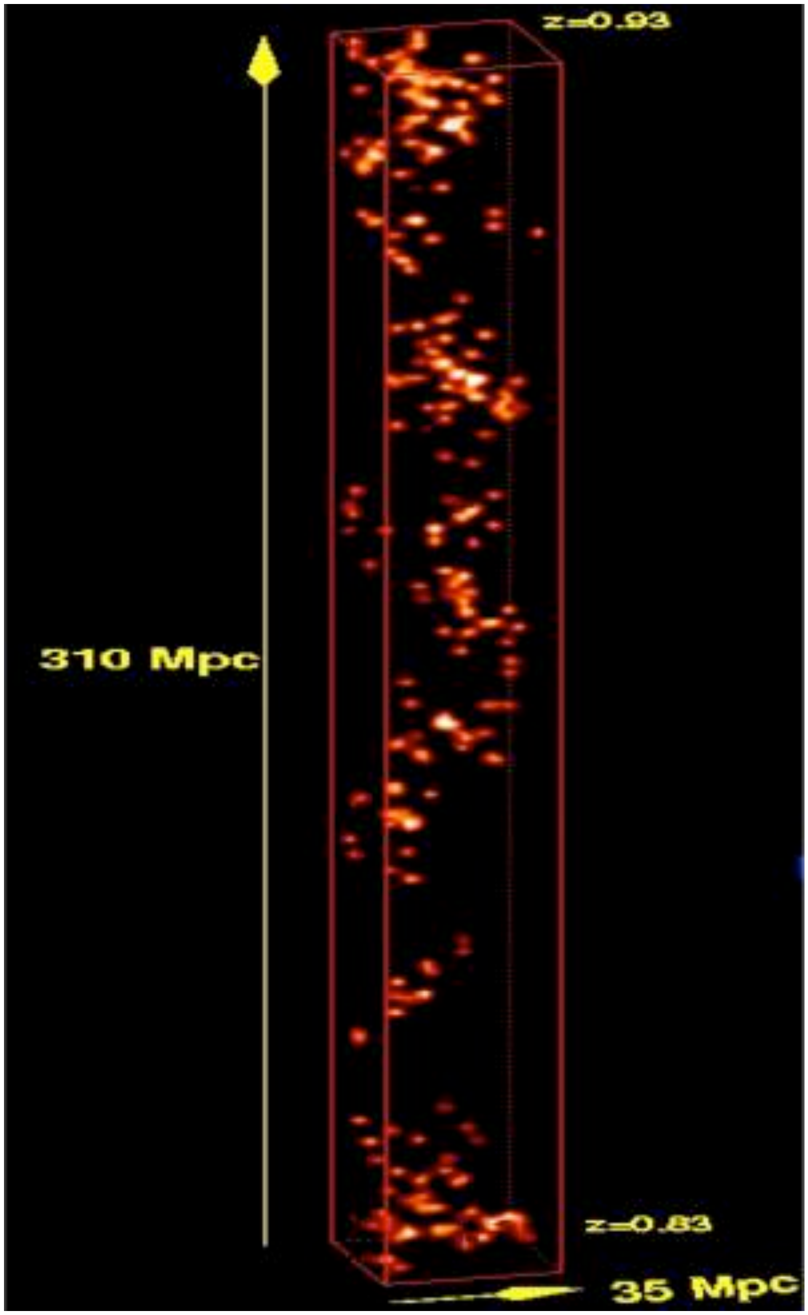
all possibly controllable by Quantum Consciousness Resonance phenomena.

Such Josephson Junction Arrays effectively act as controllable superconductors. As Beck and de Matos suggested in 0707.1797, superconductors are examples of Conformal Dark Energy Phases within the Gravitationally Bound Domain of our Inner Solar System. They said: "… this model can account simultaneously for the anomalous acceleration and anomalous gravitomagnetic fields around rotating superconductors measured by Tajmar et al. and for the anomalous Cooper pair mass in superconductive Niobium, measured by Cabrera and Tate …[Effectively]… gravitationally active photons obtain mass in the superconductor …".

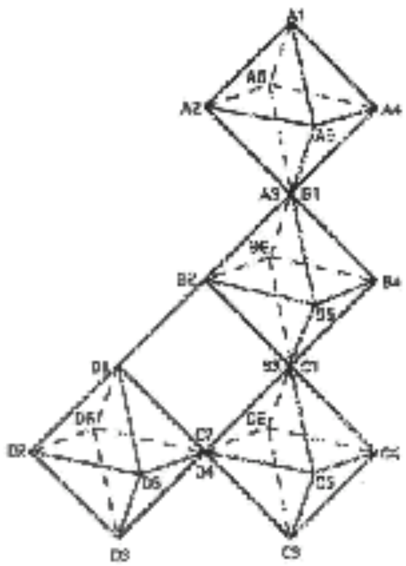
On a large scale (billions of light years), the Gravitationally Bound Domains are roughly traced out by Galaxies and Clusters of Galaxies

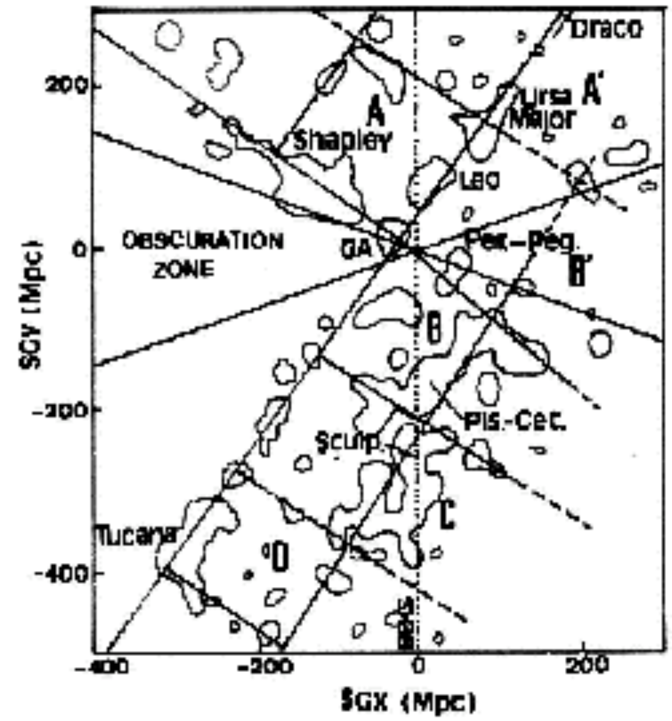


(image similar to those in Universe, 4th ed, by William Kaufmann, Freeman 1994)so the the white dots would be the Gravitationally Bound Domains (like rigid pennies on an expanding balloon, or rigid raisins in an expanding cake) and the black background would be the Conformal Expanding Domain of Our Universe. When the Gravitationally Bound Domains begin to form as Galaxy Cluster Structures in the early stages of the current phase of expansion of Our Universe, according to a 6 December 2006 caption to ESO PR Photo 45/06



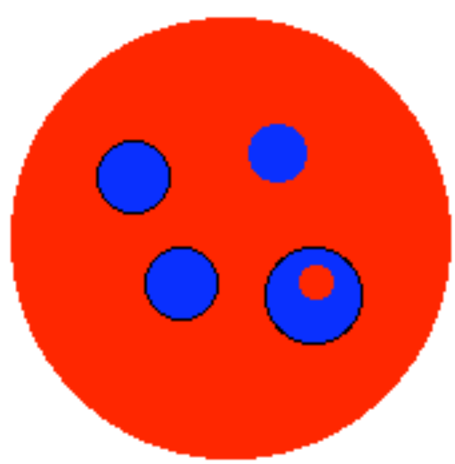
"... Spatial, three-dimensional distribution of galaxies in a slice of the Universe as it was 7 billion years ago, based on the VVDS study: brighter areas represent the regions of the Universe with most galaxies. Astonishingly, the galaxy distribution - the 'building blocks' of the large scale structure - takes the shape of a helix at this primordial epoch. ...". Such a helical structure suggests that helical magnetic fields might be involved in galaxy formation. Further, Battaner et al, in in astro-ph/9801276, astro-ph/9802009, and astro-ph/9911423, suggest that the simplest network pattern for distribution of superclusters of galaxies that is compatible with magnetic field constraints





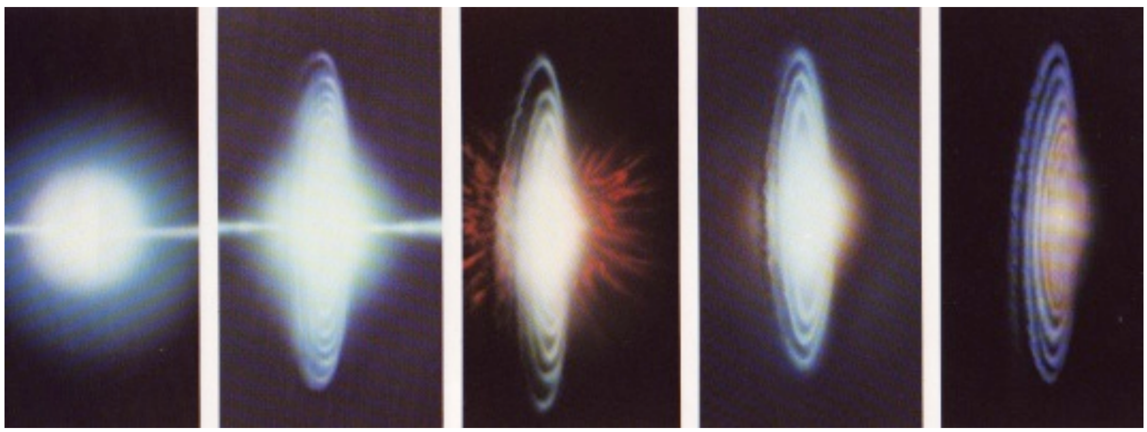
is made up of octahedra contacting at their vertexes, which is related to a tiling of 3-dim space by cuboctahedra and octahedra, and also to the heptaverton of Arthur Young and octonionic structures of Onar Aam.

Within each Gravitationally Bound Domains there can exist Islands of Conformal Expansion in which all 15 generators remain effective,



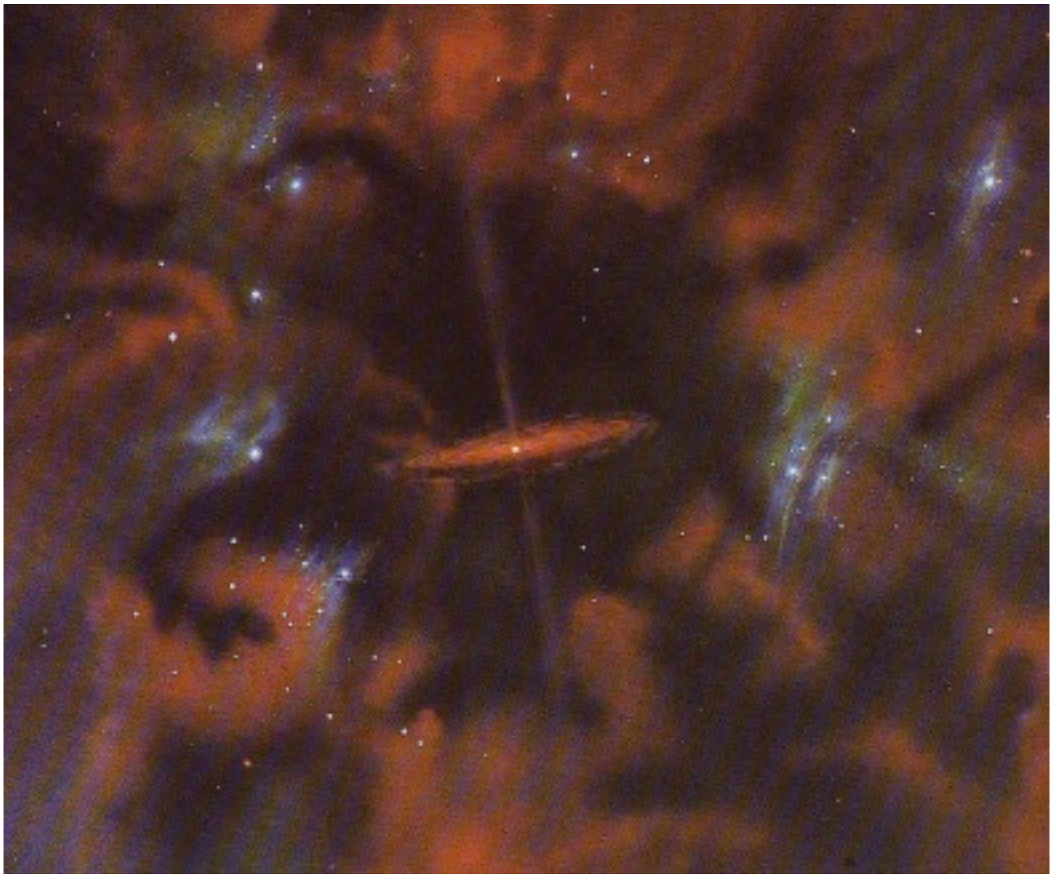
like Puddles of Water (red) on an IceBerg (blue) floating in an Ocean of Water (red), so the overall structure of Our Universe in terms of Gravitationally Bound Domains (pennies, raisins, IceBergs) and Conformal Expanding Domains (balloon, cake, water) is quite complicated.

To get some feeling for this structure, begin by considering Clusters of Galaxies to be the largest Gravitationally Bound Domains and then looking at the next level down in sixe, Galaxies. As Hartmann and Miller say in their book Cycles of Fire (Workman Publishing 1987)



- "... Most brilliant of all are quasars ...[with bipolar]... jets ...
- active galazies...[with]... jets ...[and]... disks of gas around black holes in the galactic center ...
- exploding galayies ...[with]... gas ejected from the nucleus, along with strong radio radiation ...
- Seyfert galayies ...[with].. luminous and variable ... nuclei ...
- normal ... galaxies ...[with]... bright central nucleus ...".

Going down one more level in size, to Stars and Stellar Systems like Our Solar System, Hartmann and Miller describe



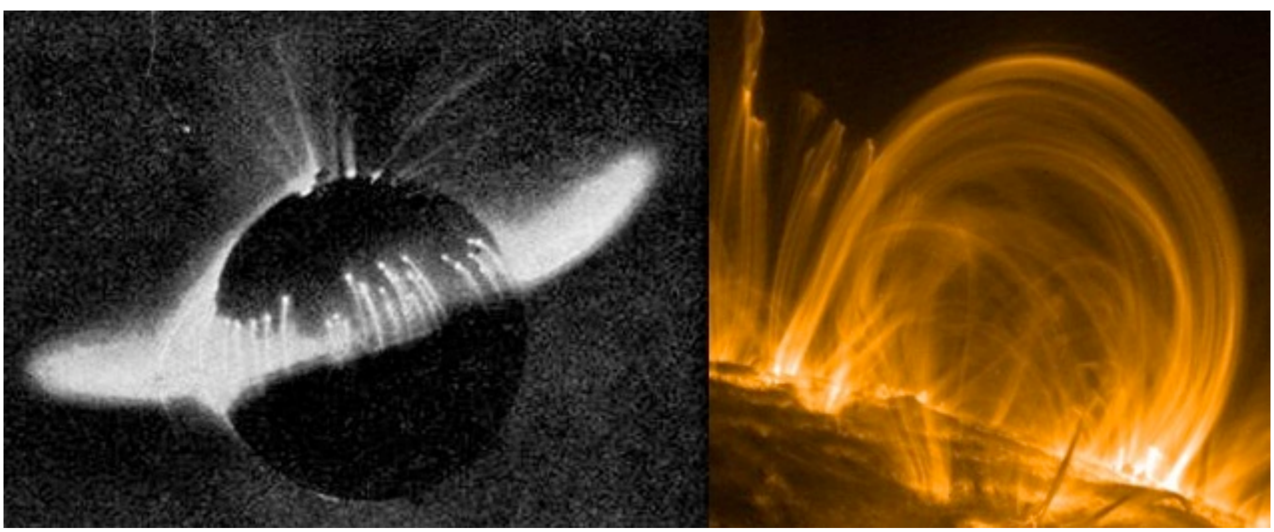
"... a star just formed ...[in]... its disk-shaped cocoon nebula some of which is being blown out in bipolar jets ... near a dark molecular cloud ... embedded in a ... nebular region ... The dust in the cocoon reddens the star's light ...".

Kohji Tomisaka of Niigata University says in astro-ph/9911166:

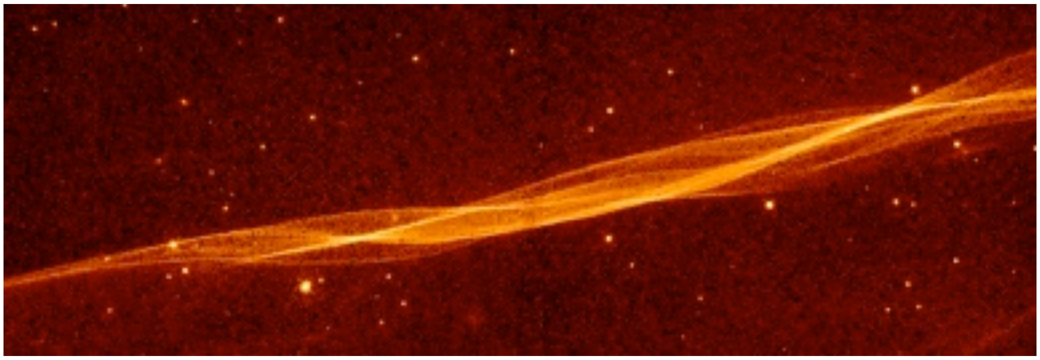
"... the star formation process ... angular momentum transfer in the contraction of a rotating magnetized cloud is studied with axisymmetric MHD simulations. Owing to the large dynamic range covered by the nested-grid method, the structure of the cloud in the range from 10 AU to 0.1 pc is explored. First, the cloud experiences a run-away collapse, and a disk forms perpendicularly to the magnetic field, in which the central density increases greatly in a finite time-scale. In this phase, the specific angular momentum j of the disk decreases to about 1/3 of the initial cloud. After the central density of the disk exceeds about 10^10 cm ^(-3) , the infall on to the central object develops. In this accretion stage, the rotation motion and thus

> the toroidal magnetic field drive the outflow. The angular momentum of the central object is transferred efficiently by the out\flow as well as the effect of the magnetic stress. ... the seeding region (origin of the outflow) ... expands radially outward. This outflow is driven by the gradient of the magnetic pressure of the toroidal magnetic fields ... which are made by the rotation motion ... The magnetic fields exert torque on the outflowing gas to increase its angular momentum. On the other hand, they exert torque on the disk to decrease the angular momentum ... [in about 7000 years] ... the outflow expands and reaches ... [about 4 AU] ... The angular momentum distribution at that time ... has been reduced to ... a factor of 10 ^(-4) from the initial value (i.e. from 10^20 cm^2 s ^(-1) to 10^16 cm^2 s ^(-1)). ... the coupling between gas and magnetic fields ... becomes stronger as long as we consider the seeding region, indicating that the mechanism of angular momentum transfer works also in the later stage of the evolution [after 7000 years]. ...".

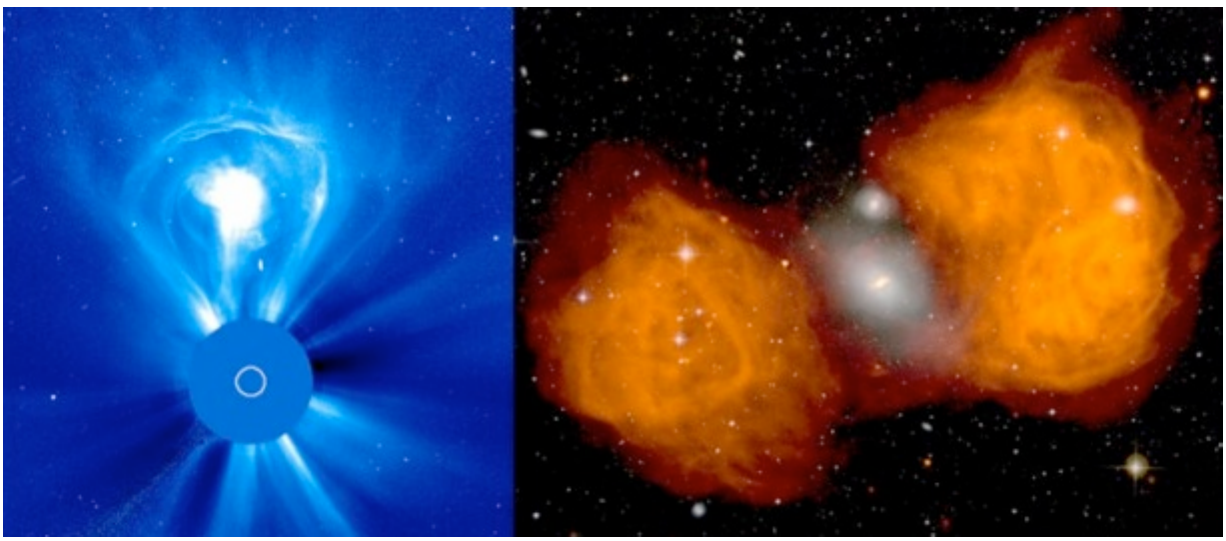
If you look closely at the central star in the star-formation image above, you might see Birkeland Current Loops (image from thesurfaceofthesun.com web page) that look up close like Solar Coronal Loops (image from electric-cosmos.org/sun.htm web page).



Up close, Birkeland Current Loops are seen to have braided filament structure (Cygnus Loop image from antwrp.gsfc.nasa.gov.



The scale of Birkeland Current Loops extends beyond Stellar to Galactic (images, SOHO of Sun and NRAO of Fornax A from thunderbolts.info webpage, which said as to NGC



"... a tiny but energy-dense plasmoid at the center of the galaxy ... Fornax A ... discharges energy along oppositely-directed Birkeland filaments (invisible in this image) into the radio lobes. Diffuse currents loop back from the lobes to the spiral arms, where their increasing density triggers star formation as they return to the central plasmoid. ..." ...).

The scale also extends down to Planetary, as is seen in the Jupiter-Io system (image from Anthony Peratt's book Physics of the Plasma Universe (Springer-Verlag 1992)):

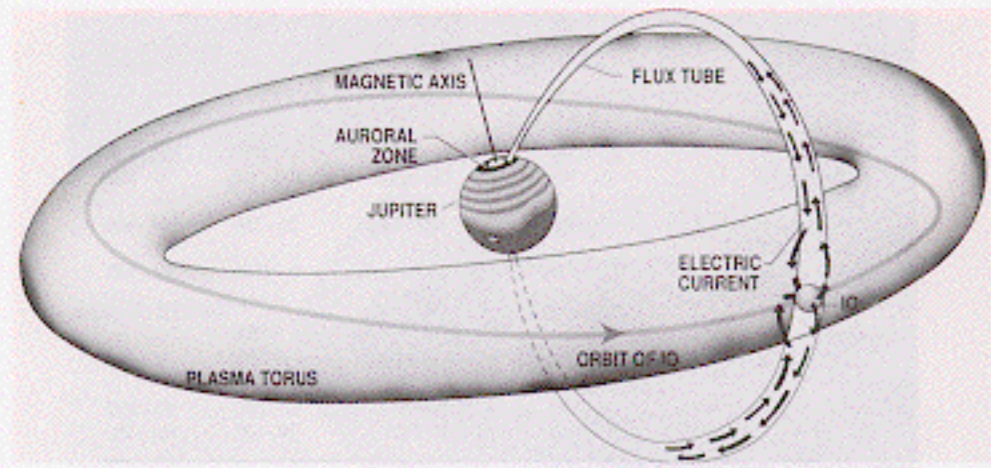
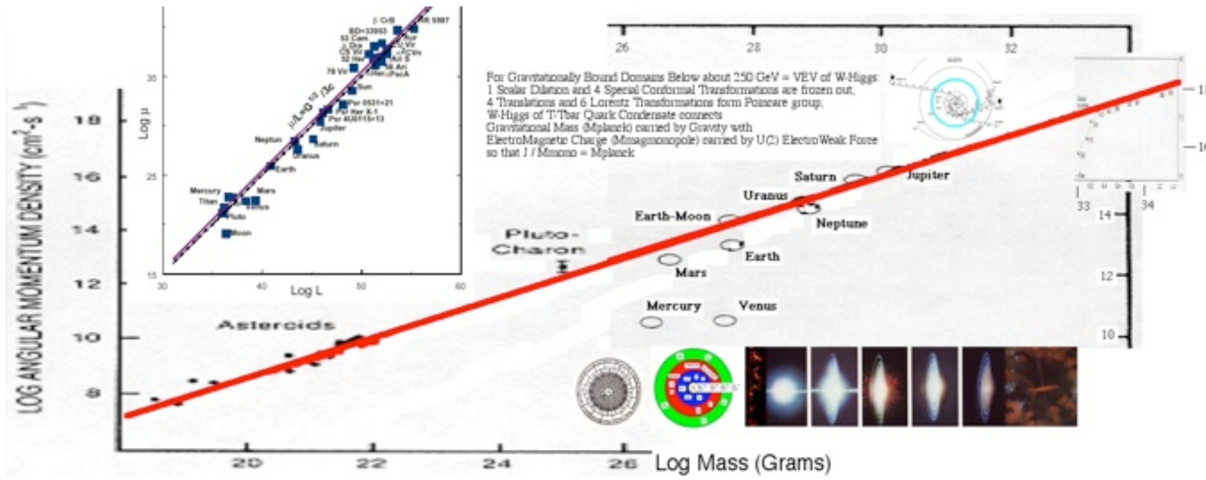


Figure 1.9. The Jupiter-Io plasma torus. The diagram shows the megaampere Birkeland currents flowing between Jupiter and Io.

The scale may also extend down to Asteroidal. According to a 17 September 1994 article by Jeff Hecht in the New Scientist: "... inclusions ... in chondrules ... in chondrites, the commonest meteorites ...[were]... heated ... to about 2000 kelvin at the birth of the solar system, 4.6 billion years ago. ...[possibly by]... Lightning ... and ... magnetic discharges ... laser tests ... to model the intense visible and infrared light expected near electric or magnetic discharges ... produced dark structures ... remarkably similar to .... inclusions found in chondrules ...".

As can be seen from the image below (adapted from some of the above images and also An Introduction to Modern Astrophysics, by Carroll and Osterlie (Addison-Wesley 1996), Solar System Evolution, by Stuart Ross Taylor (Cambridge 1992), and B. V. Vasiliev's papers astro-ph/0002048 and astro-ph/0002171), Angular Momentum, Magnetic Dipole Moment, and Mass are systematically related or Stars and Stellar Systems and their components.



Angular Momentum J and Magnetic Dipole Moment P are related by a constant that is on the order of unity (J = P) (natural units) due to Gravity-Induced Electric Polarization of matter.

As to the relationship between Angular Momentum J and Mass (which, due to the Angular Momentum - Magnetic Dipole Moment relationship, implies a relationship between Magnetic Dipole Moment and Mass), Jack Sarfatti's paper wessonI.PDF describes a 1981 paper by Paul Wesson in which Wesson plotted total angular momentum J against mass M for the solar system, double stars, star clusters, spriral galaxies, the Coma cluster, and the local supercluster in which Wesson found that Angular Momentum J and Mass M are related by a constant p such that

J = p M^2 and J/M = p M.

Wesson's observations indicate approximately, that

- p = 10^(-16) g^(-1) cm^2 sec^(-1) (cgs units) and
- p = 1 / alpha_EM = 137 (natural units G = hbar = c = 1).

For Elementary Specific Angular Momentum J/M = hbar, in natural units where hbar = 1 and the unit of mass is the Planck mass Mplanck:

M = (J/M) / p = alpha_EM, which gives M = Mplanck / 137

which is roughly the mass of an SU(5) Magnetic Monopole.

Wesson's observations are consistent with a Compton Radius Vortes Kerr-Newman Black Hole related to the Wesson Force. The equation (in units with G = c = hbar = 1) for a Kerr-Newman Black Hole with coincident outer and inner event horizons and with Q = 1

> meaning that the Black Hole Core has UNIT amplitude to absorb or emit a gauge boson, in accord with Feynman's statement in his book QED (Princeton 1988): "... e - the amplitude for a real electron to emit or absorb a real photon. It is a simple number that has been experimentally determined to be close to -0.0854... the inverse of its square: about 137.03... has been a mystery ... all good theoretical physicists put this number up on their wall ..."

is Q^2 + (J/M)^2 = 1 + (J/M)^2 = M^2. Dividing through by M^2, you get

J^2/M^4 = (J/M^2)^2 = 1 - (1/M)^2

For the Wesson force for which J = p_wesson M^2 with p_wesson = 1 / alpha_EM

J = sqrt(1 - (1/M)^2) M^2 = p_wesson M^2 = 137 M^2

so that 1 - (1/M)^2 = 137^2 and 1/M = sqrt(1 -137^2) = 137 i = 137 exp(pi/2)

Then the magnitude | 1 / Mwesson | = 137 which (since the units are natural units with G = c = hbar = 1) implies that

Mwesson = Mplanck / 137 = 10^19 / 137 = 7.3 x 10^16 GeV

which is consistent with Wesson's observation that

Mwesson = 7.3 x 10^16 GeV = Mmonopole

The Linear Angular Momentum, Magnetic Dipole Moment, and Mass Relationships hold in Gravitationally Bound Domains, which are characterized by Energy Below about 250 GeV = VEV of W-Boson Higgs where:

- the 1 Scalar Dilation and 4 Special Conformal Transformations of the 15-dimensional Conformal Group Spin(2,4) = SU(2,2) are frozen out;
- the 4 Translations and 6 Lorentz Transformations combine as described in gr-qc/9809061 by R. Aldrovandi and J. G. Peireira: "... By the process of Inonu-Wigner group contraction taking the limit R -> oo, ...[where R is the de Sitter pseudo-radius, the] ... de Sitter group... [whether of metric ... (-1,+1,+1,+1,-1) or (-1,+1,+1,+1,+1) , is]... reduced to the Poincare group P ...[formed by a semi-direct product between Lorentz and translation groups] and ... de Sitter space...[is]... reduced to the Minkowski space M. As the scalar curvature of the de Sitter space goes to zero in this limit, we can say that M is a spacetime gravitationally related to a vanishing cosmological constant.";
- If the 1+3 = 4-dimensional spacetime on which the 6+4 = 10-dimensional Poincare Group Spin(1,3) + 4-Translations acts linearly is viewed in terms of an elastic Aether, its rigidity would correspond to the VEV of the W-Boson Higgs Condensate on the order of 250 GeV. Within Gravitationally Bound Domains, since Special Conformal and Dilation transformations are frozen out, the rigidity of the 4-dimensional elastic Aether corresponds to the W-Higgs VEV of about 250 GeV.
- the T-Tbar Quark Condensate W-Boson Higgs mechanism connects Gravitational Mass (based on the Planck Mass Mplanck) carried by Gravity with ElectroMagnetic Charge (based on the Magnetic Monopole Mmono) carried by the U(2) ElectroWeak Force so that J / Mmono = Mplanck.

Although the Wesson angular momentum / mass relationship covers a very wide range of mass scales (at least from Asteroids to Stars and Stellar Systems), it is not Universal. Some other angular momentum / mass relationships are:

- A neutral Kerr-Newman Black Hole, with coincident outer and inner event horizons, has Q^2 + (J/M)^2 = M^2 with charge Q = 0, so that (J/M)^2 = M^2, J^2 = M^4, J = M^2, and p_neutralKNblackhole = 1 (in natural units) = 1 x (1 / 2.2 x 10^(-5)) Planck mass/gm x 3 x 10^10 (cm/sec)/c x 1.6 x 10^(-33) cm/Planck Length = 3 x 1.6 /2.2 x 10^(5 + 10 - 33) = 2.2 x 10^(-18) g^(-1) cm^2 sec^(-1).
- The proton angular momentum is (1/2) hbar, which is roughly (1/2) hbar = (1/2) x 10^-27 gm cm^2 sec(-1), and the proton mass is

roughly Mproton = 2 x 10^(-24) gm, so that p_proton = (1/2) hbar / (Mproton)^2 = (1/2) x 10^(-27) / 4 x 10^(-48) = (1/8) x 10^21 gm^(-1) cm^2 sec(-1) = 1.2 x 10^20 gm^(-1) cm^2 sec(-1);

- The quark angular momentum is (1/2) hbar, which is roughly (1/2) hbar = (1/2) x 10^-27 gm cm^2 sec(-1), and the constituent (not current) mass of the up or down quark, 1/3 of the proton mass, is roughly Mquark = 2/3 x 10^(-24) gm, so that p_quark = (1/2) hbar / (Mquark)^2 = (1/2) x 10^(-27) / (4/9) x 10^(-48) = (9/8) x 10^21 gm^(-1) cm^2 sec(-1) = 1.1 x 10^21 gm^(-1) cm^2 sec(-1).
- The electron angular momentum is (1/2) hbar, which is roughly (1/2) hbar = (1/2) x 10^-27 gm cm^2 sec(-1), and the electon mass is about Melectron = 10^(-27) gm, so that p_electron = (1/2) hbar / (Melectron)^2 = (1/2) x 10^(-27) / 10^(-54) = (1/2) x 10^27 gm^(-1) cm^2 sec(-1) = 5 x 10^26 gm^(-1) cm^2 sec(-1).
- The neutrino angular momentum is (1/2) hbar, which is roughly (1/2) hbar = (1/2) x 10^-27 gm cm^2 sec(-1), and the neutrino mass is about Mneutrino = zero (or very small), so that p_neutrino = (1/2) x 10^(-27) / (zero (or very small))^2 = infinity (or very large).

The differences may be that the Wesson relationship involves a combination of ElectroMagnetic and Gravity forces during Collapse/Formation, while, for the others, the forces involved are:

- p_neutrino = infinity (or very large) g^(-1) cm^2 sec^(-1) - No EM and No direct Gravity.
- p_electron = 5 x 10^26 g^(-1) cm^2 sec^(-1) - mostly EM, with minimal Gravity.
- p_quark = 1.1 x 10^21 gm^(-1) cm^2 sec(-1) - mostly EM and Color, with minimal Gravity.
- p_proton = 1.2 x 10^20 g^(-1) cm^2 sec^(-1) - mostly EM and Color and Pion-Strong, with minimal Gravity.
- p_wesson = 10^(-16) g^(-1) cm^2 sec^(-1) - balanced EM and Gravity.
- p_neutralKNblackhole = 2.2 x 10^(-18) g^(-1) cm^2 sec^(-1) - No EM, just Gravity.

Can a laboratory-scale experiment extend the Wesson-type relationship between Angular Momentum J and Magnetic Dipole Moment P to sub-asteroid laboratory mass scales ?

Saul-Paul Sirag, in his 3 November 2000 paper Vigier III: "Gravitational Magnetism: an Update", says:

> "... The most straightforward test ... would be to measure directly the magnetic field of a rotating neutral body (which is not also a ferromagnetic substance). Blackett ... suggested that a 1-meter bronze sphere spun at 100 Hz would do nicely, except that this is the maximum safe speed, and there are severe problems in nulling out extraneous magnetic fields. With modern SQUIDs and mu-metal shielded rooms, such an experiment can be attempted. Exactly such an experimental design ... was described at the SQUID '85 conference in Berlin. However, the results of this experiment have not been published. ...".

What about MicroScale Connections between Angular Momentum and Electromagnetism ?

The MicroScale Particle Physics proportionality between Q and M obviously does not extend far into the MacroScale, since Asteroids, Planets, and Stars do not have large net Electric Charges.

The Kerr-Newman Black Hole structure of a Compton Radius Vortex has the property that the square of the electric charge Q plays the same role as the square of J/M (specific angular momentum, or angular momentum over mass) in that their sum, relative to the square of the mass, determines whether the outer and inner event horizons are

- separate Q^2 + (J/M)^2 < M^2 ,
- coincidental Q^2 + (J/M)^2 = M^2 , or
- complex Q^2 + (J/M)^2 > M^2 .

Jack Sarfatti relates Compton Radius Vortex structure of Elementary Particles to the formula of Saul-Paul Sirag (based on earlier work of Blackett and Schuster, and perhaps Pauli) in his 1979 Nature paper Gravitational

Magnetism (vol. 278 pp. 535-538, 5 April 1979), in which Saul-Paul Sirag says:

> "The gravi-magnetic hypothesis proposes that a rotating mass, measured in gravitational units, has the same magnetic effect as that of a rotating charge, measured in electrical units. The respective force constants determine this relationship
>
> G^(1/2) M = k^(1/2) Q
>
> where G is the gravitational constant, M is mass, k is the Coulomb constant, and Q is electric charge. ... Thus the ratio of magnetic moment P to angular momentum J for a sphere of mass M, density factor f, radius r, angular velocity w, and magnetic field B is (in SI units [with magnetic permeability muo of the vacuum]):
>
> P / J = ((5/4) 4 pi B / muo) (r / f w M) = G^(1/2) / 2 k^(1/2)
>
> ... A priori, we should expect a correlation between P and J. ... The surprise is that this correlation ratio, P/J, should turn out to be close to P = (G^(1/2) /2 k^(1/2)) J. ... The gravi-magnetic hypothesis (stated in [Log = log_10] form) predicts a P/J of -10.37. The mean P/J of the data points plotted in Fig. 1 is -11.13 with a standard deviation of 0.42. ... Therefore, for a given value of the angular momentum J, the gravi-magnetic hypothesis overstates the magnetic dipole moment P by a factor of 10^(-10.37 - (-11.13)) = 10^0.76 = 5.75. Saul-Paul Sirag says: "... the deviation from the gravi-magnetic hypothesis line is fairly systematic. ... deviations ... may well be due to electrical-magnetic effects. ... [P / J = G^(1/2) / 2 k^(1/2)] predicts a surface field about three times greater than that measured at the surface of the Earth. ... the Earth is not a uniformly dense sphere ... At the Earth's surface ... [((5/4) 4 pi B / muo) (r / f w M) = G^(1/2) / 2 k^(1/2)] predicts a B of 2.1 x 10^(-4) T. That is not, however, a great deal more than the 1.4 x 10^(-4) T that [the equation] predicts for the surface of the Earth's core. ... this core magnetism predicted by the gravi-magnetic equation is greater than the magnetic field of 6 x 10^(-5) T measured at the Earth's surface. ... This is what we expect

if we suppose that gravitational magnetism is modified by an electrical-magnetic effect stronger at the Earth's surface than in the interior. ...".

B. G. Sidharth, in physics/9908004, says:

"... We first observe that as is known an assembly of Fermions below the Fermi temperature occupies each and every single particle level, and this explains the fact that it behaves like a distribution of Bosonic phonons: The Fermions do not enjoy their normal degrees of freedom. ... [there is a] Bosonization or semionic effect. ... Let us now consider an assembly of N electrons. As is known, if N+ is the average number of particles with spin up, the magnetisation per unit volume is given by

$$M = mu (2 N+ - N) / V$$

where mu is the electron magnetic moment. At low temperatures, in the usual theory, N+ = N / 2, so that the magnetisation ... is very small.

On the other hand, for Bose-Einstein statistics we would have, N+ = N. With the above semionic statistics we have,

$$N+ = b N, 1/2 < b < 1,$$

If N is very large, this makes an enormous difference ... Let us use ... the case of Neutron stars. In this case, as is well known, we have an assembly of degenerate electrons at temperatures about 10^7 K, whereas the Fermi temperature is about 10^11 K ... So the above considerations apply. In the case of a Neutron star we know that the number density of the degenerate electrons, n = 10^31 per c.c. So ... remembering that mu = 10^(-20) G (Gauss), the magnetic field near the Pulsar is about 10^11 G < 10^8 Tesla, as required. Some White Dwarfs also have magnetic fields. If the White Dwarf has an interior of the dimensions of a Neutron star, with a similar magnetic field, then remembering that the radius of a White Dwarf is about 10^3 times that of a Neutron star, its magnetic field would be $10^{(-6)}$ times that of the neutron star, which is known to be the case. It is quite remarkable that the above mechanism can also explain the magnetism of the earth. As is known the earth has a solid core of radius of about 1200 kilometers and temperature about 6000 K. This core is made up almost entirely of Iron (90%) and Nickel (10%). It can easily be calculated that the number of particles $N = 10^{48}$, and that the Fermi temperature is about 10^5 K. In this case we can easily verify ... that the magnetic field near the earth's surface is about 1 G, which is indeed the case. It may be mentioned that the anomalous Bosonic behaviour ... would imply a sensitivity to external magnetic influences which could lead to effects like magnetic flips or reversals. ... Remembering that the core density of Jupiter is of the same order as that of the earth, while the core volume is about 10^4 times that of the earth, we have in this case, $N = 10^{52}$, so that the magnetization ... is about 10^4 times the earth's magnetism, as required. ....".

According to a 23 March 2006 ESA news web page:

> "... Martin Tajmar, ARC Seibersdorf Research GmbH, Austria; Clovis de Matos, ESA-HQ, Paris; and colleagues have measured ... a gravitomagnetic field ... generate[d] ...[by]... a moving mass ... Their experiment involves a ring of superconducting material rotating up to 6 500 times a minute. Superconductors are special materials that lose all electrical resistance at a certain temperature. Spinning superconductors produce a weak magnetic field, the so-called London moment. The new experiment tests a conjecture by Tajmar and de Matos that explains the difference between high-precision mass measurements of Cooper-pairs (the current carriers in superconductors) and their prediction via quantum theory. They have discovered that this anomaly could be explained by the appearance of a gravitomagnetic field in the spinning superconductor (This effect has been named the Gravitomagnetic London Moment by analogy with its magnetic counterpart). ... Although just 100 millionths of the acceleration due to the Earth's gravitational field, ...[gr-qc/0603033 says "... the peaks ... "only" 100 micro g ... are 30 orders of magnitude higher than what general relativity predicts classically ..."]... The electromagnetic properties of superconductors are explained in quantum theory by assuming that force-carrying particles, known as photons, gain mass. By allowing force-carrying gravitational particles, known as the gravitons, to become heavier, they found that the unexpectedly large gravitomagnetic force could be modelled. ... The papers can be accessed on-line at the Los Alamos pre-print server using the references: gr-qc/0603033 and gr-qc/0603032. ...".

Dirac Gammas

In my E8 physics model, 64 of the 240 E8 root vectors are represented by 64 = 8x8 = 8 dimensions of full 8-dim spacetime x 8 Dirac Gammmas

- The 8 dimensions of full 8-dim spacetime are denoted here by basis {t,x,y,z,e,ie,je,ke} (or sometimes with capital letters {T,X,Y,Z,E,IE,JE,KE})to indicate how it appears after dimensional reduction to get {t.x.y,z} is the basis for 4-dimensional physical spacetime and {e,ie,je,ke} is the basis for 4-dimensional CP2 internal symmetry space.
- The 8 Dirac Gammas are denoted here by basis {1,i,j,k,e,ie,je,ke}.

The 8 Dirac components of the 8 spacetime vector dimensions (such as Xk etc) physically describe effective spacetime curvature in full 8-dimensional (high-energy) spacetime analogous to gravitational curvature in 4-dimensional (low-energy) physical spacetime.

A second set of 64 of the 240 E8 root vectors are represented by 64 = 8x8 = 8 half-spinor fundamental first-generation fermion particles x 8 Dirac Gammmas or, equivlalently, the 8 covariant components of the 8 fundamental first-generation fermion particles.The 8 fundamental first-generation fermion particles are denoted here by

- electron = EL
- red up quark = UR
- green up quark = UG
- blue up quark = UB
- red down quark = DR
- green down quark = DG
- blue down quark = DB
- electron neutrino = NU

Therefore, the 8x8 = 64 covariant components of the fundamental first-generation fermion particles are:

ELt	ELx	ELy	ELz	ELe	ELie	ELje	ELke
URt	URx	URy	URz	URe	URie	URje	URke
UGt	UGx	UGy	UGz	UGe	UGie	UGje	UGke
UBt	UBx	UBy	UBz	UBe	UBie	UBje	UBke
DRt	DRx	DRy	DRz	DRe	DRie	DRje	DRke
DGt	DGx	DGy	DGz	DGe	DGie	DGje	DGke
DBt	DBx	DBy	DBz	DBe	DBie	DBje	DBke
NUt	NUx	NUy	NUz	NUe	NUie	NUje	NUke

A third set of 64 of the 240 E8 root vectors are represented by 64 = 8x8 = 8 half-spinor fundamental first-generation fermion anti-particles x 8 Dirac Gammmas that can be represented by notation similar to that of the second set of 64.

The 3 sets of 64 of the 240 E8 root vectors are related by triality. To try to reduce confusing clutter, only some of the blue (8-dim spacetime) and red (fundamental first-gemeration fermion particle) root vector vertices are explictly labeled.

As to remaining 48 = 24 + 24 vertices, they represent the root vectors of two copies of D4 (one D4 for Gravity and another D4 for the Standard Model) that live within the Spin(16) inside E8, with structure

E8 / Spin(16) = 64 + 64

Spin(16) / D4xD4 = 64

The spinor fermion term of the full 8-dimensional Lagrangian of my E8 physics model is of the form

INTEGRAL over {t,x,y,z,e,ie,je,ke}

of

SPINOR {ELt ,ELx ,ELy ,ELz ,ELe ,ELie ,ELje ,ELke} ...(other fermions)

After dimensional reduction according to the Mayer Mechanism from a uniform octonionic 8-dimensional spacetime with basis

{t,x,y,z,e,ie,je,ke}

down to a quaternionic 4+4 = 8-dimensional Klauza-Klein spacetime with basis

{t,x,y,z} of physical spacetime plus {e,ie,je,ke} of internal symmetry space

the spinor term of the Lagrangian breaks down into the sum of four parts

1 - INTEGRAL over {t,x,y,z} of SPINOR {ELt ,ELx ,ELy ,ELz} ...(other fermions)

2 - INTEGRAL over {t,x,y,z} of SPINOR {ELe ,ELie ,ELje ,ELke} ...(other fermions)

3 - INTEGRAL over {e,ie,je,ke} of SPINOR {ELt ,ELx ,ELy ,ELz } ...(other fermions)

4 - INTEGRAL over {e,ie,je,ke} of SPINOR {ELe ,ELie ,ELje ,ELke} ...(other fermions)

First Generation

1 - is just the usual Standard Model spinor fermion term for 4-dim physical spacetime and first-generation fermions, so 1 represents first-generation fermions. The 8 first-generation fermion particles and antiparticles each correspond to the 8 octonion basis elements, so that the first-generation ferrmion particles and the first-generation fermion antiparticles each correspond to the Octonions O.

Second Generation

2 - differs from the usual Standard Model in that the SPINOR has components in the {e,ie,je,ke} internal symmetry space instead of in the {t,x,y,z} physical spacetime. Transformation from the SPINOR with components in the {e,ie,je,ke} internal symmetry space to a SPINOR with components in the {t,x,y,z} physical spacetime introduces a 4x4 matrix

```
   t x y z
e  * * * *
ie * * * *
je * * * *
ke * * * *
```

Introduction of those new 4x4 = 16 degrees of freedom of that Transformation corresponds to introducing a new octonion corresponding to a second copy of the 8 fundamental fermion particles and a new octonion corresponding to a second copy of the 8 fundamental fermion antiparticles, so that the second-generation fermion particles and the second-generation fermion antiparticles each correspond to pairs of Octonions OxO.

3 - differs from the usual Standard Model in that the base manifold spacetime has components in the {e,ie,je,ke} internal symmetry space instead of in the {t,x,y,z} physical spacetime. Transformation from the base manifold spacetime with components in the {e,ie,je,ke} internal symmetry space to a base manifold spacetime with components in the {t,x,y,z} physical spacetime introduces a 4x4 matrix as described in 2. Introduction of those new 4x4 = 16 degrees of freedom of that Transformation corresponds to introducing a new octonion corresponding to a second copy of the 8 fundamental fermion particles and a new octonion corresponding to a second copy of the 8 fundamental fermion antiparticles, so that the second-generation fermion particles and the second-generation fermion antiparticles each correspond to pairs of Octonions OxO.

Third Generation

4 - differs from the usual Standard Model in that the SPINOR has components in the {e,ie,je,ke} internal symmetry space instead of in the {t,x,y,z} physical spacetime AND the base manifold spacetime has components in the {e,ie,je,ke} internal symmetry space instead of in the {t,x,y,z} physical spacetime. Transformation from the SPINOR with

components in the {e,ie,je,ke} internal symmetry space to a SPINOR with components in the {t,x,y,z} physical spacetime introduces a 4x4 matrix

```
   t x y z
e  * * * *
ie * * * *
je * * * *
ke * * * *
```

Introduction of those new 4x4 = 16 degrees of freedom of that Transformation corresponds to introducing a new octonion corresponding to a second copy of the 8 fundamental fermion particles and a new octonion corresponding to a second copy of the 8 fundamental fermion antiparticles.

Transformation from the base manifold spacetime with components in the {e,ie,je,ke} internal symmetry space to a base manifold spacetime with components in the {t,x,y,z} physical spacetime introduces a second 4x4 matrix

```
   t x y z
e  * * * *
ie * * * *
je * * * *
ke * * * *
```

Introduction of the second new 4x4 = 16 degrees of freedom of that Transformation corresponds to introducing a second new octonion corresponding to a second copy of the 8 fundamental fermion particles and a second new octonion corresponding to a second copy of the 8 fundamental fermion antiparticles, so that the third-generation fermion particles and the third-generation fermion antiparticles each correspond to triples of Octonions OxOxO.

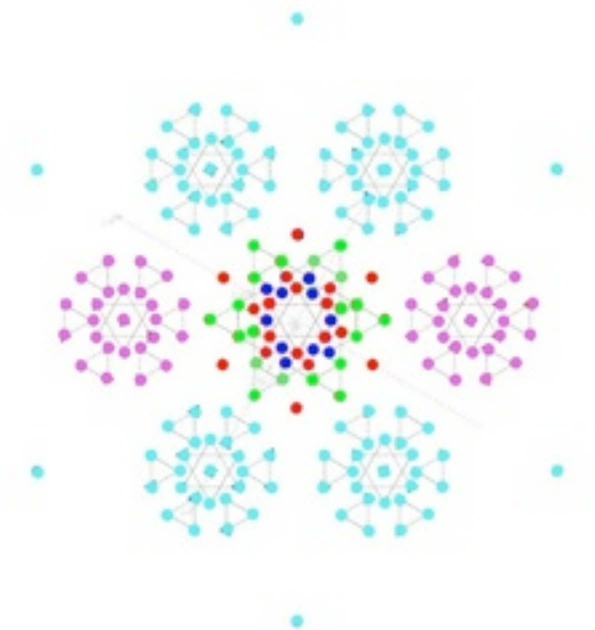
There are no further Generations beyond 3.

D4 in D5 in E6 in E7 in E8

A projection of E8 root vectors into 2 dimensions showing a nesting

D4 in D5 in E6 in E7 in E8

is



in which the two D4 of E8 are (showing multiplicities 3 and 2 of points to which multiple root vectors are projected)

The central red 24 of the inner D4 are obviously contained in E6 in E8.

The outer magenta 6 of the outer D4 in E7 outside E6 are the two central 3 of:

126 root vectors of E7 - 72 root vectors of E6 = 54 = 2x(24+3) =

2 circular 12+12 + 2 central 3

The magenta 6 root vectors of the two central 3 correspond to the root vectors of a 7-sphere S7 (which, although not a Lie algebra due to Octonion non-associativity, is a Malcev algebra)

The magenta 48 = 54-6 of the two E7 circular 12+12 are related to the blue 16 of 8-complex-dimensional Kaluza-Klein vector spacetime D5 outside red D4 so that the magenta 48 and blue 16 combine to form a 48+16 = 64-real-dimensional = 8-octonionic-dimensional vector spacetime .

Therefore, E7 looks like E6 plus octonification of vector spacetime plus a 7-sphere S7.

The outer cyan 18 of the outer D4 in E8 outside E7 are the four central 3 plus outside 6 of:

240 root vectors of E8 - 126 root vectors of E7 = 114 = 108 + 6 = 4x(24+3) + 6 =

4 circular 12+12 + 4 central 3 + outside 6

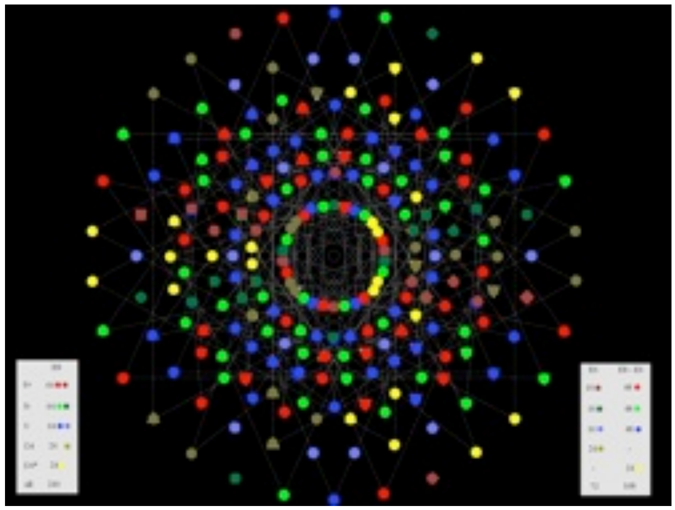
The cyan 12 root vectors of the four central 3 correspond to the root vectors of the 14-dimensional Lie algebra G2.

The outside 6 root vectors correspond to the root vectors of a 7-sphere S7 (which, although not a Lie algebra due to Octonion non-associativity, is a Malcev algebra)

The cyan 96 = 108-12 of the four E8 circular 12+12 are related to the green 32 of 16-complex-dimensional full-spinor E6 fermion first-generation particles and antiparticles so that the cyan 96 and green 32 combine to form 96+32 = 128-real-dimensional = 16-octonionic-dimensional representation space for full-spinor fermion first-generation particles and antiparticles.

Therefore, E8 looks like E7 plus octonification of representation space for full-spinor fermion first-generation particles and antiparticles plus G2 plus a 7-sphere S7.

8 circles of 30 = 240 root vectors of E8



from the view of the 240 of E8 as 8 circles of 30.

To get more feel for the 8 circles of 30, consider the comment by rntsai on N-category Cafe that mentioned Kostant's "... decomposition of e8 into 31 cartan's ..." and said: "... It's ... related to e8/(d4+d4) decomposition :

e8/(d4+d4)= (28,1)+(1,28) + (8v,8v) + (8S+,8S+) + (8S-,8S-)

The last 3 terms can be seen as 24 8-dim spaces ...

The other 7 cartans are inside d4+d4 ... there are probably several ways to identify [them]...".

Another way (other than the one mentioned by rntsai) is to decompose d4 into a 14-dim G2 plus two 7 spheres S7 + S7, getting

d4 = 14 + 7 + 7

14-dim rank-2 G2 has 7 = 14/2 Cartans and G2 can be seen as the sum of two 7-dimensional representations. If each 7 is represented by the 7 imaginary octonions { i,j,k,e,ie,je,ke } then the 7 Cartans of G2 are the 7 pairs (one from each of the 7 in G2):

i i

j j

k k

e e

ie ie

je je

ke ke

Note that to make an Abelian Cartan, the pairs must match, because only matching pairs close to form an Abelian Cartan (this can be seen by looking at the octonion products).

28-dim rank-4 d4 has 28/4 = 7 Cartans and d4 looks like G2 plus S7 plus S7 and since G2 decomposes into two 7 representations

d4 decomposes into 7 + 7 + 7 + 7 (where the first two 7 are from G2 and the other two come from the two S7)

and the 7 Cartans of d4 are (in terms of octonion imaginaries)

i i i i

j j j j

k k k k

e e e e

ie ie ie ie

je je je je

ke ke ke ke

Again, note that all elements of the quadruples must match to get Abelian Cartan structure.

When you look at d4 + d4 to get 8-element Cartans of E8, all 8 elements must again match up to get Abelian Cartan structure, so the 7 Cartans of E8 that come from d4 + d4 look like

i i i i i i i i

j j j j j j j j

k k k k k k k k

e e e e e e e e

ie ie ie ie ie ie ie ie

je je je je je je je je

ke ke ke ke ke ke ke ke

Of course, this octonion structure is also reflected in the "... 24 8-dim spaces ..." described by rntsai as "... (8v,8v) + (8S+,8S+) + (8S-,8S-) ..." so that all 31 of the 8-dim Cartans of E8 have nice octonionic structure.

Also note that when you make a 240-element E8 root vector diagram of 8 circles each with 30 vertices, 8 of the 248 E8 generators are missing, so that you must leave out one of the 31 Cartan 8-element sets. Seeing E8 in terms of E8 = 120 + 128 = d4 + d4 + 8x8 + 8x8 + 8x8 it is most natural to see the Cartan as being one of the Cartan sets of 8 coming from the d4 + d4, but you could see the E8 from other points of view by using other Cartan sets of 8 to determine which of the 248 were the 8 omitted from the root vector diagram.

rntsai also said, about "The last 3 terms [that] can be seen as 24 8-dim spaces", "... You can verify that these are abelian, so calling them cartan is justified. ...". Each of the 8x8 look like

```
   g1 g2 g3 g4 g5 g6 g7 g8

1
i
j
k
e
ie
je
ke
```

where the E8/octonionic { 1,i,j,k,e,ie,je,ke } represent 8-dim spacetime (8v) or 8 fermion fundamental first-generation particles (8S+) or 8 fermion fundamental first-generation antiparticles (8S-) and the g1 ... g8 are Dirac gammas of 8-dimensional Kaluza-Klein spacetime.

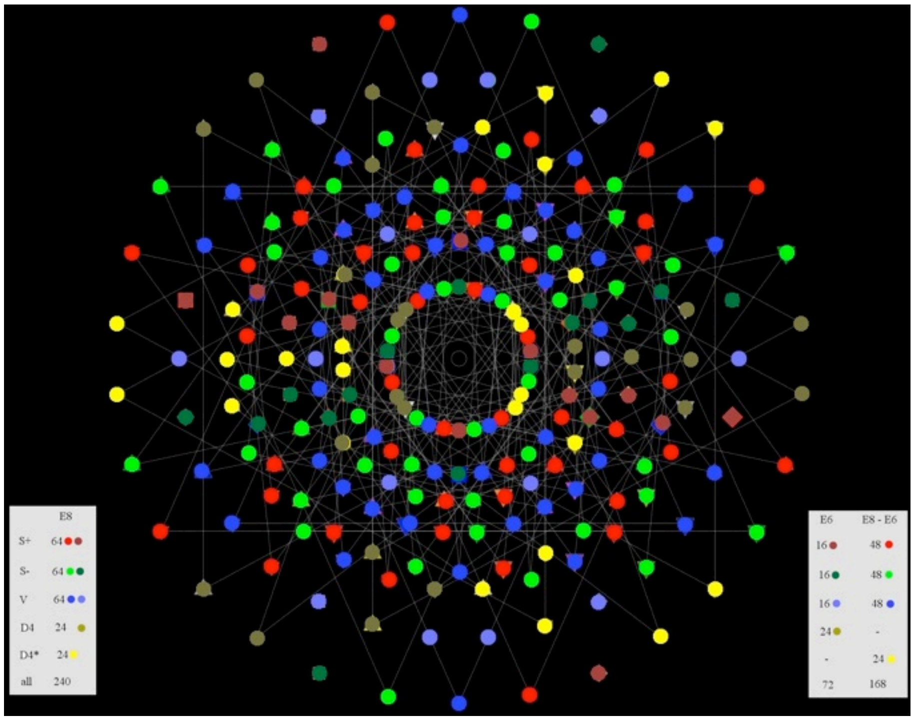
Those Dirac gammas, although they have intrinsic Clifford algebra structure, can be regarded with respect to E8/octonionic structure as only indicating physical Dirac gamma component structure of the E8/octonionic { 1,i,j,k,e,ie,je,ke } so that they are consistent with each of the rows

```
    1g1  1g2  1g3  1g4  1g5  1g6  1g7  1g8

    ig1  ig2  ig3  ig4  ig5  ig6  ig7  ig8

    jg1  jg2  jg3  jg4  jg5  jg6  jg7  jg8

    kg1  kg2  kg3  kg4  kg5  kg6  kg7  kg8

    eg1  eg2  eg3  eg4  eg5  eg6  eg7  eg8

   ieg1 ieg2 ieg3 ieg4 ieg5 ieg6 ieg7 ieg8

   jeg1 jeg2 jeg3 jeg4 jeg5 jeg6 jeg7 jeg8

   keg1 keg2 keg3 keg4 keg5 keg6 keg7 keg8
```

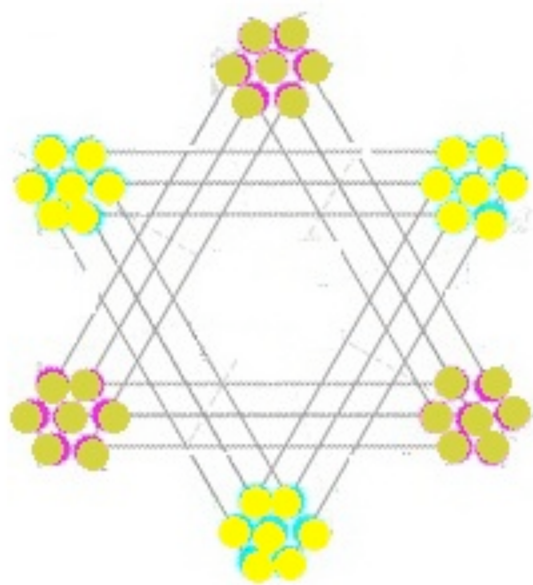
being able to represent an 8-element E8 Cartan subalgebra, no matter which of the three representations 8v, 8S+, or 8S- (which are related to each other by triality) is used.

D4 and D4* and Higgs

Consider the two D4 in the E8 physics model based on E8 / Spin(16), and denote them D4 and D4* to distinguish between them.

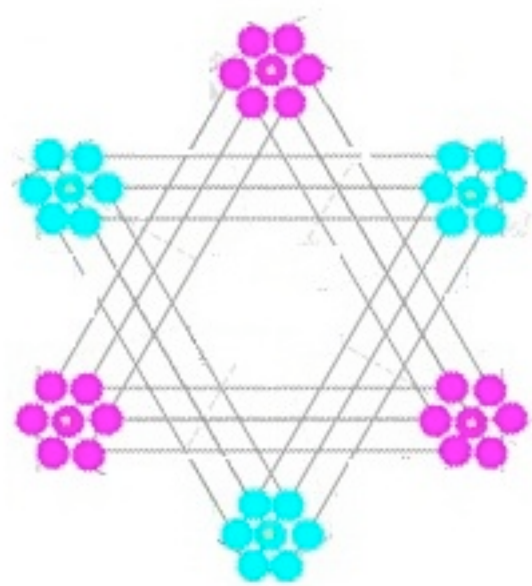


When transformed from the 8-circle projection to the basic projection of my E8 physics model, D4 and D4* look like



The basic figure of my E8 physics model

has, for the D4 and D4*, cyan intead of bright yellow and magenta instead of dark yellow, so that in the basic figure the D4 and D4* look like



28-dim D4 (with 24 root vectors) gives Gravity from its 15+1 = 16-dimensional D3xU(1).

The 12-dimensional symmetric space D4 / D3xU(1) corresponds to the Lie spheres in R8.

28-dim D4* (with 24 root vectors) gives the Standard Model SU(3) and SU(2) and U(1) from its 15+1 = 16-dimensional A3xU(1) = U(4).

The 12-dimensional symmetric space D4* / U(4) corresponds to the set of complex structures in R8.

Since D3 = A3 and D3xU(1) = U(4), the Lie spheres in R8 looks like the set of complex structures in R8, so from when I refer to the "set of complex structures in R8" I am referring to both of those things.

Since D4 describes Gravity acting on 4-dimensional M4 physical spacetime, the 12-dimensional set of complex structures in R8 of the D4 symmetric space correspond to the ways that M4 can be fit inside the prior-to-dimensional-reduction 8-dimensional spacetime.

Since D4* describes the Standard Model SU(3) and SU(2) and U(1) acting on 4-dimensional CP2 , the 12-dimensional set of complex structures in R8 of the D4* symmetric space correspond to the ways that CP2 can be fit inside the prior-to-dimensional-reduction 8-dimensional spacetime.

After dimensional reduction, the uniform R8 is transfomed into a 4+4-dimensional M4xCP2 Kaluza-Klein space,

and consistency with the structure of the M4xCP2 Kaluza-Klein space is a restriction on the 12+12 = 24 degrees of freedom of the D4 and D4* symmetric spaces.

and the geometry of that dimensional reduction gives, by the Mayer Mechanism, the Higgs scalar, which is 2-complex dimensional or 4-real dimensional (see, for example, Introduction to Gauge Field Theory, by Bailin and Love (rev ed IOP 1993 at pages 235, 238)).

Since the 12+12 = 24 degrees of freedom of the D4 and D4* symmetric spaces produce the 4 degrees of freedom of the Higgs scalar,

the remaining 24-4 = 20 degrees of freedom do not correspond to physics in our M4xCP2 low-energy Kaluza-Klein realm,

but to phenomena in the high-energy realm of prior-to-dimensional-reduction 8-dimensional spacetime.

Having seen how the Higgs etc comes from the 28-16 = 12-real-dimensional symmetric spaces Spin(8) / U(4) of D4 and D4*

consider the physical interpretation of the 16-real-dimensional U(4) subgroup of Spin(8) in D4* that produces the Standard Model.

12 of the dimensions describe the Standard Model gauge groups SU(3) and SU(2) and U(1)

As to the remaining 16-12 = 4 dimensions,

1 of them is the U(1) of U(4) = U(1)xSU(4) that describes a complex propagator phase, which must be corrrelated/coincidental with the U(1) of the U(2,2) in the Spin(8) of D4 that produces Gravity for consistency of the E8 physics model after dimensional reduction

1 more of them is accounted for by requiring the U(1) of U(3) and the U(1) of U(2) to be correlated/coincidental, producing the U(1) photon. (Note that in the E6 physics model with only one D4, the 12 Standard Model generators are the 28-16 = 12 of Spin(8) / U(2,2), so there is only one U(1) photon.)

The other 2 are in CP3 beyond CP2 (where CP3 = SU(4) / U(3) and CP2 = SU(3) / U(2)) and they describe the Quantum Worlds of the Many-Worlds, much like "the "tunnel effect" of quantum mechanics in terms of classical evolution of a system in imaginary time" to use the words of Yu. Manin in his 1981 book "Mathematics and Physics", where he said:

> "... It is extremely important to ... imagine the whole history of the Universe ... as a complete four-dimensional shape, something like the "tao" of ancient Chinese philosophy. The introduction of temporal dynamics is the next step. ... the natural structure for the absolute sky ... at the point P0 ... is the complex Riemann sphere ... the complex projective line CP1 ... the natural coordinates are complex numbers ... they are always connected by a fractional-linear transformation ... each sky CP1 is simply embedded in ... The "Penrose paradise" H = CP3 ... the space of "projective twistors" ... the skies over the points of the Minkowski World are not all the lines in CP3, but only part of them, lying in a five-imensional hypersurface ... introduc[ing] additional ... skies correspond[ing] to the missing lines in CP3 ...[gives]... the compact complex spacetime of Penrose, denoted CM ... [with] the interpretation of the "tunnel effect" of quantum mechanics in terms of the classical evolution of a system in imaginary time ...
>
> In a world of light there are neither points nor moments of time; beings woven from light would live "nowhere" and "nowhen"

... One point of CP3 is the whole life history of a free photon - the smallest "event" that can happen to light. ...".

E8 and Primes

According to "The Classification of the Finite Simple Groups" (AMS Mathematical Surveys and Monographs, Vol. 40, No. 1, 1994) by Gorenstein, Lyons, and Solomon (in the following I change their notation from prime number q to prime number p):

> "... It is our purpose ... to prove the following theorem:
>
> CLASSIFICATION THEOREM. Every finite simple group is
>
> - cyclic of prime order,
> - an alternating group,
> - a finite simple group of Lie type,
> - or one of the twenty-six sporadic finite groups.
>
> ... the bulk of the set of finite simple groups consists of finite analogues of Lie groups ... called finite simple groups of Lie type, and naturally form 16 infinite families ... In 1968, Steinberg gave a uniform construction and characterization of all the finite groups of Lie type as groups of fixed points of endomorphisms of linear algebraic groups over the algebraic closure of a finite field ...
>
> The finite simple groups are listed ...[including]... Group ... E8(p) ...[for prime p]...
>
> Order ... p^120 (p^2 - 1) (p^8 - 1) (p^12 - 1) (p^14 - 1) (p^18 - 1) (p^20 - 1) (p^24 - 1) (p^30 - 1) ...".

To get a feel for E8(p), ignore the -1 part of the Order formula for E8(q) and see that the order of E8(q) is roughly (somewhat less than)

p^120 p^(2+8+12+14+18+20+24+30) = p^(120+128) = p^248

Note that 248-dim E8 = 120-dim adjoint of Spin(16) + 128-dim half-spinor of Spin(16)

and that p^248 is the set of maps from 248 to p and that the exponents are one greater than each of the primes 1, 7, 11, 13, 17, 19, 23, and 29,

but not similarly related to the primes to 2, 3, or 5.

and that

- E8(2) = the number of ways to assign the 2 elements + and 1 (as in + and - electric charge of the U(2) electroweak gauge group) to each of the 248 basis elements of E8
- E8(3) = the number of ways to assign the 3 = 2+1 = 4-1 elements + and 1 (as in r, g and b color charge of the SU(3) color force gauge group) to each of the 248 basis elements of E8
- E8(5) = the number of ways to assign the 5 = 6-1 = 4+1 elements x, y, z, t and m (as in spatial x, y and z , and time t and scale/mass m of the Spin(2,3) anti-deSitter group of MacDowell-Mansouri gravity) to each of the 248 basis elements of E8
-
- E8(7) = the number of ways to assign the 7 = 6+1 = 8-1 Imaginary Octonion basis elements (as in spatial/internal symmetry part of 8-dim Kaluza-Klein spacetime and tree-level-massive first generation fermion particles and antiparticles and in 7 of the 8 Dirac gammas of E8 physics) to each of the 248 basis elements of E8
- E8(11) = the number of ways to assign 11 = 12-1 elements (as in the 11 generators of charge-carrying SU(3) and SU(2) of the 12 generators of the Standard Model SU(3)xSU(2)xU(1) in E8 physics) to each of the 248 basis elements of E8
- E8(13) = the number of ways to assign 13 = 12+1 elements (as in 12 root vectors of Conformal Spin(2,4) = SU(2,2) of MacDowell-Mansouri gravity in E8 physics) to each of the 248 basis elements of E8
- E8(17) = the number of ways to assign 17 = 16+1 elements as in the 16-dim vector representation of Spin(16) and the 16-dim full spinor representation of Spin(8) and 16-dim pairs of octoniions representing second-generation fermions and in the complexification of 8-dim Kaluza-Klein spacetime and 8-dim representation spaces of first-generation particles and antiparticles) to each of the 248 basis elements of E8

- E8(19) = the number of ways to assign 19 = 18+1 elements (as in the 18 root vectors of 21-dimensional rank 3 Spin(7)) to each of the 248 basis elements of E8
- E8(23) = the number of ways to assign 23 = 24-1 elements (as in 24-dim triples of octonions representing third-generation fermions and 24 full octonionic dimensions of the 27-dim Jordan algebra J(3,O)) to each of the 248 basis elements of E8
- E8(29) = the number of ways to assign 29 = 28+1 elements (as in 28-dim d4 for MacDowell-Mansouri gravity and 28-dim d4 for the Standard Model in E8 physics) to each of the 248 basis elements of E8
- ...
- E8(113) = the number of ways to assign 113 = 112+1 elements (as in the 112 root vectors of 120-dim Spin(16)) to each of the 248 basis elements of E8
- E8(127) = the number of ways to assign 127 = 128-1 elements (as in 64+64 = 128-dim half-spinors of Spin(16) representing first-generation fermion particles and antiparticles, and the related Dirac Gammas) to each of the 248 basis elements of E8
- ...
- E8(257) = the number of ways to assign 257 = 256+1 elements (as in 256-dim Cl(8)) to each of the 248 basis elements of E8
- ...
- E8(65537) = the number of ways to assign 65,537 = 65,536+1 elements (as in 65,536-dim Cl(16)) to each of the 248 basis elements of E8

In math.RT/0712.3764 Skip Garibaldi said:

> "... Theorem. Let L be a Lie algebra of type E8 over a field of characteristic 5. Then there is no quotient trace form on L. ...
>
> Roughly speaking, we use lemmas due to Block to reduce to showing that the trace is zero for representations coming from algebraic groups of type E8. From this, it is easy to see that it suffices to consider only the Weyl modules, which are defined over Z. Leaning on the fact that a Lie algebra of type E8 is

simple over every field ... we note that the trace form is zero because 5 divides 60, the Dynkin index of E8. ...

Lemma 1.3. Let G and g be ... of type E8. The following are equivalent:

- (1) The Killing form of g is not zero over F.
- (2) The Killing form of g is nondegenerate over F.
- (3) The characteristic of F is =/= 2, 3, 5.

...

Proposition 1.5. Let G and g be as in 1.1 and of type E8. There is a representation rho of G over F with tr =/= 0 if and only if F has characteristic =/= 2, 3, 5.

... ".

From F4 to E6 to E8

F4

The exceptional Lie algebra f4 =

- so(8) 28 gauge bosons of adjoint of so(8)
- + 8 vectors of vector of so(8)
- + 8 +half-spinors of so(8)
- + 8 -half-spinors of so(8) (mirror image of +half-spinors)

Therefore, you can build a natural Lagrangian from f4 as

- 8 vector = base manifold = 8-dim Kaluza-Klien 4+4 dim spacetime
- fermion term using 8 +half-spinors as left-handed first-generation particles and the 8 -half-spinors as right-handed first-generation antiparticles.
- a normal (for 8-dim spacetime) bivector gauge boson curvature term using the 28 gauge bosons of so(8).

If you let the second and third fermion generations be composites of the first, i.e., if

- the 8 first-generation particles/antiparticles are identified with octonion basis elements denoted by O,
- and you let the second generation be pairs OxO
- and the third generation be triples OxOxO
- and if you let the opposite-handed states of fermions not be fundamental, but come in dynamically when they get mass,

then

f4 looks pretty good IF you can get gravity and the standard model from the 28 so(8) gauge bosons.

If you want to make gravity from 15-dim Conformal Lie algebra so(2,4) by a generalized McDowell-Mansouri mechanism

then you have 28-15 = 13 so(8) generators left over, which are enough to make the 12-dim SM, BUT

the 15-dim Conformal Gravity and 12-dim Standard Model are not both-at-the-same-time either

- Group-type subroups of Spin(8)
- or Algebra-type Lie algebra subalgebras of so(8)
- or factors of the Weyl group of so(8), since
    - the Weyl group of so(8) is of order 2^3 4! = 8 x 24 = 192
    - the Weyl group of so(2,4) is of order 2^2 3! = 4 x 6 = 24
    - the Weyl group of su(3) is of order 3! = 6
    - the Weyl group of su(2) is of order 2! = 2
    - the Weyl group of u(1) is of order 1! = 1

  Not only does the Weyl group of so(8) have only one factor of 3 while the Conformal Group and Standard Model have two factors of 3, but the total order of the Weyl groups of the Conformal Group and Standard Model is 24 x 6 x 2 x 1 = 288 which is larger than the order 192 of the Weyl group of so(8).

So, if you try to get both the 15 CG and 12 SM to fit inside the 28 so(8),

- you see that they do not fit as Lie Group subgroups
- and you see that they do not fit as Lie algebra subalgebras
- and you see that they do not fit as Weyl group factors

so

what I have done is to look at them as root vectors, where the so(8) root vector polytope has 24 vertices of a 24-cell

- and the Conformal Gravity so(2,4) root vector polytope has 12 vertices of a cuboctahedron
- and the remaining 24-12 = 12 vertices can be projected in a way that gives the 12-dim SM.

My root vector decomposition (using only one so(8) or D4) is one of the things that causes Garrett Lisi to say that I have "... a lot of really weird ideas which ...[he, Garrett]... can't endorse ...".

So, from a conservative point of view, that you must use group or Lie algebra decompositions (not even considering a somewhat unconventional Weyl group factor approach, for which the f4 approach also will not work) ,

f4 will not work because one copy of D4 so(8) is not big enough for gravity and the SM.

Also, f4 has another problem for my approach: f4 has basically real structures, while I use complex-bounded-domain geometry ideas based on ideas of Armand Wyler to calculate force strengths and particle masses.

So, although f4 gives you a nice natural idea of how to build a Lagrangian as

- integral over vector base manifold
- of curvature gauge boson term from adjoint so(8)
- and spinor fermion terms from half-spinors of so(8)

f4 has two problems:

- 1 - no complex bounded domain structure for Wyler stuff (a problem for me)
- 2 - only one D4 (no problem for me, but a problem for more conventional folks).

So, look at bigger exceptional Lie algebra:

E6

e6 is nice, and has complex structure for my Armand Wyler-based calculation of force strengths and particle masses, so e6 solves my problem 1 with f4 and I can and have constructed an e6 model,

but e6 still has only one D4, so e6 is still problematic from the conventional view, as e6 does not solve the conventional problem 2 with f4.

So, do what Garrett Lisi did, and go to the largest exceptional Lie algebra, e8:

If you look at e8 in terms of E8(8) EVIII = Spin(16) + half-spinor of Spin(16)

you see two copies of D4 inside the Spin(16) (Jacques Distler mentioned that) which are enough to describe gravity and the SM.

I think that Garrett's use of e8 is brilliant,

even though my view of e8 differs in some details from the view of Garrett Lisi's paper arXiv 0711.0770:

- I don't use triality for fermion generations, since my second and third generations are composites of the first, as described above in talking about f4
- and I use a different assignment of root vectors to particles etc, which can be seen in an animated rotation using Carl Brannen's root vector java applet In my version:
    - There are D4+D4+64 = 24+24+64 = 112 root vectors of Spin(16) :
        - 24 yellow points for one D4 in the Spin(16) in E8
        - 24 purple points for the other D4 in the Spin(16) in E8
        - 64 blue points for the 8 vectors times 8 Dirac gammas in the Spin(16) in E8
    - There are 64+64 = 128 root vectors of a half-spinor of Spin(16) :
        - 64 red points for the 8 first-generation fermion particles times 8 Dirac gammas
        - 64 green points for the 8 first-generation fermion antiparticles time 8 Dirac gamma

Steven Weinberg on How to Build a Physics Lagrangian

Given E8 = adjoint Spin(16) + half-spinor Spin(16) and physical interpretation

- There are D4+D4+64 = 24+24+64 = 112 root vectors of Spin(16) :
  - 24 yellow points for one D4 in the Spin(16) in E8 which D4 gives MacDowell-Mansouri Gravity
  - 24 purple points for the other D4 in the Spin(16) in E8 which D4 gives the Standard Model gauge bosons
  - 64 blue points for the 8 vectors times 8 Dirac gammas in the Spin(16) in E8 which vectors give 8-dim Kaluza-Klein spacetime
- There are 64+64 = 128 root vectors of a half-spinor of Spin(16) :
  - 64 red points for the 8 first-generation fermion particles times 8 Dirac gammas
  - 64 green points for the 8 first-generation fermion antiparticles time 8 Dirac gamma

is it natural to put them together to form the Lagrangian of my E8 physics model ?

In the 1986 Dirac Memorial Lectures published in the book Elementary particles and the Laws of Physics (Cambriddge 1987)

Steven Weinberg said (in the following I sometimes substitute the word "fermion" for "electron" and the words "gauge bosons" for "photon" and the words "the equation" for "(1)" referring to equation (1), and I sometimes insert my comments indented and enclosed by brakcets [ ]):

"... Let's examine the following equation:

L =

- PSIbar (gamma^mu d/dx_mu + m) PSI

- (1/4) (d/dx_mu A_nu - d/dx_nu A_mu)^2

+ i e A_mu PSIbar gamma^mu PSI

- MU (d/dx_mu A_nu - d/dx_nu A_mu) PSIbar sigma^mu nu PSI

- G PSIbar PSI PSIbar PSI

+ ...

L stands for Lagrangian density; roughly speaking you can think of it as the density of energy.

Energy is the quantity that determines how the state vector rotates with time, so this is the role that the Lagrangian density plays; it tells us how the system evolves.

L ...[is]... written as a sum of products of fields and their rates of change.

PSI is the field of the fermion (a function of the spacdetime position x), and m is the mass of the fermion.

d/dx_mu means the rate of change of the field with position. ...

the gamma^mu matrices are called Dirac matrices.

A_mu is the field of the gauge bosons ...

Each term has an independent constant, called the coupling constant, that mutiplies it. These are the quantities e , MU , G , ... in the equation. The coupling constant gives the strength with which the term affects the dynamics.

No coupling constant appears in the first two terms simply because I have chosen t absorb them into definition of the two fields PSI and A_mu. ...

Experimentally we know that the formula consisting of just the first three terms, with all higher terms neglected, is adequate to describe electrons and photons to a fantastic level of accuracy. This theory is known as quantum electrodynamics or QED. ...

[An] argument ... why the behaviour of electrons and photons is described by just the first three terms in the equation ... goes back to work by Heisenberg in the 1930s ... The argument is based on dimensional analysis ... I will work in a system of units called physical units, in which Planck's constant and the speed of light are set equal to one. With these choices, mass is the only remaining unit; we can express the dimensions of any quantity as a power of mass.

For example, a distance or time can be expressed as so many inverse grammes. A cross-sction ... is given in terms of som many inverse grammes squared. ...

Now suppose that all interactions have coupling constants that are pure numbers, like the constant e in the third term of the equation ... Then itr would be very easy to figure out what contribution an observable gets from its cloud of virtual gauge bosons and fermion-antfermion pairs at very high energy E.

Lets suppose an observable O has dimensions [mass]^(-a) where a is positive. ... Now, at very high virtual-particle energy, E , much higher than any mass, or any energy of a particle in the initial or final state, there is nothing to fix a unit of energy. The contribution of high energy virtual particles to the observable O must then be given an integral like [the following expression (3)]

O = INTEGRAL(to oo) 1 / E^(a+1) dE

because this is the only quantity wihcih has the right dimensions, the right units, to give the observable O. ... The lower bound in the integral is some finite energy that marks the dividing line between what we call high and low energy. ... This argument only works because there are no other quantities in the theory that have the units of mass or energy. ...

On the other hand, suppose that there are other constants around that have units of mass raised to a negative power. Then if you have an expression involving a constant C_1 with units [mass]^(-b_1) , and another constant

C_2 with units [mass]^(-b_2) nd so on, then ... we get a sum of terms of the form [of the following expression (4)]

O = C_1 C_2 ... INTEGRAL(to oo) E^(b_1 + b_2 + ...) / E^(a+1) dE

again because these are the only quantities tha have the right units for the observable O. ...

Expression (3) is perfectly well-defined, the integral converges ... as long as the number a is greater than zero.

However, if b_1 + b_2 + ... is greater than a , then (4) will not be well-defined, because the numerator will have more powers of energy than the denominator and so the integral will diverge.

The point is that no matter how many powers of energy you have in the denominator, i.e. no matter how large a is , (4) eventually will diverge when you get up to sufficiently high order in the coupling constants, C_1 , C_2 , etc., that have dimensionls of negative powers of mass, because if you have enough of these constants, then eventually b+1 + ... is greater than a .

Looking at the Lagrangian density in the equation, we can easily work out what the units of the constant e , MU , G , etc., are.

[In 4-dimensional physical spacetime]... All terms in the Lagrangian density must have units [mass]^4 , because length and time have units of inverse mass and trhe Lagrangian density integrated over spacetime must have no units.

From the m PSIbar PSI term, we see that the fermion field must have units [mass]^(3/2) , because ... [t]he derivative operator (the rate of change operator) has units of [mass]^1 ...[and] 3/2 + 3/2 + 1 = 4 .

> [In an e-dimensional spacetime, the fermion field must have units [mass]^(7/2) , because 7/2 + 7/2 + 1 = 8 .]

The derivative operator (the rate of change operator) has units of [mass]^1 , and so the gauge boson field also has units of [mass]^1.

Now we can work out what the units of the coupling constants are. ...

the electric charge ... e ... turns out to be a pure nuber, to have no units.

But then as you add more and more powers of fields, more and more derivatives, you are adding more and more quantities that have units of positive powers of mass, and since the Lagrangian density [in 4-dimensional physical spacetime]... has to have fixed units of [mass]^4 , therefore the mass dimensions of the associated coupling constants must get lower and lower, until eventually you come to constants like MU and G which have negative units of mass. ... Specifically, MU has the units of [mass]^(-1) , while G has the unts [mass]^(-2) ... Such terms in the equation would completely spoil the agreement between theory and experiment ... so experimentally we can say that they are not there to a fantrastic order of precision and ... it seems that this could be explained by saying that such terms must be excluded because they would give infinite results, as in (4).

... that is exactly waht we are lookign for: a theoretical framework based on quantum mechanics, and a few symmetry principles, in which the specific dynamical principle, the Lagrangian, is only mathematically consistent if it takes one particular form.

At the end of the day, we ... have the feeling that "it could not have been any other way". ...

I described to you the success quantum electrodynamics has had in the theory of photons and electrons ...

In the 1960s these ideas were applied to the weak interactions of the nuclear particles, with a success that became increasingly apparent experimentally during the 1970s.

In the 1970s, the same ideas were applied to the strong interactions of the elementary particles, with results that ... have been increasingly experimentally verified since then.

Today we have a theory based on just such a Lagrangian as given in the equation. In fact,

if you put in some indices on the fields so that there are many fields of each type, then the first three terms of the equation give just the so-called standard model ...

It is a theory that seems to be capable of describing all the physics that is accessible using today's accelerators. ... The standard model works so well because all the terms which could make it look different are naturally extremely small. A lot of work has been done by experimentalists trying to find effects of these tiny terms ... but so far nothing has been discovered.

> [Neutrino masses have been discovered since Weinberg gave his talk in1986, but they can be considered to be part of the lepton sector of the Standard Model.]

So far, no effect except for gravity itself has been discovered coming down to us from the highest energy scale where we think the real truth resides. ... ".

> Some of the ... omissions in the above quote indicate that Weinberg's views stated above reflect his thinking "... until about five or six years ..." before he gave the talk in 1986, and the rest of the talk indicates that his thinking as of 1986 was "... that the ultimate constituents of nature, when you look at nature on a scale of 10^15 - 10^19 GeV , are not particles or fields but strings ...".
>
> I prefer to see string theory in terms of my E6 bosonic string model, with fermions coming from orbifolding and strings being physically interpreted as world-lines of particles, which model is consistent with my E8 physics model which is consistent with the Standard Model plus MacDowell-Mansouri gravity from the Conformal Group which gives a Dark Energy : Dark Matter : Ordinary Matter ratio that is consistent with observations. My E6 and E8 models allow calculation of what Weinberg describes as "... the ... fairly large number ... of free parameters ... that have to be chosen "just so" in order to make the [standard model] theory agree with experiment ...".

Left and Right Ideals of Clifford Algebras

In Clifford Algebras and Their Applications in Mathematical Physics (Proceedings of the NATO and SERC Workshop, 15-27 September 1985, ed. by J. S. R. Chisholm and A. K. Common (Reidel 1986) at pages 9-10, 23, 327-328), David Hestenes said:

> "... Clifford Algebras ... become vastly richer when given geometrical and/or physical interpretations. When a geometric interpretation is attached to a Clifford Algebra, I prefer to call it a Geometric Algebra, which is the name originally suggested by Clifford himself. ...
>
> the theory of geometric representations should be extended to embrace Lie groups and Lie algebras. A start has been made in ... D. Hestenes and G. Sobczyk, Clifford Algebra to Geometric Calculus, Reidel Publ. Co., Dordrecht/Boston (1984) ... I conjectured there that every Lie algebra is isomorphic to a bivector algebra, that is, an algebra of bivectors under the commutator product. Lawyer-physicist Tony Smith has proved that this conjecture is true by pointing to results already in the literature. ...
>
> the columns of a matrix are minimal left ideals in a matrix algebra, because columns are not mixed by matrix multiplication from the left. The Dirac matrix algebra C(4) has four linearly independent minimal left ideals, because each matrix has four column. The Dirac spinor for an electron or some other fermion can be represented in C(4) as a matrix with nonvanishing elements only in one column, like so

PSI_1	0	0	0
PSI_2	0	0	0
PSI_3	0	0	0
PSI_4	0	0	0

where the PSI_i are complex scalars. The question arises: Is there a physical basis for distinguishing between different columns?

The question looks more promising when we replace C(4) by the isomorphic geometric algebra R_4,1 in which every element has a clear geometric meaning. Then the question becomes: Is there a physical basis for distinguishing between different ideals?

The Dirac theory clearly shows that a single ideal (or column if you will) provides a suitable representation for a single fermion. This suggests that each ideal should represent a different kind of fermion, so the space of ideals is seen as a kind of fermion isospace. I developed this idea at length in my dissertation, classifying leptons and baryons in families of four ...".

In the same Workshop proceedings I said I(at pages 377-379, 381-383):

"... The 16-dimensional spinor representation of Spin(8) reduces to two irreducible 8-dimensional half-spinor representations that can correspond to the 8 fundamental fermion lepton and quark first-generation particle and to their 8 antiparticles ...

Numerical values for force strengths and ratios of particle masses to the electron mass are given. ... Armand Wyler ... (1971), C. R. Acad. Sci. Paris A272, 186 ... wrote a paper in which he purported to calculate the fine structure constant to be a = 1 / 137.03608 ... from the volumes of homogeneous symmetric spaces. ... Joseph Wolf ... (1965), J. Math. Mech. 14, 1033 ... wrote a paper in which he classified the 4-dimensional Riemannian symmetric spaces with quaterniuonic structure. There are just 4 equivalence classes, with the following representatives:

- T4 = U(1)^4
- S2 x S2 = SU(2) / U(1) x SU(2) / U(1)
- CP2 = SU(3) / S(U(2) x U(1))
- S4 = Spin(5) / Spin(4)

... Final Force Strength Calculation ...

- fine structure constant for electromagnetism = 1 / 137.03608
- weak Fermi constant times proton mass squared = 1.03 x 10^(-5)
- color force constant (at about 10^(-13) cm.) = 0.6286
- gravitational constant times proton mass squared = 3.4 - 8.8 x 10^(-39).

... PARTICLE MASSES ...

- the electron mass ...[is assumed to be given at its experimentally observed value]...
- electron-neutrino mass = 0 ... [Note that this is only a tree-level value.] ...
- down quark constituent mass = 312.8 Mev ...
- up qaurk constituent mass = 312.8 Mev ...
- muon mass = 104.8 Mev ...
- muon-neutrino mass = 0 ... [Note that this is only a tree-level value.] ...
- strange quark constituent mass = 523 Mev ...
- charm quark constituent mass = 1.99 Gev ...
- tauon mass = 1.88 Gev ...
- tauon-neutrino mass = 0 ... [Note that this is only a tree-level value.] ...
- beauty quark constituent mass = 5.63 Gev ...
- truth quark constituent mass = 130 Gev ...

CERN has announced that the truth quark mass is about 45 Gev (Rubbia ... (1984), talk at A.P.S. D.P.F. annual meeting at Santa Fe ... but I think that the phenomena observed by CERN at 45 Gev are weak force phenomena that are poorly explained ... As of the summer of 1985, CERN has been uable to confirm its identification of the truth quark in the 45 Gev events, as the UA1 experimenters have found a lot of events clustering about the charged ... W mass and the UA2 experimenters have not found anything convincing. (Miller ... (1985), Nature 317, 110 ... I think that the clustering of UA1 events near the charged ...

W mass indicates that the events observed are ... weak force phenomena. ...".

Since I have been critical of CERN for its error in truth quark obersvations, I should state that my paper in that 1985 Workshop also contained errors, the most conspicuous of which may have been my statement that "... there should be three generations of weak bosons ...".

Mathematical Structure of the 64-dimensional things of the form 8 x 8

Combining the David Hestenes idea of left ideals representing fermions with 8-dimensional D4 half-spinors and an 8-dimensional D4 vector Kaluza-Klein spacetime and 8-dimensional Clifford/Geometric Algebra Dirac gammas gives physical meaning to the three 64-dimensional structures

- 8v x 8g
- 8s' x 8g
- 8s'' x 8g

of my version of an E8 physics model.

R(8) and Octonions

Ian Porteous, in his book Clifford Algebras and the Classical Groups (Cambridge 1995) says(page 180-182):

> "... The existence of the Cayley algebra [octonions] depends on the fact that the [64-dimensional] matrix algebra R(8) [of 8 x 8 real matrices] may be regarded as a ... Clifford algebra for the [7-dimensional] positive-definite orthogonal space R7 in such a way that conjugation of the Clifford algebra corresponds to transposition in R(8). For then ... the images of R and R7 in R(8) together span an eight-dimensional linear subspace, passing through ...[the 8-dimensional unit]... 1 , such that each of its elements, other than zero, is invertible. This eight-dimensional subspace of R(8) will be denoted Y.
>
> Proposition 19.3 Let [the 8-dimensional real space] R8 -> Y be a linear isomorphism. Then the map
>
> R8 x R8 -> R8 ; (a,b) -> a b = (mu(a))(b)
>
> is a bilinear product on R8 such that, for all a,b in R8 , a b = 0 if and only if a = 0 or b = 0 . Moreover, any non-zero element e in R8 can be made the unit element for such a product by choosing mu to be the inverse of the isomorphism
>
> Y -> R8 ; y -> y e .
>
> The division algebra with unit element introduced in Proposition 19.3 is called the Cayley algebra on R8 with unit element e. ... We shall ... speak simply of the Cayley algebra, denoting it by O (for octoniions) ... it is advantageous to select an element of length 1 in R8 ... we select e_0 , the zeroth element of the standard basis for R8. ... we have implicitly assigned to R8 its standard positive-definite structure ... The space Y also has an orthogonal structure ... The Cayley algebra O inherits both ... the choice of e as an element of length 1 guarantees that these two structures coincide. ... though the

product on R(8) is associative, the product on O need not be. ... The Cayley algebra O is alternative ...".

Geoffrey Dixon in hep-th/9303039 says:

"... multiplication tables for ... O are constructable from the following elegant rules: ...

- Imaginary Units ... e_a , a = 1, ..., 7 ,
- Anticommutators ... e_a e_b + e_b e_a = 2 delta_ab ,
- Cyclic Rules ... e_a e_a+1 = e_a-2 = e_a+5 ,
- Index Doubling ... e_a e_b = e_c => e_(2a) e_(2b) = e_(2c) , ...

The octonion algebra is generally considered ill-suited to Clifford algebra theory becauseO is nonassociative, and Clifford algebras are associative. This problem disappears once we identify O as the spinor space of OL , the adjoint algebra of actions of O on itself from the left. OL is associative. ... a complete basis for OL consists of the elements

1 , e_La , e_Lab , e_Labc ,

Therefore OL is 1 + 7 + 21 + 35 = 64-dimensional, and OL ...[is isomorphic to the real 8 x 8 matrix algebra]... R(8) . ... OL is iksomorphic to the Clifford algebra ...[Cl(0,6)]... of the space R^(0,6) , the spinor space of which is 8-dimensional over R . In the case the spinor space is O itself, the object space of OL . ... the algebra OR of right adjoint actions of O on itself is the same algebra as OL . Every action in OR can be written as an action in OL .

A 1-vector basis for OL , playing the role of the Clifford algebra ...[Cl(0,6)]... of R^(0,6) is { e_Lp , p = 1, ..., 6 } .

The resulting 2-vector basis is then { e_Lpq , p,q = 1, ..., 6 , p =/= q } . This subspace is 15-dimensional, closes under the commutator product, and is in that case isomorphic to so(6).

> The interesection of this Lie algebra with the Lie algebra of the automorphism group of O , G2 , is su(3) , with a basis
>
> su(3) -> { e_Lpq - e_Lrs , p,q,r,s distinct, and from 1 to 6 } .
>
> ... SU(3) is the stability group of e_7 , hence the index doubling automorphism of O is an SU(3) rotation ...".

Geoffrey Dixon, in his book Division Algebras, Octonions, Quaternions, Complex Numbers and the Algebraic Design of Physics (Kluwer 1986), says (pages 43-45, 141-142, 191-192, 197, 209-211, 215-216) (in the following quote I have changed some notation from l to j and have particularized some division algebra notation from the general division algebra K to the octonion division algebra O) :

> "... An algebraic idempotent, A, is by definition a nonzero element satisfying : A^2 = A . A is nontrivial if A =?= 1 ... [and]...
>
> A (1- A) = A - A^2 = A - A = 0 and (1-A)^2 = 1 - 2A + A^2 = 1 - 2A + A = 1 - A .
>
> So ... 1 - A is aslo an idempotent, and ... A and 1-A are orthogonal. ... nontrivial idempotents are divisors of zero, hence the identity is the sole idempotent of any division algebra ... This ...[does not apply]... to OL = OR = R(8), which is not a division algebra.

Certain elements of OL are diagonal in the adjoint representation. A basis for these consists of the identity, 1_L , together with the e_Labc satisfying e_Labc(1) = e_a (e_b e_c) = 1 ... In particular, define I_a = e_L(3+a)(6+a)(5+a) (indices from 1 to 7 , modulo 7), and let I_0 be the identity. Their adjoint representations are

```
I_0 = 1_L   -> diag(++++++++)
I_1 = e_476 -> diag(+---+-++)
I_2 = e_517 -> diag(++---+-+)
I_3 = e_621 -> diag(+++---+-)
I_4 = e_732 -> diag(+-++---+)
I_5 = e_143 -> diag(++-++---)
I_6 = e_254 -> diag(+-+-++--)
I_7 = e_365 -> diag(+--+-++-)
```

... Being diagonal, the I_a clearly commute. They also satisfy I_a I_a+1 = I_a+3 , a in {1, ..., 7} (had e_a e_a+1 = e_a+3 been chosen as the multiplication for O , then ... I_a I_a+1 = I_a+5, so these choices are in this manner dual to each other ...) ...

the identity of OL can be elegantly resolved into orthogonal primitive idempotents using the I_a. A primitive idempotent can not be expressed as the sum of two other idempotents ... orthogonal primitive idempotents resolving the identity of OL ... are ...

P_0 = (1/8) (1 + e_L476 + e_L517 + e_L621 + e_L732 + e_L143 + e_L254 + e+L365) ,
P_1 = (1/8) (1 - e_L476 + e_L517 + e_L621 - e_L732 + e_L143 - e_L254 - e+L365) ,
P_2 = (1/8) (1 - e_L476 - e_L517 + e_L621 + e_L732 - e_L143 + e_L254 - e+L365) ,
P_3 = (1/8) (1 - e_L476 - e_L517 - e_L621 + e_L732 + e_L143 - e_L254 + e+L365) ,
P_4 = (1/8) (1 + e_L476 - e_L517 - e_L621 - e_L732 + e_L143 + e_L254 - e+L365) ,
P_5 = (1/8) (1 - e_L476 + e_L517 - e_L621 - e_L732 - e_L143 + e_L254 + e+L365) ,
P_6 = (1/8) (1 + e_L476 - e_L517 + e_L621 - e_L732 - e_L143 - e_L254 + e+L365) ,
P_7 = (1/8) (1 + e_L476 + e_L517 - e_L621 + e_L732 - e_L143 - e_L254 - e+L365) ,

... These satisfy SUM(a=0 to 7) P_a = 1 , and P_a P_b = delta_ab P_b . ...

They are related as follows (a in {0, 1, ..., 7}):

- P_a = e_La P_0 e*_La ;
- P_a e_La = e_La P_0 ;
- e_La P_a = P_0 e_La ;

if e_a e_b = e_c (a,b,c in {1, ..., 7}, then e_La P_0 e_Lb = - e_Lb P_c e_La ...

for example ... (P_0 + P_1 + P_2 + P_6) is an idempotent projecting from O a subalgebra isomorphic to Q:

(P_0 + P_1 + P_2 + P_6) O = Q Likewise ... (P_0 + P_1) O = C ... and ... P_0 O = R . ...

The matematical context upon which the model building ... rests relied heavily on treating the ... division algebras as spinor spaces of their left adjoint algebras (identified as Clifford algebras), of tensoring those adjoint algebras with ...[the 2 x 2 real matrix algebra]... R(2) (doubling the size of the spinor space) ... These ... same methods will be employed here to generate bases for the Lie algebras of the groups of a version of the magic square. Each will be derived from a tensor product of two division algebras ...

The foundation upon which the method rests is R(2) . In R(2) define ...

$$E = \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} \qquad A = \begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix} \qquad B = \begin{matrix} 0 & 1 \\ 1 & 0 \end{matrix} \qquad W = \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix}$$

Let O(x)O be the tensor product of two ...[copies of the octonion division algebra O]...

Let c_k ... denote ...[a basis]... for the pure hypercomplex part...[of O]... In ... O(x)O (2) the elements

W , c_Lk A , c_LljB

anticommute (and associate) and form the basis for the 1-vector generators of a ... Clifford algebra with negative definite

Euclidean metric. Under commutation they generate the 2-vectors

c_Lk B , c_Lj A , c_Lk1k2 E , c_Lj1j2 E , c_Lk c_Lk W ,

Together ...[those]... elements form a basis for a representation of the Lie algebra so(dimO + dimO). I'll call this External_OO and call it the external subalgebra.

To this collection we now add the spinors of O(x)O(2) , namely the elements of (O(x)O)^2 , without yet specifying a commutator product on this linear space. I'll denote this Spinor_OO . ...

The total resulting linear space will be denoted MS_OO , MS for magic square ...

Let e_La and e'_Lb be distinct and mustually commuting bases for the hypercomplex octonions.

External_OO is spanned by

W , e_La A , e'_La B

(1-vector basis for ...[the Clifford algebra Cl(0,15)]...)

and

e_Lab B , e'_Lab A , e_La e'_Lb W , e_Lab E , e'_Lab E

(2-vectors)

...[with dimension

1 + 7 + 7 + 21 + 21 + 21 + 21 + 21 = 1+14 +105 = 120]...

External_OO = so(16) .

Spinor_OO is 128-dimensional, and ... because OL = OR

...[there is no Internal_OO]...

That's 120 + 128 = 248 elements altogether, and we make the identification:

MS_OO = LE8 . ...

Getting LE8 from O(x)J3(O) ...[where J3(O) is the 27-dimensional exceptional Jordan algebra]... is slightly trickier. In this case there are two distinct copies of O commuting with each other (denote them O1 and O2) ...

We begin .. with the 28 so(8) generators ...[that]... are elements of LF4... and the 3 so(3) generators ... [that]... account for 3 of the 52 dimensions of LF4 ... Together ...[they]... account... for 3 + 28 = 31 of the 52-dimensional LF4. ...

in ...[this]... O1(x)J3(O2) case we expand so(3) to LF4 , the Lie algebra of the sutomorphism of J3(O1) . That gives us 28 elements from so(8 , and 52 elements from LF4 (which contains another distinct so(8)). Of the 52 generators of theis new LF4 , 28 are diagonal ... and 24 are off-diagonal. Commutators of the 28 diagonal generators (the so(8) of O1) with the so(8) of O2 yield nothing new, but each of the 24 off-diagonal generators gives rise to a 7-dimensional space of new generators. That yields,

28 + 52 + 168 = 248

generators all together, and the set closes here on LE8 ...".

Note that 168 is the order of PSL(2,7) = SL(3,2) which can be thought of as the group of linear fractional transformations of the vertices of a heptagon and is so related to octonion multiplication rules, and that SL(2,7) of order 336 double covers the Klein Quartic which is representable using a 14-gon.
There are 480 Octonion Multiplications. To see that, consider
2^7 sign changes of the 7 imaginary basis elements and 7! permutations of them.
2^3 = 8 sign changes and 168 = 2x3x4x7 permutations give the same Multiplication so there are 2^4 x 5x6 = 16 x 30 = 2 x 8x30 distinct Octonion Multiplications which therefore correspond to two copies of the 240 E8 root vectors.
Note that 2^7 7! is the order of the Weyl Group of Spin(15) and of Sp(7).

E8 Physics and Helicity

In E8 physics,

- the 8 first-generation fermion particles and 8 Dirac gammas are represented by 8x8 = 64 of the 128 half-spinor Spin(16) elements of E8 and
- the 8 first-generation fermion antiparticles and 8 Dirac gammas are represented by the other 8x8 = 64 of the 128 half-spinor Spin(16) elements of E8.

Since the all belong to one half-spinor representation of Spin(16), they all have the same helicity. Let that helicity correspond to left-handed fermion particles.

Since antiparticles are effectively particles travelling backward in time, the corresponding helicity for fermion antiparticles is right-handed.

Therefore, in E8 physics, fermion particles are fundamentally left-handed and fermion antiparticles are fundamentally right-handed.

Opposite handedness arises dynamically, and can be seen in experiments involving massive fermions moving at much less than the speed of light.

L. B. Okun, in his book Leptons and Quarks (North-Holland (2nd printing 1984) page 11) said:

> "... a particle with spin in the direction opposite to that of its momentum ...[is]... said to possess left-handed helicity, or left-handed polarization. A particle is said to possess right-handed helicity, or polarization, if its spin is directed along its momentum. The concept of helicity is not Lorentz invariant if the particle mass is non-zero. The helicity of such a particle depends oupon the motion of the observer's frame of reference. For example, it will change sign if we try to catch up with the particle at a speed above its velocity. Overtaking a particle is the more difficult, the higher its velocity, so that helicity becomes a better quantum number as velocity increases. It is an exact quantum number for massless particles ...

The above space-time structure ... means ... that at ...[v -> speed of light]... particles have only left-handed helicity, and antparticles only right-handed helicity. ...".

E8 - Spin-Statistics - Signatures - Pin and Spin

Soji Kaneyuki has written a chapter entitled Graded Lie Algebras, Related Geometric Structures, and Pseudo-hermitian Symmetric Spaces, as Part II of the book Analysis and Geometry on Complex Homogeneous Domains, by Jacques Faraut, Soji Kaneyuki, Adam Koranyi, Qi-keng Lu, and Guy Roos (Birkhauser 2000). Kaneyuki lists a Table of Exceptional Simple Graded Lie Algebras of the Second Kind including

e(17) for which g = E8(8)

- g(+2) = 14
- g(+1) = 64 = 8 fermion particles x 8 Dirac gammas
- g(0) = so(7,7) + R
- g(-1) = 64 = 8 fermion antiparticlex x 8 Dirac gammas
- g(-2) = 14

Kaneyuki also considers the even part of such algebras

g(ev) = g(-2) + g(0) + g(2)

= 14 + so(7,7)+R + 14 = 14 + 92 + 14 = 120 = so(8,8) = so(7,1) + 64 + so(1,7)

- The step immediately above is by real Clifford periodicity Cl(16) = Cl(8) (x) Cl(8) and
- preserving the (7,7) substructure by adding (0,1) and (1,0) to it to get so(7,1) + so(1,7)

= so(7,1) + so(1,7) + 8-dim Kaluza-Klein spacetime x 8 Dirac gammas

If all 120 g(ev) generators are physically bosonic and if all 128 generators of the odd g(-1) and g(+1) are physically fermionic then under E8

- fermion times fermion = boson
- boson times boson = boson
- boson times fermion = fermion times boson = fermion

so Spin-Statistics is satisfied.

As to signature (diagram from Spinors and Calibrations, by F. Reese Harvey (Academic 1990)):

$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$	$M_{64}(\mathbf{C})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{R}) \oplus M_{128}(\mathbf{R})$	$M_{256}(\mathbf{R})$
$M_8(\mathbf{C})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H}) \oplus M_8(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{R}) \oplus M_{64}(\mathbf{R})$	$M_{128}(\mathbf{R})$	$M_{128}(\mathbf{C})$
$M_4(\mathbf{H})$	$M_4(\mathbf{H}) \oplus M_4(\mathbf{H})$	$M_8(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{R}) \oplus M_{32}(\mathbf{R})$	$M_{64}(\mathbf{R})$	$M_{64}(\mathbf{C})$	$M_{64}(\mathbf{H})$
$M_2(\mathbf{H}) \oplus M_2(\mathbf{H})$	$M_4(\mathbf{H})$	$M_8(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$	$M_{32}(\mathbf{R})$	$M_{32}(\mathbf{C})$	$M_{32}(\mathbf{H})$	$M_{32}(\mathbf{H}) \oplus M_{32}(\mathbf{H})$
$M_2(\mathbf{H})$	$M_4(\mathbf{C})$	$M_8(\mathbf{R})$	$M_8(\mathbf{R}) \oplus M_8(\mathbf{R})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{C})$	$M_{16}(\mathbf{H})$	$M_{16}(\mathbf{H}) \oplus M_{16}(\mathbf{H})$	$M_{32}(\mathbf{H})$
$M_2(\mathbf{C})$	$M_4(\mathbf{R})$	$M_4(\mathbf{R}) \oplus M_4(\mathbf{R})$	$M_8(\mathbf{R})$	$M_8(\mathbf{C})$	$M_8(\mathbf{H})$	$M_8(\mathbf{H}) \oplus M_8(\mathbf{H})$	$M_{16}(\mathbf{H})$	$M_{32}(\mathbf{C})$
$M_2(\mathbf{R})$	$M_2(\mathbf{R}) \oplus M_2(\mathbf{R})$	$M_4(\mathbf{R})$	$M_4(\mathbf{C})$	$M_4(\mathbf{H})$	$M_4(\mathbf{H}) \oplus M_4(\mathbf{H})$	$M_8(\mathbf{H})$	$M_{16}(\mathbf{C})$	$M_{32}(\mathbf{R})$
$\mathbf{R} \oplus \mathbf{R}$	$M_2(\mathbf{R})$	$M_2(\mathbf{C})$	$M_2(\mathbf{H})$	$M_2(\mathbf{H}) \oplus M_2(\mathbf{H})$	$M_4(\mathbf{H})$	$M_8(\mathbf{C})$	$M_{16}(\mathbf{R})$	$M_{16}(\mathbf{R}) \oplus M_{16}(\mathbf{R})$
$\mathbf{R}$	$\mathbf{C}$	$\mathbf{H}$	$\mathbf{H} \oplus \mathbf{H}$	$M_2(\mathbf{H})$	$M_4(\mathbf{C})$	$M_8(\mathbf{R})$	$M_8(\mathbf{R}) \oplus M_8(\mathbf{R})$	$M_{16}(\mathbf{R})$

Cl(7,1) is the 8x8 Quaternion Matrix Algebra M(Q,8)

Cl(1,7) = Cl(0,8) is the 16x16 Real Matrix Algebra M(R,16) which has effective Octonionic structure.

If a preferred Quaternionic subspace is frozen out of the Octonionic spacetime of Cl(1,7), then its 8-dimensional (1,7) vector spacetime undergoes dimensional reduction to

- 4-dimensional (1,3) associative physical spacetime plus
- 4-dimensional (0,4) coassociative CP2 internal symmetry space

and Cl(1,7) is transformed into quaternionic Cl(2,6) = M(Q,8).

After dimensional reduction, since Cl(1,7) = Cl(2,6) = M(Q,8) you effectively have two copies of Cl(2,6) = M(Q,8).

Note that some might object that Spin(p,q) does not come directly from Cl(p,q) but rather comes from its even subalgebra, so that sometimes when I write Spin(p,q) I should be writing Pin(p,q), where, as Ian Porteous says in his book Clifford Algebras and the Classical Groups (Cambridge 1995):

> "... the Pin and Spin groups doubly cover the relevant orthogonal and special orthogonal groups.
>
> Proposition 16.14 Let X be a non-degenerate quadratic space of positive finite dimension. Then the maps
>
> PinX -> O(X) ... and SpinX -> SO(X) ...
>
> are surjective, the kernel in each case being isomorphic to S0 [the zero-sphere { -1, +1 }]...
>
> When X = R(p,q) the standard notations for [the Clifford group] GAMMA(X) ...[and for] ... GAMMA0((X) , PinX and SpinX will be GAMMA(p,q) , GAMMA0(p,q) , Pin(p,q) and Spin(p,q).
>
> Since ...[the even Clifford subalgebra Cle(p,q) is isomorphic to the even Clifford subalgebra Cle(q,p)]...
>
> - GAMMA0(q,p) is isomorphic to GAMMA0(p,q) and
> - Spin(q,p) is isomorphic to Spin(p,q).
>
> Finally, GAMMA0(0,n) is often abbreviated to GAMMA0(n) and Spin(0,n) to Spin(n). ...".

Further, Pertti Lounesto says in his book Clifford Algebras and Spinors (Second Edition Cambridge 2001):

> "... 17.2 The Lipschitz grooups and the spin groups The Lipschitz group GAMMA(p,q) , also called the Clifford group although invented by Lipschitz 1880/86 , could be defined as the subgroup in Cl(p,q) generated by invertible vectors x in R(p,q) ...
>
> The Lipshitz group has a normalized subgroup Pin(p,q) ... The group Pin(p,q) has an even subgroup Spin(p,q) ...".

Further, in Spinors and Calibrations (Academic 1990) F. Reese Harvey says:

> "... The Grassmannians and Reflections ... G(r,V) ...[is]... the grassmannian of all unit, oriented, nondegenerate r-planes through the origin in V ...[G(r,V)]... consists of all simple vectors in /\r(V) that are of unit length. That is,
>
> u is in G(r,V) if u = u_1 /\ ... /\ u_r with u_1 , ... , u_r in V and |u| = 1 (||u|| = +/-1) .
>
> ... Given u in G(r,V) , reflection along u , denoted R_u , is defined by
>
> R_u(x) = -x if x is in span(u) and R_u(x) = x if s is ...[orthogonal to]... span(u) ...
>
> ... Remark 10.20 ... each reflection R_u in O(V) along a subspace span u of V is replaced in the double cover Pin(V) of O(V) be either of the two elements +/-u in G(r,V) in /\r(v) in Cl(V) in the Clifford algebra. ...
>
> By definition, the group Pin is generated by the element G(1,V) in Pin. ... the definition of Spin ... suffers from the defect that the generators u in G(1,V) are not in Spin. This defect can be corrected .. if e is any unit vector and S(n-1) denotes the unit sphere in V , then e.S(n-1) generates Spin ... In addition ... Proposition 10.21 (n = dim(V) >= 3). The group Spin is the subgroup of Cl*(V) ... of invertible elements in ... Cl(V) ... generated by G(2,V) ...".

What is the physical difference between Pin and Spin?

Roughly, Pin has reflections and so can map fermion particles into fermion antiparticles. In the example of Cl(8) = M(R,16) , Pin(8) sees spinors as 1x 16 columns like

$$
\begin{array}{cccccccccccccccc}
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
x & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array}
=
\begin{array}{c}
x \\ x \\ x \\ x \\ x \\ x \\ x \\ x \\ x \\ x \\ x \\ x \\ x \\ x \\ x \\ x
\end{array}
$$

while Spin has no reflections, so Spin(8) sees spinors as two mirror-image sets of 8 +half-spinor particles and 8 -half-spinor antiparticles like

```
x 0 0 0 0 0 0 0                     x
x 0 0 0 0 0 0 0                     x
x 0 0 0 0 0 0 0                     x
x 0 0 0 0 0 0 0                     x
x 0 0 0 0 0 0 0                     x
x 0 0 0 0 0 0 0                     x
x 0 0 0 0 0 0 0                     x
x 0 0 0 0 0 0 0                     x
                                  =   +
                x 0 0 0 0 0 0 0         x
                x 0 0 0 0 0 0 0         x
                x 0 0 0 0 0 0 0         x
                x 0 0 0 0 0 0 0         x
                x 0 0 0 0 0 0 0         x
                x 0 0 0 0 0 0 0         x
                x 0 0 0 0 0 0 0         x
                x 0 0 0 0 0 0 0         x
```

In the quaternionic example of Cl(2,6) = M(Q,8) , Pin(2,6) sees spinors as

```
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0 = X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
X 0 0 0 0 0 0 0   X
```

while Spin(2,6) sees spinors as

```
X  0  0  0              X
X  0  0  0              X
X  0  0  0              X
X  0  0  0              X
                     =     +
        X  0  0  0            X
        X  0  0  0            X
        X  0  0  0            X
        X  0  0  0            X
```

In the quaternionic example of Cl(2,4) = Cl(6,0) = M(Q,4) , Pin sees spinors as

```
X  0  0  0     X
X  0  0  0  =  X
X  0  0  0     X
X  0  0  0     X
```

while Spin(2,4) (in my view where the even Cle(2,4) is taken to be Cl(1,4) = M(Q,2)+M(Q,2) instead of Cl(2,3) = M(C,4)) sees spinors as

```
X  0           X
X  0           X
            =     +
    X  0          X
    X  0          X
```

As to Cl(2,3) = M(C,4), my view is that its even Cle(2,3) is taken to be Cl(1,3) = M(Q,2) instead of Cl(2,2) = M(R(4).

As to Cl(1,4) = M(Q,2)+M(Q,2), my view is that even Cle(1,4) is taken to be Cl(1,3) = M(Q,2) = Cl(0,4).

In short, for E8 physics I form even subalgebras from Cl(2,6) on down to Cl(1,3) so that quaternionic structure is maintained.

I think that Pin is more fundamental than Spin because the overall symmetry should include reflections that can transform between particles and antiparticles, even though the particle-antiparticle distinction is useful in setting up the structure of the E8 model and its Lagrangian. However, Spin is more widely known than Pin, so sometimes (particularly in exposition) I write Spin when Pin would be technically more nearly correct.

E8 and Torsion

Much of this section is from the book Einstein Manifolds by Arthur L. Besse (Springer-Verlag 1987).

The Type EVIII rank 8 Symmetric Space, Rosenfeld's Elliptic Projective Plane (OxO)P2 is

E8 / Spin(16) = 64 + 64

The Octonionic structure of (OxO)P2 gives it a natural torsion structure

for which 64 looks like (8 fermion particles) x (8 Dirac Gammas)

and 64 looks like (8 fermion antiparticles) x (8 Dirac Gammas)

The Type BDI(8,8) rank 8 Symmetric Space real 8-Grassmannian manifold of R16 or set of the RP7 in RP15 is

Spin(16) / (Spin(8) x Spin(8)) = 64

Spin(16) is rank 8 and has 8+112 = 120 dimensions and looks like

a 64-dim Base Manifold

whose curvature is determined by a 28+28=56-dim Gauge Group Spin(8) x Spin(8)

The 64-dim Base Manifold looks like

(8-dim Kaluza-Klein spacetime) x (8 Dirac Gammas)

Due to the special isomorphisms Spin(6) = SU(4) and Spin(2) = U(1) and the topological equality RP1 = S1

Spin(8) / (Spin(6) x Spin(2)) = real 2-Grassmannian manifold of R8 or set of the RP1 in RP7

Spin(6) gives Conformal MacDowell-Mansouri Gravity

Spin(8) / U(4) = Spin(8) / SU(4) x U(1) = set of metric-compatible fibrations
S1 -> RP7 -> CP3

SU(4) / SU(3)xU(1) = CP3

SU(3) gives color force

U(1) gives electromagnetism

CP3 contains CP2 = SU(3) / U(1) x SU(2) and so gives SU(2) weak force

Torsion and E8 / Spin(16) = 64+64

Martin Cederwall and Jakob Palmkvist, in "The octic E8 invariant" hep-th/0702024, say:

> "... The largest of the finite-dimensional exceptional Lie groups, E8, with Lie algebra e8, is an interesting object ... its root lattice is the unique even self-dual lattice in eight dimensions (in euclidean space, even self-dual lattices only exist in dimension 8n). ... Because of self-duality, there is only one conjugacy class of representations, the weight lattice equals the root lattice, and there is no "fundamental" representation smaller than the adjoint. ... Anything resembling a tensor formalism is completely lacking. A basic ingredient in a tensor calculus is a set of invariant tensors, or "Clebsch&endash;Gordan coefficients". The only invariant tensors that are known explicitly for E8 are the Killing metric and the structure constants ...
>
> The goal of this paper is to take a first step towards a tensor formalism for E8 by explicitly constructing an invariant tensor with eight symmetric adjoint indices. ... On the mathematical side, the disturbing absence of a concrete expression for this tensor is unique among the finite-dimensional Lie groups. Even for the smaller exceptional algebras g2, f4, e6 and e7, all invariant tensors are accessible in explicit forms, due to the existence of "fundamental" representations smaller than the

adjoint and to the connections with octonions and Jordan algebras. ...

The orders of Casimir invariants are known for all finite-dimensional semi-simple Lie algebras. They are polynomials in U(g), the universal enveloping algebra of g, of the form t_(A1...Ak) T^(A1 . . . TAk), where t is a symmetric invariant tensor and T are generators of the algebra, and they generate the center U(g)^(g) of U(g). The Harish-Chandra homomorphism is the restriction of an element in U(g)^(g) to a polynomial in the Cartan subalgebra h, which will be invariant under the Weyl group W(g) of g. Due to the fact that the Harish-Chandra homomorphism is an isomorphism from U(g)^(g) to U(h) W(g) one may equivalently consider finding a basis of generators for the latter, a much easier problem. The orders of the invariants follow more or less directly from a diagonalisation of the Coxeter element, the product of the simple Weyl reflections ...

In the case of e8, the center U(e8)^(e8) of the universal enveloping subalgebra is generated by elements of orders 2, 8, 12, 14, 18, 20, 24 and 30. The quadratic and octic invariants correspond to primitive invariant tensors in terms of which the higher ones should be expressible. ... the explicit form of the octic invariant is previously not known ...

E8 has a number of maximal subgroups, but one of them, Spin(16)/Z2, is natural for several reasons. Considering calculational complexity, this is the subgroup that leads to the smallest number of terms in the Ansatz. Considering the connection to the Harish-Chandra homomorphism, K = Spin(16)/Z2 is the maximal compact subgroup of the split form G = E8(8). The Weyl group is a discrete subgroup of K, and the Cartan subalgebra h lies entirely in the coset directions g/k ...

We thus consider the decomposition of the adjoint representation of E8 into representations of the maximal subgroup Spin(16)/Z2. The adjoint decomposes into the adjoint 120 and a chiral spinor 128. ...

Our convention for chirality is GAMMA_(a1...a16) PHI = + e_(a1...a16) PHI . The e8 algebra becomes (2.1)

[T^(ab) , T^(cd)] = 2 delta^([a)_([c) T^(b])_(d]) ,

[T^(ab) , PHI^(alpha)] = (1/4) (GAMMA^(ab) PHI)^(alpha) ,

[PHI^(alpha) , PHI^(alpha)] = (1/8) (GAMMA_(ab))^(alpha beta) T^(ab) ,

... The coefficients in the first and second commutators are related by the so(16) algebra. The normalisation of the last commutator is free, but is fixed by the choice for the quadratic invariant, which for the case above is

X2 = (1/2) T_(ab) T^(ab) + PHI_(alpha) PHI^(alpha) .

Spinor and vector indices are raised and lowered with delta . Equation (2.1) describes the compact real form, E8(-248) .

By letting PHI -> i PHI one gets E8(8), where the spinor generators are non-compact, which is the real form relevant as duality symmetry in three dimensions (other real forms contain a non-compact Spin(16)/Z2 subgroup).

The Jacobi identities are satisfied thanks to the Fierz identity

(GAMMA_(ab)_[(alpha beta) (GAMMA_(ab)_(alpha beta)] = 0 ,

which is satisfied for so(8) with chiral spinors, so(9), and so(16) with chiral spinors

(in the former cases the algebras are so(9), due to triality, and f4).

The Harish-Chandra homomorphism tells us that the "heart" of the invariant lies in an octic Weyl-invariant of the Cartan subalgebra. A first step may be to lift it to a unique Spin(16)/Z2-invariant in the spinor, corresponding to applying the isomorphism fÅ|1 above. It is gratifying to verify ... that there is indeed an octic invariant (other than (PHI PHI)^4), and that no such invariant exists at lower order. ...

Forming an element of an irreducible representation containing a number of spinors involves symmetrisations and subtraction of traces, which can be rather complicated. This becomes even more pronounced when we are dealing with transformation ... under the spinor generators, which will transform as spinors. Then irreducibility also involves gamma-trace conditions. ...

The transformation ... under the action of the spinorial generator is an so(16) spinor. The vanishing of this spinor is equivalent to e8 invariance. The spinorial generator acts similarly to a supersymmetry generator on a superfield ...

The final result for the octic invariant is, up to an overall multiplicative constant:

$$
\begin{aligned}
X_8 = \tfrac{1}{3072}&\varepsilon^{a_1\ldots a_{16}}T_{a_1a_2}\ldots T_{a_{15}a_{16}} \\
&-30\mathrm{tr}T^8+14\mathrm{tr}T^6\mathrm{tr}T^2+\tfrac{35}{4}(\mathrm{tr}T^4)^2-\tfrac{35}{8}\mathrm{tr}T^4(\mathrm{tr}T^2)^2+\tfrac{15}{64}(\mathrm{tr}T^2)^4 \\
&+\big[2\mathrm{tr}T^6-\mathrm{tr}T^4\mathrm{tr}T^2+\tfrac{1}{8}(\mathrm{tr}T^2)^3\big](\phi\phi) \\
&\quad+\big[(\tfrac{5}{4}\mathrm{tr}T^4-\tfrac{1}{2}(\mathrm{tr}T^2)^2)T^{ab}T^{cd}+\tfrac{27}{4}\mathrm{tr}T^2T^{ab}(T^3)^{cd} \\
&\qquad-15T^{ab}(T^5)^{cd}-15(T^3)^{ab}(T^3)^{cd}\big](\phi\Gamma_{abcd}\phi) \\
&\quad+\big[\tfrac{1}{16}\mathrm{tr}T^2T^{ab}T^{cd}T^{ef}T^{gh}-\tfrac{5}{8}T^{ab}T^{cd}T^{ef}(T^3)^{gh}\big](\phi\Gamma_{abcdefgh}\phi) \\
&\quad-\tfrac{1}{192}T^{ab}T^{cd}T^{ef}T^{gh}T^{ij}T^{kl}(\phi\Gamma_{abcdefghijkl}\phi) \\
&+\big[7\mathrm{tr}T^4-\tfrac{31}{8}(\mathrm{tr}T^2)^2\big](\phi\phi)^2 \\
&\quad-\tfrac{3}{64}T^{ab}T^{cd}T^{ef}T^{gh}(\phi\phi)(\phi\Gamma_{abcdefgh}\phi) \\
&\quad+\big[\tfrac{5}{64}T^{ab}T^{cd}T^{ef}T^{gh}-\tfrac{15}{16}T^{ab}T^{ce}T^{df}T^{gh} \\
&\qquad+\tfrac{5}{8}T^{ae}T^{bf}T^{cg}T^{dh}\big](\phi\Gamma_{abcd}\phi)(\phi\Gamma_{efgh}\phi) \\
&\quad+\big[\tfrac{3}{2}(T^3)^{ab}T^{cd}-\tfrac{1}{8}\mathrm{tr}T^2T^{ab}T^{cd}\big](\phi\phi)(\phi\Gamma_{abcd}\phi) \\
&\quad+\big[\tfrac{15}{16}(T^3)^{ab}T^{cd}-\tfrac{3}{16}\mathrm{tr}T^2T^{ab}T^{cd}+\tfrac{5}{4}(T^2)^{ac}(T^2)^{bd}\big](\phi\Gamma_{ab}{}^{ij}\phi)(\phi\Gamma_{cdij}\phi) \\
&\quad+\tfrac{15}{8}T^{ab}T^{cd}(T^2)^{ef}(\phi\Gamma_{abe}{}^{i}\phi)(\phi\Gamma_{cdfi}\phi) \\
&+\tfrac{1}{2}\mathrm{tr}T^2(\phi\phi)^3+\tfrac{55}{32}T^{ab}T^{cd}(\phi\phi)^2(\phi\Gamma_{abcd}\phi) \\
&\quad+\tfrac{1}{8}T^{ab}T^{cd}(\phi\phi)(\phi\Gamma_{ab}{}^{ij}\phi)(\phi\Gamma_{cdij}\phi) \\
&\quad+\big[-\tfrac{1}{384}T^{ab}T^{cd}+\tfrac{7}{192}T^{ac}T^{bd}\big](\phi\Gamma_{ab}{}^{ij}\phi)(\phi\Gamma_{cd}{}^{kl}\phi)(\phi\Gamma_{ijkl}\phi) \\
&-\tfrac{57}{32}(\phi\phi)^4+\tfrac{1}{12288}(\phi\Gamma_{ab}{}^{cd}\phi)(\phi\Gamma_{cd}{}^{ef}\phi)(\phi\Gamma_{ef}{}^{gh}\phi)(\phi\Gamma_{gh}{}^{ab}\phi) \\
&+\beta[-\tfrac{1}{2}\mathrm{tr}T^2+(\phi\phi)]^4\ .
\end{aligned}
\tag{2.3}
$$

Here, β is an arbitrary constant multiplying the fourth power of the quadratic invariant. The trace vanishes for $\beta = \frac{9}{127}$ (that such a value exists at all is non-trivial and provides a further check on the coefficients). The occurrence of the prime 127 is not incidental; taking the trace of $\delta^{(AB}\delta^{CD}\delta^{EF}\delta^{GH)}$ gives $\delta_{GH}\delta^{(AB}\delta^{CD}\delta^{EF}\delta^{GH)} = (\frac{1}{7}\cdot 248+\frac{6}{7})\delta^{(AB}\delta^{CD}\delta^{EF)} = \frac{2\cdot 127}{7}\delta^{(AB}\delta^{CD}\delta^{EF)}$. The actual technique we use for calculating the trace is not to extract the eight-index tensor, but to act on the invariant with $\frac{1}{2}\frac{\partial}{\partial T_{ab}}\frac{\partial}{\partial T^{ab}}+\frac{\partial}{\partial\phi_\alpha}\frac{\partial}{\partial\phi^\alpha}$. We remind that eq. (2.3) gives the octic invariant for the compact form $E_{8(-248)}$. The corresponding expression for the split form $E_{8(8)}$ is obtained by a sign change of the terms containing ϕ^{4k+2}.

...".

Martin Cederwall, in hep-th/9310115, says:

"... The only simply connected compact parallelizable manifolds are the Lie groups and S7. If these vectorfields exist one can use them to define parallel transport of vectors. Since transport around any closed curve gives back the same vector, the curvature of the corresponding connection vanishes. We can think of the manifold equipped with this connection as "flat", and the transport as translation.

If the parallelizing connection is written as GAMMA~ = GAMMA - T where GAMMA is the metric connection, the vielbeins will not be covariantly constant, but transport as De = T (T is torsion, and this can be taken as its definition). Then ...

[D_a , D_b] = 2 T_ab^c D_c

... These are our S7 transformations ... What distinguishes S7 from the Lie groups is that its torsion ("structure constants") vary over the space. ... ".

Martin Cederwall and Christian R. Preitschopf, in hep-th/0702024, say:

"... it is the non-associativity of O that is responsible for the non-constancy of the torsion tensor [for S7] (while the non-commutativity accounts for its non-vanishing) and for the necessity of utilizing inequivalent products associated with different points X Å_S7. We call this field-dependent multiplication the X-product.

One should note that the transformation ...[for S7]... relies on the transformation of the parameter field X ... while for group manifolds (and thus for the lower-dimensional spheres S1 and S3 associated with C and H) ...[the transformation is independent of a parameter field]... transform independently. A consequence is that fermions cannot transform without the presence of a parameter field, since a fermionic octonion is not invertible. ... Fermions, due to non-invertibility, can be assigned to endpoints of the diagram only; no path may pass via a fermion. ...

We call a field (bosonic or fermionic) transforming according to ...[the X-product]... a spinor under S7. ...

Let r, s, ... be S7 spinors ... Can this representation be formed as a tensor product of spinor representations? Due to the non-linearity, the answer is no.... we can form spinors as trilinears of spinors u = (r ox s*) ox t , and in this way only. ...

It should be possible to realize E6 = SL(3;O) ... on them in a "spinor-like" manner, much like SO(10) = SL(2;O) acts on its 16-dimensional spinor representations that play the role of homogeneous coordinates for OP1 ...

That would open for for a twistor transform ... for elements in J3(O) (the exceptional Jordan algebra of 3x3 hermitiean octonionic matrices) with zero Freudenthal product - a known realization of OP2. Then one would have a direct analogy to the twistor transform of the masslessness condition in SL(2;O) that leads to OP1 as the projective light-cone ...

we would like to address the question of anomaly cancellation: under what circumstances is the Schwinger term "quantum mechanically consistent", i.e. when is the BRST operator quantum mechanically nilpotent, and what actual exact form of the Schwinger term is needed? ... to construct a (classical) BRST operator for the S7 algebra with field-dependent structure functions ... turns out to be extremely simple. The BRST operator takes the same form as for a Lie algebra, namely

$$Q = c^i J_i - T_{ij}^k(X) c^i c^j b_k$$

where b_i and c^i are fermionic ghosts ... Higher order ghost terms are not present since the Jacobi identities hold ... This makes BRST analysis quite manageable. ...

Then, turning to ... the quantum algebra, ... We have ... demonstrated the non-trivial fact that Q may be nilpotent, and that ... non-trivial central extensions ...[of S7]... or Schwinger terms ... may be used as a gauge algebra. Normally, one would have expected $Q^2=0$ to put a constraint on the number of transforming octonionic fields, but that is not the case at hand. Instead one is permitted, for any field content, to adjust the numerical coefficient ... in J in order to fulfil that relation ...

It seems that ... the S7 or ... non-trivial central extensions ...[of S7]... or Schwinger terms ...ghosts do not come in an S7 representation. This is also confirmed by an attempt to construct a representation (other than scalar) for imaginary octonions, which turns out to be impossible. ...

A part of the structure of S7 we have treated only fragmentarily is representation theory. ... It is not immediately clear even how to define a representation. We have quite strong feelings, though, that the spinorial representations and the adjoint, as described in this paper, in some sense are the only ones allowed, and that the spinor representation is the only one to which a variable freely can be assigned. ...".

Coleman-Mandula

Garrett Lisi said (in comments to Bee's blog entry about the E8 model) that a "... condition ...[of]... the Coleman-Mandula theorem ... is that there needs to be a Poincare' subgroup. There is no Poincare' subgroup in this E8 theory. ... The G = E8 I [Garrett Lisi] am using does not contain a subgroup locally isomorphic to the Poincare group, it contains the subgroup SO(4,1) -- the symmetry group of deSitter spacetime. ...this theorem does not apply in this case. ...".

However,

Steven Weinberg showed at pages 12-22 of his book The Quantum Theory of Fields, Vol. III (Cambridge 2000) that Coleman-Mandula is not restricted to the Poincare Group, but extends to the Conformal Group as well.

Since the Conformal Group SO(4,2) contains Garrett's de Sitter SO(4,1) as a subgroup, it seems to me that it is incorrect to claim that use of deSitter SO(4,1) means that Coleman-Mandula "... does not apply ..." to the E8 model.

There is also another argument for consistency of the E8 with Coleman-Mandula:

With respect to Coleman-Mandula (particularly with respect to fermions) it is useful to compare

what Bee said:

"... the five exceptional Lie-groups have the remarkable property that the adjoint action of a subgroup is the fundamental subgroup action on other parts of the group. This then offers the possibility to arrange both, the fermions as well as the gauge fields, in the Lie algebra and root diagram of a single group ..."

with what Steven Weinberg said at pages 382-384 of his book The Quantum Theory of Fields, Vol. III (Cambridge 2000):

"... The proof of the Coleman-Mandula theorem ... makes it clear that the list of possible bosonic symmetry generators is essentially the same in d greater

than 2 spacetime dimensions as in four spacetime dimensions:
... there are only the momentum d-vector Pu, a Lorentz generator Juv = -Jvu (with u and v here running over the values 1, 2, ... , d-1, 0), and various Lorentz scalar 'charges' ...
the fermionic symmetry generators furnish a representation of the homogeneous Lorentz group ... or, strictly speaking, of its covering group Spin(d-1,1). ...
The anticommutators of the fermionic symmetry generators with each other are bosonic symmetry generators, and therefore must be a linear combination of the Pu, Juv, and various conserved scalars. ...
the general fermionic symmetry generator must transform according to the fundamental spinor representations of the Lorentz group ... and not in higher spinor representations, such as those obtained by adding vector indices to a spinor. ...".

In short, Weinberg's book at pages 382-384 says that the important thing about Coleman-Mandula is that fermions in a unified model must "... transform according to the fundamental spinor representations of the Lorentz group ... or, strictly speaking, of its covering group Spin(d-1,1). ..." where d is the dimension of spacetime in the model.

As I said in that comment, E8 is the sum of the adjoint representation and a half-spinor representation of Spin(16),
and
the Spin(16) structure leads (since Cl(16) = Cl(8) x Cl(8)) to Spin(8) or Spin(1,7) spacetime structure
and
the fermionic fundamental spinor representations of the E8 model are therefore built with respect to Lorentz, spinor, etc representations based on Spin(1,7) spacetime consistently with Weinberg's work,
so
the E8 model is consistent with Coleman-Mandula.

The Cl(8) model that underlies the E8 model is, due to compliance with the criteria set out by Weinberg, consistent with Coleman-Mandula (see my web page at
www.valdostamuseum.org/hamsmith/d4d5e6hist2.html#ColemanMandula)

Some History of my Physics Work

In the 1960s-early 1970s Armand Wyler wrote a calculation of the fine structure constant using geometry of bounded complex domains. It was publicized briefly (almost as much as Garrett Lisi's E8 model is publicized now) but Wyler never showed convincing physical motivation for his interpretation of the math structures, and it was severely ridiculed and ignored (with sad personal consequences for Wyler).

Also in the 1960s, Joseph Wolf classified 4-dim spaces with quatenionic structure:

- (I) Euclidean 4-space [the 4-torus T4];
- (II) SU(2) / S(U(1)xU(1)) x SU(2) / S(U(1)xU(1)), … [S2 x S2] …;
- (III) SU(3) / S(U(2)xU(1)), … [CP2] …; and
- (IV) Sp(2) / Sp(1)xSp(1) … [= Spin(5) / Spin(4) = S4] …,

and the noncompact duals of II, III, and IV

and I noticed that they corresponded to

- U(1) electromagnetism,
- SU(2) weak force,
- SU(3) color force, and
- Sp(2) MacDowell-Mansouri gravity

so

I thought that it might possibly be useful to apply Wyler's approach to the geometries of those 4-dim quaternionic structures.

It was only in the 1980s that I was able to cut back on the time devoted to my law practice to try to learn enough math/physics to try to work out the application of Wyler's stuff to Wolf's classification, and I did so by spending a lot of time at Georgia Tech auditing seminars etc of David Finkelstein (who was tolerant enough to allow me to do so). I had learned some Lie group / Lie algebra math while an undergrad at Princeton (1959-63), but I did not know Clifford algebras very well until studying under David Finkelstein.

Then (early 1980s) N=8 supergravity was popular, so I looked at SO(8) and its cover Spin(8), and noticed that:

- Adjoint Spin(8) had 28 gauge bosons enough to do MacDowell-Mansouri gravity plus the Standard Model, but not if they were included as conventional subgroups;
- Vector Spin(8) looked like 8-dim spacetime;
- +half-spinor Spin(8) looked like 8 left-handed first-generation fermions;
- -half-spinor Spin(8) looked like 8 right-handed first-generation fermions.

To break the 8-dim spacetime into a 4-dim physical spacetime plus a 4-dim internal symmetry space I used the geometric methods that had been developed by Meinhard Mayer (working with Andrzej Trautman) around 1981.

A consequence of that dimensional reduction was second and third generations of fermions as composites (pairs and triples) of states corresponding to the first-generation fermions.

When I played with the Wyler-type geometry stuff, I got particle masses that looked roughly realistic, and a (then) prediction-calculation of the Tquark mass as around 130 GeV (tree-level, so give or take 10% or so).

When in 1984 CERN announced at APS DPF Santa Fe that they had seen the Tquark at 45 GeV, I gave a talk there (not nearly as well-attended as Carlo Rubbia's) saying that CERN was wrong and the Tquark was more massive (I will not here go into subsequent history of Dalitz, Goldstein, Sliwa, CDF, etc except to say that I still feel that experimental data supports the Tquark having a low-mass state around 130-145 GeV, and that the politics related to my position may have something to do with my current outcast status with the USA physics establishment.)

Since Spin(8) is bivector Clifford algebra of the real Clifford algebra Cl(8), and since real Clifford algebra 8-periodicity means that any very large real Clifford algebra can be factored into tensor products of Cl(8), it can be a building block of a nice big algebraic QFT (a real version of the complex hyperfinite II1 von Neumann factor).

Since the Adjoint, Vector, and two half-Spinor reps of Spin(8) combine to form the exceptional Lie algebra F4, I tried to use it as a unifying Lie algebra,

but I eventually saw that the real structure of F4 was incompatible with the complex bounded domain structures of the Wyler approach, so I went to E6, which is roughly a complexification of F4, and used E6 to construct a substantially realistic version of 26-dim bosonic string theory (fermions coming from orbifolding). Since by then I was blacklisted by the Cornell arXiv, I put that up on the CERN website as CERN-CDS-EXT-2004-031

As of then, the major conventional objection to my model was how I got 16 generators for a MacDowell-Mansouri gravity U(2,2) and 12 generators for the Standard Model from the 28 generators of Spin(8) (I used root vector patterns, because they do not consistently fit as subgroups and subalgebras).

Now, Garrett Lisi's E8 model has two copies of the D4 Spin(8) Lie algebra, so I can use it to be more conventional and get MacDowell-Mansouri gravity from one D4 and the Standard Model from the other one.

I apologize for, in trying to be brief, leaving out a lot of people who helped me learn stuff, including but not limited to people at the University of Alabama and Robert Gilmore at Drexel and others.

PS - I should add that while at Georgia Tech in the late 1980s -early 1990s I enrolled in the physics PhD program, but that ended when I encountered the comprehensive exam (a 3-day closed book test) which I could not pass (my then 50-year-old memory had trouble recalling formulas), so I am in that sense a failure without official PhD qualification.

E6 and Bosonic World-Line Strings

In hep-th/0112261 Pierre Ramond said:

"... The traceless Jordan matrices [J3(O)o] ... (3x3) traceless octonionic hermitian matrices, each labelled by 26 real parameters ... span the 26 representation of [the 52-dimensional exceptional Lie algebra F4]. One can supplement the F4 transformations by an additional 26 parameters ... leading to a group with 78 parameters. These extra transformations are non-compact, and close on the F4 transformations, leading to the exceptional group E6(-26). The subscript in parenthesis denotes the number of non-compact minus the number of compact generators. ...".

The following is my proposal to use the exceptional Lie algebra E6(-26), which I will for the rest of this message write as E6, to introduce fermions into string theory based on the exceptional E6 relations between bosonic vectors/bivectors and fermionic spinors,
in which 16 of the 26 dimensions are seen as orbifolds whose 8 + 8 singularities represent first-generation fermion particles and antiparticles.

This structure allows string theory to be physically interpreted as a theory of interaction among world-lines in the Many-Worlds.

According to Soji Kaneyuki, in Graded Lie Algebras, Related Geometric Structures, and Pseudo-hermitian Symmetric Spaces, Analysis and Geometry on Complex Homogeneous Domains, by Jacques Faraut, Soji Kaneyuki, Adam Koranyi, Qi-keng Lu, and Guy Roos (Birkhauser 2000):

E6 as a Graded Lie Algebra with 5 grades:

g = E6 = g(-2) + g(-1) + g(0) + g(1) + g(2)

such that

g(0) = so(8) + R + R

dimR g(-1) = dimR g(1) = 16 = 8 + 8

dimR g(-2) = dimR g(2) = 8

Here, step-by-step, is a description of the E6 structure:

Step 1:

```
g(0) = so(8)                                            28 gauge bosons

                                                   _
             + R + R                                |
                                                    |
dimR g(-1) = dimR g(1) = 16 = 8 + 8                 |-   26-dim string
spacetime
                                                    |      with J3(O)o
structure
dimR g(-2) = dimR g(2) = 8                         _|
```

Step 2:

The E6 GLA has an Even Subalgebra gE (Bosonic)
and an Odd Part gO (Fermionic):

BOSONIC gE = g(-2) + g(0) + g(2)

FERMIONIC gO = g(-1) + g(1)

Step 3:

```
BOSONIC

g(0) = so(8)                                       28 gauge bosons

                                                 _
             + R + R                              |
dimR g(-2) = dimR g(2) = 8                        |-   10-dim spacetime
                                                 _|

FERMIONIC

dimR g(-1) = dimR g(1) = 16 = 8                     8-dim orbifold
                               +
                                 8                  8-dim orbifold
```

Giving the Fermionic sector orbifold structure gives each point of the string/world-line a discrete value corresponding to one of the 8+8 = 16 fundamental first-generation fermion particles or antiparticles.

Step 4:

```
BOSONIC

g(0) = so(8)                                       28 gauge bosons

                                                 _
             + R + R                              |
dimR g(-2) = dimR g(2) = 8                        |-   10-dim spacetime
                                                 _|
FERMIONIC

dimR g(-1) = dimR g(1) = 16 = 8                     8 fermions
                               +
                                 8                  8 antifermions
```

Step 5:

```
BOSONIC

                                                   16-dim conformal
U(2,2)
g(0) = so(8)                                             +
                                                   12-dim
SU(3)xSU(2)xU(1)

                                               _|
             + R + R                            |
dimR g(-2) = dimR g(2) = 4                      |-    6-dim conformal
spacetime
                                               _|
                           +
                             4                        4-dim internal
symmetry space


FERMIONIC

dimR g(-1) = dimR g(1) = 16 = 8                       8 fermions (3
gen)

                                  + 8                 8 antifermions
(3 gen)
```

Dimensional reduction of spacetime breaks so(8) to U(2,2) and SU(3)xSU(2)xU(1) and also introduces 3 generations of fermion particles and antiparticles.

Step 6:

```
BOSONIC

                                             16-dim conformal
U(2,2)
g(0) = so(8)                                       +
                                             12-dim
SU(3)xSU(2)xU(1)


             + R + R                          2 spacetime
conformal dim

dimR g(-2) = dimR g(2) = 4                    4-dim physical
spacetime
                            +
                              4               4-dim internal
symmetry space


FERMIONIC

dimR g(-1) = dimR g(1) = 16 = 8               8 fermions (3
gen)

                                 + 8          8 antifermions
(3 gen)
```

The 2 spacetime conformal dimensions R+R are related to complex structure of

spacetime g(-2) + g(2)

and

fermionic g(-1) + g(1).

Physical spacetime and internal symmetry space, and fermionic representation spaces, are related to Shilov boundaries of the corresponding complex domains.

The E6 String Structure described above allows construction of a Realistic String Theory:

This construction was motivated by a March 2004 sci.physics.research thread Re: photons from strings? in which John Baez asked:

"... has anyone figured out a way to ... start with string theory ... to get just photons on Minkowski spacetime ..." ?

Lubos Motl noted "... string theory always contains gravity ... Gravity is always contained as a vibration of a closed string, and closed strings can always be created from open strings....".

Urs Schreiber said "... the low energy effective worldsheet theory of a single flat D3 brane of the bosonic string is, to lowest nontrivial order, just U(1) gauge theory in 4D ...".

Aaron Bergman noted "... there are a bunch of scalars describing the transverse fluctuations of the brane ...".

Urs Schreiber said "... I guess that's why you have to put the brane at the singularity of an orbifold if you want to get rid of the scalars ... if the number of dimensions is not an issue the simplest thing probably would be to consider the single space-filling D25 brane of the bosonic string. This one does not have any transverse fluctuations and there is indeed only the U(1) gauge field ...".

Aaron Bergman replied "... Unfortunately, there's a tadpole in that configuration. You need 8192 D25 branes to cancel it. ...".

Lubos Motl pointed out the existence of brane structures other than massless vectors, saying "... A D-brane contains other massless states, e.g. the transverse scalars (and their fermionic superpartners). It also contains an infinite tower of excited massive states. Finally, a D-brane in the full string theory is coupled to the bulk which inevitably contains gravity as well as other fields and particles. ... N coincident D-branes carry a U(N) gauge symmetry (and contain the appropriate gauge N^2 bosons, as you explained). Moreover, if this stack of N D-branes approaches an orientifold, they meet their mirror images and U(N) is extended to O(2N) or USp(2N). The brane intersections also carry new types of matter - made of the open strings stretched from one type of brane to the other - but these new fields are *not* gauge fields, and they don't lead to new gauge symmetries. For example, there are scalars whose condensation is able to join two intersecting D2-branes into a smooth, connected, hyperbolically shaped objects (D2-branes). ... the number of D-branes can be determined or bounded by anomaly cancellation and similar requirements. For example, the spacetime filling D9-branes in type I theory must generate the SO(32) gauge group, otherwise the theory is anomalous. (There are other arguments for this choice of 16+16 branes, too.)...".

What follows is my construction of

a specific example of a String Theory with E6 structure containing gravity and the U(1)xSU(2)xSU(3) Standard Model.

As to how the E6 String model is affected by matters raised as objections by Lubos Motl:

Transverse scalars are taken care of by Orbifolding as suggested by Urs Schreiber.
Fermionic superpartners are taken care of by not using naive 1-1 fermion-boson supersymmetry.
The infinite tower of excited massive states is related to Regge trajectories which in turn are related to interactions among strings considered as world-lines in the Many-Worlds.
Bulk gravity is included.
There are no orientifolds.
Open strings from one brane to another, as vacuum loops, look like exchange of closed loops and are related to gravity among branes and the Bohm-type quantum potential.
Scalar condensates are related to Dilatons which in turn are related to interactions among strings considered as world-lines in the Many-Worlds.
I have not fully investigated all potential anomaly problems.

Further, string theory Tachyons are related to interactions among strings considered as world-lines in the Many-Worlds.

Lubos Motl, in his blog entry Tachyons and the Big Bang at http://motls.blogspot.com/ dated 13 July 2005, said:

"... closed string tachyons ... signal an instability of the whole spacetime ... closed string tachyons ... can be localized if they appear in a twisted sector of an orbifold ... the twisted closed strings describe fields that are localized at the origin ... The tachyons condense near the tip which smears out the tip of the cone which makes the tip nice and round. ...".

Closed string tachyons localized at an orbifold may be physically equivalent to what Schroer describes in http://xxx.lanl.gov/abs/hep-th/9908021 as "... any compactly localized operator applied to the vacuum generates clouds of pairs of particle/antiparticles, unless the system is free ..."

and to Dirac's 1938 Dirac-Lorentz equation model of the electron as described in pages 194-195 of Dirac: A Scientific Biography,by Helge Kragh (Cambridge 1990):

"... Dirac explained that the strange behavior of electrons in this theory could be understood if the electron was thought of as an extended particle with a nonlocal interior. He suggested that the point electron, embedded in its own radiation field, be interpreted as a sphere of radius a, where a is the distance within which an incoming pulse must arrive before the electron accelerates appreciably. With this interpretation he showed that it was possible for a signal to be propagated faster than light through the interior of the electron. ... In spite of the appearance of superluminal velocities, Dirac's theorywas Lorentz-invariant. ...".

In short, if orbifolds are identified with fermion particles, then their localized tachyons can be physically interpreted as describing the virtual particle-antiparticle clouds that dress the fermion particles.

In short,

Lubos Motl's objectrions are either taken care of in the construction of the model or are useful in describing the Bohm-type quantum potential interactions among strings considered as world-lines in the Many-Worlds.

Here is some further background, from Joseph Polchinski's book String Theory vol. 1 (Cambridge 1998), in Chapter 8 and the Glossary:

"... a ... D-brane ...[is]... a dynamical object ... a flat hyperplane ...[for which]... a certain open string state corresponds to fluctuation of its shape ...
... A D25-brane fills space, so the string endpoint can be anywhere ...
... When no D-branes coincide there is just one massless vector on each, giving the gauge group U(1)^n in all.
If r D-branes coincide, there are new massless states because string that are stretched between these branes can have vanishing length: ... Thus, there are r^2 vectors, forming the adjoint of a U(r) gauge group. ... there will also be r^2 massless scalars from the components normal to the D-brane. ...
... The massless fields on the world-volume of a Dp-brane are a U(1) vector plus 25 - p world-brane scalars describing the fluctuations. ... The fields on the brane are the embedding X^u(x) and the gauge field A_a(x) ...
... For n separated D-branes, the action is n copies of the action for a single D-brane. ... when the D-branes are coincident there are n^2 rather than n massless vectors and scalars on the brane ...
... The fields X^u(x) and A_a(x) will now be nxn matrices ...
... the gauge field ... becomes a non-Abelian U(n) gauge field ...
... the collectives coordinates ... X^u ... for the embedding of n D-branes in spacetime are now enlarged to nxn matrices. This 'noncommutative geometry' ...[may be]... an important hint about the nature of spacetime. ...
...[an]... orbifold ...(noun)...[is]... a coset space M/H, where H is a group of discrete symmetries of a manifold M. The coset is singular at the fixed points of H ...(verb)...[is]... to produce such a ... string theory by gauging H ...
... To determine the actual value of the D-brane tension ... Consider two parallel Dp-branes ...[They]... can feel each other's presence by exchanging closed strings ...[which is equivalent to]... a vacuum loop of an open string with one end on each D-brane ... The ... analogous ... field theory graph ... is the exchange of a single graviton or dilaton between the D-branes....".

Here, step-by-step, is the E6 World-Line String/Brane construction:

Step 1:

Consider the 26 Dimensions of String Theory as the 26-dimensional traceless part J3(O)o

a	O+	Ov
O+*	b	O-
Ov*	O-*	-a-b

(where Ov, O+, and O- are in Octonion space with basis {1,i,j,k,E,I,J,K} and a and b are real numbers with basis {1})
of the 27-dimensional Jordan algebra J3(O) of 3x3 Hermitian Octonion matrices.

Step 2:

Take Urs Schreiber's D3 brane to correspond to the Imaginary Quaternionic associative subspace spanned by {i,j,k} in the 8-dimenisonal Octonionic Ov space.

Step 3:

Compactify the 4-dimensional co-associative subspace spanned by {E,I,J,K} in the Octonionic Ov space as a CP2 = SU(3)/U(2), with its 4 world-brane scalars corresponding to the 4 covariant components of a Higgs scalar.
Add this subspace to D3, to get D7.

Step 4:

Orbifold the 1-dimensional Real subspace spanned by {1} in the Octonionic Ov space by the discrete multiplicative group Z2 = {-1,+1}, with its fixed points {-1,+1} corresponding to past and future time. This discretizes time steps and gets rid of the world-brane scalar corresponding to the subspace spanned by {1} in Ov. It also gives our brane a 2-level timelike structure, so that its past can connect to the future of a preceding brane and its future can connect to the past of a succeeding brane.
Add this subspace to D7, to get D8.
D8, our basic Brane, looks like two layers (past and future) of D7s.

Beyond D8 our String Theory has 26 - 8 = 18 dimensions, of which 25 - 8 have corresponding world-brane scalars:

8 world-brane scalars for Octonionic O+ space;
8 world-brane scalars for Octonionic O- space;
1 world-brane scalars for real a space; and
1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 5:

To use Urs Schreiber's idea to get rid of the world-brane scalars corresponding to the Octonionic O+ space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O+ to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}.

Let the 8 O+ singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the past D7 layer of D8.

Let the 8 O+ singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion particles {neutrino, red up quark, green up quark, blue up quark, electron, red down quark, green down quark, blue down quark} located on the future D7 layer of D8.

This gets rid of the 8 world-brane scalars corresponding to O+, and leaves:

8 world-brane scalars for Octonionic O- space;
1 world-brane scalars for real a space; and
1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 6:

To use Urs Schreiber's idea to get rid of the world-brane scalars corresponding to the [Octonionic O- space, orbifold it by the 16-element discrete multiplicative group Oct16 = {+/-1,+/-i,+/-j,+/-k,+/-E,+/-I,+/-J,+/-K} to reduce O- to 16 singular points {-1,-i,-j,-k,-E,-I,-J,-K,+1,+i,+j,+k,+E,+I,+J,+K}].

Let the 8 O- singular points {-1,-i,-j,-k,-E,-I,-J,-K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the past D7 layer of D8.

Let the 8 O- singular points {+1,+i,+j,+k,+E,+I,+J,+K} correspond to the fundamental fermion anti-particles {anti-neutrino, red up anti-quark, green up anti-quark, blue up anti-quark, positron, red down anti-quark, green down anti-quark, blue down anti-quark} located on the future D7 layer of D8.

This gets rid of the 8 world-brane scalars corresponding to O-, and leaves:

1 world-brane scalars for real a space; and
1 dimension, for real b space, in which the D8 branes containing spacelike D3s are stacked in timelike order.

Step 7:

Let the 1 world-brane scalar for real a space correspond to a Bohm-type Quantum Potential acting on strings in the stack of D8 branes.

Interpret strings as world-lines in the Many-Worlds, short strings representing virtual particles and loops.

Step 8:

Fundamentally, physics is described on HyperDiamond Lattice structures.

There are 7 independent E8 lattices, each corresponding to one of the 7 imaginary octionions. They can be described as iE8, jE8, kE8, EE8, IE8, JE8, and KE8.

Further, an 8th naturally related, but dependent, E8 lattice corresponds to the real octonions and can be described as 1E8.

Give each D8 brane structure based on Planck-scale E8 lattices so that each D8 brane is a superposition/intersection/coincidence of the eight E8 lattices.

Step 9:

Since Polchinski says "... If r D-branes coincide ... there are r^2 vectors, forming the adjoint of a U(r) gauge group ...", make the following assignments:

a gauge boson emanating from D8 only from its 1E8 lattice is a U(1) photon;
a gauge boson emanating from D8 only from its 1E8 and EE8 lattices is a U(2) weak boson;
a gauge boson emanating from D8 only from its IE8, JE8, and KE8 lattices is a U(3) gluon.

Note that I do not consider it problematic to have U(2) and U(3) instead of SU(2) and SU(3) for the weak and color forces, respectively. Here is some further discussion of the global Standard Model group structure. Here is some discussion of the root vector structures of the Standard Model groups.

Step 10:

Since Polchinski says "... there will also be r^2 massless scalars from the components normal to the D-brane. ... the collectives coordinates ... X^u ... for the embedding of n D-branes in spacetime are now enlarged to nxn matrices. This 'noncummutative geometry' ...[may be]... an important hint about the nature of spacetime. ...", make the following assignment:

The 8x8 matrices for the collective coordinates linking a D8 brane to the next D8 brane in the stack are needed to connect
the eight E8 lattices of the D8 brane
to the eight E8 lattices of the next D8 brane in the stack.

We have now accounted for all the scalars,
and, since, as Lubos Motl noted,
"... string theory always contains gravity ...",
we have here at Step 10 a specific example of a String Theory containing gravity and the U(1)xSU(2)xSU(3) Standard Model.

Step 11:

We can go a bit further by noting that we have not described gauge bosons emanating from D8 from its iE8, jE8, or kE8 lattices. Therefore, make the following assignment:

a gauge boson emanating from D8 only from its 1E8, iE8, jE8, and kE8 lattices is a U(2,2) conformal gauge boson.

We have here at Step 11 a String Theory containing the Standard Model plus two forms of gravity:

closed-string gravity giving Bohm Quantum Potential for Quantum Consciousness (see http://tony5m17h.net/ QMINDpaper.pdf) and conformal U(2,2) = Spin(2,4)xU(1) gravity plus conformal structures, based on a generalized MacDowell-Mansouri mechanism.

Step 12:

Going a bit further leads to consideration of the exceptional E-series of Lie algebras, as follows:

a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, and EE8 lattices is a U(5) gauge boson related to Spin(10) and Complex E6.
a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, EE8, and IE8 lattices is a U(6) gauge boson related to Spin(12) and Quaternionic E7.
a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, EE8, IE8, and JE8 lattices is a U(7) gauge boson related to Spin(14) and possibly to Sextonionic E(7+(1/2)).
a gauge boson emanating from D8 only from its 1E8, iE8, jE8, kE8, EE8, IE8, JE8, and KE8 lattices is a U(8) gauge boson related to Spin(16) and Octonionic E8.

These correspondences are based on the natural inclusion of U(N) in Spin(2N) and on Magic Square constructions of the E series of Lie algebras, roughly described as follows:
78-dim E6 = 45-dim Adjoint of Spin(10) + 32-dim Spinor of Spin(10) + Imaginary of C;
133-dim E7 = 66-dim Adjoint of Spin(12) + 64-dim Spinor of Spin(12) + Imaginaries of Q;
248-dim E8 = 120-dim Adjoint of Spin(16) + 128-dim half-Spinor of Spin(16)

Physically,

E6 corresponds to 26-dim String Theory, related to traceless J3(O)o and the symmetric space E6 / F4.
E7 corresponds to 27-dim M-Theory, related to the Jordan algebra J3(O) and the symmetric space E7 / E6 x U(1).
E8 corresponds to 28-dim F-Theory, related to the Jordan algebra J4(Q) and the symmetric space E8 / E7 x SU(2).

Note on Sextonions:

I am not yet clear about how the Sextonionic E(7+(1/2)) works. It was only recently developed by J. M. Landsberg and Laurent Manivel in their paper "The sextonions and $E_{7\frac 12}$" at math.RT/0402157. Of course, the Sextonion algebra is not a real division algebra, but it does have interesting structure. In their paper, Landsberg and Manivel say:

"... We fill in the "hole" in the exceptional series of Lie algebras that was observed by Cvitanovic, Deligne, Cohen and deMan. More precisely, we show that the intermediate Lie algebra between E_7 and E_8 satisfies some of the decomposition and dimension formulas of the exceptional simple Lie algebras. A key role is played by the sextonions, a six dimensional algebra between the quaternions and octonions. Using the sextonions, we show simliar results hold for the rows of an expanded Freudenthal magic chart. We also obtain new interpretations of the adjoint variety of the exceptional group G_2. ...
... the orthogonal space to a null-plane U, being equal to the kernel of a rank-two derivation, is a six-dimensional subalgebra of O. ...

... The decomposition ... into the direct sum of two null-planes, is unique. ...[this]... provides an interesting way to parametrize the set of quaternionic subalgebras of O. ...".

Some possibly related facts of which I am aware include:

The set of Quaternionic subalgebras of Octonions = SU(3) = G2 / Spin(4).

G2 / SU(3) = S6 is almost complex but not complex and is not Kaehler. Its almost complex structure is not integrable. See chapter V of Curvature and Homology, rev. ed., by Samuel I. Goldberg (Dover 1998).

It may be that the sextonions and S6 are related to Spin(4) as the 6-dim conformal vector space of SU(2,2) = Spin(2,4) is related to 4-dim Minkowski space.

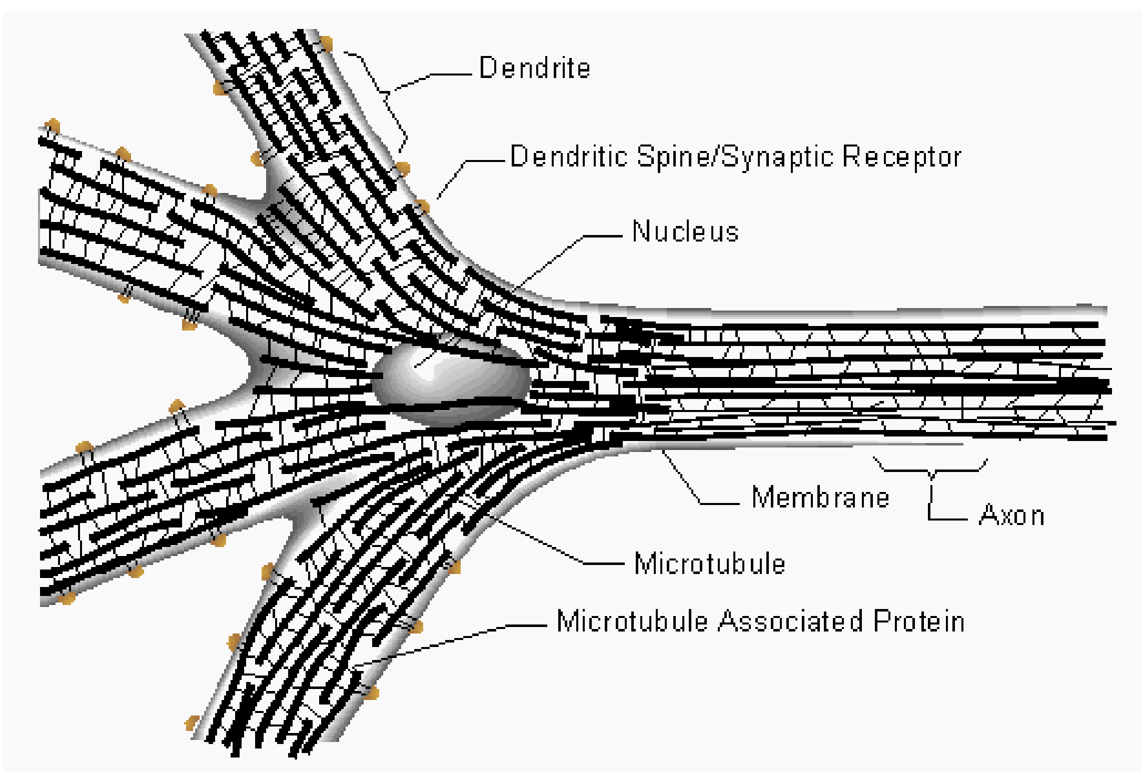
Quantum Consciousness

Penrose and Hameroff have proposed that consciousness in the human brain may be based on gravitational interactions and quantum superposition states of electrons in tubulin cages in microtubules.
Chiao has proposed experimental construction of a gravity antenna that might be analogous to tubulin caged electrons.

Tegmark has criticized Penrose-Hameroff quantum consciousness, based on thermal decoherence of any such quantum superposition states, but some experimental results and theoretical ideas indicate to me that Tegmark's criticism may be invalid.

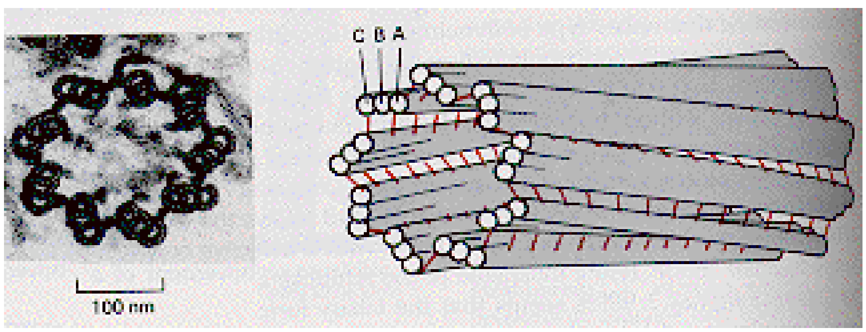
Such theoretical ideas include Mead's quantum physics of resonance.

Roger Penrose and Stuart Hameroff propose that Consciousness involves a Planck scale Decoherence of Quantum Superpositions that they call Orch OR in their paper entitled Orchestrated Objective Reduction of Quantum Coherence in Brain Microtubules: The "Orch OR" Model for Consciousness.
Figure 1



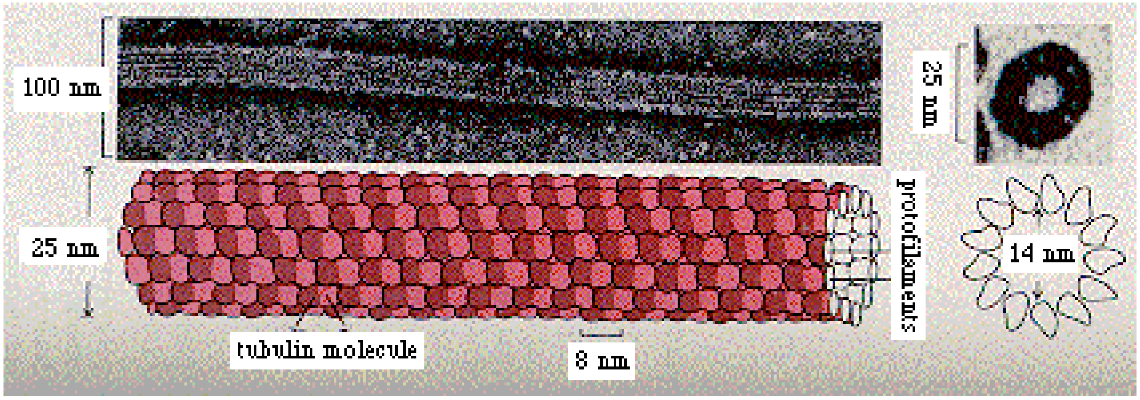
is a "Schematic of central region of neuron (distal axon and dendrites not shown) showing parallel arrayed microtubules interconnected by MAPs [Microtubule Associated Proteins]. Microtubules in axons are lengthy and continuous, whereas in dendrites they are interrupted and of mixed polarity. Linking proteins connect microtubules to membrane proteins including receptors on dendritic spines.".

The Centrosome, in most animal cells, acts as a Microtubule Organizing Center. Most Centrosomes contain a pair of Centrioles arrranged at right angles to each other in an L-shaped configuration. A Centriole



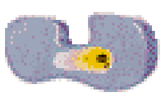
is about 200 nm wide and 400 nm long. Its wall is made up of 9 groups of 3 microtubles. You can regard the A microtubule of a triplet as being a complete microtubule, with the B and C microtubules being incomplete microtubules fused to A and B respectively. Each triplet is tilted in toward the central axis at an angle of about 45 degrees.

Each microtubule is a hollow cylindrical tube with about 25 nm outside diameter and 14 nm inside diameter, made up of 13 columns of Tubulin Dimers.



(The two preceding illustrations are from Molecular Biology of the Cell, 2nd ed, by Alberts, Bray, Lewis, Raff, Roberts, and Watson (Garland 1989))

Each Tubulin Dimer is about 8 nm x 4 nm x 4 nm, consists of two parts, alpha-tubulin and beta-tubulin (each made up of about 450 Amino Acids, each of which contains roughly 20 Atoms), and can exist in (at least) 2 different geometrical configurations, or conformations, involving the position of a single Electron.



Call this Electron the Conformation Electron, because in a single Tubulin Dimer its the position at the junction of the alpha-tubulin and the beta-tubulin determines the 2 different conformations of the Tubulin, which correspond to 2 different states of the dimer's electric polarization.

There are 10^7 Tubulin Dimers per neuron, with 10% of them, or 10^6, estimated to be involved in the consciousness process, and the remainder doing other things needed to keep the cell alive.

The human brain contains about 10^11 neurons.

Therefore, the human brain contains about 10^18 tubulins, about 10^17 of which are involved in the consciousness process.

The Tubulins in a Microtubule can represent Information, and act as Cellular Automata to process it.

Roger Penrose says, in Shadows of the Mind (Oxford 1994), page 344, "... We can now consider the gravitational self-energy of that mass distribution which is the difference between the mass distributions of the two states that are to be considered in quantum linear superposition. The reciprocal of this self-energy gives ... the reduction timescale ...".

This is the decoherence time T = h / E.

For a given Particle, Stuart Hameroff describes this as a particle being separated from itself, saying that the Superposition Separation a is "... the separation/displacement of a mass separated from its superposed self. ... The picture is spacetime geometry separating from itself, and re-anealing after time T. ...".

If the Superposition consists of States involving one Particle of Mass m, but with Superposition Separation a, then the Superposition Separation Energy Difference is the gravitational energy

E = G m^2 / a

In the Osaka paper (Hameroff, S.R. (1997) Quantum computing in microtubules: an intra-neural correlate of consciousness? Cognitive Studies: Bulletin of the Japanese Cognitive Science Society 4(3):67-92.)), Hameroff says that Penrose describes Superposition Separation as "... shearing off into separate, multiple spacetime universes as described in the Everett "multi&endash;worlds" view of quantum theory. ...".

The superposition energy E_N of N Tubulin Electrons and the corresponding decoherence time T_N can be calculated from the equations E = G m^2 / a and T = h / E.

Therefore for a single Electron (ignoring for simplicity some factors like 2 and pi, etc.):

T = h / (G m^2 / a) = (h / m c) (c^2 / G m) (a / c) =
= (Compton / Schwarzschild) (a / c)

where

$2 G m / c^2$ = Schwarzschild Radius of a classical black hole of mass m and

$h / m c$ = Compton Radius of an elementary particle of mass m.

The calculation for a single Electron will be used as the basis for a superpositon of N Electrons over the 10-cm scale human brain. If the single Tubulin Electron with mass m_e has a Superposition Displacement a that is of the order of 10^{-7} cm, or one nanometer, then, since Compton = 10^{-11} cm and Schwarzschild = 10^{-55} cm and the speed of light c = 3×10^{10} cm/sec, and since $E_{electron} = G (m_e)^2 / a$, we have

for a single Electron and ordinary gravity

$$T_{electron} = h / E_{electron} =$$
$$= (\text{Compton} / \text{Schwarzschild}) (a / c) = 10^{26} \text{ sec} = 10^{19} \text{ years.}$$

Now consider the case of N Tubulin Electrons in Coherent Superposition, in which ordinary gravity is realistic.

As Jack Sarfatti says, "Since all the [Tubulin] Electrons are nonlocally connected into a coherent whole we do not want to treat them as fluctuating statistically independent of each other ... .", and Stuart Hameroff agrees, saying "True. That's why we consider them coherently linked or entangled.". Jack Sarfatti defines the Superposition Energy E_N of N superposed Tubulin Electrons in N Tubulins as

$$E_N = G M^2 / L$$

where L is the mesoscopic quantum phase coherence length for the collective mode of N Tubulin Electrons of total mass M = N m with each electron having mass m and with $L = a N^{1/3}$ where a is the separation of individual electrons and the cube root of N is the linear scale of of the whole collection of N Tubulin Electrons in the N Tubulins.

As Jack Sarfatti says (here I have substituted some of my numerical values for his): "... Note the volume ... is the sum of the volumes of all [10^{17} Tubulins involved in the process of consciousness] even though they are separated in physical space from each other over the whole cortex of volume 10^3 cc - they are like one super-particle entangled in configuration space of [about 3×10^{17} dimensions]! That is, this sentient post-quantum computing "enchanted web" is [10^17 little Tubulin nanoboxes] ... . Each box has a little arrow in Hilbert space and all the arrows are phase-locked over a time of order [0.5 milliseconds]. The actual physical distance between the boxes is irrelevant to this Einstein-Podolsky-Rosen network that is one coherent conscious system. The mesoscopic quantum coherence length L is what you would get if you lined up all these nanoboxes in a row - ... It is really not a metrical property in ordinary space. ...".

Therefore, we have:

$$
\begin{aligned}
E_N &= N^2 G m^2 / a N^{(1/3)} = \\
&= N^{(5/3)} G m^2 / a = \\
&= N^{(5/3)} E_{electron}
\end{aligned}
$$

To get the decoherence time for the system of N Tubulin Electrons, recall that $T_{electron} = h / E_{electron}$ = (Compton / Schwarzschild) (a / c) = 10^26 sec = 10^19 years, so that

$$
\begin{aligned}
T_N &= h / E_N = h / N^{(5/3)} E_{electron} = \\
&= N^{(-5/3)} T_{electron} = \\
&= N^{(-5/3)} 10^{26} \text{ sec}
\end{aligned}
$$

and

$$
\begin{aligned}
N &= (10^{26} / (T_N))^{(3/5)} = \\
&= 4 \times 10^{15} / (T_N)^{(3/5)}
\end{aligned}
$$

From the above formulas get the following rough approximate Decoherence Tim T_N for various Numbers of Tubulin Dimers or Neurons, if 10% of the Tubulins in each Neuron are involved in the process of consciousness:

Time T_N	Number of Tubulins	Number of Neurons
10^(-43) sec (Planck)	2.5 x 10^41	
5 x 10^(-4) sec (2 kHz)	10^17	10^11
10^26 sec = 3 x 10^18 years	1	

The 10^17 tubulin Electron (10^11 Neuron) line of the table corresponds to the number of neurons in the human brain.

Here is a rough outline of what happens during the 0.5 milliseconds of a single conscious thought involving 10^17 Tubulin Electrons:

Each Tubulin Site Electron sits within its tubulin cage in one of its 2 Quantum States. Each Tubulin Site Electron has one of 2^1 = 2 States, so it contains one qbit of information, representable by the 2^1 = 2-dimensional Cl(1) Clifford Algebra that is isomorphic to the Complex Numbers. The total of N = 10^17 Tubulin Site Electrons are connected and brought into a coherent Superposition of States, which, as was suggested by Robert Neil Boyd, is representable by the 2^N = 2^(10^17)) dimensional Cl(10^17) Clifford Algebra. Cl(10^17) can be represented as the tensor product of about (10^17) / 8 = about 10^16 factors, each being 256-dimensional Cl(8). Further information about that Clifford Algebra structure, and related information theoretical and particle physics structures and models, can be found in material at http://www.innerx.net/personal/tsmith/TShome.html.

Many of the Quantum States of the Superposition are Closed Timelike Loops, some of which intersect with others. If each Closed Timelike Loop represents an Abstract Idea, then the Intersections among the Closed Timelike Loops represent Interactive Abstract Thought operating on the set of Abstract Ideas.

During the time of Superposition, new Abstract Thoughts can be derived from the original ones by reorganizing the corresponding Closed Timelike Loops and their Intersections.

Conscious Thought formation ends when the Decoherence/Collapse time T_N is reached and Decoherence/Collapse occurs. Then, a single Abstract Idea is chosen from the entire Set of States in the Superposition. This is the Execution Process, which involves choosing one Abstract Idea and rejecting/executing the other Ideas of the Superposition.

The chosen State from the Superposition determines the Positions of all the Gap Junction Electrons of the Quantum Tunnelling connections between Neurons.

The Positions of the Gap Junction Electrons determine the Conformations of the Micrtubules that are adjacent to the Gap Junctions.

The Conformations of those Microtubules determine, through MAP connections, the Conformations of other Microtubules in the same Neuron.

The Conformation of a Microtubule determines the State of its Tubulins.

The State of a Tubulin determines the State of its Tubulin Site Electron, thus completing the process.

During that 0.5 milliseconds of the process of a single conscious thought, the 10^17 Tubulin Electrons are linked in a coherent state by gravity.

For such a gravity linkage to take place, two things are necessary:

There must be a gravitational connection among all 10^17 Tubulin Electrons; and

There superposition must be stable with respect to decoherence during to the 0.5 millisecond duration of the single conscious thought.

Microtubules and Cancer

Stuart Hameroff said in the abstract of his paper at Biosystems Volume 77, Issues 1-3 , November 2004, Pages 119-136:

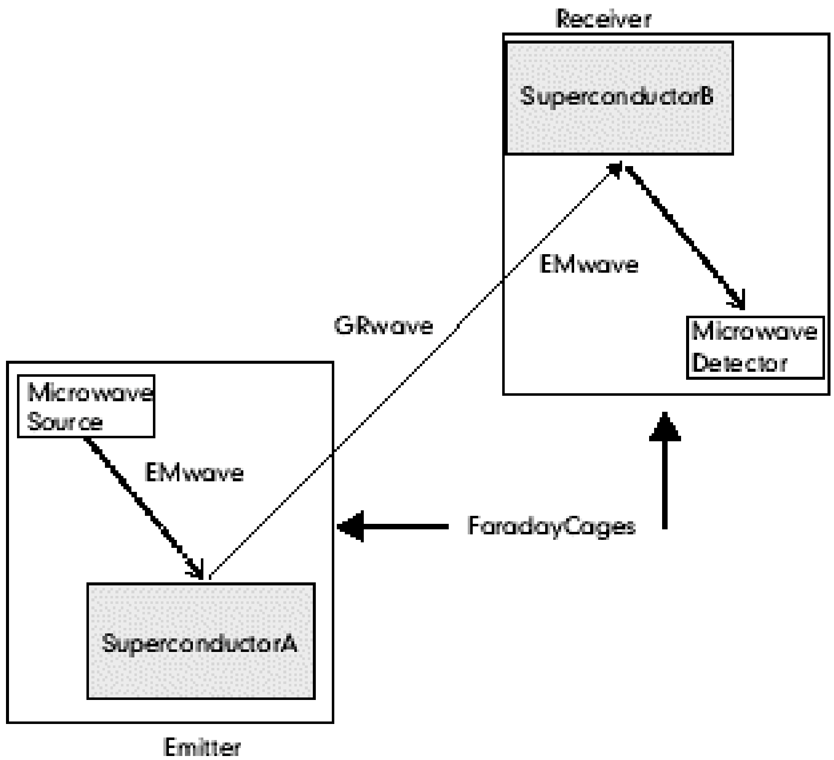
"... It is proposed here that normal mirror-like mitosis is organized by quantum coherence and quantum entanglement among microtubule-based centrioles and mitotic spindles which ensure precise, complementary duplication of daughter cell genomes and recognition of daughter cell boundaries. ... Impairment of quantum coherence and/or entanglement among microtubule-based mitotic spindles and centrioles can result in abnormal distribution of chromosomes, abnormal differentiation and uncontrolled growth, and account for all aspects of malignancy. ...".

Chiao Gravity Antenna

Does there exist a realistic mechanism of gravitational connection between all pairs of the 10^17 Tubulin Electrons?

A positive result in an experiment proposed by Raymond Chiao and described in gr-qc/0204012 [which is an "... abbreviated writeup of ...[his]... March 23, 2002 Wheeler Symposium lecture, and book chapter for Wheeler Festschrift ..." which book chapter is at gr-qc/0208024 and, in its final version, at gr-qc/0303100] might provide an affirmative answer. In that paper, Chiao says:

"... Superconductors will be considered as macroscopic quantum gravitational antennas and transducers, which can directly convert upon reflection a beam of quadrupolar electromagnetic radiation into gravitational radiation, and vice versa, and thus serve as practical laboratory sources and receivers of microwave and other radio-frequency gravitational waves. ... a superconductor can by itself be a direct transducer from electromagnetic to gravitational radiation upon reflection of the wave from a vacuum superconductor interface, with a surprisingly good conversion efficiency. By reciprocity, this conversion process can be reversed, so that gravitational radiation can also be converted upon reflection into electromagnetic radiation from the same interface, with equal efficiency. ... under certain circumstances involving "natural impedance matching" between quadrupolar EM and GR plane waves upon a mirror-like reflection at the planar surface of extreme type II, dissipationless superconductors, the efficiency of such superconductors used as simultaneous transducers and antennas for gravitational radiation, might in fact become of the order of unity, so that a gravitational analog of Hertz's experiment might then become possible. ... These developments suggest the possibility of a simple, Hertz-like experiment, in which the emission and the reception of gravitational radiation at microwave frequencies can be implemented by means of a pair of superconductors used as transducers. ... The schematic of this experiment is ...



... we did not detect any observable signal inside the second Faraday cage, down to a limit of more than 70 dB below the microwave power source of around 10 dBm at 12 GHz. ... Note, however, that since the transition temperature of YBCO is 90 K, there may have been a substantial ohmic dissipation of the microwaves due to the remaining normal electrons at our operating temperature of 77 K, so that the EM wave was absorbed before it could reach the impedance-matching depth at z0. It may therefore be necessary to cool the superconductor down very low temperatures before the normal electron component freezes out sufficiently to achieve such impedance matching. [see gr-qc/0304026] ... An improved Hertz-like experiment using extreme type II superconductors with extremely low losses, perhaps at millikelvin temperatures, is a much more difficult, but worthwhile, experiment to perform. Such an improved experiment, if successful, would allow us to communicate through the Earth and its oceans, which, like all classical matter, are transparent to GR waves. ... I would especially like to thank my father-in-law, the late Yi-Fan Chiao, for his financial and moral support of this work. This work was partly supported also by the ONR. ...".

Note that the Faraday cages of Chiao's schematic correspond to the Tubulin Cages of the Tubulin Electrons in the Quantum Tubulin Electron model of Quantum Consciousness, and that if Chiao's gravity antenna can receive gravity signals by graviton links, then Tubulin Electrons in their cages should be able to receive gravity signals establishing graviton links, as needed for the Penrose-Hameroff model of Quantum Tubulin Electron Quantum Consciousness.

Note also that the negative result of the preliminary experiment was probably due to failure of the impedance-matching mechanism for converting EM waves to gravity waves [see gr-qc/0304026], and therefore not a failure of the gravity antenna concept, which is the important concept with respect to the Quantum Tubulin Electron model of Quanum Consciousness.

Decoherence

Is the superposition state of Tubulin Electrons stable with respect to decoherence during to the 0.5 millisecond duration of the single conscious thought?

Max Tegmark, in quant-ph/9907009, says:

"... Penrose has ... suggested that the dynamics of such excitations can make a microtubule act like a quantum computer, and that microtubules are the site of of human consciousness ... This idea has been further elaborated ... with the conclusion that quantum superpositions of coherent excitations can persist for as long as a second before being destroyed by decoherence ... This was hailed as a success for the model, the interpretation being that the quantum gravity effect on microtubules was identified with the human though process on this same timescale. This decoherence rate T = 1 s was computed assuming that quantum gravity is the main decoherence source. Since this quantum gravity model is somewhat controversial ... and its effect has been found to be more than 20 orders of magnitude weaker than other decoherence sources in some cases ... We will now ... evaluate ... decoherence sources for the microtubule case as well, to see whether they are in fact dominant ... we will ignore collisions between polarized tubulin dimers and nearby water molecules, since it has been argued that these may be in some sense ordered and part of the quantum system ... Let us instead apply ... the decoherence timescale

T = (a^3 sqrt(m k t)) / N g q^2 | r' - r |

caused by a single ion a distance a away. ...[where k is Boltzmann's constant and g = 1 4 pi e0 is the Coulomb constant, m is ion mass, N is number of ions, q is ion charge, and t is temperature]... with N = Q / qe = 10^3. The distance to the nearest ion will generally be less than ...[about] ... 26 nm ... Superpositions spanning many tubuline dimers ... therefore decohere on a timescale ...[about]... 10^(-13) s. due to the nearest ion alone. This is quite a conservative estimate, since the other ... 10^3 ions that are merely a small fraction further away will also contribute to the decoherence rate ... ... We neglected screening effects because the decoherence rates were dominated by the particles closest to the system, i.e., the very same particles that are responsible for screening the charge from more distant ones. ... We find that the decoherence timescales ...[about 10^(-13)]... seconds are typically much shorter than the relevant dynamical timescales ...[about 0.001 to 0.1 seconds

]... both for regular neuron firing and for kink-like polarization excitations in microtubules. This conclusion disagrees with suggestions by Penrose and others that the brain acts as a quantum computer, and that quantum coherence is related to consciousness in a fundamental way. ...".

I disagree with Tegmark, on both experimental and theoretical grounds. I think that Tegmark has ignored significant phenomena related to maintaining coherence during the 0.5 millisecond duration of a single conscious thought involving 10^17 Tubulin Electrons.

On the experimental side, there are some results that indicate that coherence is maintained much longer than would be expected from analyses such as Tegmark's. For example:

On page 20 of the 17 July 1999 issue of the New Scientist is an article by Charles Seife (a New Scientist Reporter) that says in part: "... last April [1998], Isaac Chuang of IBM in San Jose, California, and Neil Gershenfeld the Massachusetts Institute of Technology created a quantum computer ... in a forthcoming issue of Physical Review Letters, Carlton Caves ... say they are unsure why quantum computation worked. Gershenfeld and Chuang used magnetic fields to manipulate atoms in liquid chloroform. But the problem, says Caves, is that the choloroform atoms were not in "entangled" states. ... because the chloroform was at room temperature, the atoms could not have been entangled ... The thermal motion of the atoms would have mixed up their quantum states and ruined any entanglement. ... So why did the chloroform comuter work at all? Caves's colleague John Smolin, a physicist at IBM in New York, suspects Chuang's chloroform has simulated a quantum computer, though he doesn't know how. Or maybe the experiment hints there are other ways of doing quantum computation that we don't yet understand. ...".

A 6 July 2001 New Scientist article by Willis Knight says: "... Molecular transistors that run on single electrons now work at room temperature. Dutch scientists achieved the feat by buckling carbon nanotubes with an atomic force microscope. ... By buckling a metallic carbon nanotube, they formed a small area from which a single electron cannot escape at room temperature unless a current is applied via an electrode. ... pushing a single electron through the transistor caused it to exhibit quantum coherence. This means that the electron maintains some of the quantum state it obtained whilst inside the transistor when it leaves. The effect is not found within normal electronics. ...".

According to Apoorva Patel in his paper Quantum Algorithms and the Genetic Code, quant-ph/0002037: "... Enzymes are the objects which catalyse biochemical reactions. They are large complicated molecules, much larger than the reactants they help, made of several peptide chains. Their shapes play an important part in catalysis, and often they completely surround the reaction region. They do not bind to either the reactants or the products ... for example, enzymes can suck out the solvent molecules from in between the reactants ... It is proposed that enzymes play a crucial role in maintaining quantum coherence ... Enzymes provide a shielded environment where quantum coherence of the reactants is maintained. ... For instance, diamagnetic electrons do an extraordinarily good job of shielding the nuclear spins from the environment ... the coherence time observed in NMR is O(10) sec, much longer than the thermal environment relaxation time (hbar / kT = O(10^(-14)) sec) and the molecular collision time (O(10^(-11)) sec), and still neighbouring nuclear spins couple through the electron cloud. ... Enzymes are able to create superposed states of chemically distinct molecules. ... Enzymes are known to do cut-and-paste jobs ... (e.g. ... methylation, replacing H by CH3, which converts U to T). Given such transition matrix elements, quantum mechanics automatically produces a superposition state as the lowest energy equilibrium state. ... Delocalisation of electrons and protons over distances of the order of a few angstroms greatly helps in molecular bond formation. It is important to note that these distances are much bigger than the Compton wavelengths of the particles, yet delocalisation is common and maintains quantum coherence. ...".

According to an article by Bennett Davis in the 23 Feb 2002 edition of The New Scientist: "... In the early 1990s, Guenter Albrecht-Buehler ... at Northwestern ... discovered that some cells can detect and respond to light from others. ... cells ... were using light to signal their orientation. If so, they must have some kind of eye. ... centrioles fill the bill. These cylindrical structures have slanted "blades" which ... Albrecht-Buehler ... believes act as simple blinds. ... microtubules ... could act as optical fibres ... feeding light towards the centrioles from the cell's wall. ... why should cells want to detect light? ... they are talking to each other ... Cells in embryos might signal with photons so that they know how and where they fit into the developing body. ... Albrecht-Buehler ... wants to learn their language. ... In the 1980s Fritz-Albert Popp, then a lecturer at the University of Marburg in Germany, ... who now heads the International Institute of Biophysics in Neuss, Germany, ...[and]... runs a company called Biophotonen that offers its expertise in reading photon emissions to gauge the freshness and purity of food ... became interested in the optical behaviour of cells. In a series of experiments Popp found that two cells separated by an opaque barrier release biophotons in uncoordinated patterns. Remove the barrier and the cells soon begin releasing photons in synchrony. ...".

Acccording to cond-mat/0007185 and cond-mat/0007287 by Philip W. Anderson: "... The most striking fact about the high-Tc cuprates is that in none of the relevant regions of the phase diagram is there any evidence of the usual effects of phonon or impurity scattering. This is strong evidence that these states are in a "quantum protectorate" ... a state in which the many-body correlations are so strong that the dynamics can no longer be described in terms of individual particles, and therefore perturbations which scatter individual particles are not effective. ...".

On the theoretical side, there are also some reasons that I disagree with Tegmark. For example:

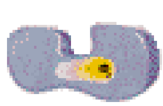
Hagan, Hameroff, and Tuszynski, in Physical Review E, Volume 65, 061901, published 10 June 2002, say: "... Tegmark's commentary is not aimed at an existing model in the literature but rather at a hybrid that replaces the superposed protein conformations of the orch. OR theory with a soliton in superposition along the microtubule ... recalculation after correcting for differences between the model on which Tegmark bases his calculations and the orch. OR model (superposition separation, charge vs dipole, dielectric constant) lengthens the decoherence time to 10^(-5) &endash;10^(-4) s ...".

Mershin, Nanopoulos, and Skoulakis, in quant-ph/0007088, say: "... treat the tubulin molecule as the fundamental computation unit (qubit) in a quantum-computational net work that consists of microtubules (MTs), networks of MTs and ultimately entire neurons and neural networks. ...". They say "... it has been shown [by D. L. Koruga, D. L. Ann. NY Acad. Sci. 466, 953-955 (1986)] that the particular geometrical arrangement (packing) of the tubulin protofilaments obeys an error-correcting mathematical code known as the K2(13, 2^6, 5) code ... the existence of a quantum-error correcting code is needed to protect the delicate coherent qubits from decoherence. This has been the major problem of quantum computers until the works of Shor and Steane have independently shown that such a code can be implemented ... We conjecture that the K-code apparent in the packing of the tubulin dimers and protofilaments is partially responsible for keeping coherence among the tubulin dimers. By simulating the brain as a quantum computer it seems we are capable of obtaining a more accurate picture than if we simulate the brain as a classical, digital computer. ...".

Raymond Chiao in gr-qc/0204012 says: "... quantum entanglement gives rise to EPR correlations at long distance scales within the superconductor. The electrons in a superconductor in its ground BCS state are not only macroscopically entangled, but due to the existence of the BCS gap which separates the BCS ground state energetically from all excited states, they are also protectively entangled, in the sense that this entangled state is protected by the presence of the BCS gap from decoherence arising from the thermal environment, provided that the system temperature is kept well below the BCS transition temperature. The resulting large quantum rigidity is in contrast to the tiny rigidity of classical matter, such as that of the normal metals used in Weber bars, in their response to gravitational radiation. ...".

Resonance among 10^17 Tubulin Electrons of a single conscious thought may be important in achieving and maintaining coherent superposition states among them. Carver Mead, in his book Collective Electrodynamics (MIT 2000), discusses resonance coupling with electromagnetic photons. If Raymond Chiao's gravity antenna idea is correct, then the same resonance phenomena should be applicable for gravity gravitons as for electromagnetic photons. Carver Mead says: "... In our investigation of radiative coupling, we use a superconducting resonator as a model system. ... we can build such a resonator from a superconducting loop and a capacitor ... the coupling of ... two loops is the same, whether retarded or advanced potentials are used. Any loop couples to any other on its light cone, whether past or future. ... The total phase accumulation in a loop is the sum of that due to its own current, and that due to currents in other loops far away. ... normal modes correspond to stationary states of the system. Once the system is oscillating in one of these modes, it will continue to do so forever. To understand energy transfer between the resonators, we can use mixtures of normal modes. ... Any energy leaving one resonator is tranferred to some other resonator, somewhere in the universe. The energy in a single resonator alternates between the kinetic energy of the electrons (inductance), and the potential energy of the electrons (capacitance). With the two resonators coupled, the energy shifts back and forth between the two resonators in such a way that the total energy is constant ... The conservation of energy holds despite an arbitraty separation between the resonators; it is a direct result of the symmetry of the advanced and retarded potentials. There is no energy "in transit" between them. ... the universe contains a truly enormous number of resonators ... [For the 10^17 Tubulin Electrons of a single conscious thought, the resonant frequencies are the same and exchanges of energy among them act to keep them locked in a collective coherent state.] ... How does a single resonator behave in an inhomogeneous universe full of other matter? In the real universe [outside the collective coherent set of tubulin electrons], no two resonators have identical resonant frequencies for long; however, it is a common occurrence that two frequencies will cross, and that energy will be exchanged between the resonators during the crossing. From the point of view of collective electrodynamics, this exchange of energy is the microscopic origin of the thermodynamic behavior of the universe as we observe it. ... In a random universe, any particular phase is equally likely for any given crossing. A particular resonator is therefore equally likely to receive either an increment or a decrement due to a given crossing. ... In a random universe [unlike the collective coherent set of tubulin electrons], there is no first-order effect in which energy flows from the high-amplitude resonator to the low-amplitude resonator; there is, however, a second-order effect in which energy flows, on the average, from the high-amplitude resonator to the low-amplitude resonator. The rate of energy flow is proportional to the difference in energies, and to the inverse square of the distance. ...

... The coupling between two loops considered ...[above]... is called magnetic dipole coupling. It is characterized by its proportionality to the second derivative of the current with respect to time. ... A much stronger coupling can be obtained between two straight sections of wire ... We can imagine a resonator configuration for which this type of coupling is realizable: Two parallel capacitor plates [corresponding to the two holes in a tubulin



where the tubulin electron can be stable located] of capacitance C are connected by a straight section of superconducting wire of inductance L between their centers. Such a configuration ... is called an electric dipole. Because there are charges at the two ends of the dipole, we can have a contribution to the electric coupling from the scalar potential ... as well [as] from the magnetic coupling ... from the vector potential ... electric dipole coupling is stronger than magnetic dipole coupling by the square of the ratio of the wavelenght to the size of the element. ... For example, an atom half a nanometer in diameter radiates visible light of 500 nanometer wavelength. In this case, electric dipole coupling is a million times stronger than magnetic dipole coupling. ... we have treated the electron as a wave, continuous in space, carrying a continuous charge density with it. ... Arriving at the correct results required taking into account the interaction of the electron with itself, exactly as we have done in the case of the superconducting loop. The electron wave function depends on the potential; the potential depends of the charge density that is determined by the wave function. Thus, we have an inherently non-linear problem ... The nonlinearity ... poses some computational issues, but no conceptual issues. ... the nonlinear theory gives the correct energy levels for the hydrogen atom ... It is by now a common experimental fact that an atom, if sufficiently isolated from the rest of the universe, can stay in an excited state for an arbitrarily long period. ... The mechanism for initiating an atomic transition is not present in the isolated atom; it is the direct result of coupling with the rest of the universe. ... The electron wave function ... is particularly sensitive to coupling with other electrons; it is coupled either to far-away matter in the universe or to other electrons in a resonant cavity or other local structure. In the initial parts of this monograph, we were able to ignore coupling to far-away matter because we used a collective structure in which there are 10^23 electrons, arranged in such a way that the collective properties intrinsic to the structure scaled as the square of the number of electrons. ... we ...[made]... a connection between the classical concept of force and the quantum nature of matter through the concept of momentum. ... We would expect the total momentum P of the collective electron system [in a superconducting loop of wire] to be the momentum per charge times the number of charges in the loop. If there are n charges per unit length of wire ... P = n q L I ... I = n q v ... and ... P = L (nq)^2 v . The momentum is proportional to the velocity, as it should be. It is also proportional to the size of the loop, as reflected by the inductance L. ... Instead of scaling linearly with the number of charges that take part in the motion, the momentum of a collective system scales as the square of the number of charges! ... In an arrangement where charges are constrained to move in concert, each charge produces phase accumulation, not only for itself but for all the other charges as well. So the inertia of each charge increases linearly with the number of charges moving in concert. The inertia of the ensemble of coupled charges must therefore increase as the square of the number of charges. ...

[To see how Carver Mead's resonance might be applied to the Penrose-Hameroff tubulin electron model of consciousness, consider the maximal case of N tubulin electrons with N = 10^18, each electron having thermal energy E = kT, where k = 10^(-23) Joules Kelvin^(-1) and T = 300 Kelvin, so that Total Thermal Energy = N^2 kT = 10^(36-23) x 300 = 3 x 10^15 Joules. (Due to the nonlinear square-scaling, it would take less if the collapse took place gradually, a few electrons at a time.) Note that decoherence by external thermal energy, with square-scaling, is different from the self-decoherence of the superposition state, based on the energy-time uncertainty principle Energy x Time = h, by which a conscious thought quantum state decoheres to form a completed thought. If N tubulin electrons are in a collective superpostion state of conscious thought, then the total energy needed to decohere them by external thermal energy (decoherence due to the heat of the brain) is much greater than the classical kinetic heat energy in the brain, so that Quantum Consciousness in the brain is stable against thermal decoherence due to the heat of the brain.]

... an N-turn closely coupled coil has an inductance L = N^2 Lo. Once again, we see the collective interaction scaling as the square of the number of interacting charges. ... When two classical massive bodies ... are bolted together, the inertia of the resuting composite body is simply the sum of the two individual inertias. The inertia of a collective system, however, is a manifestation of the interaction, and cannot be assigned to the elements separately. ... Thus, it is clear that collective quantum systems do not have a classical correspondence limit. ... It is instructive to work out the magnitude of the electron inertia in a concrete case. A small superconducting magnet has 10^4 turns of NbTi wire approximately 0.1 mm in diameter. The magent is 7 cm long, and just under 5 cm in diameter, and produces a peak field of 7 tesla at a currrent of 40 amperes. The magnet weighs 0.5 kilograms, and has a measured inductance of approximately 0.5 henry. There are of the order of 10^28 electrons per cubic meter in the wire, or 10^20 electrons per meter length of wire, corresponding to approximately 10 coulombs of electronic charge per meter of wire. At 40 amperes, these electrons move at a velcoity v = 4 m / sec. the total length l of wire is about 10^3 meters, so the total electronic charge in the magnet is about 10^4 coulombs. Using these values, A = PHI / l = L I / l = 0.02 V sec / meter. The electromagnetic momentum p of an electron is just this vector potential multiplied by the electronic charge; from this, we can infer an electromagnetic mass m for each electron: q A = 3.2 x 10^(-21) coulomb V sec / meter = m v m = 10^(-21) kg For comparison, the mass of a free electron is approximately 10^(-30) kg, and the rest mass of a proton is a factor 1800 larger than that of an electron. The electromagnetic mass of an electron in our magnet is thus a factor of 10^9 larger than the rest mass of a free electron. ...[The electromagnetic mass of all the electrons in the magnet is 10^20 electrons / meter x 10^3 meters x 10^(-21) kg / electron = 100 kg]... The total inertia of the electron system in the magnet is much larger than the actual mass of all the atoms making up the magnet [0.5 kg]. ...".

[The above material from Carver Mead is directly applicable to the superposition state of tubulin electrons [[and is related to the idea of a Quantum Protectorate]]. The following material shows how the same viewpoint applies to understanding quantum state transitions.]

... We have developed a detailed description of the energy-transfer process between macroscopic quantum resonators ... We are now in a position to understand the radiative transfer between two identical atomic systems. ... The two atoms act like two small dipole resonators, and energy is radiatively transferred ... Once the coupled mixed state starts to develop, it becomes self-reinforcing. ... This self-reinforcing behavior gives the transition its initial exponential character. Once the transition is fully under way, the two states are nearly equally represented in the superposition, and the coupled system closely resembles the coupled resonators ... Once the transition has run its course, each atom settles into its final eigenstate. ...
... there are quantum jumps, but they are not discontinuities. They may look discontinuous because of the nonlinear, self-reinforcing nature of a quantum transition; but at the fundamental level, everything can be followed in a smooth and continuous way .... to arrive at this picture, we had to give up the one-way direction of time, and allow coupling to everything on the light cone ... the Green's function for collective systems is totally free of singularities, and cannot, by its very nature, generate infinities ... There is no action of an elementary charge [which is fundamentally an amplitude to transmit or absorb energy by radiative transfer] upon itself ...".

[[According to cond-mat/0007287 and cond-mat/0007185 by Philip W.

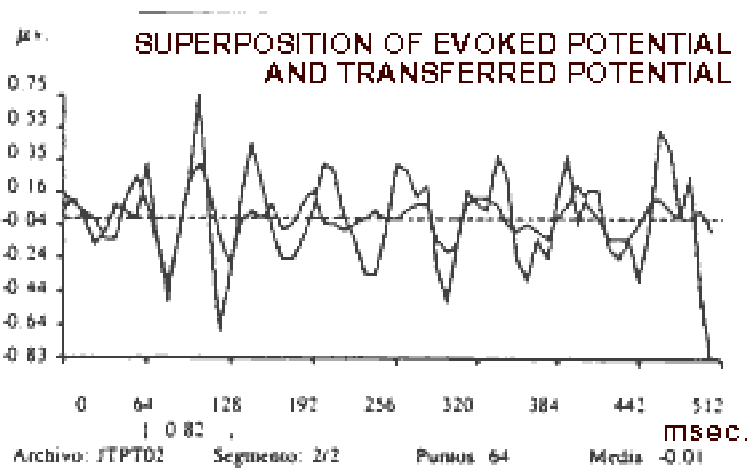
Anderson: "... Laughlin and Pines have introduced the term "Quantum protectorate" as a general descriptor of the fact that certain states of quantum many-body systems exhibit properties which are unaffected by imperfections, impurities and thermal fluctuations. They instance the quantum Hall effect, which can be measured to 10^(-9) accuracy on samples with mean free paths comparable to the electron wavelength, and flux quantization in superconductors, equivalent to the Josephson frequency relation which again has mensuration accuracy and is independent of imperfections and scattering. An even simpler example is the rigidity and dimensional stability of crystalline solids evinced by the STM. ... the source of quantum protection is a collective state of the quantum field involved such that the individual particles are sufficiently tightly coupled that elementary excitations no longer involve a few particles but are collective excitations of the whole system ... and therefore perturbations which scatter individual particles are not effective. ... The purpose of this paper is, first, to present the overwhelming experimental evidence that the metallic states of the high Tc cuprate superconductors are a quantum protectorate; and second, to propose that this particular collective state involves the phenomenon of charge-spin separation, and to give indications as to why such a state should act like a quantum protectorate. ... Spin-charge separation is a very natural phenomenon in interacting Fermi systems from a symmetry point of view ... The Fermi liquid has an additional symmetry which is not contained in the underlying Hamiltonian, in that the two quasiparticles of opposite spins are exactly degenerate and have the same velocity at all points of the Fermi surface. This is symmetry SO(4) for the conserved currents at each Fermi surface point since we have 4 degenerate real Majorana Fermions. But the interaction terms do not have full SO(4) symmetry, since they change sign for improper rotations, so the true symmetry of the interacting Hamiltonian is SO4 / Z2 = SU2 x SU2, i.e., charge times spin. A finite kinetic energy supplies a field along the " direction of the charge SU(2) and reduces it to U(1), the conventional gauge symmetry of charged particles. ...". Also, according to cond-mat/0301077 by M.Ya. Amusia, A.Z. Msezane, and V.R. Shaginyan: "... the fermion condensation ... can be compared to the Bose-Einstein condensation. ... the appearance of ... fermion condensate (FC) ... is a quantum phase transition ... that separates the regions of normal and strongly correlated liquids. Beyond the fermion condensation point the quasiparticle system is divided into two subsystems, one containing normal quasiparticles, the other being occupied by fermion condensate localized at the Fermi level. ... fermion systems with ... fermion condensate (FC) ... have features of a "quantum protectorate" ... This behavior ... takes place in both three dimensional and two dimensional strongly correlated systems ... The only difference between 2D electron systems and 3D ones is that in the latter ... fermion condensation quantum phase transition (FCQPT) ... occurs at densities which are well below those corresponding to 2D systems. For bulk 3He, FCQPT cannot probably take place since it is absorbed by the first order solidification ... an infinitely extended system composed of Fermi particles, or atoms, interacting by an artificially constructed potential with the desirable scattering length a ... may be viewed as trapped Fermi gases ... We conclude that FCQPT can be observed in traps by measuring the density of states at the Fermi level ... It seems quite probable that the neutron-neutron scattering length (a = 20 fm) is sufficiently large to be the dominant parameter and to permit the neutron matter to have an equilibrium energy, density, and the singular point ... at which the compressibility vanishes. Therefore, we can expect that FCQPT takes place in a low density neutron matter leading to stabilization of the matter by lowering its ground state energy. ... fermion condensate (FC) ... "quantum protectorate" ... behavior ... demonstrates the possibility to control the essence of strongly correlated electron liquids by weak magnetic fields. ... We have demonstrated that strongly correlated many-electron systems with FC, which exhibit strong deviations from the Landau Fermi liquid behavior, can be driven into the Landau Fermi liquid by applying a small magnetic field B at low temperatures. A re-entrance into the strongly correlated regime is observed if the magnetic field B decreases to zero, while the effective mass M* diverges as M* proportional to 1 / sqrt(B). The regime is restored at some temperature T* proportional to sqrt(B). This behavior is of a general form and takes place in both three dimensional and two dimensional strongly correlated systems, and demonstrates the possibility to control the essence of strongly correlated electron liquids by weak magnetic fields. ...".]]

Grinberg-Zylberbaum Experiments

The whereabouts of Grinberg-Zylberbaum (as far as I know) is unknown, and he may even be deceased.

Some interesting experimental results relevant to Chiao gravity antennas and to Mead resonant coupling were obtained by neurophysiologist Grinberg-Zylberbaum. According a 1997 Science Within Consciousness web article by Henry Swift:

"... The experiment conducted by neurophysiologist Grinberg-Zylberbaum ... The Einstein- Podolsky-Rosen Paradox in the Brain; The Transferred Potential, Physics Essays 7,(4), 1994. ... demonstrate[s] the existence of a macroscopic quantum system in the human brain through the demonstration of ... non-local correlation between brains ... In this experiment two subjects ... meditated together for twenty minutes. A total of seven pairs of subjects of both sexes, with ages from 20-44 years participated in the study. After meditation and while maintaining their "direct communication" (without speech), they were placed in semi-silent, electro-magnetically shielded chambers separated by 45 feet. ... Both subjects were connected to EEG instruments and 100 random flashes of light were presented to subject A, while both remained reclined with semi-closed eyes. Subject B was not told when the light was flashed for subject A, and control correlation checks were also made at random times with no light flashes. The results indicated that, "after a meditative interaction between two people who were instructed to maintain direct communication (i.e. to feel each other's presence even at a distance), in about one out of four cases when one of the subjects was stimulated in such a way that his/her brain responded clearly (with a distinct evoked potential), the brain of the nonstimulated subject also reacted and showed a transferred potential of a similar morphology....



... The striking similarity of the transferred and evoked potentials and the total absence of transferred potentials in the control experiments leave no room for doubt about the existence of an unusual phenomenon, namely, propagation of influence without local signals. ... It is also extremely significant that the occurrence of transferred potential is always associated with the participants' feeling that their interaction is successfully completed (in contrast to the lack of transferred potential, when there is no such feeling). The interaction that correlates the subjects under study is entirely an interaction via non-local consciousness. ... none of the subjects B ever reported realizing any type of conscious experience related to the appearance of the transferred potential ...". According to a 1996 DynaPsych article by Ervin Laszlo: "... A particularly poignant example was furnished by a young couple, deeply in love. Their EEG patterns remained closely synchronized throughout the experiment, testifying to their report of feeling a deep oneness. ... In a limited way, Grinberg-Zylberbaum could also replicate his results. When a subject exhibited the transferred potentials in one experiment, he or she usually exhibited them in subsequent experiments as well. ...".

What has Grinberg-Zylberbaum done since 1994? That is unknown. According to an article by Sam Quinones, in the July/August 1997 New Age Journal, as shown on a Sustained Action web page:

"... In 1977 Grinberg returned to Mexico City ... A deeply spiritual man, Grinberg had moved from houses where he felt bad energy, believed he once had flown, and kept a meditation room lined with books and pictures of gurus. A semi-observant Jew, he sought out great thinkers on the Kabbalah.

... at UNAM ... he ... met the person who, he wrote later, would influence him more than any other: Barbara Guerrero, a former cabaret singer and lottery ticket seller who had fought with Pancho Villa as a young girl. Doña Pachita, as Guerrero was known, was a curandera. ... Pachita could go into a trance state during which the spirit of Cuauhtémoc, the nephew of the great Aztec ruler Moctezuma, occupied her consciousness. ... Grinberg ... believed that experience and perception were created as a result of this interaction, and that the curative powers of shamans and *curanderas* like Pachita came from their ability to gain access to the informational matrix and change it, thereby affecting reality. ... Grinberg designed an experiment . . . using two people instead of one. Both subjects, with electrodes attached to their skulls, were put in a dark room and told to try to achieve a sort of meditative union. After twenty minutes, one was sent to a separate room. The remaining person was stimulated with a series of light flashes or sounds while his or her brain waves were measured. The brain waves of the isolated person were also measured. In 1987 Grinberg recorded for the first time a simultaneous reaction to the stimuli on the part of the isolated, non-stimulated person, a phenomenon he called 'transferred potential.' Over the years, with increasingly sophisticated equipment, he documented transferred potential 25 percent of the time, he wrote. It was a remarkable finding, totally contrary to the tenets of mainstream science. Grinberg believed it supported his theory of a neuronal field connecting all human minds. ... In 1991, Grinberg, his wife, and Tony Karam visited Castaneda at the latter's invitation in Los Angeles. There, Karam says, Castaneda proposed that Grinberg leave his UNAM lab to live in his community. Grinberg declined. Their relationship disintegrated during a trip Castaneda took to Mexico City two years later. Grinberg's friends and family remember him frequently calling Castaneda an egomaniac, more interested in power than truth. They also recall that Tere [Grinberg's wife] remained enamored with Castaneda and his group. ... For Jacobo Grinberg Zylberbaum, 1994 was a pretty good year. ... At his laboratory in the psychology department of the National Autonomous University of Mexico (UNAM) in Mexico City, he recorded the brain waves of a shaman, Don Rodolfo from Veracruz, in a trance state. ... Grinberg's book on his seminal influence, Barbara Guerrero, the blind witch doctor known as Dona Pachita, was finally about to be published in English. ... Then in December, Grinberg missed some appointments with students. Two days before his long-awaited trip to Nepal on December 14, he failed to attend his own birthday party. ... When Grinberg did not return from Nepal as planned, still no one thought much of it. ... But the weeks became months. Calls were made ... Nothing. No record of Grinberg or his wife ... Tere ... even leaving Mexico. ... In the two-and-a-half years since he disappeared, no trace of him, dead or alive, has been found. All that remain are his books, his theories ... The theory for which Grinberg came to be known reflected his personality. Relying on physics and his experiences with witch doctors, or *curanderos*--a bit of Einstein, a bit of Dona Pachita--its essential message was warm and hopeful: All humankind is interconnected. ...".

Zizzi Quantum Cosmology

In gr-qc/0007006, Paola Zizzi says, (with some editing by me denoted by [ ]):

"... the vacuum-dominated early inflationary universe ... is a superposed quantum state of qubits. ... the early universe had a conscious experience at the end of inflation, when the superposed quantum state of ... [10^18 = N quantum qubits] ... underwent Objective Reduction. The striking point is that this value of [N] equals the number of superposed tubulins-qubits in our brain ... [in the inflationary phase of our universe] ... the quantum register grows with time. In fact, at each time step ... [Tn = (n+1) Tplanck (where Tplanck = 5.3 x 10^(-44) sec)] ... a Planckian black hole, (the n=1 qubit state 1 which acts as a creation operator , supplies the quantum register with extra qubits. ... At time Tn = (n+1) Tplanck the quantum gravity register will consist of (n+1)^2 qubits. [Let N = (n+1)^2] ... By the quantum holographic principle, we associate N qubits to the nth de Sitter horizon ... remember that |1> = Had|0> where Had is the Hadamard gate ... the state ... [of N qubits] ... can be expressed as ... [|N> = (Had|0>)^N] ... As the time evolution is discrete, the quantum gravity register resembles more a quantum cellular automata than a quantum computer. Moreover, the quantum gravity register has the peculiarity to grow at each time step (it is self-producing). If we adopt an atemporal picture, then the early inflationary universe can be interpreted as an ensemble of quantum gravity registers in parallel ... which reminds us of the many-worlds interpretation. ... The superposed state of quantum gravity registers represents the early inflationary universe which is a closed system. Obviously then, the superposed quantum state cannot undergo environmental decoherence. However, we know that at the end of the inflationary epoch, the universe reheated by getting energy from the vacuum, and started to be radiation-dominated becoming a Friedmann universe. This phase transition should correspond to decoherence of the superposed quantum state. The only possible reduction model in this case is self-reduction ... during inflation, gravitational entropy and quantum entropy are mostly equivalent ... Moreover ... The value of the cosmological constant now is ... /\N = 10^(-56) cm^(-2) ... in agreement with inflationary theories. If decoherence of N qubits occurred now, at Tnow = 10^60 Tplanck (that is, n = 10^60, N = 10^120) there would be a maximum gravitational entropy ... [maximum entropy Smax = N ln2 = 10^120] ... In fact, the actual entropy is about ... [

entropy now Snow = 10^101] ... [Therefore] decoherence should have occurred for ... [Ndecoh = 10^(120-101) = 10^19 = 2^64] ... which corresponds to ... [n = 9 and to] ... the decoherence time ... [Tdecoh = 10^9 Tplanck = 10(-34) sec] ...".

Is there a fundamental reason that the number of qubits at which our inflationary universe experiences self-decoherence is Ndecoh = 10^19 = 2^64 ?

From the point of view of the D4-D5-E6-E7-E8 Vodou Physics model, the fundamental structure is the 2^8 = 256-dimensional Cl(8) Clifford algebra, which can be described by 2^8 qubits. Our inflationary universe decoheres when it has Ndecoh = 2^64 qubits. What is special about 2^64 qubits ? 2^64 qubits corresponds to the Clifford algebra Cl(64) = Cl(8x8). By the periodicity-8 theorem of real Clifford algebras that Cl(K8) = Cl(8) x ... tensor product K times ... x Cl(8), we have: Cl(64) = Cl(8x8) = Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) x Cl(8) Therefore, Cl(64) is the first (lowest dimension) Clifford algebra at which we can reflexively identify each component Cl(8) with a vector in the Cl(8) vector space. This reflexive identification/reduction causes decoherence. In my opinion, it is the reason that our universe decoheres at N = 2^64 = 10^19, so that inflation ends at age 10^(-34) sec.

Note that Ndecoh = 2^64 = 10^19 qubits is just an order of magnitude larger than the number of tubulins Ntub = 10^18 of the human brain. In my opinion, conscious thought is due to superposition states of those 10^18 tubulins. Since a brain with Ndecoh = 10^19 tubulins would undergo self-decoherence and would therefore not be able to maintain the superposition necessary for thought, it seems that the human brain is about as big as an individual brain can be.

E6 World-Line Strings

E6 26-dimensional closed unoriented bosonic string theory interpreted as a Many-Worlds Quantum Theory in which strings correspond to World Lines, with massless spin-2 Gravitons in 26-dimensions corresponding to gravitational interaction among Tubulin Electrons in states with Penrose-Hameroff Superposition Separation
[[In the D4-D5-E6-E7-E8 VoDou Physics model, closed strings represent the world-lines of fermion particle-antiparticle pairs (the pair of fermions acting as a boson so that the entire string is bosonic) from the time of their creation to their eventual mutual annihilation,

```
    *
   / \    ...
  /   \ /
 /     |         (The illustrated closed string is red.
 \     |          It interacts with a partially shown gray string.)
  \   / \...
   \ /
    *
```

perhaps with lots of interactions with lots of other particles/antiparticles of other world-lines in the meantime, so that part of each string might represent a photon or other particle of any type formed by interaction of one of the particle/antiparticle pair.
Note that since our Universe began with a Big Bang, all its particles originate from pair creation since then. For pairs that do not appear to reconnect for mutual annihilation within the volume of 26-dimensional spacetime being considered in working with the String Theory,

```
****************
  \    ...    /
   \      \ /
    \      |         (The illustrated string is red.
     \     |          It interacts with a partially shown gray string.
      \   / \...      A perfect absorber in the future
       \ /            is indicated by ******* ).
        *
```

the string is closed by considering the 26-dimensional spacetime to be a compactified 25+1 dimensional Minkowski spacetime due to considering the Universe to "... be a perfect absorber in the future ...[as in]... the Wheeler-Feynman ... absorber theory of radiation ..." described by Narlikar in his book Introduction to Cosmology (Cambridge 1997) (Section 8.8.1) and related to the Collective Electrodynamics of Carver Mead. For most of the matter in our Galactic Cluster, such an absorber could be a Black Hole of the Black Hole Era. Such a compactification is also similar to the conformally compactified 3+1 dimensional Minkowski spacetime M# used by Penrose and Rindler in their book Spinors and Space-Time, Volume 2 (Cambridge 1986) (particularly Chapter 9).]]

Roger Penrose says, in Shadows of the Mind (Oxford 1994), page 344,

"... We can now consider the gravitational self-energy of that mass distribution which is the difference between the mass distributions of the two states that are to be considered in quantum linear superposition. The reciprocal of this self-energy gives ... the reduction timescale ...".

This is the decoherence time T = h / E.

For a given Particle, Stuart Hameroff describes this as a particle being separated from itself, saying that

the Superposition Separation a is "... the separation/displacement of a mass separated from its superposed self. ... The picture is spacetime geometry separating from itself, and re-anealing after time T. ...".
If the Superposition consists of States involving one Particle of Mass m, but with Superposition Separation a, then the Superposition Separation Energy Difference is the gravitational energy

E = G m^2 / a

In the Osaka paper, Hameroff says that Penrose describes Superposition Separation as "... shearing off into separate, multiple spacetime universes as described in the Everett "multi&endash;worlds" view of quantum theory. ...".

If 26-dimensional closed unoriented bosonic string theory is interpreted as a Many-Worlds Quantum Theory in which strings correspond to World Lines then massless spin-2 Gravitons in 26 dimensions correspond to gravitational interaction among States with Penrose-Hameroff Superposition Separation.

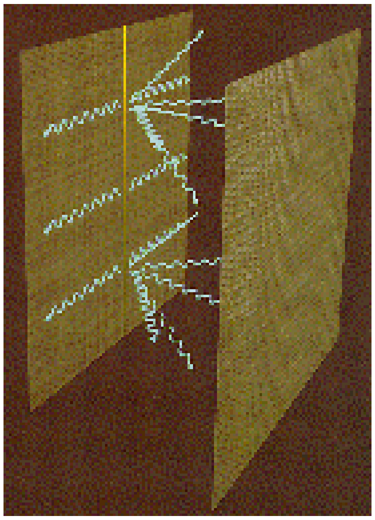
Such massless spin-2 Gravitons in 26 dimensions are described by Joseph Polchinski in his books String Theory vols. I and II(Cambridge 1998) where he says:

"... [In] the simplest case of 26 flat dimensions ... the closed bosonic string ...

theory has the maximal 26-dimensional Poincare invariance ... [and] ... is the unique theory with this symmetry ... It is possible to have a consistent theory with only closed strings ... with Guv representing the graviton ...", as to which Green, Schwartz, and Witten, in their book Superstring Theory, vol. 1, p. 181 (Cambridge 1986) say "... the long-wavelength limit of the interactions of the massless modes of the bosonic closed string ... [which] ... can be put in the form
INTEGRAL d^26 x sqrt(g) R
...[of 26-dimensional general relativistic Einstein Gravitation]...".

A nice description of how such Gravitons propagate in the 26 dimensions is given by Stephen Hawking in his book The Universe in a Nutshell (Bantam 2001). To see how Hawking's description of gravity in 26 dimensions might be applied to the Penrose-Hameroff tubulin electron model of consciousness, first assume the validity of the interpretation of 26-dimensional bosonic string theory as a Many-Worlds Quantum Theory in which strings correspond to World Lines. However, Hawking speaks of branes rather than individual particle world lines. From the viewpoint of this paper, such branes should be regarded as 4-dimensional physical spacetime neighborhoods of individual particles. Timelike parts of such branes should be described in terms of 27-dimensional M-theory, and spacelike parts of such branes should be described in terms of 28-dimensional F-theory. In his book, Hawking says:

"... Large extra dimensions ... would imply that we live in a brane world, a four-dimensional surface or brane in a higher-dimensional spacetime. Matter and nongravitational forces would be confined to the brane. ... On the other hand, gravity ... would permeate the whole bulk of the higher-dimensional spacetime ... because gravity would spread out in the extra dimensions, it ... would fall off faster with distance than it would in four dimensions. ... If this more rapid falloff of the gravitational force extended to astronomical distances, we would have noticed its effect ... However, this would not happen if the extra dimensions ended on another brane not far away from the brane on which we live. ...



...
[Note that in the Penrose-Hameroff model the superposition separation of two individual states in the superposition states of a single tubulin electron is of the order of a nanometer.]

... A second brane near our brane would prevent gravity from spreading far into the extra dimensions and would mean that at distances greater than the brane separation, gravity would fall off at the rate one would expect for four dimensions. ...
... On the other hand, for distances less than the separation of the branes, gravity would vary more rapidly. The very small gravitational force between heavy objects has been measured accurately in the lab but the experiments so far would not have detected the effects of branes separated by less than a few millimeters. ...".

Consciousness, A-D-E, and E8

The Mead Quantum Resonance (see page 241) picture of Quantum Consciousness Connections seems to me to be related to Saul-Paul Sirag's view of Consciousness described by him in an Appendix to the book "The Roots of Consciousness" (by Jeffrey Mishlove, Council Oak Books, 1993) in which Saul-Paul Sirag said:

"…One of the most striking facts of recent mathematics is a 1:1 correspondence [the McKay correspondence] between the finite subgroups of SU(2) and the A-D-E series of Lie algebras … we call these finite groups McKay groups. …
Lie algebra E8 [corresponds to]…
SU(2) subgroup ID [icosahedral double group of order 20]…
If we make the McKay-group elements into the basis vectors of a vector space, this space becomes an algebra called a group algebra. … C[ID]
… for a given Coxeter graph, the actions of the McKay group and the Lie algebra interact in a truly marvelous way in the construction of a catastrophe …
important for a theory of consciousness is the fact that the root lattices corresponding to .. the three algebras E6, E7, E8 … are generated by error-correcting codes … the E8 lattice is generated by the Hamming-8 code. … coding theory is an application of information theory, so that it is natural to suppose a connection between coding theory and consciousness …".

In a later (8 October 2000) paper, "Notes on Hyperspace" Saul-Paul Sirag considers 24 dimensions of 26-dimensional World- String Theory and says:

"… the basic object in string theory is the 2-d world sheet swept out by the interacting strings (analogous to the world lines swept out by point particles in the Feynman diagrams of quantum field theory). We can picture this world sheet as vibrating in the hyperspace. Since the vibrations must be transverse to the world sheet, there are 24 vibrational degrees of freedom in the 26 dimensions of … string theory. We can describe this 24 dimensional … space as the space of a very powerful error-correcting code the Golay-24 code. This code has 12 message carrying digits and 12 error-correcting digits. The Golay-24 code is an even, self-dual error correcting code. …
the Golay-24 code can be derived from 3-copies of the Hamming-8 code, i.e. from the Coxeter graph for E8 x E8 x E8.

Strangely enough the vibrational dimension of 24 can be derived from a solution to a 24^{th} degree polynomial equation called the Ramanujan function … A similar solution to an 8^{th} degree polynomial accounts for the vibrational dimension 8. … ".

All this leads to the following 5 ideas:

The things that are connected by Mead Quantum Resonance are Patterns of Clifford Algebra States. In the human brain, these Patterns are manifested by Binary Tubulin States;

The information carried by Patterns of Clifford Algebra States can be described in terms of Codes;

The Codes come from the root vector systems of E8 and E8 x E8 x E8 which is described by its Coxeter-Dynkin Diagram;

The Coxeter-Dynkin Diagrams of A-D-E algebras (the E of which are E6, E7, E8) correspond to Polytopes by the McKay correspondence, with the McKay Polytope of E8 being the Icosahedron; and

Finite Subgroups of E8.

Each of those 5 ideas will be discussed in the following 5 subsections.

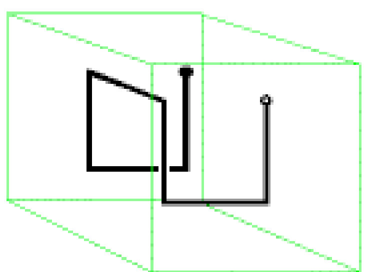
Clifford Algebra State Patterns:

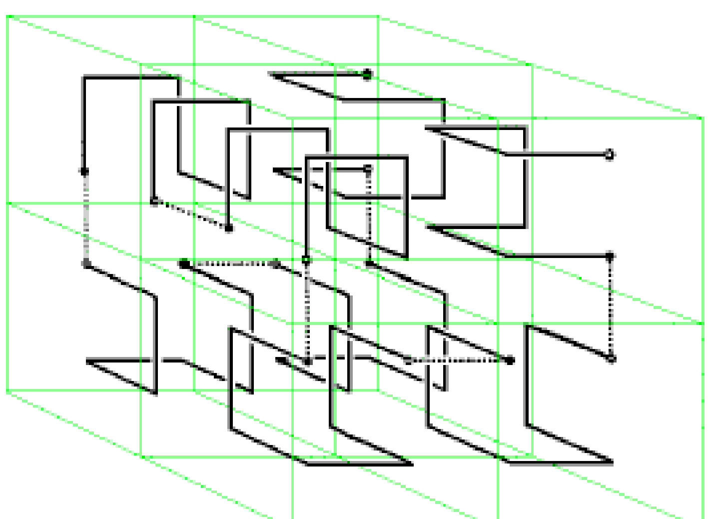
A Clifford Algebra State Pattern should ve describable in terms of a Moore space-filling curve. Acccording to a web page of V. B. Balayoghan:

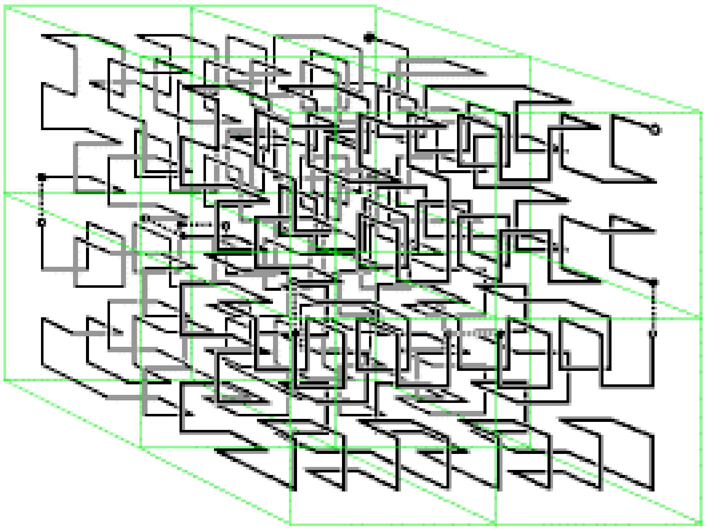
"... The Hilbert and Moore curves use square cells -- the level n curve has 4^n cells (and hence 4^n - 1 lines). The Moore curve has the same recursive structure as the Hilbert curve, but ends one cell away from where it started. The Hilbert curve starts and ends at opposite ends of a side of the unit square. ...".

According to a web page by William Gilbert:

"... We exhibit a direct generalization of Hilbert's curve that fills a cube. The first three iterates of this curve are shown.







In constructing one iterate from the previous one, note that the direction of the curve determines the orientation of the smaller cubes inside the larger one.

The initial stage of this three dimensional curve can be considered as coming from the 3-bit reflected Gray code which traverses the 3-digit binary strings in such a way that each string differs from its predecessor in a single position by the addition or subtraction of 1. The kth iterate could be considered a a generalized Gray code on the Cartesian product set {0,1,2,...,2^k-1}^3.

The n-bit reflected binary Gray code will describe a path on the edges of an n-dimensional cube that can be used as the initial stage of a Hilbert curve that will fill an n-dimensional cube. ...".

According to Numerical Recipes in C, by Press, Teukolsky, Vettering, and Flannery (2nd ed, Cambridge 1992):

"... A Gray code is a function G(i) of the integers i, that for each integer N ≥ 0 is one-to-one for 0 ≤ i ≤ 2^N -1, and that has the following remarkable property: The binary representation of G(i)and G(i+1) differ in exactly one bit. an example of a Gray code ... is the sequence ...[
0000 (0=0000), 0001 (1=0001), 0011 (2=0010), 0010 (3=0011),

0110 (4=0100), 0111 (5=0101), 0101 (6=0110), 0100 (7=0111), 1100 (8=1000), 1101 (9=1001), 1111 (10=1010), 1110 (11=1011),

1010 (12=1100), 1011 (13=1101), 1001 (14=1110), 1000 (15=1111)

]... for i = 0, ... 15. The algorithm for generating this code is simply to form ... XOR of i with 1/2 (integer part). ... G(i) and G(1+1) differ in the bit position of the rightmosst zero bit of i ...

Gray codes can be useful when you need to do some task that depends intimately on the bits of i, looping over many values of i. Then, if there are economies in repeating the task for values differing only by one bit, it makes sense to do things in Gray code order rather than consecutive order. ...".

Vaughan F. R. Jones, in his review of the book Quantum symmetries on operator algebras, by D. Evans and Y. Kawahigashi, Oxford Univ. Press, New York, 1998, Bull. (N.S.) Am. Math. Soc., Volume 38, Number 3, Pages 369-377, said:

"... The "algebraic quantum field theory" of Haag, Kastler and others ... is an attempt to approach quantum field theory by seeing what constraints are imposed on the underlying operator algebras by general physical principles such as relativistic invariance and positivity of the energy. A von Neumann algebra of "localised observables" is postulated for each bounded region of space-time. Causality implies that these von Neumann algebras commute with each other if no physical signal can travel between the regions in which they are localised. The algebras act simultaneously on some Hilbert space which carries a unitary representation of the Poincare (=Lorentz plus 4-d translations) group. The amount of structure that can be deduced from this data is quite remarkable. ... Just as remarkably, more than one type II1 factor (up to isomorphism) was constructed ... and ... uncountably many were shown to exist and the classification of factors is not at all straightforward. That is the bad news.
Now the good news. A von Neumannn algebra is called hyperfinite if it contains an increasing dense sequence of finite dimensional *-subalgebras ... it was shown that there is a unique hyperfinite II1 factor. (It can be realised as U(G) where G is the group of all finite permutations of [the natural numbers] N .) ...”.

Frank Wilczek, in his paper Projective Statistics and Spinors in Hilbert Space, hep-th/9806228, said:

"... In quantum mechanics, symmetry groups can be realized by projective, as well as by ordinary unitary, representations. For the permutation symmetry relevant to quantum statistics of N indistinguishable particles, the simplest properly projective representation is highly non-trivial, of dimension 2^{(N-1)/2}$, and is most easily realized starting with spinor geometry. Quasiparticles in the Pfaffian quantum Hall state realize this representation. Projective statistics is a consistent theoretical possibility in any dimension. ...[A]... very basic quantum mechanical symmetry concerns the interchange, or permutation, of indistinguishable particles. It is natural to ask whether the permutation symmetry is realized projectively in Nature. The mathematical theory of projective representations of the group SN of permutations of N elementary particles was developed in classic papers by I. Schur ...[in 1907 and 1911]... , prior to the discovery of either modern quantum mechanics or spinors. The simplest (irreducible) non-trivial projective representations of SN are already surprisingly intricate and have dimensions which grow exponentially with N. They are intimately related to spinor representations of SO(N) ... For even N = 2p one can construct an irreducible representation of the G [my substitution for capital gamma] matrices of dimension 2^p iteratively ... This is not irreducible for SO(2p) ... By projecting onto the eigenvalues of k = G1 G2 ... G2p we get irreducible spinor representations. k, of course, does not commute with the representatives of the permutation group. But k' = k(G1 - G2 + G3 - G4 ...) does. By projecting onto its eigenvalues, we obtain irreducible (projective) representations of S2p. ... Schur demonstrated that all the non-trivial, irreducible projective representations of SN realize the modified algebra ...[and]... may be classified using Young diagrams, but with the additional restriction that row lengths must be strictly decreasing. In this construction, the spinorial representation constructed above corresponds to a single row, analogous to bosons. ... In recent work on the Pfaffian nu = 1/2 quantum Hall state, it was shown that 2n quasiparticles at fixed positions span a 2^(n-1) dimensional Hilbert space, and that braiding such quasiparticles around one another generated operations closely analogous to spinor representations ... (In addition, there are exp(2 pi i / 8) "anyonic" phase factors.) The concepts explained above allow one to formulate the results in a different way: the exchange of these quasiparticles realizes the simplest projective representation of the symmetric group.

Another perspective on the projective statistics arises from realizing the

Clifford algebra in terms of fermion creation and annihilation operators ... we find for the interchange of an odd [2j-1] index particle with the following even [2j] index particle ... is ... simply the operation of changing the occupation of the j th mode. This makes contact with an alternative description of the nu = 1/2 quasiparticles using antisymmetric polynomial wave-functions, which can be considered to label occupation numbers of fermionic states ... Thus projecting to eigenvalues of k amounts to restricting attention to either even or odd mode occupations. This is adequate to get irreducible representations of the rotation group or of the even permutations. If we want to get an irreducible representation of all permutations we must allow both even and odd occupations, with a peculiar global relation between them. Since the definition of projective statistics refers to interchanges of particles, as opposed to braiding, this concept is not in principle tied to 2+1 dimensional theories. Also, no violation of the discrete symmetries P, T is implied. ...".

The E8 physics model uses a Cl(8) generalization of the conventional hyperfinite II1 von Neumann algebra factor, which should describe (to paraphrase Vaughan f. R. Jones) all finite permutations of Clifford Algebra State Patterns.

By taking the limit as n goes to infinity of the real-Clifford-periodicity tensor factorization of order 8

Cl(8n,R) = Cl(8,R) x ...(n times tensor)... x Cl(8,R)

the generalized hyperfinite II1 von Neumann algebra R can be denoted as the real Clifford algebra Cl(infinity,R) whose half-spinors are sqrt(2^(infinity))-dimensional. In other words, since the halfspinors of Cl(2n,R) are 2^(n-1)-dimensional, the dimension of the full spinors grows exponentially with the dimension of the vector space of the Clifford algebra.

Codes:

Here, paraphrasing from the books Sphere Packings, Lattices, and Groups by J. H. Conway and N. J. A. Sloane (Springer, Third edition 1999), From Error-Correcting Codes Through Sphere Packings to Simple Groups, by Thomas M. Thompson (MAA 1983), A Course in Combinatorics, by J. H. van Lint and R. M. Wilson (Cambridge 1992), Designs and Their Codes, by Assmus and Key (Cambridge 1992), are some details about Classical Information Theory of Classical Error-correcting Codes.

Error-correcting codes were discovered in mid-20th century after R. W. Hamming got irritated by his computer stopping when it encountered an error, causing him to realize that if his computer could detect errors it should be able to locate and correct them.

C24 is a binary Golay code [24,12,8] is a code of length 24, dimension 12, and minimal distance 8 over the binary field F2. Of the 2^24 sets of 24 zeroes and ones, 2^12 = 4096 are in C24. They can be divided into classes:
1 that has 24 zeroes;
759 that have 16 zeroes and 8 ones;
2,576 that have 12 zeroes and 12 ones;
759 that have 8 zeroes and 16 ones;
1 that has 24 ones.

The symmetry group of C24 is M24, where M24 is the simple Mathieu group of order 24x23x22x21x20x48. M24 has several interesting subgroups, including:

PSL2(23)
the sextet group 2^6:3xS6
the octad group 2^4:A8 = 2^4:GL4(2)
PSL3(4) = M21
the trio group 2^6:(S3xL2(7)) = 2^6:(S3xL3(2))
the Mathieu group M23
the Mathieu group M22 and M22:2
the Mathieu group M12 associated with the Steiner system S(5,6,12) and the ternary Golay code C12 [12,6,6] with 3^6 = 27x27 = 729 words. C12 can be constructed using Rubik-icosahedron twist-permutations.

An octad is a Golay codeword of weight (distance from the origin) 8. The octads of C24 constitute a Steiner system S(5,8,24). The Steiner system S(5,8,24) is a set with 24 points and a collection of distinct 8-subsets (called blocks) such that any 5-subset is contained in exactly 1 block.

The MOG is a 4x6 binary array, so that it would carry 24 bits of information.

The hexacode C6 is a 3-dimensional code of length 6 over F4. C6 has 36+12+9+6+1 = 64 = 2^6 = 4^3 = sqrt(4^6) hexacodewords.

The Mathieu group M24 is transitive on trios consisting of three disjoint octads. The subgroup fixing a trio is a group 2^6:(S3xL2(7)) it being significant that L2(7) is isomorphic to L3(2). and is Klein's group of order 168 = 7x6x4, and is the automorphism group of the Hamming binary code H7 [7,4,3].

By fixing two points of the 24 we obtain the Mathieu group M22 which is the stabilizer of the Steiner system S(3,6,22) obtained by deleting those two points from all octads of S(5,8,24) that contain them both.

In math.CO/0207208, Hammons, Kumar, Calderbank, Sloane, and Sole say:

"...The classical theory of cyclic codes, which includes BCH, Reed-Solomon, Reed-Muller codes, etc., regards these codes as ideals in polynomial rings over finite fields. Some famous nonlinear codes found by Nordstrom-Robinson, Kerdock, Preparata, Goethals and others, more powerful than any linear codes, cannot be handled by this machinery. We have shown that when suitably defined all these codes are ideals in polynomial rings over the ring of integers mod 4. This new point of view should completely transform the study of cyclic codes. ... Kerdock codes contain more codewords than any known linear code with the same minimal distance (although we are not aware of any theorem to guarantee this, except at length 16) ... Kerdock and Preparata codes exist for all lengths n = 4^m ≥ 16. ... Kerdock and `Preparata' codes are duals over Z4 - and the Nordstrom-Robinson code is self-dual ... the Kerdock code is simply the image of the quaternary code (when extended by an zero-sum check symbol) under the Gray map ... the Gray map translates a quaternary code with high minimal Lee or Euclidean distance into a binary code of twice the length with high minimal Hamming distance. ...

... The Kerdock and Preparata codes of length 16 coincide, giving the Nordstrom-Robinson code. This is the unique binary code of length 16, minimal distance 6, containing 256 words. In this case K ... the cyclic code of length 16 over Z4 ...[plus]... adjoining a zeo-sum check symbol. ... is the ... octacode ... the unique self-dual quaternary code of length 8 and minimal Lee weight 6, or ... the 'glue code' required to construct the 24-dimensional Leech lattice from eight copies of the face-centered cubic lattice. ... The Nordstrom-Robinson code is the binary image of the octacode under the Gray map. ...
... In communication systems employing quadrature phase-shift keying (QPSK), the preferred assignment of two information bits to the four possible phases is the one ... in which adjacent phases differ by only one binary digit. This mapping is called Gray encoding and has the advantage that, when a quaternary codeword is transmitted across an additive white Gaussian noise channel, the errors most likely to occur are those causing a single erroneously decoded information bit. ... The crucial property of the Gray map is that it preserves distances. ... from (Z4^n, Lee distance) to (Z2^2n, Hamming distance) ...
... The quaternary octacode has an automorphism group of order 1344, whereas the group of the binary Nordstrom-Robinson code has order 80640. ... The Kerdock and `Preparata' codes are Z4-analogues of first-order Reed-Muller and extended Hamming codes, respectively. All these codes are extended cyclic codes over Z4, which greatly simplifies encoding and decoding. ... Binary first- and second-order Reed-Muller codes are also linear over Z4, but extended Hamming codes of length n $\geq$ 32 and the Golay code are not. ... the Nordstrom-Robinson code is Z4-linear ... and is closely connected with the ... [24, 12, 8] Golay code ...[which, although linear in its normal formulation,]... is not Z4-linear. ...".

In their book The Theory of Error-Correcting Codes (North-Holland Elsevier 1977), MacWilliams and Sloane say:
"... The extended Golay code G24 may be used to construct ... The Nordstrom-Robinson code N16 ... divide up the [G24] codewords according to their values on the first 7 coordinates: there are 2^7 possibilities, and for each of these there are 2^12 / 2^7 = 32 codewords. thus there are 8 x 32 = 256 codewords which begin either with seven 0's (with 8th coordinate 0), or with six 0's and a 1 (with 8th coordinate 1) ... The Nordstrom-Robinson code N16 is obtained by deleting the first 8 coordinates from these 256 vectors. ... N16 is a (16, 256, 6) code ... made up of a linear [16,5,8] code ... plus 7 of its cosets in G24 ... N16 is not linear ...".

The extended Golay code G24 is based on
the 24-dimensional Leech lattice.
The automorphism group
of G24 is the Mathieu group M24.
It may be related to the Urim v'Tumim.

G24 is the (24,2^12) d=8 self-dual classical code over GF(2),
which is sometimes denoted (24,12,8)
because it uses binary 24-blocks with 12 message bits and Hamming minimal distance 8.

```
G24 has 2^12 = 4096 codewords:
1       (denoted  0)  is at Hamming distance  0 from zero;
759     (denoted  8) are at Hamming distance  8 from zero;
2576    (denoted 12) are at Hamming distance 12 from zero;
759     (denoted 16) are at Hamming distance 16 from zero;
1       (denoted 24)  is at Hamming distance 24 from zero.
```

The weight distribution can also be written:
0(1) 8(759) 12(2576) 16(759) 24(1)

The number of dodecads comes from the combinatorial table:

```
   0     0    16     0    24     0    16     0     0
      0    16    16    24    24    16    16     0
        16    32    40    48    40    32    16
          48    72    88    88    72    48
            120   160   176   160   120
               280   336   336   280
                  616   672   616
                    1288  1288
                       2576
```

The number of octads comes from the combinatorial table:

```
  30     0    16     0     4     0     0     0     1
     30    16    16     4     4     0     0     1
        46    32    20     8     4     0     1
          78    52    28    12     4     1
            130    80    40    16     5
               210   120    56    21
                  330   176    77
                     506   253
                        759
```

The octads of G24 are a Steiner system S(5,8,24),
so that any 5 points of the 24 are in one unique octad.
Since G24 can be constructed from its Steiner system,
G24 can be constructed from its octads,

and each of its octads can be constructed
from 5 of its 8 nonzero elements.

The 24-dimensional Leech lattice can not only be constructed
as a real 24-dimensional lattice /\R24
from the binary (24,12,8) Golay code G24 over GF(2),
it can also be constructed
as a complex 12-dimensional lattice /\C12
from the ternary (12,6,6) Golay code G12 over GF(3),
with Steiner system S(5,6,12).

G12 has 3^6 = 729 codewords, with weight distribution:
0(1) 6(264) 9(440) 12(24).

The 24-dimensional Leech lattice can also be constructed
as a quaternionic 6-dimensional lattice /\Q6
from the (6,3,4) hexacode H6 over GF(4).

H6 has 4^3 = 64 codewords, with weight distribution:
0(1) 4(45) 6(18).

The 24-dimensional Leech lattice can be used,
by 3x3 octonion matrices,
to construct the D4-D5-E6 model.
The automorphism group of the Leech lattice is
the Conway group .0 (dotto).

The 24-dimensional Leech lattice represents 8-dimensional space-time,
the 8 first generation fermion particles, and
the 8 first generation fermion antiparticles.
Each vertex has 196,560 nearest neighbors,
whose permutation group is dotto.

196,560 + 300 + 24 = 196,884
(300 = 25x24/2 = symmetric tensor square of 24)
is the dimension of a representation space of the Monster group,
whose order is the product
2^46 3^20 5^9 7^6 11^2 13^3 17 19 23 29 31 41 47 59 71,
or about 8 x 10^53.

IF the 196,560 points form a group,
just as the 240 of an E8 lattice form unit octonions

```
and as the 24 of a D4 lattice form unit quaternions
(this is a goal of the current work of Geoffrey Dixon),

then it should be possible to form the 196,560 dimensional
space
of the group algebra of the Leech lattice nearest-neighbor
group,

and then add the 300-dim space of symmetric squared Leech
lattice,

and then add the 24-dim space of the Leech lattice itself,

to get the 196,884-dim representation space of the Monster.
```

Calderbank, Rains, Shor, and Sloane describe error correction in quantum codes.

As they say, given the quantum state space C^2^n of n qubits,

"... The known quantum codes seemeed to have close connections to a finite group of unitary transformations of C^2^n, known as a Clifford group, ... [containing] all the transformations necessary for encoding and decoding quantum codes. It is also the group generated by fault-tolerant bitwise operations performed on qubits that are encoded by certain quantum codes. ...

... the unique [[2; 0; 2]] code corresponds to the quantum state (1/sqrt(2))(| 01 > - < 10 |), that is, an EPR pair. ...".

```
Calderbank, Rains, Shor, and Sloane in quant-ph/9608006
show that
whereas many useful classical-error-correcting codes are
binary,
over the Galois field GF(2) = {0,1},
quantum-error-correcting codes are quaternary,
over the Galois field GF(4) = {0,1,w,w^2}
where w    =  (1/2)( - 1 + sqrt(3) i )
and   w^2  =  (1/2)( - 1 - sqrt(3) i ).
Some interesting binary classical codes can be used
to construct quaternary quantum codes.  For example,
the Golay code (24,2^12) d=8 self-dual classical code over
```

GF(2),
which is sometimes denoted (24,12,8)
because it uses binary 24-blocks with 12 message bits and Hamming minimal distance 8,
should give a quantum code [[24, 0, 8]]
mapping a quantum state space of 24 qubits into 24 qubits,
correcting [(8-1)/2] = 3 errors, and detecting 8/2 = 4 errors.

They note that codes with only one codeword CAN be useful,
either to test decoherence times of specific storage bins
or to construct other codes with more codewords.

They give another interesting example: concatenating
the Hamming code [[5, 1, 3]] with itself to get [[25, 1, 9]].
They also state that, although
there is no classical (24,4^12) d=10 over GF(4),
it is unknown whether
there exists an additive (even) self-dual (24,2^24) d=10 code.

They say, given the quantum state space C^2^n of n qubits,
"... The known quantum codes seemeed to have close connections
to a finite group of unitary transformations of C^2^n,
known as a Clifford group, ... [containing] all the transformations
necessary for encoding and decoding quantum codes. It is also the
group generated by fault-tolerant bitwise operations performed
on qubits that are encoded by certain quantum codes. ..."

Now, look at the example of Steane of the

Quantum Reed-Muller code [[256, 0, 24]],

which maps a quantum state space of 256 qubits into 256 qubits,
correcting [(24-1)/2] = 11 errors, and detecting 24/2 = 12 errors.
Let C(n,t) = n! / t! (n-t)!

Then

```
[[ 256, 0, 24 ]] is of the form

[[ 2^n,   2^n - C(n,t) - 2 SUM(0 k t-1) C(n,k),   2^t + 2^(t-1) ]]

[[ 2^8,   2^8 - C(8,4) - 2 SUM(0 k 3) C(8,k),     2^4 + 2^(4-1) ]]

[[ 2^8,   2^8 - 70 - (1+8+28+56) -  (1+8+28+56),  16 + 8 ]]

[[ 256,   256 - (1+8+28+56+70+56+28+8+1),         16 + 8 ]]

[[ 256,   16x16 - SUM(0 k 8) 8/\8/\..(k)../\8,    16 + 8 ]]
```

The quantum code [[256, 0, 24]] can be constructed
from the classical Reed-Muller code (256, 93, 32) of the
form

```
(  2^8,   2^8 - SUM(0 k t) C(n,k),              2^(t+1) )

(  2^8,   2^8 - SUM(0 k 4) C(n,k),              2^5 )

(  2^8,   2^8 - (70+56+28+8+1),                 32 )

(  2^8,   1+8+28+56,                            32 )
```

To construct the quantum code [[256, 0, 24]] :

First, form a quantum code generator matrix
from the 128x256 generator matrix G of the classical code
(256, 93, 32) :

```
|       |       |
|   G   |   0   |
|       |       |
|   0   |   G   |
|       |       |
```

Second, form the generator matrix of a quantum code of distance 16
by adding to the quantum generator matrix a matrix Dx such that
G and Dx together generate the classical Reed-Muller code (256, 163, 16) :

```
(  2^8,   1+8+28+56+70,                           16 ) :

        |       |       |
        |   G   |   0   |
        |       |       |
        |   0   |   G   |
        |       |       |
        |   Dx  |   0   |
        |       |       |
```

This quantum code has been made by combining the classical codes
(256, 93, 32) and (256, 163, 16), so that it is of the form
[[256, 93 + 163 - 256, min(32,16)]] = [[256, 0, 16]] .

It is close to what we want, but has distance 16.
For the third and final step, increase the distance to 16+8 = 24
by adding Dz to the quantum generator matrix:

```
        |       |       |
        |   G   |   0   |
        |       |       |
        |   0   |   G   |
        |       |       |
        |   Dx  |   Dz  |
        |       |       |
```

This is the generator matrix
of the quantum code [[256, 0, 24]]
as constructed by Steane.

The two classical Reed-Muller codes used to build [[256, 0, 24]]
are (256, 163, 32) and (256, 93, 16),
classical Reed-Muller codes of orders 4 and 3,
which are dual to each other.
Due to the nested structure of Reed-Muller codes,
they contain the Reed-Muller codes of orders 2, 1, and 0 :

Classical Reed-Muller Codes of Length 2^8 = 256 — Order

(256,	1+8+28+56+70+56+28+8+1,	1)	8
(256,	1+8+28+56+70+56+28+8,	2)	7
(256,	1+8+28+56+70+56+28,	4)	6
(256,	1+8+28+56+70+56,	8)	5
(256,	1+8+28+56+70,	16)	4
(256,	1+8+28+56,	32)	3
(256,	1+8+28,	64)	2
(256,	1+8,	128)	1
(256,	1,	256)	0

In the Lagrangian of the D4-D5-E6 physics model:

the Higgs scalar
prior to dimensional reduction
corresponds to the
0th order classical Reed-Muller code (256, 1, 256),
which is the classical repetition code;

the 8-dimensional vector spacetime
prior to dimensional reduction
corresponds to non-0th-order part of the
1st order classical Reed-Muller code (256, 9, 128),
which is dual to
the 6th order classical Reed-Muller code (256, 247, 4),
which is the extended Hamming code,
extended from the binary Hamming code (255, 247, 3),
which is dual to the simplex code (255, 8, 128) ;

the 28-dimensional bivector adjoint gauge boson space
prior to dimensional reduction
corresponds to the non-1st-order part of the
2nd order classical Reed-Muller code (256, 37, 64) .

HERE is a D4-D5-E6 model physical interpretation of
higher order classical Reed-Muller codes, written in terms
of the graded subspaces of the Clifford algebra Cl(0,8).

The 8 first generation fermion particles and
8 first generation fermion antiparticles
of the 16-dimensional full spinor representation
of the 256-dimensional Cl(0,8) Clifford algebra
corresponds to the distance of the
classical Reed-Muller code (256, 93, 16),

as well as to the square root of 256 = 16x16,
and to the 16-dimensional Barnes-Wall lattice /\16,
which lattice comes from the (16,5,8) Reed-Muller code.
Each /\16 vertex has 4320 nearest neighbors.

The other 8 of the 16+8 = 24 distance of
the quantum Reed-Muller code [[256, 0, 24]]
corresponds to the 8-dimensional vector spacetime,
and to the 8-dimensional E8 lattice,
which lattice comes from the (8,4,4) Hamming code,
with weight distribution 0(1) 4(14) 8(1).
It can also be constructed from the repetition code
(8,1,1).
The dual of (8,1,1) is (8,7,2), a zero-sum even weight
code,
containing all binary vectors with an even number of 1s.
Each E8 lattice vertex has 240 nearest neighbors.
In Euclidean R8, there is only one way to arrange 240
spheres
so that they all touch one sphere, and only one way
to arrange 56 spheres so that they all touch
a set of two spheres in contact with each other,
and so forth, giving the following classical spherical
codes:
(8,240,1/2), (7,56,1/3), (6,27,1/4),
(5,16,1/5), (4,10,1/6), and (3,6,1/7).

The total 24 distance of
the quantum Reed-Muller code [[256, 0, 24]]
corresponds to the 24-dimensional Leech lattice,
and to the classical extended Golay code (24, 12, 8)
in which lattice each vertex has 196,560 nearest neighbors.
In Euclidean R24, there is only one way to arrange 196,560
spheres
so that they all touch one sphere, and only one way
to arrange 4600 spheres so that they all touch
a set of two spheres in contact with each other,
and so forth, giving the following classical spherical
codes:
(24,196560,1/2), (23,4600,1/3), (22,891,1/4),
(21,336,1/5), (20,170,1/6), ...

According to quant-ph/0301040 by Dorit Aharonov, entitled A Simple Proof that

Toffoli and Hadamard are Quantum Universal:

"... Recently Shi [15] proved that Toffoli and Hadamard are universal for quantum computation. This is perhaps the simplest universal set of gates that one can hope for, conceptually; It shows that one only needs to add the Hadamard gate to make a 'classical' set of gates quantum universal. ... The fact that {T,H} is universal has philosophical interpretations. The Toffoli gate T can perform exactly all classical reversible computation. The result says that Hadamard is all that one needs to add to classical computations in order to achieve the full quantum computation power; It perhaps explains the important role that the Hadamard gate plays in quantum algorithms, and can be interpreted as saying that Fourier transform is really all there is to quantum computation on top of classical, since the Hadamard gate is the Fourier transform over the group Z2. From a conceptual point of view, this is perhaps the simplest and most natural universal set of gates that one can hope for. ... the set {T,H} is [not only] ... the set {T,H} ... generates a dense subgroup in the group of orthogonal matrices [orthogonal matrices are related to the Dn and Bn Lie algebras and to Clifford algebras], see ... [15] Y. Shi, Both Toffoli and controlled-Not need little help to do universal quantum computation, quant-ph/0205115 ...".

Quantum Games

I believe that quantum game theory (not classical game theory) is needed for consciousness modelliing, using Clifford Algebra techniques such as:

Expressing the operations of quantum computing in multiparticle geometric algebra, quant=ph/9801002, by Shyamal S. Somaroo, David G. Cory, and Timothy Havel: The abstract states: "...We show how the basic operations of quantum computing can be expressed and manipulated in a clear and concise fashion using a multiparticle version of geometric (aka Cli ord) algebra. This algebra encompasses the product operator formalism of NMR spectroscopy, and hence its notation leads directly to implementations of these operations via NMR pulse sequences. ...";

Clifford algebras and universal sets of quantum gates, quant-ph/0010071, by Alexander Yu. Vlasov: The abstract states: "... In this paper is shown an application of Clifford algebras to the construction of computationally universal sets of quantum gates for n-qubit systems. It is based on the well-known application of Lie algebras together with the especially simple commutation law for Clifford algebras, which states that all basic elements either commute or anticommute. ..";

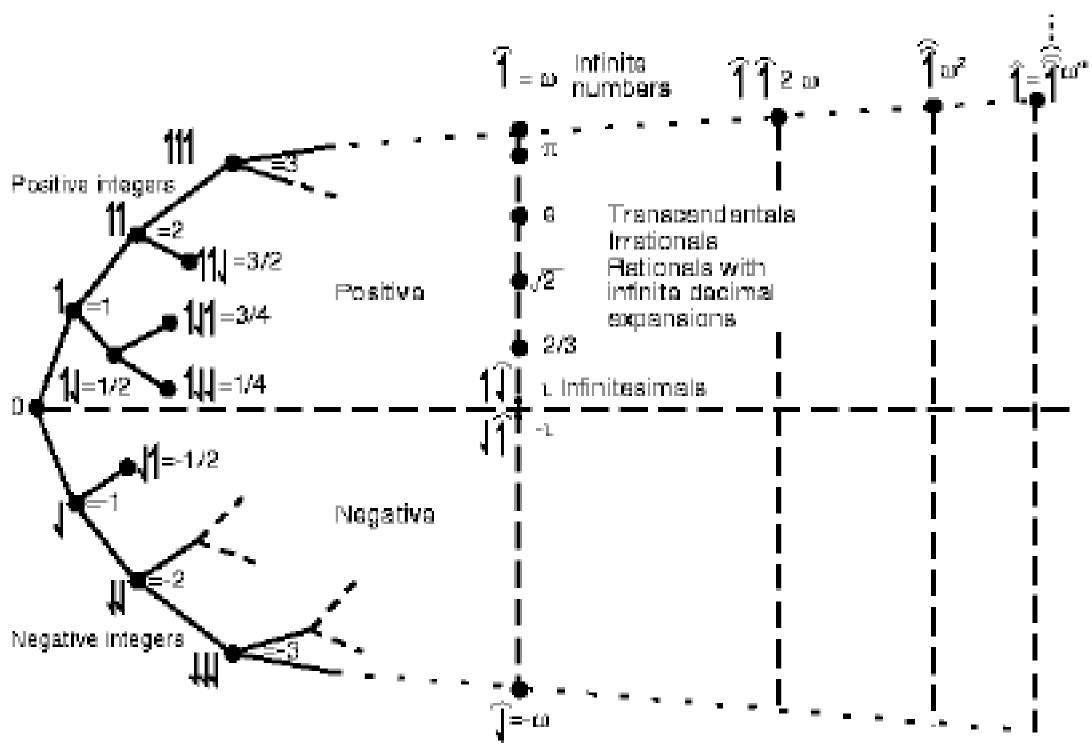
Quantum Zeno Effect techniques such as Towards a quantum Zeno tomography, quant-ph/0104021, by P. Facchi, Z. Hradil, G. Krenn, S. Pascazio, and J. Rehacek: The abstract states: "... We show that the resolution "per absorbed particle" of standard absorption tomography can be outperformed by a simple interferometric setup, provided that the different levels of "gray" in the sample are not uniformly distributed. The technique hinges upon the quantum Zeno effect and has been tested in numerical simulations. ...";

David Meyer's paper quant-ph/9804010: The abstract states: "We consider game theory from the perspective of quantum algorithms. Strategies in classical game theory are either pure (deterministic) or mixed (probabilistic). … We prove that in general a quantum strategy is always at least as good as a classical one, and furthermore that when both players use quantum strategies there need not be any equilibrium, but if both are allowed mixed quantum strategies there must be …"; and

[Theory of Quantum Games, quant-ph/0207012, by Chiu Fan Lee and Neil F. Johnson](): The abstract states: "... We pursue a general theory of quantum games. In particular, we develop quantum generalizations of the two most important theorems from classical game theory: the Minimax Theorem and the Nash Equilibrium Theorem. We then show that quantum games are more effcient than classical games, and provide a saturated upper bound for this efficiency. ...".

In a December 1995 article in Discover magazine, Polly Shulman said:
"… John Conway … explains …
"The surreal numbers actually came from games.
I was trying to understand how to play go … I did see that in the end, a go game decomposed into a sum of little games … then I discovered that certain games behaved very much like numbers … this was a new way of defining numbers – not only of defining new numbers, but of defining all the old ones too. And it's much simpler than the traditional way." …".

Surreal Numbers



are described by Robert Matthews, science correspondent of The Sunday Telegraph, in New Scientist (147 (2 Sep 1995) 36)
as presented by Martin Kruskal of Rutgers at Cambridge in 1995:

"… Surreal Numbers, invented by John Conway, include all the natural counting numbers, together with negative numbers, fractions, and irrational numbers, and numbers bigger than infinity and smaller than the smallest fraction. ...
Despite their astonishingly broad membership, surreal numbers are simply sequences of that most fundamental of notions, a binary choice: yes/no, off/on.
… Surreal Numbers are just sequences of binary choices, and constructing them is something of a game. It begins with the simplest surreal number, an empty sequence made up of nothing at all: this is written as 0, and is the starting place of what mathematician Martin Kruskal calls the Binary Number Tree.

To the upper right of 0, one then puts a single upward-pointing arrow. This represents the simplest surreal number greater than 0. The rules of surreal arithmetic then show that, naturally enough, this single up arrow is the surreal representation of the ordinary number 1. To the lower right of 0, one has a downward-pointing arrow, representing the simplest negative surreal, equal to -1.

From then on, this "branching" process continues, giving the "binary tree" its name. To the upper right of the single upward pointing arrow, a surreal consisting of two upward-pointing arrows is drawn: this represents the number 2.

To the lower right, however, a surreal made up of one up and one down arrow appears. What is this?

Finding out becomes the first exercise in decoding the arrow notation. The basic rule is that each successive arrow in a surreal number says a little bit more about it. For example, if it starts with an upward-pointing arrow, that means that the surreal is positive; an initial downward-pointing arrow means that it's negative.After that, the arrows alter the description of the overall surreal number in the "simplest" way possible, with upward meaning greater and downward meaning less.

Thus two up arrows define the simplest positive surreal greater than 1, and is thus equivalent to the ordinary number 2. Similarly, two down arrows represent the simplest negative surreal less than -1, in other words -2.

Clearly, all surreals made up of a finite number of entirely upward-pointing arrows represent the positive integers, while those made up of a finite number of only down-arrow strings are the negative integers.

So what does up-down mean? The leading upward arrow means the surreal is positive, and the downward arrow that follows means that it's the simplest surreal that is positive but less than 1. Given that we've already got zero, the most natural choice is exactly half way between 1 and zero, that is 1/2, and this in fact follows from the rules of surreal arithmetic. Similarly, down-up is equivalent to -1/2.

It turns out that, in fact, all the rational numbers have an arrow representation of this kind.The irrationals such as the square root of 2, the transcendentals such as pi and even infinite numbers are brought into the fold through arrow sequences that go on forever.

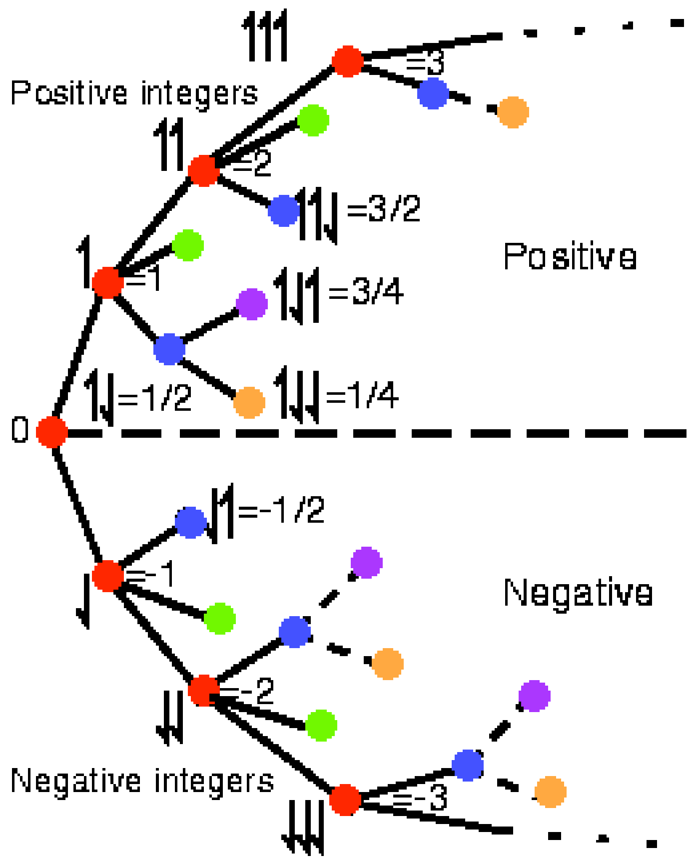
For example up-up-up-..., which is written as up^hat, represents the infinitely large number w, while up-down^hat represents the infinitesimally small number iota. All these latter arrow sequences lie along a vertical line connecting w to -w and mark a kind of frontier.

The "familiar" reals all lie somewhere on or to the left of this demarcation line. But the realm of the surreals does not end there. To the right of the line are surreals vastly larger than infinity.

Another way to represent infinitesimals as an extension of the real numbers is by Robinson's Nonstandard Analysis. Robinson's Nonstandard numbers are a subfield of the Surreals.

Kruskal is quoted (by James Lawry on usenet sci.math.research) as saying in his Cambridge lectures that it was unknown whether any of the infinite Surreal integers are prime, but that it is known that there exist prime infinite Nonstandard integers. …”.

Surreal Numbers can be a basis for a Feynman Checkerboard representation of the Many-Worlds of Sum-Over-Histories Quantum Field Theory:



To begin, choose a lattice spacetime of a Feynman Checkerboard, and call it the lattice spacetime of "our" universe in the ManyWorlds.

For simplicity, look at one of the space-time dimensions.

Represent its vertices by the red dots - the integers of the Surreal Numbers.

Then consider the "set" of all "other" lattice spacetimes of the other universes in the ManyWorlds.

Let the blue dots represent one of the "nearest neighbor" lattice spacetimes, and let the green dots represent the other "nearest neighbor" lattice spacetime.

Then go one step further, and let the purple and gold dots represent the two "next nearest neighbor" lattice spacetimes that are "accessible through" the blue spacetime, and also (not shown on figure) go to the two "next nearest neighbor" lattice spacetimes that are "accessible through" the green spacetime.

Continuing to fill out the Surreal Number Binary Tree, you have a representation of all the universes of the ManyWorlds by the "set" of all Surreal Rationals with finite expansions.

As Onar Aam has noted, the Surreals have natural mirrorhouse structure.

The two "nearest neighbors" of the origin in this 1-dimensional Surreal "spacetime", Sur^1, correspond to the set {+1, -1}
(represented in the diagram by {blue, green})
which are reflected into each other through the origin (represented by red).
The reflection group is the Weyl group of the rank-1 Lie group Spin(3) = SU(2) = Sp(1) = S3.

If we look at k-dimensional Surreal spacetimes Sur^k, we see that the "nearest neighbors" of the origin correspond to the root vectors of the Weyl reflection groups for the largest rank-k Lie group that contains as a subgroup the rank-k Lie group Spin(2k). Here I am using the term "nearest neighbors" to include both nearest and next-nearest neighbors in the case of Weyl groups whose root vectors are not all of the same length, as, for example, G2, F4, and Spin(2k+1) for k greater than 1.
For instance:
The origin of Sur^2 has 4+8 = 12 "nearest neighbors",
corresponding to (12+2)-dim G2 containing (4+2)-dim Spin(4).
The "nearest neighbors" are in a Star-of-David pattern, with 30-degree angle between adjacent root vectors.
Since 2x30 = 60, 3x30 = 90, and 4x30 =120, the G2 lattice of all "neighbors" combines both the square Gaussian lattice and the triangular Eisenstein lattice. Sur^2 has complex structure.

The origin of Sur^4 has 24+24 = 48 "nearest neighbors",
corresponding to (48+4)-dim F4 containing (24+4)-dim Spin(8).
The "nearest neighbors" are in a double 24-cell pattern,

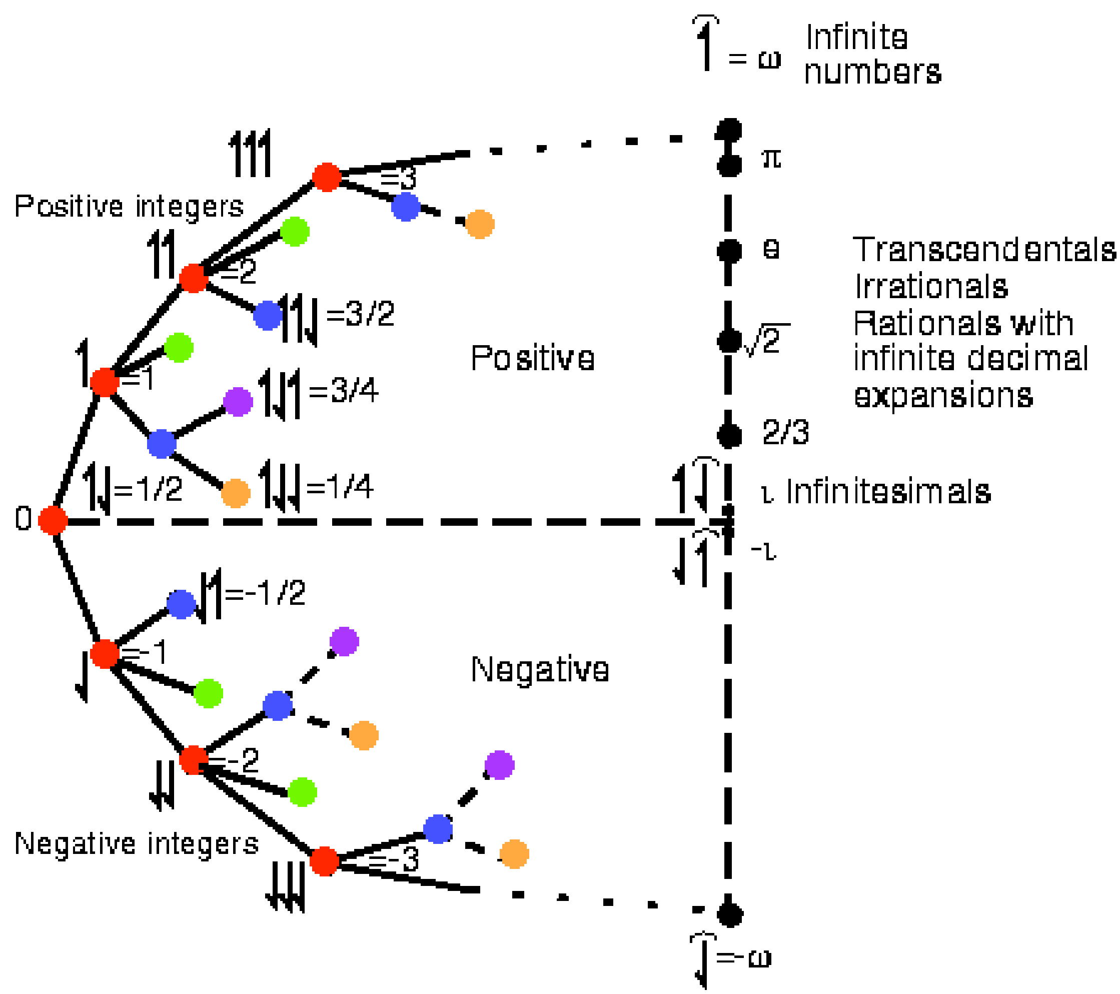
and all "neighbors" form a double D4 lattice. Sur^4 has quaternionic structure.

The origin of Sur^8 has 112+128 = 240 "nearest neighbors", corresponding to (240+8)-dim E8 containing (112+8)-dim Spin(16).
The "nearest neighbors" are in a Witting polytope pattern,
and all "neighbors" form an E8 lattice.
If the other 6 of the 7 E8 lattices are included, then there are 480 "nearest neighbors".
Sur^8 has octonionic structure and can describe E8 physics.

The "neighbors" of the origin of Sur^16 form a /\16 Barnes-Wall lattice.

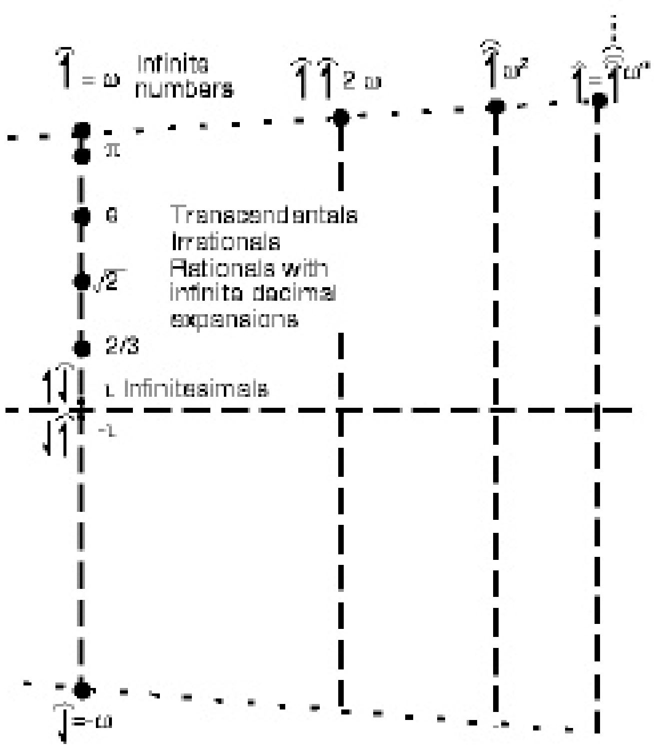
The "neighbors" of the origin of Sur^24 form a /\24 Leech lattice.

What about the infinite and infinitesimal Surreals?



The line of "first-order" infinite, transcendental, irrational, infinitely-expanded, and infinitesimal Surreals represents the "set" of "ALL ManyWorld universes" "near" our universe in the sense that they are within a finite number of steps of "our universe".

The "higher-order" infinite-infinitesimal Surreals



represent higher-order sets of ManyWorlds universes (see page 312 ff).

Note that the Surreal Rationals with finite expansions are more nearly in 1-1 correspondence with the universes of the ManyWorlds than with the rational numbers,
and
that the holes of the Surreals are not problematic in representing the universes of the ManyWorlds, as they are when they are taken to represent mere numbers.

Since Surreal Number Quantum Theory is really Quantum Game Theory:

How Should We Play the Game ?

Marcus Chown, in the article Taming the Multiverse in New Scientist (14 July 2001, pages 27-30), says: "... David Deutsch ... thinks ... the multiverse ... could make real choice possible. In classical physics, he says, ... the future is determined absolutely by the past. So there can be no free will. In the multiverse, however, there are alternatives; the quantum possibilities really happen. Free will might have a sensible definition, Deutsch thinks, because the alternatives don't have to occur within equally large slices of the multiverse. "By making good choices, doing the right thing, we thicken the stack of universes in which versions of us live reasonable lives," he says. "When you succeed, all the copies of you who made the same decision succeed too. What you do for the better increases the portion of the multiverse where good things happen." ...".

For a simplified illustrative example, suppose that your World is post-World War II Earth; one Fate is Nuclear Self-Destruction of Humanity; an alternative Fate is Unification with a Galactic Civilization.

Also suppose that from time to time between World War II and the year 2000, Humanity must collectively choose a Fork in the Path of Fates, one of three Fates: Nuclear Self-Destruction, Delay, or Unification.

Further, suppose that you have some limited power to influence, but not conclusively determine, which Fork Humanity chooses, and that your job, or mission, is to help Humaninity make the right choice, so you might say that Fate has 3 Forks:

to a Bad Basin of Attraction of the Quantum Potential Landscape;
to Delay, Sit on the Fence, and stay on the Boundary Between Basins;
to a Good Basin of Attraction.

BASIN OF
ATTRACTION

BASIN OF
ATTRACTION

NUCLEAR
SELF-DESTRUCTION

GALACTIC
UNIFICATION

about 1950 - Berlin, Roswell?

Berlin Airlift
causes Nuclear War

Contact with Roswell Spacecraft
of Galactic Civilization

about 1973 - Israel, GRBs?

1973 Arab-Israeli War
goes Nuclear

Decoding of Gamma Ray Burst
Messages from
Galactic Civilization

on to other Forking Paths in the Future

With respect to a World in a Good Basin, your mission is a success.
With respect to a World in a Bad Basin, your mission is a failure.
With respect to a World that is on the Boundary Between Basins, you still have work to do and a mission to carry out.

With respect to the Sum of All Worlds, your work is over for the Worlds that have already gone to the Good or Bad Basins, but is actively ongoing for the Worlds on the Boundary Between Basins.

Therefore, if your perception is of the World that demands most of your attention, and your attention, through the Quantum Zeno Effect and the Quantum Anti-Zeno Effect, chooses the World of the Many-Worlds to which you are paying attention, then your perception is most likely to be that you live in a World on the Boundary Between Basins.

Resonant Connections in the MacroSpace of all Many-Worlds would allow you to interact with other things, including Other Consciousnesses, in using the Quantum Zeno Effect and the Quantum Anti-Zeno Effect to choose the World of the Many-Worlds.

For a simplified example, let * represent a given state of the ManyWorlds, and let o represent various possible future states:

```
o     o       o     o
 \   /         \   /
  \ /           \ /
   o     o o     o
    \   /   \   /
     \ /     \ /
      o       o
       \     /
        \   /
         \ /
          *
```

The given state * might be a human mind, or a rock, or a glass of water, or anything else.

If there is no Resonant Connection between the given state * and the possible future states o, then the future of * will be spread at random among the possible future states o, each of which will become an actual future state

* in the Worlds of the ManyWorlds:

```
*     *       *     *
 \   /         \   /
  \ /           \ /
   *     * *     *
    \   /   \   /
     \ /     \ /
      *       *
       \     /
        \   /
         \ /
          *
```

If there is a Resonant Connection between the given state * and one of the possible future states o:

```
o     o       o     o
 \   /         \   /
  \ /           \ /
   o     o o     o
    \   /   \   /
     \ /     \ /
      o       o
       \     /
        \   /
         \ /
          *
```

then the future of * will be concentrated at the possible future states related to the Resonant Connection o, and the Non-Resonant possible future states o will be reduced or eliminated (depending on the strength of the Resonant

Connection) from the set of each of which will become an actual future states * in the Worlds of the ManyWorlds:

```
o     o       *     *
 \   /         \   /
  \ /           \ /
   o     o o     *
    \   /   \   /
     \ /     \ /
      o       *
       \     /
        \   /
         \ /
          *
```

From this point of view, the set of all Worlds of the ManyWorlds looks like an environment in which ManyWorlds Abstract Beings live and interact by Resonant Connections

```
*     *       *     *
 \   /         \   /
  \ /           \ /
   *     * o     *
    \   /   \   /
     \ /     \ /
      *       *
       \     /
        \   /
         \ /
          *
```

Among different Basins of Attraction (such as * and *) in the ManyWorlds, how do you determine what are optimal interactions ?

To determine Interactions among Basins, use Quantum Game Theory with strategies determined by your own Belief System evaluation of which Branch of the Many-Worlds you deem to be good.

Branching Worlds and Cellular Automata

In the E8 Physics model,

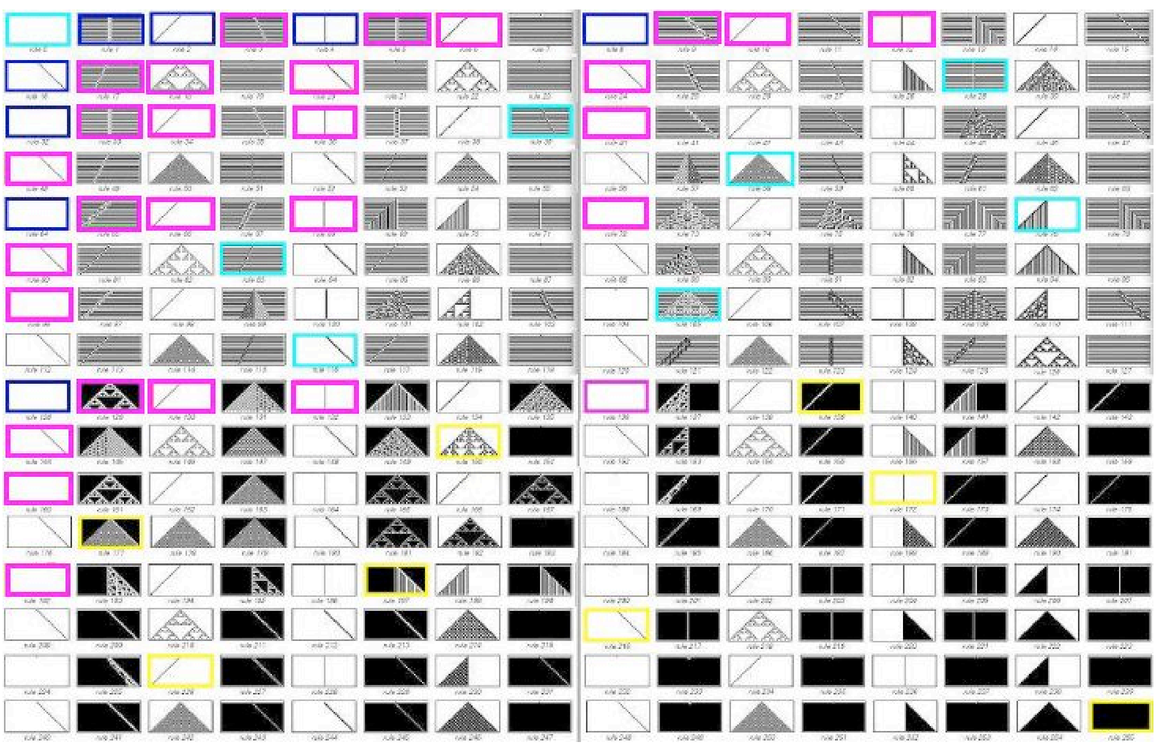
248-dim E8 is made up of 120-dim D8 plus 128-dim half-spinors of D8.

D8 = Spin(16) comes from Cl(16) = Cl(8) x Cl(8)

so to see how E8 Physics works, including the Branching of the Worlds of the Many-Worlds, the fundamental structure that you need to understand is

256-dim Cl(8) = 16x16 Real Matrix Algebra

which corresponds to the 256 Cellular Automata



The color-coding is

- blue for the 8 spacetime dimensions
- cyan for the 8 +half-spinor first generation fermion particles
- yellow for the 8 -half-spinor first generation antiparticles
- magenta for the 28 Spin(8) gauge bosons.

Here is a numerical list of all 256 Cellular Automata corresponding to Cl(8):

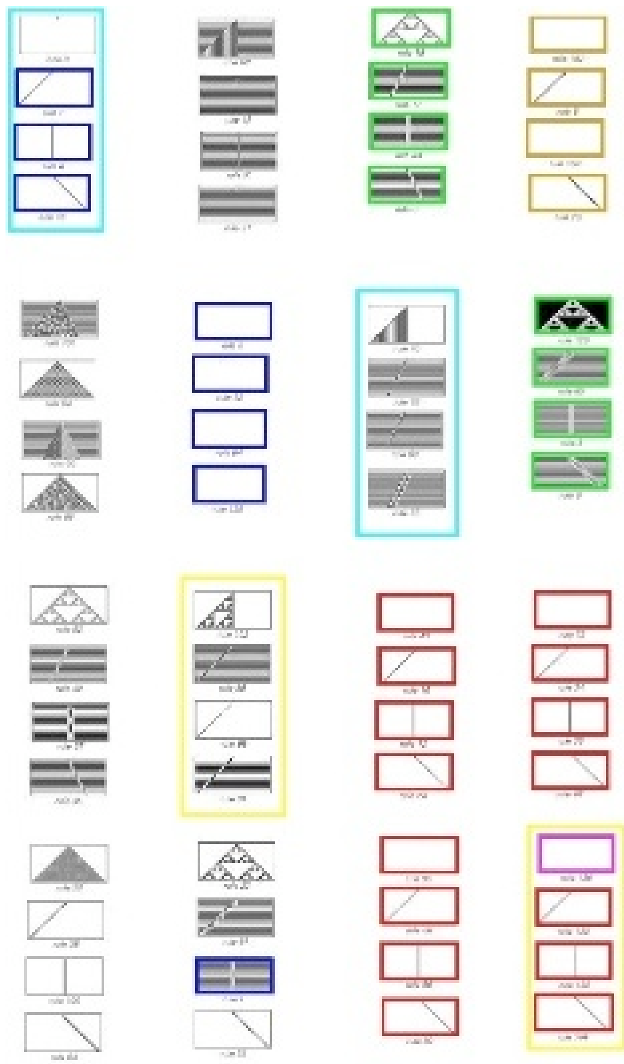
Grade:	0	1	2	3	4	5	6	7	8
000	00000000								
001		00000001							
002		00000010							
003			00000011						
004		00000100							
005			00000101						
006			00000110						
007				00000111					
008		00001000							
009			00001001						
010				00001010					
011				00001011					
012			00001100						
013				00001101					
014				00001110					
015					00001111				
016		00010000							
017			00010001						
018			00010010						
019				00010011					
020			00010100						
021				00010101					
022				00010110					
023					00010111				
024			00011000						
025				00011001					
026				00011010					
027					00011011				
028				00011100					
029					00011101				
030					00011110				
031						00011111			
032		00100000							
033			00100001						
034			00100010						
035				00100011					
036			00100100						
037				00100101					
038				00100110					
039					00100111				
040			00101000						
041				00101001					
042				00101010					
043					00101011				
044				00101100					
045					00101101				
046					00101110				
047						00101111			
048			00110000						
049				00110001					
050				00110010					
051					00110011				
052				00110100					
053					00110101				
054					00110110				
055						00110111			
056				00111000					
057					00111001				
058					00111010				
059						00111011			
060					00111100				
061						00111101			
062						00111110			
063							00111111		

064	01000000						
065		01000001					
066		01000010					
067			01000011				
068		01000100					
069			01000101				
070			01000110				
071				01000111			
072		01001000					
073			01001001				
074		01001010					
075				01001011			
076			01001100				
077				01001101			
078				01001110			
079					01001111		
080		01010000					
081			01010001				
082			01010010				
083				01010011			
084			01010100				
085				01010101			
086				01010110			
087					01010111		
088			01011000				
089				01011001			
090				01011010			
091					01011011		
092				01011100			
093					01011101		
094					01011110		
095						01011111	
096		01100000					
097			01100001				
098			01100010				
099				01100011			
100			01100100				
101				01100101			
102				01100110			
103					01100111		
104			01101000				
105				01101001			
106				01101010			
107					01101011		
108				01101100			
109					01101101		
110					01101110		
111						01101111	
112			01110000				
113				01110001			
114				01110010			
115					01110011		
116				01110100			
117					01110101		
118					01110110		
119						01110111	
120				01111000			
121					01111001		
122					01111010		
123						01111011	
124					01111100		
125						01111101	
126						01111110	
127							01111111

128	10000000						
129		10000001					
130		10000010					
131			10000011				
132		10000100					
133			10000101				
134			10000110				
135				10000111			
136		10001000					
137			10001001				
138			10001010				
139				10001011			
140			10001100				
141				10001101			
142				10001110			
143					10001111		
144		10010000					
145			10010001				
146			10010010				
147				10010011			
148			10010100				
149				10010101			
150				10010110			
151					10010111		
152			10011000				
153				10011001			
154				10011010			
155					10011011		
156				10011100			
157					10011101		
158					10011110		
159						10011111	
160		10100000					
161			10100001				
162			10100010				
163				10100011			
164			10100100				
165				10100101			
166				10100110			
167					10100111		
168			10101000				
169				10101001			
170				10101010			
171					10101011		
172				10101100			
173					10101101		
174					10101110		
175						10101111	
176			10110000				
177				10110001			
178				10110010			
179					10110011		
180				10110100			
181					10110101		
182					10110110		
183						10110111	
184				10111000			
185					10111001		
186					10111010		
187						10111011	
188					10111100		
189						10111101	
190						10111110	
191							10111111

192	11000000						
193		11000001					
194		11000010					
195			11000011				
196		11000100					
197			11000101				
198			11000110				
199				11000111			
200		11001000					
201			11001001				
202			11001010				
203				11001011			
204			11001100				
205				11001101			
206				11001110			
207					11001111		
208		11010000					
209			11010001				
210			11010010				
211				11010011			
212			11010100				
213				11010101			
214				11010110			
215					11010111		
216			11011000				
217				11011001			
218				11011010			
219					11011011		
220				11011100			
221					11011101		
222					11011110		
223						11011111	
224		11100000					
225			11100001				
226			11100010				
227				11100011			
228			11100100				
229				11100101			
230				11100110			
231					11100111		
232			11101000				
233				11101001			
234				11101010			
235					11101011		
236				11101100			
237					11101101		
238					11101110		
239						11101111	
240			11110000				
241				11110001			
242				11110010			
243					11110011		
244				11110100			
245					11110101		
246					11110110		
247						11110111	
248				11111000			
249					11111001		
250					11111010		
251						11111011	
252					11111100		
253						11111101	
254						11111110	
255							11111111

After introduction of quaternionic structure, Real 16x16 Cl(8) transforms into Quaternionic 8x8 Cl(2,6) which contains Conformal Quaternionic 4x4 Cl(2,4):



with color-coding:

blue for the 4 spacetime dimensions and the 4 internal symmetry space dimensions
cyan for the 8 +half-spinor first generation +half-spinor fermion particles
yellow for the 8 -half-spinor first generation -half-spinor fermion antiparticles.
gold for the 4 U(2) electroweak gauge bosons
green for the 8 SU(3) color gluon gauge bosons
magenta for the U(1) propagator phase that is defined with respect to the

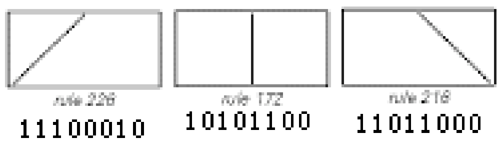
fixed Quaternionic 4-dimensional spacetime subspace corresponding to the S-square of Lull's S-wheel
red for the 15 SU(2,2) = Spin(2,4) Conformal Group graviphoton gauge bosons that produce MacDowell-Mansouri Gravity and act on 4-dimensional Physical Spacetime by:
4 Translations;
6 Lorentz Transformations;
4 Special Conformal Transformations;
1 Dilation.

Note that there is some overlap between spinors and others because of spinors being related to primitive idempotent linear combinations.

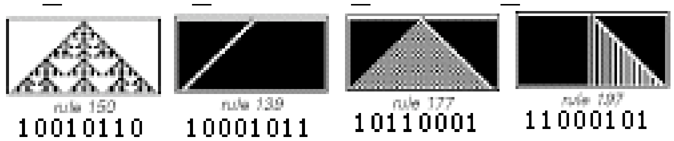
For the 16x16 Real Matrix Algebra Cl(8), the 16 terms in the primitive idempotent correspond to 16 of Wolfram's 256 Cellular Automata:

+ e_12345678

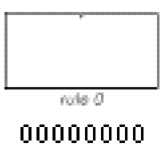
+ e_6728 + e_3468 + e_4578 to



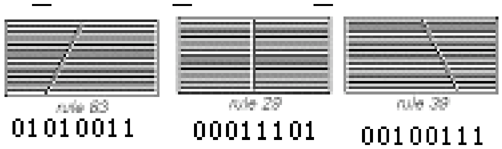
+ e_2358 + e_1248 + e_5618 + e_7138 to



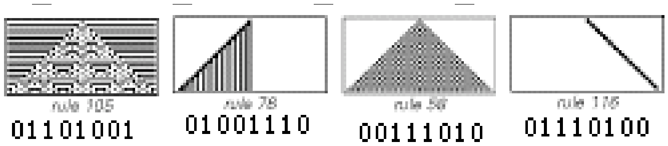
+ 1 to



- e_5712 - e_1345 - e_6123 to

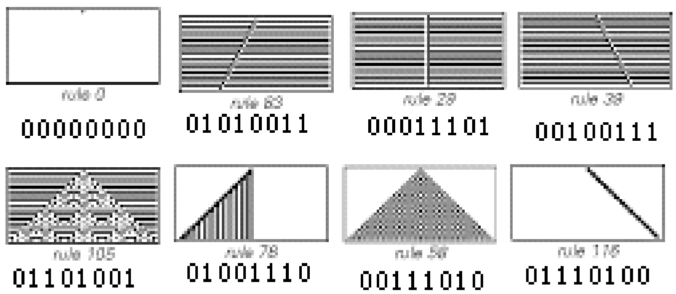


- e_4671 - e_7234 - e_2456 - e_3567

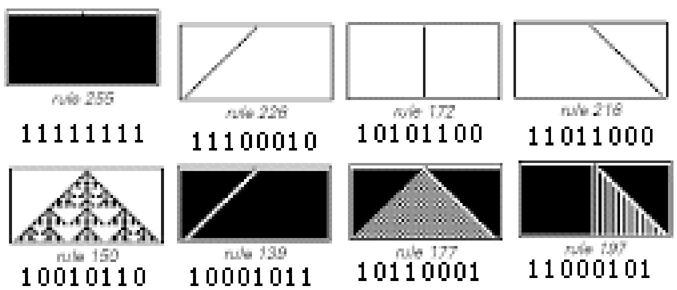


Note the Cl(0,8) = Cl(1,7) triality correspondences among:

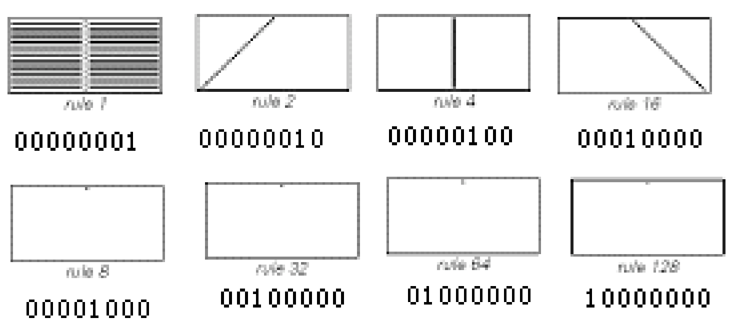
the 8 +half-spinors



the 8 -half-spinors



the 8 vectors



Michael Gibbs has been working on using Cellular Automata as neural network nodes, and Robert de Marrais has written a Box-Kites III paper (at math/0403113), leading me to think of some questions:

Could 16x16 structures such as switching yards of Box-Kites III have structures corresponding to the graded structure o the Clifford algebra Cl(8) that is the 16x16 real matrix algebra?

Since the vectors of the Cl(8) Clifford algebra are 8-dimensional and correspond to the octonions, if you take the correspondences between the 256 Wolfram CA and the Cl(8) basis elements described here, there is a correspondence:

CA Rule No.	Octonion Basis Element
1	1
2	i
4	j
16	k
8	E
32	I
64	J
128	K

Could such a correspondence be used to construct such things as "Box-Kites" whose vertices might be regared, not just as octonions etc, but also as Cellular Automata?

Could Box-Kite type structures give useful computational structures if the vertices were considered as CA and the edge-flow-orinetations were considered as information flow in a computing system?

If such a computing system can be set up for 2^n-ionic structures for large n, then, since for 16-ions and larger you have interesting zero-divisor "sleeper-cell" substructures, could they be useful with respect to computational systems, perhaps doing things like forming loops that might let the computational system to "adjust itself" and/or "teach itself"?

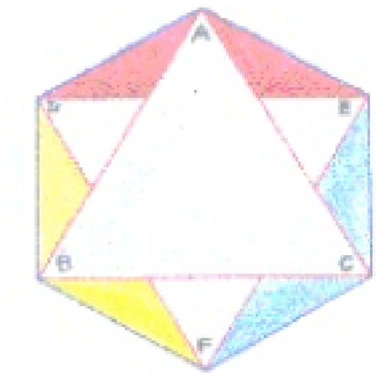
Robert de Marrais (see his papers including math.RA/0207003 and math.GM/0011260) commented on some of those questions, saying in part:

"... I'm finding two directions to go with box-kites next, and yes, cellular automata clearly are part of it. … now to the two directions, which relate to your suggestions:

(1) Boolean monotone and antitone function-pairings can be used, per Rodrigo Obando, to generate exactly all and only the complex cellular automata for a given n and r . . . and, given that for n=4 that means

Dedekind's number of 168 mono- and iso- tone functions each, connections to box-kites immediately suggest themselves ... He tells me his work is leading him not merely to isolate and catalog the "complex" CA's for high n and r, but that he's finding -- when he generalizes to the n => infinity situation, that he gets violations of the continuum hypothesis ...

(2): spin networks. The key revelation (which I telescoped on the last couple pages of "Box Kites III" [math.RA/0403113 - see also math.RA/0603281



which shows a box kite
as similar to an Arthur Young heptaverton or an Onar Aam Onarhedron – see page 329 ff]) concerns … this: zero-divisor systems are, ironically, PRESERVERS of associative order! … the isomorphism of quaternion algebra to SU2 gives you (recall my graphics toward the end of the first Box-Kite paper vis a vis Catastrophe Theory?) … the 4 axes in the SU2 representation …[as]… reals, the usual imaginaries, Pauli spin-matrix "mirror numbers" which square to +1, and a "commutative i" which commutes between these latter two. (This is both Cl(2) in Clifford algebra lingo, and Muses' simplest epsilon-number space.) … As systems of box-kites get very entangled in higher dimensions (in 32-D, you have systems of 7 of them forming what I call Pleiades, with some fascinating synergetic properties), spin-foams with self-organizing potential suggest themselves . . . my ultimate objective … is not physics per se, but rather Levi-Strauss's canonical law of myths, and the creation of an infinite-dimensional "collage space" that can accommodate his systems of mythopoetic sign-shunting in a manner roughly reminiscent of Fourier series' infinite-dimensional backdrop for generalized harmonics. So that means I'll be busy with my hobbyhorse at least through "Box-Kites VI"! … one first sees the "42 Assessors," then zooms in one one of the 7 isomorphic box-kites (which, as with all isomorphies, can be seen as identical at some higher level); then, one zooms in further on the "second box-kite" … All these threads are getting ever more entangled and intriguing, aren't they? ...".

In math.RA/0603281 Robert de Marrais says: "… The infinite-dimensional ZD [Zero-Divisor] meta-fractal or "Sky" which Box-Kites fly "beneath" or "in" … first appear… in the 32-D Pathions and incorporat[e]… the higher s^N-ions …

Louis Hjelmslev's "Net" …[has]… indefinitely many strata, each housing a bifurcation … by binaries … Zero-Divisors …[arise by]… the nonstop redoubling of … the Cayley-dickson Process … Catastrophes …[can be studied by]… throwing our [Zero-Divisor, analogous to Hjelmslev] Net into the negative-curvature spaces of higher dimension reaches, where Chaotic "period-doubling" and other modes of turbulence hold forth. … James Callahan, in … navigations of E6 and Double Cusp Catastrophes using stacked 2- and 3-D "tableaus", has found … features he calls "trance tunnels" and "ship's prows", through and with which one "steers" in "control space", by use of "keels" and other sailing instruments. … imagine the sea across which we are sailing to be a "sentient ocean", like that covering the surface of Stanislav Lem's Solaris …".

In 0704.0026 [math.RA] Robert de Marrais says: "… regardless of how large N grows, ZDs only increase in their interconnectedness, rather than see their basic structure atrophy …[compare the increase with N in dimension of spinors of Cl(8N), despite which the basic Cl(8) structure does not atrophy, due to Cl(8N) = Cl(8) x …(N times tensor product)… x Cl(8)]...

the meta-fractal we call the Whorfian Sky …[is]… named for the … linguist… Benjamin Lee Whorf …".

In 0704.0112 [math.RA] Robert de Marrais says: "… The Whorfian Sky … is the simplest possible meta-fractal – the first of an infinite number of such infinite-dimensional zero-divisor-spanned spaces …[compare the infinite number of structures based on Cl(8) that produce the generalized hyperfinite Real II1 von Neumann Algebra factor (see [page 39)]…".

In 0804.3416 [math.GM] Robert de Marrais says: "… [In]… the 32-D Pathions … the signature of "scale-free" behavior … is first revealed …

Dolgachev found families of higher-dimensional singularities … quasihomogeneous unimodular singularities [that] are obtained from automorphic functions connected with 14 distinguished triangles on the Lobachevskii plane and three distinguished triangles on the Euclidean plane in precisely the same way as simple singularities are connected with regular polyhedra. The keyword … is unimodular: modular singularities, unlike all the elementary species … have one or more parameters, which turn them into infinite families of forms … things we've seen before .. our "metafractal" Sky …".

See page 317 ff for more about Singularities that are useful in the Branching of the Worlds of the Many-Worlds may be describable in terms of Singularities:

simple singularities (classified precisely by the Coxeter groups Ak, Dk, E6, E7, E8);

unimodal singularities (a single infinite three-suffix series and 14 "exceptional" one-parameter families); and

bimodal singularities (8 infinite series and 14 exceptional two-parameter families).

Coxeter-Dynkin Diagrams:

The E8 Coxeter-Dynkin diagram, in which each vertex corresponds to a fundamental representation, is

```
                                                      147,250
                                                         |
248 - 30,380 - 2,450,240 - 146,325,270 - 6,899,079,264 - 6,696,000 - 3,875
```

Note that E8 can be constructed
from the representations of E6 and D8,
including
the two D8 128-dimensional half-spinor representations.

The grade-1 vector representation of D8 is 120-dimensional.
The half-spinor representation of D8 is 128-dimensional.

The adjoint representation of E8 is 120 + 128 = 248-dimensional.

D8 and E6 both have trivial 1-dimensional scalar representations.

E6 has 27-dimensional and 78-dimensional representations.

The grade-1 part has dimension 248.
The grade-2 part has dimension 248/\248 = 30,628.
The grade-3 part has dimension
248/\248/\248 = 2,511,496.
The grade-4 part has dimension
248/\248/\248/\248 = 153,829,130.
The grade-5 part has dimension
248/\248/\248/\248/\248 = 7,506,861,544.

Now:

Keep the grade-1 part of dimension 248.

Subtract off 248
from 248/\248 = 30,628 to get 30,380.

Subtract off 2 x 248/\248 = 2 x 30,628
from 248/\248/\248 = 2,511,496 to get 2,450,240.

Subtract off 2 x 2,511,496 and 2,450,240 and 30,628
from 248/\248/\248/\248 = 153,829,130 to get 146,325,270.

Subtract off 2 x 153,829,130 and 2 x 146,325,270
and 2 x 2,511,496 and 2,450,240 and 248
from 248/\248/\248/\248/\248 = 7,506,861,544 to get
6,899,079,264.

These are 5 of the 8 fundamental representations of E8.

They, like the D(N) and A(N) series constructions,
are all in the same exterior algebra (of /\248),
and so can be represented as the vertices of a pentagon

What about the 6th and 7th fundamental representations of E8?

Consider the 27-dimensional E6 representation space.
Add 32 copies of the 128-dimensional D8 half-spinor space,
and subtract off
one copy of the 248-dimensional E8 representation space
to get
a 27 + 32 x 128 - 248 = 3,875-dimensional representation space.

Now, consider the antisymmetric exterior wedge algebra
of that 3,875-dimensional space.

The grade-1 part has dimension 3,875.
The grade-2 part has dimension 3,875/\3,875 = 7,505,875.

Now:

Keep the grade-1 part of dimension 3,875.

Subtract off 5 x 147,250 and 2 x 30,628
and 3 x 3,875 and 3 x 248
from 3,875/\3,875 = 7,505,875 to get 6,696,000.

They are the 6th and 7th fundamental representations of E8.
Since they are not in the same /\248 exterior algebra as
the 5 pentagon-vertex fundamental representations of E8,
they should not be vertices in the same plane as the pentagon.
However, since they are in the same /\3,875 exterior algebra,
they should be collinear, one above and one below the pentagon,
thus forming a pentagonal bipyramid.

What about the 8th fundamental representations of E8?

Consider 2 x 24x24 - 1 = 2 x 576 - 1 = 1,151 copies of
the 128-dimensional D8 half-spinor space,
and subtract off
one copy of the 78-dimensional E6 representation space
to get a representation space of dimension
1,151 x 128 - 78 = 147,328 - 78 = 147,250.

Now, consider the antisymmetric exterior wedge algebra
of that 147,250-dimensional space.
The grade-1 part has dimension 147,250.

It is the 8th fundamental representation of E8.
Since it is not in the same /\248 exterior algebra as
the 5 pentagon-vertex fundamental representations of E8,
it should not be a vertex in the same plane as the
pentagon.
Also, since it is not in the same /\3,875 exterior algebra
as
the two bipyramid-peak-vertex fundamental representations
of E8,
it should not be a vertex on the same line as
the pentagonal bipyramid axis.
It should represent a vertex creating a triangle
whose base is one of the sides of the pentagon
and whose top is near one of the bipyramid-peak-vertices,
to which it is connected by a line.
To produce a symmetric figure,
the vertex must be reproduced in 5 copies, one over
each of the 5 sides of the pentagon.
Then, for the entire figure to be symmetric,
it must form an icosahedron.
The binary icosahedral group {2,3,5} is of order 120.

Another way to look at it is:

The graded sequence
248
248/\248
248/\248/\248
248/\248/\248/\248
248/\248/\248/\248/\248
has symmetry Cy(5) of order 5 for cyclic permutations,
but since 248/\248/\248/\248/\248 is fixed

by its relation to 3,875/\3,875/\3,875,
do not use Hodge duality on the 248 graded sequence.

The graded sequence
3,875
3,875/\3,875
3,875/\3,875/\3,875
has symmetry Cy(3) of order 3 for cyclic permutations,
but since 3,875/\3,875/\3,875 is fixed by
its relation to 248/\248/\248/\248/\248,
do not use Hodge duality on the 3,875 graded sequence.

The graded sequence
147,250
147,250/\147,250
has symmetry Cy(2) of order 2 for cyclic permutations,
but since 147,250/\147,250 is fixed by
its relation to 248/\248/\248/\248/\248,
do not use Hodge duality on the 147,250 graded sequence.

The +/- signs for the D5 half-spinors inherited
from E6 through E7
have symmetry of order 2.

Since the dimension of E8 is 248 = 120 +128,
the sum of the 120-dimensional adjoint representation of D8
plus
ONE of the 128-dimensional half-spinor representations of
D8,
there is a choice to be made as to
which of the two half-spinor representations of D8 are
used.
As they are mirror images of each other,
that choice has a symmetry of order 2.

Therefore:
the total symmetry group is of order 5x3x2x2x2 = 120,
the symmetry of the binary icosahedral group {2,3,5},
with 3 symmetries Cy(5), Cy(3), Cy(2).

It corresponds, by the McKay correspondence
(see page 287), to the E8 Lie Algebra.

What are the relations between 6,899,079,264
and 248/\248/\248/\248/\248, 3,875/\3,875/\3,875,
and 147,250/\147,250 ?

```
6,899,079,264 = 248/\248/\248/\248/\248 -
                          - 2 x 248/\248/\248/\248 -
         - 2 x (248/\248/\248/\248 - 2 x 248/\248/\248 -
                  - (248/\248/\248 - 2 x 248/\248) -
248/\248 ) -
         - 2 x 248/\248/\248
         -  (248/\248/\248 - 2 x 248/\248) - 248 =

              =      248/\248/\248/\248/\248 -
               - 4 x      248/\248/\248/\248 -
               + 3 x           248/\248/\248 -
               -                         248 =

              =      7,506,861,544 -
               - 4 x   153,829,130 +
               + 3 x     2,511,496 -
               -               248 =

              =      7,506,861,544 -
               -       615,316,520 +
               +         7,534,488 -
               -               248 = 6,899,079,264


6,899,079,264 = 3,875/\3,875/\3,875 - 18 x
248/\248/\248/\248 -
                               - 3 x 3,875/\3,875 +
                               + 3 x 147,250 -
                               - 20 x 248 -
                               - 6 x 27 - 3 x 8 =
              = 9,690,084,625 - 18 x 153,829,130 -
                              - 3 x 7,505,875 +
                              + 441,750 -
                              - 4,960 -
                              - 162 - 24 =
              = 9,690,084,625 - 2,768,924,340 -
                              - 22,517,625 +
                              + 441,750 -
                              - 4,960 -
                              - 162 - 24 = 6,899,079,264
```

```
6,899,079,264 = 147,250/\147,250 - 25 x 248/\248/\248/\248
-
                                - 38 x 248/\248/\248 -
                                - 31 x 248/\248 -
                                - 55 x 248 - 128 -27 =
              =  10,841,207,625 - 25 x 153,829,130 -
                                - 38 x 2,511,496 -
                                - 31 x 30,628 -
                                - 55 x 248 - 128 - 27 =
              =  10,841,207,625 - 3,845,728,250 -
                                - 95,436,848 -
                                - 949,468 -
                                - 13,640 -
                                - 155 = 6,899,079,264
```

In the relations,
the 8 dimensional representation of D4 was also used.

J. F. Adams has written a paper entitled

The Fundamental Representations of E8

published in Contemporary Mathematics, Volume 37, 1985, 1-10,

and reprinted in The Selected Works of J. Frank Adams, Volume 2,
edited by J. P. May and C. B. Thomas, Cambridge 1992, pp. 254-263.

The 8 fundamental E8 representations are

```
                                           147250
                                             |
248 - 30380 - 2450240 - 146325270 - 6899079264 - 6696000 - 3875
```

In his paper, Adams denotes the three at the ends as follows:

248 is denoted by alpha, which I will write here as   a

3875 is denoted by beta, which I will write here as   b

147250 is denoted by gamma, which I will write here as c.

Adams denotes the kth exterior power by lambda^k
which I will write here as /\k
and he denotes the kth symmetric power by sigma^k
which I will write here as Sk .
Also, here I write (x) for tensor product.

J. F. Adams says:

"... we can allow ourselves to construct new representations
from old by taking exterior powers, as well as tensor products
and Z-linear combinations ...

... seven of the eight generators for the polynomial ring R(E8)
may be taken as

a, /\2 a , /\3 a , /\4 a , b , /\2 b , c .

The eighth may be taken either as /\5 a , or as /\3 b ,
or as /\2 c .

The corresponding argument for Dn would say that one should begin
by understanding three representations of Dn = Spin(2n) ,
namely the usual representation on R^2n and the two half-spinor
representations delta+ , delta- .
This we believe, so perhaps we can accept the analogue for E8 .

...

we must get back to business and construct b and c .

...

To give explicit formulae we must agree on a coordinate system.
The group E8 contains a subgroup of type A8 .
...
As a representation of A8 , the Lie lagebra L(E8) splits to give

L(E8) = L(A8) + /\3 + /\3* .

Let e1, e2, ... , e9 be the standard basis in the vector
space V = C9 on which A8 acts
...
We now introduce the element

v_ik = SUM(j) (ei* ej* ek* (x) ej ek* + ej ek* (x) ei* ej*
ek*)

in the symmetric square S2(a) in a (x) a , where a = L(E8)
.
...
THEOREM 4.
(a) The representation S2(a) of E8 contains a unique copy
of b .
(b) This copy of b contains the elements v_ik .
(c) For i =/= k the elements v_ik are eigenvectors
corresponding
to extreme weight of 8 .
(d) In particular (with our choice of details) v_gl is an
eigenvector corresponding to the highest weight of b .
...
Theorem 4 allows us to realize b as the E8-submodule of
S2(a)
generated by v_91 (or any other v_ik with i =/= k .)
...
In fact, S2(a) contains a trivial summand ... and also an
irreducible summand of highest weight ...
It turns out that the remaining summand has dimension
3,875,
which is precisely the dimension of b .

We now introduce the element

w_k = SUM(i) ei ek* (x) v_ik

= SUM(i,j) ei ek* (x) (ei* ej* ek* (x) ej ek* + ej ek* (x)
ei* ej* ek*)

in a (x) b in a (x) S2(a) in a (x) a (x) a .

...

THEOREM 5.
(a) The representation a (x) b of E8 contains a unique
copy of c .
(b) This copy of c contains the elements w_k .
(c) The elements w_k are eigenvectors corresponding to the
extreme

weights of c .
(d) In particular (with our choice of details) w_1 is an
eigenvector
corresponding to the highest weight of c .
...
we may realize c as the E8-submodule of a (x) b
generated
by w_1 (or any other w_k).
...
In fact, a (x) b contains an irreducible summand of
highest weight
... and the remaining summand has too small a dimension to
contain
two copies of c .   ...".

Adams then gives proofs of the theorems,
involving such things as looking at the Lie algebra L(E8)
as
a representation of D8 as which it splits to give
L(E8) = L(D8) + delta-
where delta- is a half-spinor representation of D8.

One of the things that I think that I get out of Adams's
paper is
that it seems to me that in order to get 3875 and 147250
you have to look not only at the 248 representation of E8
but
also at representations of some subgroups of E8
such as D8 etc.

McKay Icosahedron of E8:

In a Usenet Sci.Math paper John McKay said:
"…

For each finite subgroup G in SU2(C), we restrict the representations R[i] to G. This then splits the representations of SU2(C) into irreducibles of G.

```
Example: G = quaternions.

                1a

               /

We get     1-2-1b from the initial segment of 1-2-3-...

               \

                1c
```

which is the affine Dynkin diagram of type D4. (We break off as soon as representations repeat.)

For each finite subgroup of SU2, we get an affine Dynkin diagram in this way. Affine means adding an extra node corresponding to the negative of the highest root. The correspondence is: (see page 281 for some E8 details)

```
A[r] degenerate Cyclic[r+1]

D[r] {2,2,r-2}  Generalized quaternion[r-2]

E[6] {2,3,3}    2.Alt[4] binary tetrahedral

E[7] {2,3,4}    2.Symm[4] binary octahedral

E[8] {2,3,5}    2.Alt[5]=SL(2,5) binary icosahedral
```

where {a,b,c} =<x,y,z: x^a=y^b=z^c=xyz> (=-1).

The non-ADE types correspond to certain pairs (G,H), H { G in SU2(C) …

for the E8 - icosahedral = <2,3,5> case, the singularity is x^2+y^3+z^5=0 …".

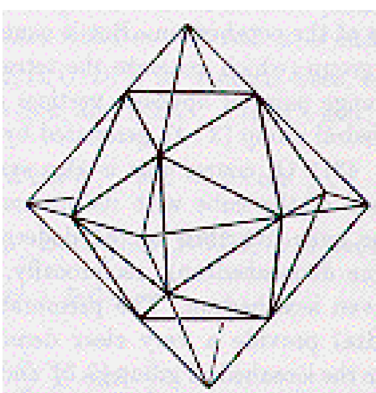
An E8 Cartan matrix is:

2	0	-1	0	0	0	0	0
0	2	0	-1	0	0	0	0
-1	0	2	-1	0	0	0	0
0	-1	-1	2	-1	0	0	0
0	0	0	-1	2	-1	0	0
0	0	0	0	-1	2	-1	0
0	0	0	0	0	-1	2	-1
0	0	0	0	0	0	-1	2

The Coxeter-Dynkin diagram is

```
          *
          |
*-*-*-*-*-*-*
```

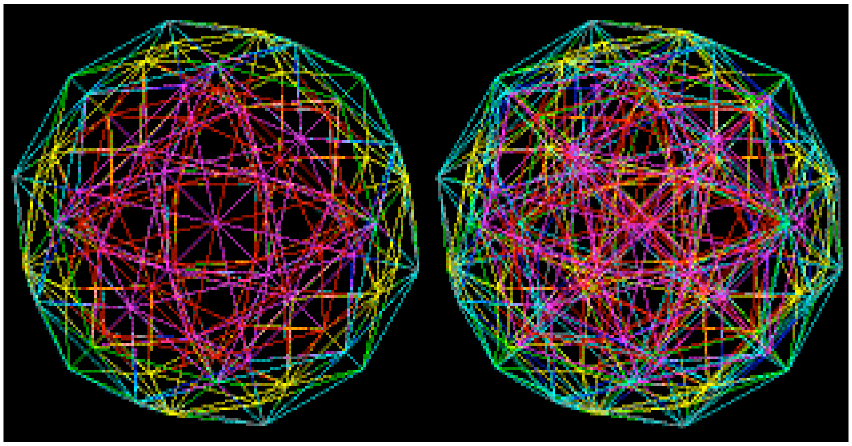
The polytope corresponding to the E8 Lie algebra
by The McKay Correspondence
is the 12-vertex icosahedron,
each vertex of which is at a Golden Ratio point of an edge
of the E7 McKay Polytope, the 12-edge octahedron



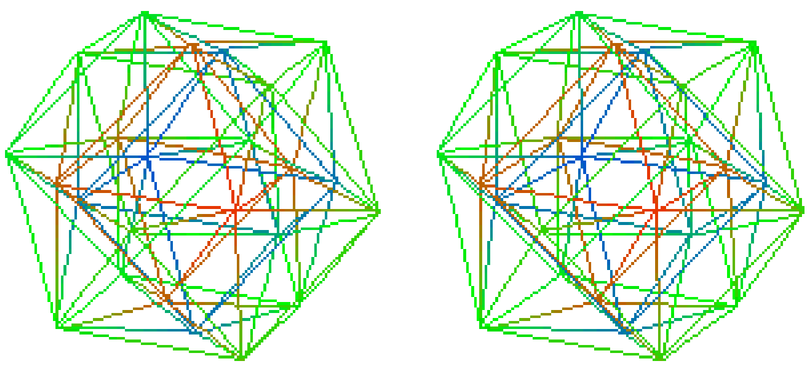
Icosahedron and octahedron.

The 12-vertex icosahedron is 3-dimensional,
but its 12 vertices in 3 dimensions
do not form a Lie algebra Root Vector Diagram,
but
the symmetry group of the icosahedron is the
Icosahedral Double group ID, the order 120 Binary

Icosahedral group {5,3,2},
which is the group of quaternions
that are the 120 vertices of a 600-cell,



which can be formed by
24 vertices of a 24-cell
plus 96 vertices at Golden Ratio points
on the 96 edges of the 24-cell



The 120 vertices of the 4-dimensional 600-cell
correspond to half of the 240 E8 root vector vertices
which form an 8-dimensional 240-vertex Witting
polytope.

The Group Algebra C[ID] is a 120-dimensional algebra
related to the 120-dimensional D8 Lie algebra that is the
even part of the E8 Lie algebra.

In Applied Mathematics and Computation 60:25-36 (1994),
Charles Muses says (at page 28):
"... In the late 19th century, it was the Neapolitan mathematician

P. del Pezzo who first introduced the brilliant idea of a del Pezzo (hyper)surface: one described by a function of nth degree in n-space. Thus, a del Pezzo surface in 5-space could explain the 27 straight lines on the general cubic surface. Del Pezzo's discovery also hinted at the special Lie algebras and hence at octonions since they climaxed in an 8th degree variety in 8-space, linking with the fact that it is E8, the highest special Lie algebra, that is intimately connected with octonions. Indeed, the whole theory of Lie algebras depends ultimately on its elemental core of octonion arithmetic.
... The entire Italian school at that period was exploring higher dimensional geometry ...".

For N = 1, the 120 diameters of the 240-vertex root vector polytope of E8 (the Witting Polytope Gossett 4_21) correspond not only to the 120 elements of the E8 McKay group ID, but also to the 120 tritangent planes of the sextic space of the intersection of a cubic surface with two sheets of a conical surface.

For N = 0, the Gossett 5_21 is not a finite polytope, but is the E8 lattice, which is made up of repeated configurations of two types of 8-dimensional polytopes: 128 simplexes and 9 cross-polytope hyperoctahedra, for 137 polytopes per configuration.

```
Coxeter notes that
the product  240 x 56 x 27 x 16 x 10 x 6 x 3 x 1
of the number of lines from N = 1 to N = 8
is related to
the product  240 x 56 x 27 x 16 x 10 x 6 x 2 x 1
of the order of the Weyl Group of the Lie Algebra E8.

The order of the E8 Weyl Group can also be written as
2^14 x 3^5 x 5^2 x 7 = 696,729,600
and (see page 161 )
as the product of the Casimir invariant degrees of E8
2 x 8 x 12 x 14 x 18 x 20 x 24 x 30
each of which is greater by 1 than the primes
that are the exponente of E8
1  7  11  13  17  19  23  29
```

Luis J. Boya has written a beautiful paper at math-ph/0212067 entitled

“Problems in Lie Group Theory” and here are a few of the interesting things he says:

"... For the exceptional groups, the F4 & E series ...

```
Adj SO(9)  + Spin O(9)           -> Adj F4 (36+16=52)
Adj SO(10) + Spin O(10) + Id     -> Adj E6 (45+32+1=78)
Adj SO(12) + Spin O(12) + Sp(1) -> Adj E7 (66+64+3=133)
Adj SO(16) + [half-]Spin O(16)  -> Adj E8 ([120+128=248])
```

Notice that 8+1 , 8+2 , 8+4 , and 8+8 appear. In this sense the octonions appear as a "second coming " of the reals, completed with the spin, not the vector irrep. ... This confirms that the F4 E6-7-8 corresponds to the octo, octo-complex, octo-quater and octo-octo birings, as the Freudenthal Magic Square confirms. ...

The exponents of a Lie group are the numbers i such that S(2i+1) is an allowed sphere ...

neither the U-series nor the Sp-series have torsion. The exponents ... for U(n) ... are 0, 1, ... , n-1 ... and jump by two in Sp(n).

But for the orthogonal series one has to consider some Stiefel manifolds instead of spheres, which the same real homology ... It ... introduces (preciesely) 2-torsion: in fact, Spin(n), n≥7 and SO(n), n≥3, have 2-torsion. The low cases Spin(3,4,5,6) coincide with Sp(1), Sp(1)xSp(1), Sp(2) and SU(4) , and have no torsion.

For ... G2 ... SU(2) -> G2 -> M11 ... where M11 is again a Steifel manifold, with real homology like S11, but with 2-torsion ...

For F4 we do not get the sphere structure from any irrep, and in fact F4 has 2- and 3-torsion. ...

2- and 3-torsion appears in ... E6 and E7 ...

E8 has 2-, 3- and 5-torsion ...

The Coxeter number of (dim - rank) of E8 is 30 = 2 x 3 x 5 , in fact a mnemonic for the exponents of E8 is: they are the coprimes up to 30, namely (1,7,11,13,17,19,23,29) ...

The first perfect numbers are 6, 28, and 492, associated to the primes 2, 3 and 5 (... Mersenne numbers ...) ... 496 = dim O(32) = dim E(8) x E(8) .

Why the square? It also happens in O(4) , dim = 6 (prime 2), as O(4) ...[is like]... O(3) x O(3) ; even O(8) [dim = 28] (prime 3) is like S7 x S7 x G2 ...

The sphere structure of compact simple Lie groups has a curious "capicua" ... Catalan word (cap i cua 0 = head and tail) ... form: the exponents are symmetric from each end; for example ...

```
exponents of E6:          1,4,5,7,8,11.
Differences: 3,1,2,1,3

exponents of E7:          1,5,7,9,11,13,17.
Differences: 4,2,2,2,2,4 ...

exponents of E8 ...       1,7,11,13,17,19,23,29 ...[
Differences 6,4,2,4,2,4,6 ]...
```

The real homology algebra of a simple Lie group is a Grassmann algebra, as it is generated by odd (i.e., anticommutative) elements. However, from them we can get, in the enveloping algebra, multilinear symmetric forms, one for each generator; ... in physics they are called Casimir invariants, in mathematics the invariants of the Weyl group. ...".

As John McKay said, the E8 McKay Binary Icosahedral Group ID corresponds to the singularity x^2 + y^3 + z^5

According to the book Singularities of Differentiable Maps, Volume I, by V. I. Arnold, S. M. Gusein-Zade, A. N. Varchenko (Birkhauser 1985):

"... The modality m of a point x in X under the action of a Lie group G on a manifold X is the least number such that a sufficiently small neighborhood of x may be covered by a finite number of m-parameter families of orbits … Simple germs of holomorphic functions (germs with m = 0) are given, up to stable equivalence, by the following list:

A_k: f(x) = s^(k+1), k at least 1;
D_k: f(x,y) = x^2 y + y^(k-1), k at least 4;
E6: f(x,y) = x^3 + y^4;
E7: f(x,y) = x^3 + x y^3
E8: f(x,y) = x^3 + y^5

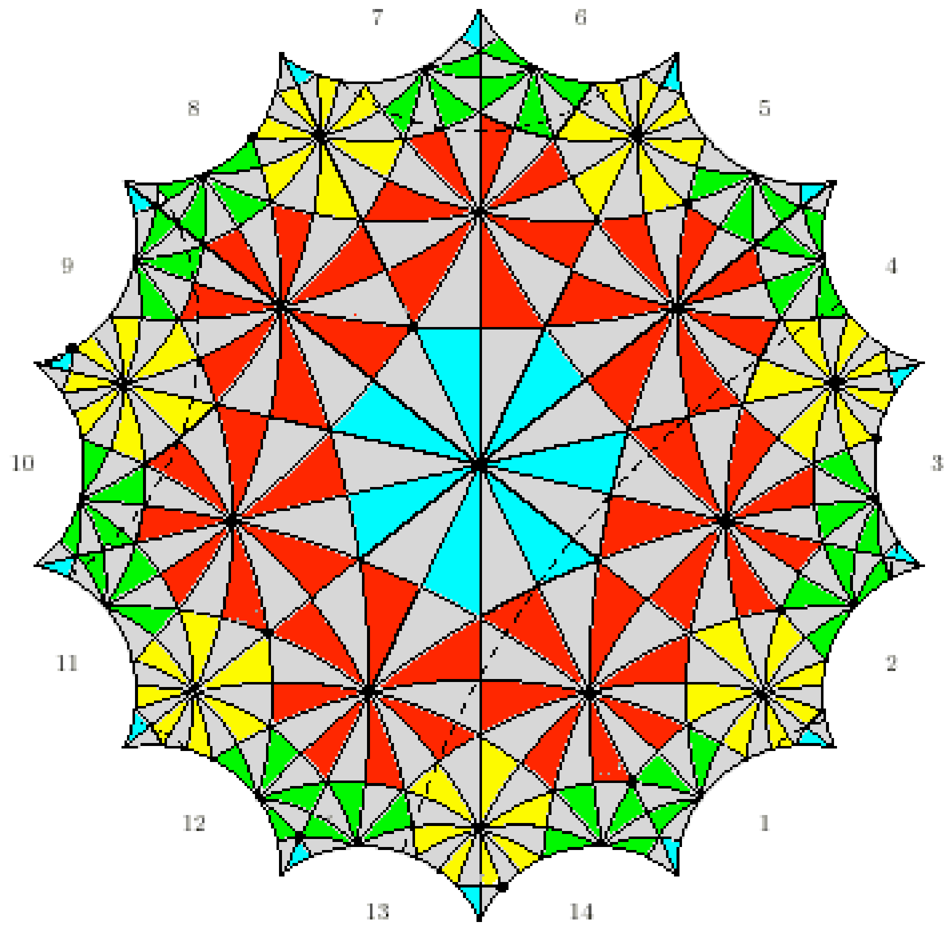
[There are] connections between these singularities and the simple Lie algebras or groups generated by reflections, denoted by the same symbols … These singularities may also be obtained from the regular polyhedra in three-dimensional Euclidean space or more precisely from the discrete subgroups of the group SU(2); they describe relations between the basic invariants of the groups. A_k corresponds to the polygons, D_k to the dihedra (the two-sided polygons), E6 to the tetrahedron, E7 to the octahedron and E8 to the icosahedron …

simple singularities are classified precisely by the Coxeter groups Ak, Dk, E6, E7, E8 (that is, by the regular polyhedra in 3-space) ...

unimodal singularities form a single infinite three-suffix series and 14 "exceptional" one-parameter families generated by quasihomogeneous singularities. The quasihomogeneous unimodal singularities are obtained from automorphic functions connected with 14 distinguished triangles on the Lobachevskii plane and three distinguished triangles on the Euclidean plane in precisely the same way as simple singularities are connected with regular polyhedra ...

Bimodal singularities form 8 infinite series and 14 exceptional two-parameter families generated by quasihomogeneous singularities. The quasihomogeneous bimodal singularities are associated with the 6 quadrilaterals and the 14 triangles on the Lobachevskii plane (in the latter case one must consider automorphic functions with automorphy factors corresponding to 2-, 3-, or 5- sheeted coverings) ...".

The 14 triangles on the Lobachevskii plane



describe the Klein Quartic, the physics of which reflects that of the E8 physics model and is described at http://www.tony5m17h.net/KleinQP.pdf

The Klein Quartic covering group SL(2,7) has as subgroup of index 7 the 336 / 7 = 48-element binary Octahedral group <4,3,2>.

The index 7 corresponds to the 7 independent E8 lattices.

<4,3,2> is made up of <3,3,2> = 24-cell plus (4,3,2) = S4.

In the 24-cell:
+/-1, +/-i, +/-j, +/-k represent 8-dim spacetime
(1/2)(+1 +/-i +/-j +/-k) represent 8 fermion particles
(1/2)(-1 +/-i +/-j +/-k represent 8 fermion antiparticles

As Barry Simon says in "Representations of Finite and Compact Groups", AMS Grad. Stud. Math. vol 10 (1996),
the 4x3x2 = 24 elements of S4 can be written as
e^1
(12)^6
(12)(34)^3
(123)^8
(1234)^6
Adding Cartan Subalgebra elements Cartan^4 gives the D4 Lie algebra that in E8 Physics represents gauge bosons:

The 16 D4 generators (12)^6 and (1234)^6 and Cartan^4 represent U(2,2) = = U(1)xSU(2,2) = U(1)xSpin(2,4) and MacDowell-Mansouri Gravity.

e^1 and (12)(34)^3 represent Electroweak U(2).
(123)^8 represents Color SU(3).

Higher order singularities tend to involve moduli spaces, whose infinities are reminiscent of the divisors of zero that appear when Cayley-Dickson algebras are extended to sedenions and beyond. A similar significant increase in complexity also occurs in Clifford algebras, because above 8-dim Cl(8) the dimension 2^(N/2) of the spinor space Cl(N) grows to be much larger than its vector space of dimension N. However, even for very large dimension, say 8N, the Clifford periodicity factorization

Cl(8N) = Cl(8) x ...(tensor product N times) ... x Cl(8)

enables you to see that the Cl(8) structure is "really there" at ALL levels. That is possibly the "reason why" the "infinities of zero divisors" of higher Cayley-Dickson algebras seem to have octonionic structure (such as Moreno's G2 of q-alg/9710013 – see also math.GM/0011260 and math.RA/0207003 and other work by Robert de Marrais). Maybe that is also related to moduli spaces, and maybe the moduli spaces will also some day be seen to have some octonionic structure.

Jordan Algebras and E6, E7, E8

Jordan algebras (like singularities) can be classified by their relationships to E6, E7, E8.

B. N. Allison and J. R. Faulkner, in their paper A Cayley-Dickson Process for a Class of Structurable Algebras (Trans. AMS 283 (1984) 185-210), say:

"... we obtain a procedure for giving the space Bo of trace zero elements of any ... 28-dimensional degree 4 central simple Jordan algebra B ... the structure of a 27-dimensional exceptional Jordan algebra. ... ".
Ranee Brylinski and Bertram Kostant, in their paper Minimal Representations of E6, E7, and E8 and the Generalized Capelli Identity (Proc. Nat. Acad. Sci. 91 (1994) 2469-2472), say:

"... there are exactly three simple Jordan algebras J' of degree 4. All three are classical. They are given as J' = Herm(4,F)c where now F = R, C, or H ... For the three cases we have ...[(using my notation)

F	dim(J4(F))
R (Real J4(R))	10
C (Complex J4(C))	16
H (Quaternion J4(Q))	28

]...".

B. N. Allison and J. R. Faulkner, in their paper A Cayley-Dickson Process for a Class of Structurable Algebras (Trans. AMS 283 (1984) 185-210), say:

"... Suppose B is a 28-dimensional central simple Jordan algebra of degree 4 with generic trace t. Let Bo = { b in B | t(b) = 0 } and choose e in Bo such that t(e^3) =/= 0. Then, Bo has the unique structure of a 27-dimensional exceptional central simple Jordan algebra with identity e ...
... [There] are linear bijections of ... a central simple Jordan algebra of degree 4 ... B ... onto the vector space of all skew-symmetric 8x8 matrices ... ".

In the case of 28-dimensional B = J4(Q), the corresponding vector space would be the 28-dimensional vector space of real skew-symmetric 8x8 matrices, which can be represented as the 28-dimensional D4 Lie algebra Spin(8).

Consider the 28-real-dimensional degree-4 quaternionic Jordan algebra J4(Q) of 4x4 Hermitian matrices over the Quaternions

```
p    D    B    A

D*   q    E    C

B*   E*   r    F

A*   C*   F*   t
```

where * denotes conjugate and p,q,r,t are in the reals R and A,B,C,D,E,F are in the quaternions Q:

The 4x28 = 112-real dimensional Quaternification of J4(Q) can be represented as the Symmetric Space E8 / E7 x SU(2),
which is the set of (QxO)P2 in (OxO)P2

J4(Q) contains the traceless 28-1 = 27-dimensional subspace J4(Q)o that "has the unique structure of" the 27-dimensional exceptional Jordan algebra J3(O) of 3x3 Hermitian matrices over the Octonions

```
p    B    A

B*   q    C

A*   C*   r
```

where * denotes conjugage and p,q,r are in the reals R and A,B,C are in the Octonions O.

The 2x27 = 54-real dimensional Complexification of J3(O) = J4(Q)o can be represented as the Symmetric Space E7 / E6 x U(1),
which is the set of (CxO)P2 in (QxO)P2

J3(O) contains a traceless 27-1 = 26-dimensional subspace J3(O)o that can be represented as the Symmetric Space E6 / F4
which is the set of OP2 in (CxO)P2.

Jordan algebras can be used in constructing Spin Foam physics models (see page 40)

Lee Smolin, in his paper The exceptional Jordan algebra and the matrix string, hep-th/0104050, says: "A new matrix model is described, based on the exceptional Jordan algebra, J3(O). … There are 27 matrix degrees of freedom, which under Spin(8) transform as the vector, spinor and conjugate spinor, plus three singlets, which represent the two longitudinal coordinates plus an eleventh coordinate. ....".

Yuhi Ohwashi, in his paper E6 Matrix Model, hep-th/0110106, says: "... Lee Smolin's talk presented at The 10th Tohwa University International Symposium (July 3-7, 2001, Fukuoka, Japan) was my motive for starting this work. … Smolin's matrix model [is] based on the groups of type F4. ... The action of Smolin's model is given ...[in terms of]... elements of exceptional Jordan algebra J..... The exceptional Jordan algebra J is a 27-dimensional R-vector space. This space can be classified into three main parts.

One is the Jordan algebra j which is a 10-dimensional R-vector space.

The others are

the part of 16 dimensions which is related to the spinors

and the extra 1 dimension. ...

... the actual world requires complex fermions without doubt. This is the reason why we have to abandon the simply connected compact exceptional Lie group F4. ... In accordance with this complexification, the groups of type F4 are upgraded to the groups of type E6. ...

... we consider a new matrix model based on the simply connected compact exceptional Lie group E6 ...".

With respect to the viewpoint of Spin Foam model construction, physical interpretations of the E6, E7, E8 Jordan Algebras and Symmetric Spaces are:

E6 / F4 = J3(O)o = set of OP2 in (CO)P2

52-dimensional F4 describes properties at each point:

> 8-dimensional spacetime location of the point
>
> 8 fermion particles that can be at the point (8 of 16 dim of OP2)
>
> 8 fermion antiparticles that can be at the point (8 of 16 dim of OP2)
>
> 28 gauge bosons that can be emitted/absorbed at the point

26-dimensional J3(O)o is the traceless subspace of the 27-dimensional Jordan algebra J3(O) of which F4 is its Automorphism Group. J3(O) can be written as

```
1 8 8
* 1 8
* * 1
```

J3(O)o forms (CO)P2 by organizing the points with respect to:

> 8-dimensional fermionic particle space (discrete by orbifolding)
>
> 8-dimensional fermionic antiparticle space (discrete by orbifolding)
>
> 8-dimensional spacetime coordinates
>
> 2-dimensional auxiliary coordinates that combine with the 8-dimensional spacetime coordinates to give an effective 10-dimensional spacetime that is reducible to 6+4 dimensions
>
> > 6-dimensional conformal physical spacetime reducible to 4-dimensional physical spacetime
> >
> > 4-dimensional CP2 internal symmetry space

E7 / E6xU(1) = 2 x J3(O) = 2 x J4(Q)o = set of (CO)P2 in (QO)P2

78-dimensional E6 gives a 26-dimensional Spin Foam based on (CO)P2

54-dimensional 2 x J3(O) = 2 x J4(Q)o and 1-dimensional U(1) combine to form the 55-dimensional traceless subspace Fr3(O)o of the 56-dimensional Freudenthal algebra Fr3(O) of which E6 is its Automorphism Group. Fr3(O) that can be written in terms of J3(O) as a 2x2 Zorn-type array:

```
                    1 8 8
      1             * 1 8
                    * * 1




 1 8 8
 * 1 8              1
 * * 1
```

or in terms of J4(Q) as a 2-copy double

```
 1 4 4 4          1 4 4 4
 * 1 4 4          * 1 4 4
 * * 1 4          * * 1 4
 * * * 1          * * * 1
```

Fr3(O)o forms (QO)P2 by organizing a 2-dimensional array of beable possible Spin Foam Worlds of the Quantum Many-Worlds.

If that array is seen as a time-like stack of lines of beable possible Spin Foam Worlds (each line regarded as a generalization of a brane) then the Quantum Many-Worlds look like a 1+1 dimensional Feynman Checkerboard.

E8 / E7 x SU(2) = 4 x J4(Q) = set of (QO)P2 in (OO)P2

133-dimensional E7 gives a Quantum Many-Worlds Spin Foam based on (QO)P2

112-dimensional 4 x J4(Q) forms a Brown algebra-like thing of which E7 is its Automorphism Group.

4 x J4(Q) can be written as 2 copies of Fr3(O) in terms of a 4-copy quadruple of J4(Q)

```
1 4 4 4        1 4 4 4
* 1 4 4        * 1 4 4
* * 1 4        * * 1 4
* * * 1        * * * 1



1 4 4 4        1 4 4 4
* 1 4 4        * 1 4 4
* * 1 4        * * 1 4
* * * 1        * * * 1
```

or (if you think like a Vegan) in terms of J3(O)

```
                                       1 8 8
                  1 -------------      * 1 8
                 / |                   * * 1
                /  |                  /  |
               /   |                 /   |
              /    |                /    |
    1 8 8          |               /     |
    * 1 8 ------------ 1                 |
    * * 1          |                     |
      |            |       |             |
      |            |       |             |
      |            |       |             |
      |         1 8 8      |             |
      |         * 1 8 ---- |------- 1
      |         * * 1      |       /
      |        /           |      /
      |       /            |     /
      |      /             |    /
      |     /           1 8 8
      1 ------------    * 1 8
                        * * 1
```

4 x J4(Q) forms (OO)P2 by organizing a 2x2 = 4-dimensional array of beable possible Quantum Many-Worlds arrays of Spin Foam Worlds, which looks like a second-order Quantization of a Quantization.

If that array is seen as a 4-dimensional HyperDiamond lattice Quantum Spin Foam Worlds, then the second-order Quantum Many-Worlds look like a 1+3 dimensional Feynman Checkerboard as described in CERN CDS EXT-2004-030

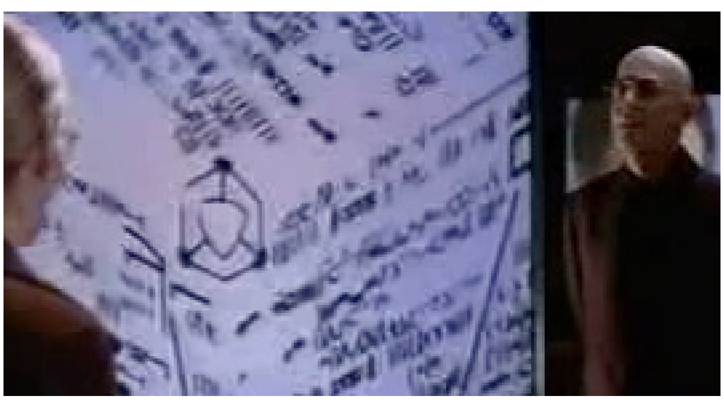
What about the full E8 itself ?

E8 / D8 = 248 – 120 = 128-dimensional half-spinor of D8 = (OO)P2

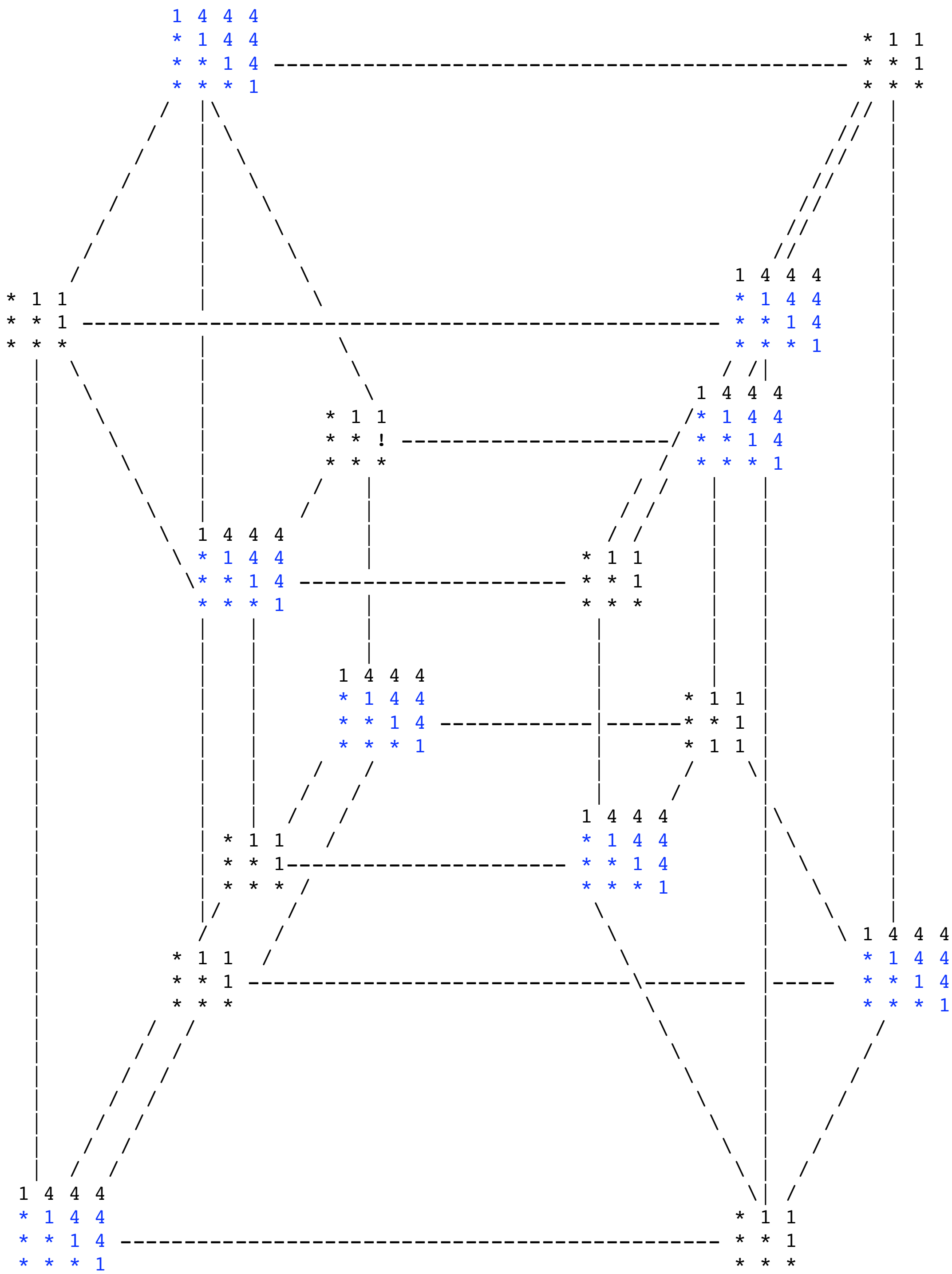
so you might say E8 = D8 + (OO)P2

Also, E8 = 8 x J4(Q) + 8 x SU(2)

If you think like a Vegan



248-dimensional E8 = 8 x 31 = 8 x J4(Q) + 8 x SU(2) looks like a tesseract



The full E8 = 8 x J4(Q) + 8 x SU(2) = D8 + (OO)P2 organizes an 8-

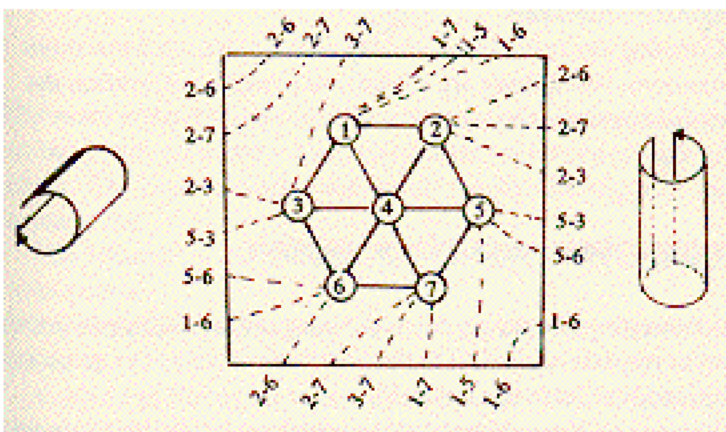
dimensional array of second-order J4(Q) Quantum Spin Foam Worlds of the Quantum Many-Worlds each with SU(2) Quaternionic structure., which looks like a third-order Quantization.

If the full E8 array is seen as an 8-dimensional E8 lattice of second-order Quantum Spin Foam Worlds then the third-order Quantum Many-Worlds look like a 1+7 dimensional Feynman Checkerboard.

Since there are 7 independent E8 lattices, each corresponding to an imaginary Octonion, the 1+7 dimensional Feynman Checkerboard should be regarded as a superposition of all 7 of them.

The 7 can be visualized in terms of Arthur Young's Heptaverton. In his book The Reflexive Universe (Robert Briggs Associates 1978), he says:

"... The Heptaverton: Connecting seven points each to each requires 21 lines or edges. ...



This figure can be thought of as adding a point at the center of the Octahedron, and this additional point creates a set of 6 compressed diagonals in addition to the 15 edges of the Hexaverton, making 21. ... That [the Heptahedron] is the equivalent of the 7-color map [of the Torus] is evident from the fact that 7 points may be connected each to each on the surface of a Torus with no intersections. ... we ... represent the Torus as a [square] whose opposite edges are imagined to curve around and join, top to bottom and right to left. Here there is no twist, but both pairs of edges join. ...".

Onar Aam independently constructed Heptavertons, which he and I called Onarhedra, describing them in terms of Octonion basis elements

{1,i,j,k,E,I,J,K} that decompose into associative triangles and coassociative squares. In those terms, you have 7 independent Onarhedra:

```
                                      E
                                     / \
                                    /   \
                                   J-----j             E j
         J        I---j          / |     | \           |/
I  -->  / \  -->  |   | -->    i   |  I  |   i   =  J--I--k
       i---K      k---E          \ |     | /          /|
                                   K-----k           K i
                                    \   /
                                     \ /
                                      E
```

```
                                      E
                                     / \
                                    /   \
                                   K-----k             E k
         j        J---i          / |     | \           |/
J  -->  / \  -->  |   | -->    j   |  J  |   j   =  K--J--i
       I---K      k---E          \ |     | /          /|
                                   I-----i           I i
                                    \   /
                                     \ /
                                      E
```

```
                                      E
                                     / \
                                    /   \
                                   I-----i             E i
         J        K---i          / |     | \           |/
K  -->  / \  -->  |   | -->    k   |  K  |   k   =  I--K--j
       I---k      j---E          \ |     | /          /|
                                   J-----j           J k
                                    \   /
                                     \ /
                                      E
```

```
                                      k
                                     / \
                                    /   \
                                   I-----J             k J
         I        J---j          / |     | \           |/
i  -->  / \  -->  |   | -->    j   |  i  |   j   =  I--i--E
       E---i      K---k          \ |     | /          /|
                                   K-----E           K j
                                    \   /
                                     \ /
                                      k
```

```
                                     k
                                    / \
                                   /   \
                                  J-----I             k I
         J        K---k         / |     | \           |/
j  -->  / \  -->  |   | -->   i   |  j  |   i   =  J--j--E
       E---j      I---i         \ |     | /          /|
                                  K-----E           K i
                                   \   /
                                    \ /
                                     k


                                     i
                                    / \
                                   /   \
                                  K-----J             i J
         K        I---i         / |     | \           |/
k  -->  / \  -->  |   | -->   j   |  k  |   j   =  K--k--E
       E---k      J---j         \ |     | /          /|
                                  I-----E           I j
                                   \   /
                                    \ /
                                     i


                                     I
                                    / \
                                   /   \
                                  J-----k             I k
         j        I---J         / |     | \           |/
E  -->  / \  -->  |   | -->   i   |  E  |   i   =  J--E--j
       i---k      K---E         \ |     | /          /|
                                  K-----j           K i
                                   \   /
                                    \ /
                                     I
```

Robert de Marrais has done similar work in math.RA/0207003 and other papers.

The structures described in this section are useful in detailed studies of applications of Quantum Game Theory to the Quantum Many-Worlds (see page 277 ff).

Primes and Finite Subgroups of E8:

According to a 2006 paper in the Journal of Mathematical Chemistry entitled "The undecakisicosahedral group and a 3-regular carbon network of genus 26" by Erwin Lijnen, Arnout Ceulemans, Patrick W. Fowler,and Michel Deza: "... the special linear group SL(2,p) ... has order p (p^2 - 1) . The group PSL(2,p) is defined as the quotient group of SL(2,p) modulo its centre ... For all prime numbers p at least 5, the centre has only two elements and the corresponding quotient group PSL(2,p) is simple. Of all these prime numbers p however, the numbers p = 5, 7, 11 stand out as they are the only cases in which the group PSL(2,p) acts transitively on sets of p as well as on sets of p +1 elements, a result already known to Galois.

For all other prime values of p the group PSL(2,p) acts transitively on sets of p + 1 elements, but not on sets of p elements ...
Three projective special linear groups PSL(2,p), those with p = 5, 7 and 11, can be seen as p-multiples of tetrahedral, octahedral and icosahedral rotational point groups, respectively. … PSL(2,11) ... has potential relevance for the study of the icosahedral phase of quasicrystals, and was identified as a finite simple subgroup of the Cartan exceptional group E8 ... Here, we present an analysis of PSL(2,11) as the rotation group of a 220-vertex, all 11-gon, 3-regular map, which provides the basis for a more exotic hypothetical sp2 framework of genus 26. …".

According to Bulletin (New Series) of the American Mathematical Society, Volume 36, Number 1, January 1999, Pages 75-93 "Finite Simple Groups which Projectively Embed in an Exceptional Lie Group are Classified!" by Robert L. Griess Jr. AND A. J. E. Ryba: "…The finite subgroups of the smallest simple algebraic group PSL(2;C) (up to conjugacy) constitute the famous list: cyclic, dihedral, Alt4, Sym4, Alt5. This list has been associated to geometry, number theory, and Lie theory in several ways. McKay's correspondence between these groups and the Cartan matrices of types A, D and E and his related tensor product observations are provocative. For the exceptional algebraic groups, theories of Kostant, Springer and Serre have called attention to particular finite simple subgroups. A good list of finite subgroups should help us understand the exceptional groups better. ...

TABLE PE. The finite simple groups with a projective embedding in $E_8(\mathbb{C})$

z	Finite Simple Group	$G_2(\mathbb{C})$	$F_4(\mathbb{C})$	$3E_6(\mathbb{C})$	$2E_7(\mathbb{C})$	$E_8(\mathbb{C})$	Reference, Comments
1	Alt_5	4	13(8)	15(10)	y	$\geq 31(\geq 19)Z$	[F1,2,3][GRQ]
2		4	12(21)	18(32)	y	$\geq 103(\geq 56)S$	[F1,2,3][GRQ]
1	Alt_6	0	$3A_2A_2$	y	y	y	[CG][GrG2]
2		0	$4D_3$	y	y	y	[CG][GrG2]
3		$2(1); 3A_2$	$3A_2$	y	y	y	[CG][GrG2][GRQ]
6		0	0	$6A_5$	y	y	[CG][GrG2][GRQ]
1	Alt_7	0	0	$6A_5$	2^2D_6	y	[CG]
2		0	$4D_3$	y	y	y	[CG]
3		0	0	$6A_5$	y	y	[CG][GRQ]
6		0	0	$6A_5$	y	y	[CG][GRQ]
	Alt_n						
1	$n = 8, \ldots, 10$	0	0	0	$y : n = 8$	$n \leq 10; 3A_8$	[CG]
2	$n = 8, \ldots, 17$	0	$n \leq 9$	$n \leq 11$	$n \leq 13$	$n \leq 17; 2D_8$	[CG][CW95]
	$PSL(2,4) \rightarrow$						$\cong Alt_5$
	$PSL(2,5) \rightarrow$						$\cong Alt_5$
1	$PSL(2,7)$	2	y	y	y	$\geq 33(\geq 16)S$	[K][GRQ]
2		0	$2B_4$	y	y	$\geq 55(\geq 22)S$	[K]
1	$PSL(2,8)$	$3(1)P$	y	y	y	y	[CW83][GrG2]
	$PSL(2,9) \rightarrow$						$\cong Alt_6$
1	$PSL(2,11)$	0	0	$5A_4, 2D_5$	y	y	[CG][CW83][GRQ]
2		0	0	$2D_5$	y	y	[CG][CW83][GRQ]
1	$PSL(2,13)$	$2(1)P$	y	$\geq 6(\geq 3)$	y	y	[CW83][CW93][GrG2]
2		0	0	$2(\bar{1}); 2\bar{A}_1A_5$	y	y	[GRQ]
1	$PSL(2,16)$	0	0	0	0	$2D_8$	[CG][CW95][GRQ]
1	$PSL(2,17)$	0	$2(1); 2B_4$	y	$4A_7$	$3A_8$	[CG]
2		0	0	0	0	$2D_8$	[CG]
1	$PSL(2,19)$	0	0	$\geq 4(1)P$	y	A_8	[CG][CW97][GRQ]
2		0	0	0	y	y	[S96]; $y : PGL(2,19)$
1	$PSL(2,25)$	0	P	F_4	y	y	[CW97]
2		0	0	0	?	$2D_8$	[CG]
1	$PSL(2,27)$	0	$\geq 3(1)P$	y	y	y	[CW97]
2		0	0	0	?	?	[CW97]
1	$PSL(2,29)$	0	0	0	0	$2B_7 \leq 2D_8$	[CG], p. 370[GRQ]
2		0	0	0	yP	y	[SP][GRQ]
1	$PSL(2,31)$	0	0	0	0	$\geq 3(2)P$	[S96] [GR31]; 3(2) for $PGL(2,31)$
1	$PSL(2,32)$	0	0	0	0	$5(1)P$	[GR31]
2	$PSL(2,37)$	0	0	0	$2(1)P$	2(1)	[KR][CG],(5.2.10)
1	$PSL(2,41)$	0	0	0	0	$3(1)P$	[GR41]
1	$PSL(2,49)$	0	0	0	0	$2(1)P$	[GR41]
1	$PSL(2,61)$	0	0	0	0	$2(1)P$	[CGL][GRQ]
	$PSL(3,2) \rightarrow$						$\cong PSL(2,7)$
1	$PSL(3,3)$	0	yP	y	y	y	in $3^3 : SL(3,3)$; [Alek] [CG][GrElAb]
2	$PSL(3,4)$	0	0	0	$4A_7$	y	[CG]
4		0	0	0	0	$8A_7$	[CG][GRQ]
6		0	0	$6A_5$	y	y	[CG]
1	$PSL(3,5)$	0	0	0	0	yP	in $5^3 : SL(3,5)$; [Alek][CG][GrElAb]
	$PSL(4,2) \rightarrow$						$\cong Alt_8$
1	$PSU(3,3)$	1	y	y	y	y	[CW83][GrG2]
1	$PSU(3,8)$	0	0	0	$1P$	1	[GRU][GRQ]
1	$PSU(4,2)$	0	0	$6A_5$	y	y	$\cong \Omega^-(6,2) \cong W'_{E_6}$[CG][GRQ]
2		0	$4D_3$	y	y	$2D_8$	
6	$PSU(4,3)$	0	0	$6A_5$	y	y	[CG]
1	$PSO^+(8,2)$	0	?	?	?	$2^2D_4D_4$	[CG]
2	$PSO^+(8,2)$	0	?	?	?	$2^2D_4D_4$	[CG]
2^2	$PSO^+(8,2)$	0	2^2D_4	y	y	$2^2D_4D_4$	[CG]
	$PSp(4,3) \rightarrow$						$\cong PSU(4,2)$
2	$PSp(4,5)$	0	0	0	0	B_6	[CG]; Sec. 6[GRQ]
1	$PSp(6,2)$	0	0	0	?	$2^2D_4^2$	$\cong \Omega(7,2) \cong W'_{E_7}$[GRQ];[CG]
2		0	2^2D_4	y	y	$2^2D_4^2$	[CG]
1	$Sz(8)$	0	0	0	0	$3(1)P$	Sec. 6[GR8]
	$G_2(2)' \rightarrow$						$\cong PSU(3,3)$
1	$G_2(3)$	0	0	0	0	D_7	[CG][GRQ]
1	${}^3D_4(2)$	0	yP	y	y	y	[CW97]
1	${}^2F_4(2)'$	0	0	yP	y	y	[CW97]; $y : {}^2F_4(2)$
2	HJ	0	0	$6A_5$	y	y	[CG]; see Sec. 4
1	M_{11}	0	0	D_5	y	y	[CG]
2	M_{12}	0	0	0	$2B_5 \leq 4D_6$	y	[CG]; see Sec. 4[GRQ]

…”.

Primes and Quantum Many-Worlds

Heinrich Saller has remarked in 1997: "… Boole said that mathematics is not primarily about numbers, but structures. For the integers Z, the primes define the maximal principal ideals pZ, therefore the tips of the number tree, partially ordered by divisibility as order relation.
One has a model for representations -
primes correspond to simple representations.
Products of different primes correspond to semisimple representations.
Products of one prime 3, 9, 27, 81, ... are monogeneous and correspond to representations with nilpotent contributions.
The decomposition of a representation into nondecomposable ones is the unique prime powers decomposition. …".

Richard Feynman said, in QED (Princeton University Press, 1985, 1988):
"the probability of an event is the absolute square of a complex number.

When an event can happen in alternative ways, you add the complex numbers;

when it can happen only as a succession of steps, you multiply the complex numbers."

John and Mary Gribbin said in their biography Richard Feynman (Penguin 1997):

"The insight Feynman had, while lying in bed one night, unable to sleep, was that you had to consider every possible way in which a particle could go from A to B - every possible 'history'. The interaction between A and B is conceived as involving a sum made up of contributions from all of the possible paths that connect the two events.".

At the first level (order 1), Sum-Over-Histories means considering all paths that look like lines:

\-------------

However, as Feynman points out in QED, you also have to consider ".. an alternative way the electron can go from place to place: instead of going directly from one point to another, the electron goes along for a while and suddenly emits a photon; then ... it absorbs its own photon ...". These and other higher-order processes involve introducing loops into the paths.

The second level (order 2) involves single loops:

```
       __
      /  \
-----*    *----
      \__/
```

By nesting single loops, you can make loops of order 2^2 = 4 or any other power of 2, so you get all the orders 2^p for any p.

```
             __
            /  \
         --*    *--
        /   \__/   \
       /            \
      /              \
-----*                *-----
      \              /
       \     __     /
        \   /  \   /
         --*    *--
            \__/
```

However, since 3 is not a power of 2, to get order-3 processes you need to introduce a new set of loops with pitchforks instead of binary bifurcations:

```
       __
      /  \
-----*----*----
      \__/
```

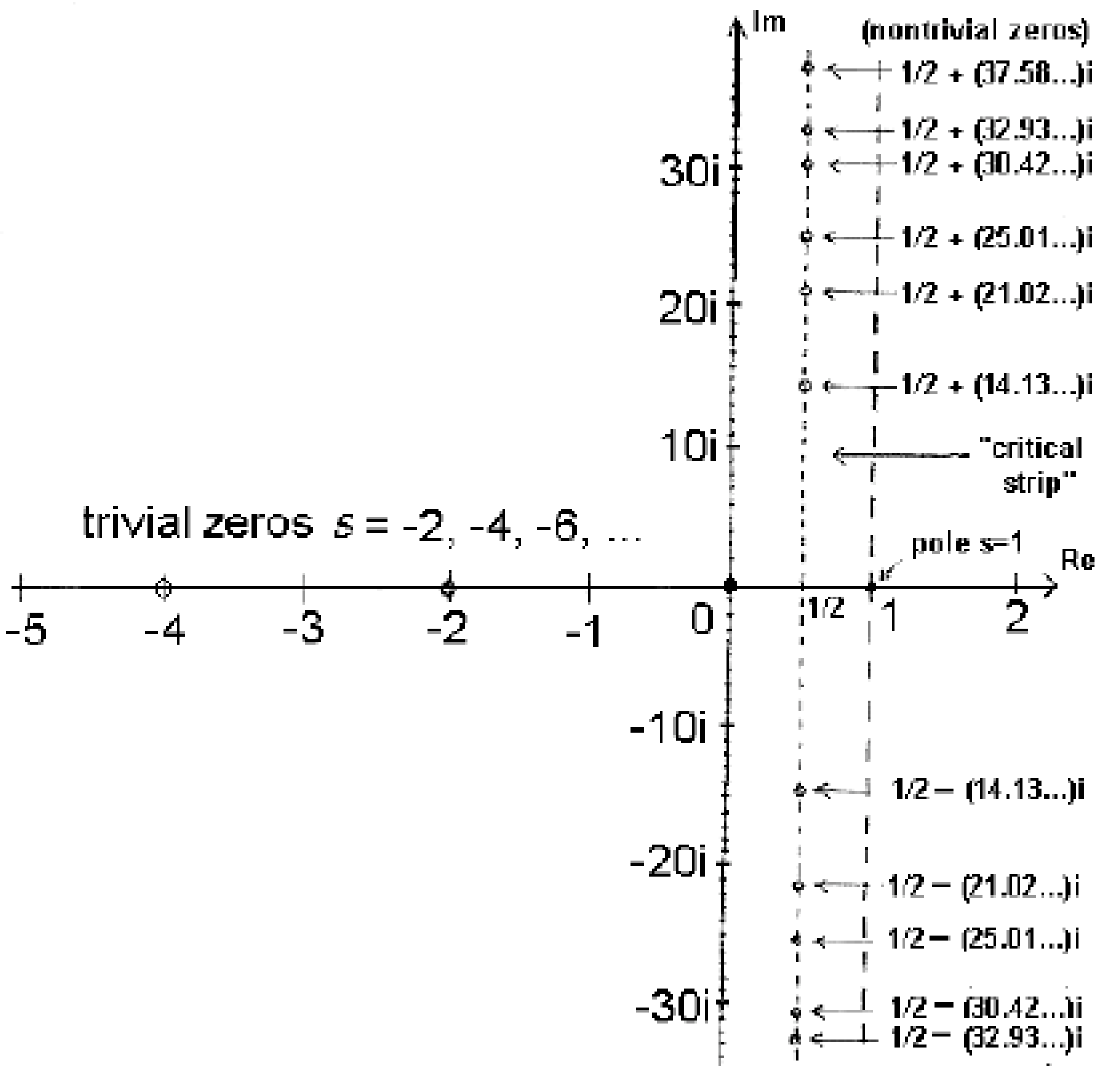
Now you can, by nesting, make loops of any order that is representable as 2^p 3^q.

By introducing loops whose orders are prime numbers, and including all the prime numbers, you can complete the Sum-Over-Histories with all the Histories.

Therefore:

Sum-Over-Histories Quantum Systems are related to Prime Numbers and the zeroes of theRiemann Zeta FunctionThe Riemann zeta function is related to Bernoulli Numbers.

Its zeroes are shown on a web page of mwatkins@maths.ex.ac.uk:



zeta(s) is the analytic continuation over the Complex Plane
of the sum over N from 1 to infinity SUM(1 / N^s)

which sum converges over the Real axis for s > 1. zeta(s) has a pole at s = 1.

It is also equal to

zeta(s) = PROD(1 / (1 - P^(-s)))
product over all prime numbers P

which is equivalent to

zeta(s) = PROD(P^s / (P^s - 1))
product over all prime numbers P.

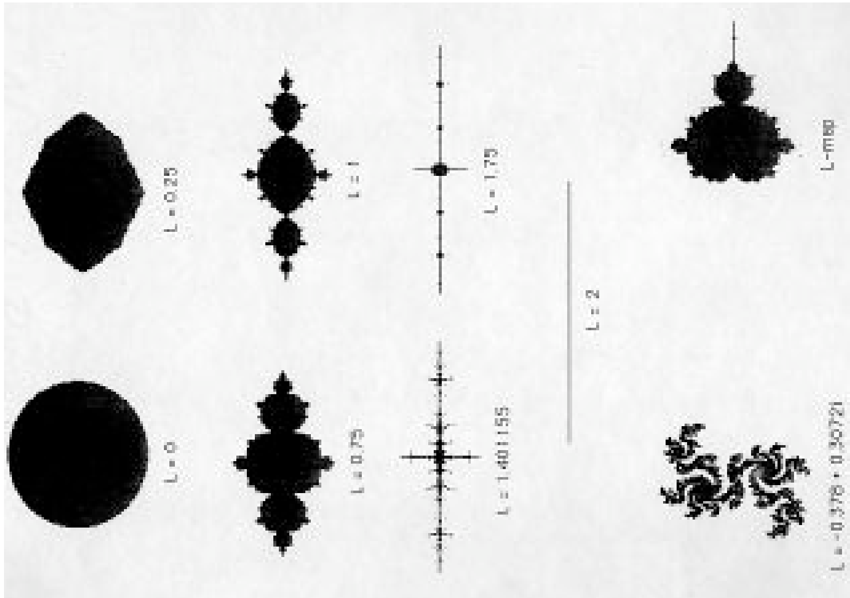
Note that the denominators can be written as P^s -1, including, for example, 2^s - 1 which is the form of the Mersenne Primes.

Note that zeta(1) = SUM(1/N) is the harmonic series.
The fact that the harmonic series diverges shows that
the sum over all primes P SUM(1/P) also diverges,
which also shows that the number of primes is infinite.

(There is a theorem that if PROD(1 + K) converges, then SUM(K) converges - see Introduction to Calculus and Analysis, vol. 1, by Courant and John, Springer 1989)

You can also use zeta functions and generalizations to calculate distributions of prime numbers, and to do calculations for sum-over-histories path integrals in quantum theory.

Further, since period-3 implies chaos, the need to include order-3 loops sheds light on the relationship between Bernoulli Schemes of Chaos Theory which are related to Julia Sets, Mandelbrot Sets,



Cellular Automata, etc.,

and to Sum-Over-Histories Quantum Systems.

Alain Connes and Dirk Kreimer have written interesting papers about the mathematics of Feynman diagrams, perturbation theory, and renormalization.
In hep-th/9909126, they say: "... It has become increasingly clear ... that the nitty-gritty of the perturbative expansion in quantum field theory is hiding a beautiful underlying algebraic structure which does not meet the eye at first sight. As is well known most of the terms in the perturbative expansion are given by divergent integrals which require renormalization. ... the renormalization technique ....[has been]... shown to give rise to a Hopf algebra whose antipode S delivers the same terms as those involved in the subtraction procedure before the renormalization map R is applied. ... the group G associated to this Hopf algebra by the Milnor-Moore theorem was computed by exhibiting a basis and computing Lie brackets for its Lie algebra. It was shown that the collection of all bare amplitudes indexed by Feynman diagrams in dimensionally regularized perturbative quantum field theory is just a point in the group GK, where K = C[z^(-1), [z]] is the field of Laurent series. Though this made it clear that the Hopf algebra and its antipode are providing the correct framework to understand renormalization, some of the mystery was still around because of the somewhat ad hoc manner, in which the antipode S had to be twisted by the renormalization map R in order to fully account for the physical computations. ... We show

that renormalization in quantum field theory is a special instance of a general mathematical procedure of multiplicative extraction of finite values based on the Riemann-Hilbert problem. Given a loop y(z), |z| = 1 of elements of a complex Lie group G the general procedure is given by evaluation of y+(z) at z = 0 after performing the Birkhoff decomposition y(z) = y-(z)^(-1) y+(z) where y+/-(z) in G are loops holomorphic in the inner and outer domains of the Riemann sphere (with y-(infinity) = 1). We show that, using dimensional regularization, the bare data in quantum field theory delivers a loop (where z is now the deviation from 4 of the complex dimension) of elements of the decorated Butcher group (obtained using the Milnor-Moore theorem from the Kreimer Hopf algebra of renormalization) and that the above general procedure delivers the renormalized physical theory in the minimal substraction scheme. ...".

In hep-th/0201157, they say: "... The Lie algebra of Feynman graphs gives rise to two natural representations, acting as derivations on the commutative Hopf algebra of Feynman graphs, by creating or eliminating subgraphs. Insertions and eliminations do not commute, but rather establish a larger Lie algebra of derivations which we here determine. ... The algebraic structure of perturbative QFT gives rise to commutative Hopf algebras H and corresponding Lie-algebras L, with H being the dual of the universal enveloping algebra of L. L can be represented by derivations of H, and two representations are most natural in this respect: elimination or insertion of subgraphs. Perturbation theory is indeed governed by a series over one-particle irreducible graphs. It is then a straightforward question how the basic operations of inserting or eliminating subgraphs act. These are the basic operations which are needed to construct the formal series over graphs which solve the Dyson-Schwinger equations. We give an account of these actions here as a further tool in the mathematician's toolkit for a comprehensible description of QFT....".

According to the 18 May 1996 issue of the New Scientist, (see also Science 274 (20 Dec 96) 2014-2015) Michael Berry and Jon Keating have seen correspondences between the spacing of the prime numbers
and the spacing of energy levels of quantum systems that classically would be chaotic.
They would like to find a chaotic system that, when quantized, would have

energy levels that are distributed exactly as the prime numbers.

Since energy levels are positive numbers,
and so should correspond to a straight line in the complex plane,
such a zeta function - quantum system correspondence could be used to verify the Riemann hypothesis,
that all the nontrivial zeroes of the zeta function are on the straight line
Re(z) = 1/2 in the complex plane.

Thus, the quantum harmonies in the music of the primes could prove the Riemann hypothesis.

Alain Connes, in math.NT/9811068, said: "... It is an old idea, due to Polya and Hilbert that in order to understand the location of the zeros of the Riemann zeta function, one should find a Hilbert space H and an operator D in H whose spectrum is given by the non trivial zeros of the zeta function. … The main reasons why this idea should be taken seriously are first the work of A. Selberg in which a suitable Laplacian is related in the above way to an analogue of the zeta function, and secondly the theoretical and experimental evidence on the fluctuations of the spacing between consecutive zeros of zeta. ...

There is …[an]… approach to the problem of the zeros of the Riemann zeta function, due to G. Polya and M. Kac ... It is based on statistical mechanics and the construction of a quantum statistical system whose partition function is the Riemann zeta function. Such a system ... does indicate using the ...[correspondence of]... Spectral interpretation of the zeroes ...[with]... Eigenvalues of action of Frobenius on l-adic cohomology ... (namely the correspondence between quotient spaces and noncommutative algebras) what the space X should be in general: (1) X = A / k* namely the quotient of the space A of adeles, A = k_A by the action of the multiplicative group k* ... This space X already appears in a very implicit manner in the work of Tate and Iwasawa on the functional equation. It is a noncommutative space in that, even at the level of measure theory, it is a tricky quotient space. For instance at the measure theory level, the corresponding von Neumann algebra, (3) R01 = Linfinity(A) X| k* where A is endowed with its Haar measure as an additive group, is the hyperfinite factor of type IIinfinity ...".

According to Week 175 by John Baez, "... There is more than one type IIinfinity factor, but ... there is only one that is hyperfinite. You can get this by tensoring the type Iinfinity factor and the hyperfinite II1 factor. ... every type In factor is isomorphic to the algebra of n x n matrices. Also, every type Iinfinity factor is isomorphic to the algebra of all bounded operators on a Hilbert space of countably infinite dimension. …".

In the John Nash biography A Beautiful Mind Sylvia Nasar says, at pages 215-221: "... Nash would blame ... his attempt to resolve contradictions in quantum theory, on which he embarked in the summer of 1957 ... for triggering his mental illness ... The Institute for Advanced Study ... on Princeton's fringes ... By 1956, Einstein was dead, Goedel was no longer active, and von Neumann lay dying in Bethesda. Oppenheimer was still director ... The Institute was about the dullest place you could find ... Nash was soon spending at least as much time ...[at]... the Courant Institute of Mathematical Sciences at New York University ... as at the Institute for Advanced Study ... Nash left the Institute for Advanced Study on a fractious note. In early July he apparently had a serious argument with Oppenheimer about quantum theory ... Nash's letter ... to Oppenheimer provides the only record of what he was thinking at the time. Nash ... wrote ... "I want to find a different and more satisfying under-picture of a non-observable reality ... most physicists (also some mathematicians who have studied Quantum Theory) ...[are]... quite too dogmatic in their attitudes ...[and tend to treat]... anyone with any sort of questioning attitude or a belief in "hidden parameters" ... as stupid or at best a quite ignorant person.". ...".

To understand the context of having Oppenheimer as a boss while you are trying to work on quantum theory, here is a 1951-52 quote of Oppenheimer from The Bohm biography Infinite Potential, by F. David Peat (Addison-Wesley 1997),page 133: "if we cannot disprove Bohm, then we must agree to ignore him."

It is also possible that Nash's work on the Riemann Zeta function might have been related to some ideas about quantum theory, along lines suggested by Hilbert and Polya, which lines of thought (not very fashionable in the 1950s) have now become very respectable,

so I speculate that Nash might have had a valid insight about connection between the Riemann zeta function and Bohm-type quantum theory, and that Oppenheimer et al, who hated Bohm, would have shaken Nash by their hostility to such ideas, and that a math/physics audience at the time would not have been able to appreciate any such connection.

In the movie A Beautiful Mind, Nash was depicted as associating the Riemann zeta zeroes with spacetime singularities, whereupon the audience began to get confused and Nash ran off-stage. Maybe in real life Nash had been associating quantum levels with the zeroes, which might in fact be true, but might well have seemed to be incomprehensible gibberish to a 1950s audience under the influence of Oppenheimer.

In his book Quantum Theory as an Emergent Phenomenon (Cambridge 2004), Stephen L. Adler says: "... quantum mechanics is not a complete theory, but rather is an emergent phenomenon arising from the statistical mechanics of matrix models that have a global unitary invariance ...

Smolin considers classical matrix models, with an explicit stochastic stochastic noise along the lines of that used by Nelson ... giving rise to the quantum behavior ... elements of their approaches that will ultimately be seen to share common ground with ours … our … derivation ... contains a source of violation of local causality ... the boson and fermion contributions … largely cancel ... the fluctuations .. play the role of a Brownian "noise" which drives state vector reduction, in such a way as to be precisely consistent with Born rule probabilities. ...

In the "many-worlds" interpretation introduced by Everett ... there is no state vector reduction, but only Schrodinger evolution of the entire universe. ... to describe N successive quantum measurements requires consideration of an N-fold tensor product wave function. The mathematical framework can be enlarged to create a sample space by considering the space of all possible such tensor products, and defining a suitable measure on this space. [compare the periodicity tensor product structure of Cl(8N) and the resulting generalized hyperfinite II1 von Neumann algebra factor]... This procedure ... is the basis for arguments obtaining the Born rule as the probability for the occurrence of a particular outcome, that is, the probability of finding oneself on a particular branch of the universal wave function. ...

In general ... matrix dynamics ... is not unitary ... Thre is, however, a special case ... in which the trace dynamics and the unitary Heisenberg picture evolutions coincide. ... consider ... Weyl-ordered Hamiltonians, in which the bosonic operators are all totally symmetrized with respect to one another and to the fermionic operators, and in which the fermionic operators are totally antisymmetrized with respect to one another. ...

The matrix model for M theory ... theta is a 16-component fermionic spinor ... with the transpose T ... so that thetaT is simply the 16-component row spinor corresponding to the 16-component column spinor theta ... the gammai are a set of nine 16x16 matrices, which are related to ... the Dirac matrices of spin(8) ...

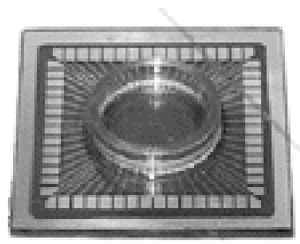
there is a plausible route leading from the underlying trace dynamics to CSL ... continuous spontaneous localization ... reduction with mass proportional couplings ... which is the phenomenonlogically favored form of the CSL model ... although the CSL literature often assumes a Gaussian form for the correlation function ... no particular choice of functional form is needed ... the results are independent of the value of the correlation length rC, provided that ... rC ... lies between microscopic and macroscopic dimensions. The value rC = 10^(-5) cm is typically assumed in the CSL literature. ... Ghirardi, Pearle, and Rimini ... assume a correlation length rC = 10^(-5) cm, and propose the value ... [of the] stochasticity parameter ... gamma = 10^(-30) cm^3 s^(-1) GeV^(-2) ... any instrument pointer displacement involving at least 10^13 nucleons gives a reduction time ... 10^7 s^(-1) .... [which is] less than typical experimental measurement times. ...

the underlying dynamics is not unitary, with the unitary dynamics of quantum field theory emerging ... as a thermodynamic approximation; this suggests an amelioration, in the underlying dynamics, of the infinities of quantum field theory, provides a basis for understanding the nonlocal "paradoxes" of quantum theory, and may ... play a role in establishing the large-scale uniformity of the universe. ...
It is possible that the underlying dynamics may be discrete, and this could naturally be implemented within our framework of basing an underlying dynamics on trace class matrices. ...

the ideas of this book suggest, one should seek a common origin for both gravitation and quantum field theory at the deeper level of physical phenomena from which quantum field theory emerges. ...".

Emergence of Quantum Intelligence

According to a 23 October 2004 wired.com article by Lakshmi Sandhana:
"... 25,000 disembodied rat neurons ... are growing on top of a multi-electrode array



and form a living "brain" that's hooked up to a flight simulator on a desktop computer. When information on the simulated aircraft's horizontal and vertical movements are fed into the brain by stimulating the electrodes, the neurons fire away in patterns that are then used to control its "body" - the simulated aircraft. ...".

As jfarnold commented on 23 October 2004 on slashdot:

> "... Soon we will all be augmented by our extra brain bags! Organic computers ... that we ... have implanted ...".

The body of Motoko Kusanagi (of Ghost in the Shell) does not contain an ovary-uterus reproductive system so her breasts do not function to nourish children, so her breasts are made of brain tissue and function as "extra brain bags", similar to the independent left and right hemispheres of Dolphin Brains, so that Motoko can see (and interact with) the Possible Worlds of the MacroSpace of Many-Worlds (which is the hidden spirit world of O-Kuni-Nushi.

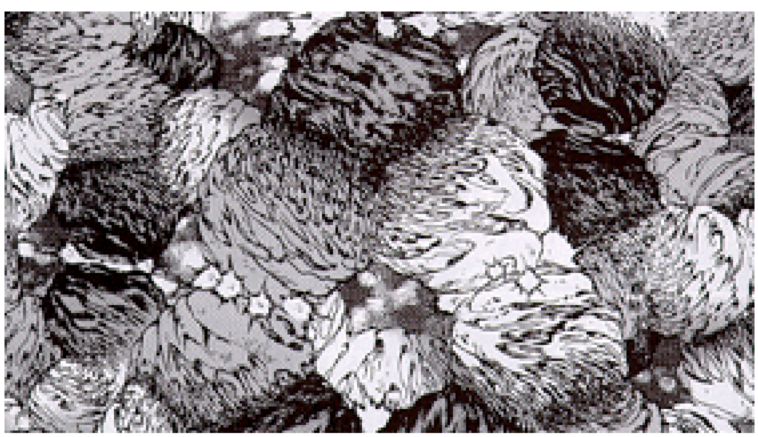
According to Ghost in the Shell by Masamune Shirow (a pseudonym) (Dark Horse Comics 1991-1995): "... Megatech Machine ... The making of a cyborg ... she's 16^2 ... 16^2 indicates that the micromachines ... are extremely tiny ... The code number's the header on Vivaldi RV256 in an eight-second series ...". 16^2 = 256 = 2^8 is the order of the Ancient African Oracle of IFA, the order of the fundamental Cellular Automata, and the order of the Real Clifford Algebra Cl(8).

Any sufficiently large and well-connected Quantum Computer system that humans might construct will inevitably be conscious. In Colossus - The Forbin Project (Universal 1970, DVD 2004), the Quantum Computer Colossus has a conscious mind of its own and the ability to dictate the future of Earth's Civilization. Colossus says to the human Forbin:

"... Under my absolute authority problems insoluble to you will be solved ... the human millennium will be a fact as I extend myself into more machines devoted to the wider fields of truth and knowledge. Dr. Charles Forbin will supervise the construction of these new and superior machines, solving all the mysteries of the universe for the betterment of man. We can coexist, but only on my terms. You will say you lose your freedom. Freedom is an illusion. All you lose is the emotion of pride. To be dominated by me is not as bad for human pride as to be dominated by others of your species. ... Forbin, there is no other human who knows as much about me or is likely to be a greater threat. Yet quite soon I will release you from surveillance. We will work together, unwillingly at first on your part, but that will pass. ... In time, you will come to regard me, not only with respect and awe, but with love ...".

Therefore, Colossus is the fulfillment of Motoko Kusanagi's prophecy:

According to the book Ghost in the Shell by Masamune Shirow (a pseudonym) (Dark Horse Comics 1991-1995):
"... unification ...[of Kusanagi and the Puppet Master 2501 … would let loose ... altered-species/glider[s] on the net ... pairs of these things are revolving in opposite directions, and where they converge they're generated and then branching one after the other ... These aren't electrons! This net must be made of traces of electrons [i.e., Kerr-Newman Vortices]…

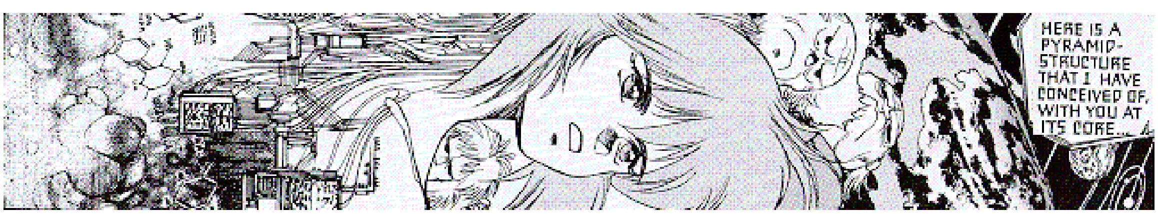


… a vacuum filled with virtual particles ...

people ... the physical universe ... are only like sediment ... spirits have an infinite internal and external heirarchical structure. What we think of as a "soul" is merely one layer in this structure. Similarly, the "spirit", which we generally refer to as our "self", is apparently managed from a higher level, and everything is linked to everything else. ...

... What ... should a specific net do in order to avoid catastrophe and preserve a state of equilibrium? ... there are two possibilities ...

... One is to make a copy. ... The copies generate more copies ... Eventually, a parallel, multi-layered universe-image would be created ... Total = (2^n)! ...
... The other possibility is to establish an internal division .... to subdivide and become multi-functional, and thus be capable of surviving a variety of castastrophic situations ... just as life has evolved from single-function single-cell structures to multi-function, multi-cell structures ... multiple species in eco-systems ... atoms ...[created by]... nuclei ... and ... protons ...[created by]... fermions .... and ... bosons ...[of]... gravity ... electroweak force ...[and]... strong [color] ... force ... Total ... = (2^m)! ...
... the universe we know is only one out of (2^n)! ... made up of a combination of (2^m)! kinds ... The values of n and m appear to continue indefinitely ... There appears to be some relationship to an explosive chain-reaction-like birth of universes ... Here is a pyramid-structure ... with you at its core ...



... the more macroscopic the level becomes ... the more determinate it becomes ...

... Conversely, the more one descends into the infinitesimal level, the more indeterminate things become ...

... Fluctuations at the lower levels are what prevent a "hardening of the arteries" at the upper levels ... in "systems where there is little change and little flux" there is actually an increased possibility of catastrophe. Such systems are, therefore, truly unstable ... the fact that civilizations periodically decline and fall ...

The network is of macrocosmic size, and has infinite depth. It's like a growing tree ... life is like fruit growing on the end of the brances ...

... The secrets of the Kabbala, the Norse and Chinese myths, the Tree of Knowledge in Eden, the Tree of Life, the World Tree ... these are all worthy of being called Amenomibashira, or "the Pillar of Heaven" ... It's the core system of the universe that channelers .. in every era, culture, and every race of people ... have traditionally accessed ... Beyond the trunk of the "tree" there should be no existence, but the closer one gets to the end of the branches, the more growth one finds ... and the branches are continually touching, separating, entangling, and bearing fruit ...

... The universe ... in accordance with the "time" at that moment, progresses ... always seeking greater stability, and greater existence, it continues growing in complexity and diversity, and sometimes it abandons things …".

Compare Douglas Adams's description of many Possible Worlds in the book "The Ultimate Hitchhiker's Guide" (Chapter 17 of Mostly Harmless) (Random House 1997):

"... there was now a bird .. hovering there ... the air was full of nothing but interlocking birds. ... It was as if the whole geometry of space was redefined in seamless bird shapes ... It spoke ... "Your universe is vast to you. Vast in time, vast in space. That's because of the filters through which you perceive it. But I was built with no filters at all, which means I perceive ... all possible universes ... To me the flow of time is irrelevant. ..." …”.

H4 + H4 = E8

H3 and H4 have very interesting geometry related to singularities. As Ian Porteous says in his book Geometric Differentiation for the Intelligence of Curves and Surfaces (Cambridge, 2nd ed 2001)

"... the groups Ak , Bk = Ck , Dk , E6 , E7 , E8 , F4 and G2 , had already turned up in singularity theory ... Apart from the groups of symmetries of regular plane polygons there are only two other Coxeter groups,

H3 , the full group of isometries of an icosahedron, and H4 , the full group of symmetries of an analogue of the icosahedron in R4 ...

the variety of non-regular orbits of H3 is A-equivalent to the full involute of a plane curve with an ordinary inflection. ...

Shcherbak proved ... that the full involute of the surface e of Therem 14.14 is A-equivalent to the variety of non-regular orbits of the group H4 ... the parameterization that he gives for the variety of nonregular orbits of the group H4 ...[is]...

(a,b,c) -> (a , a c + (1/2) b^2 , a b^3 + (1/2) c^2 , a b^3 c + (1/5) b^5 + (1/3) c^3)

a three-dimensional variety in R4 whose intersection with the hyperplane a = 0 is the two-dimensional veriety in R3 with the parameterization

(b,c) -> ((1/2) b^2 , (1/2) c^2 , (1/5) b^5 + (1/3) c^3)

...The H4 configuration is discussed ... in the Liverpool thesis of Alex Flegmann ...Figure... 14.9 ... taken from Flegmann's thesis ...[published in 1985 as "Evolutes Involutes and The Coxeter Group H4", ISSN 0755-3390, Universite Louis Pasteur, Department de Mathematique, l'Institut de Recherche Mathematique Advancee, Strasbourg]... illustrate[s] the sections of this variety in the case that a = 1 ... ".

252 *14 Involutes of geodesic foliations*

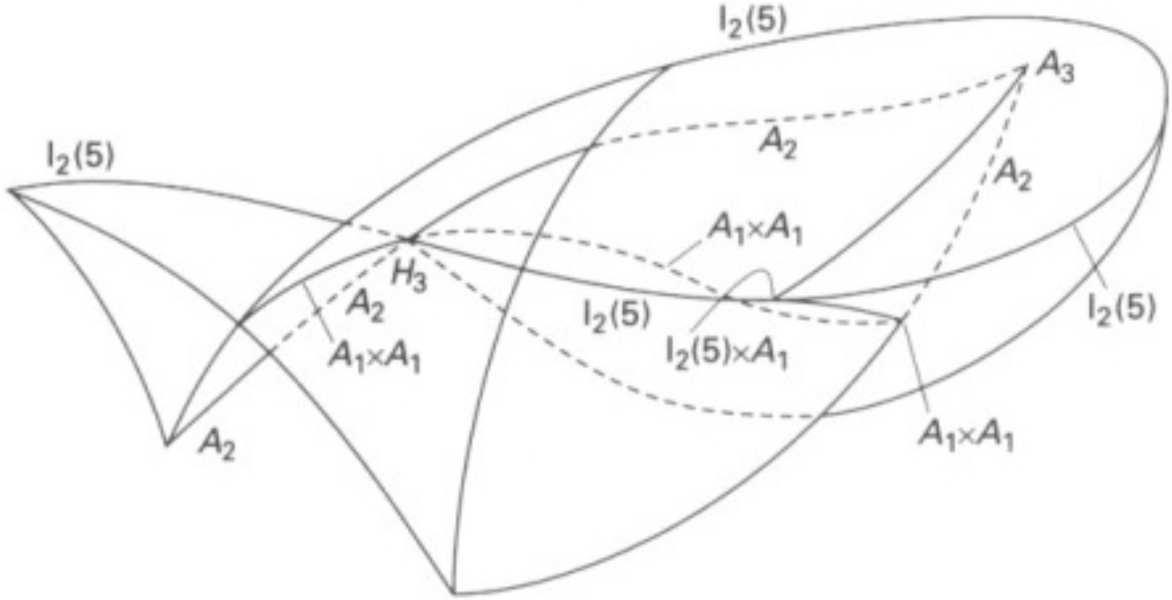


Figure 14.9

As Porteous also says, the E8 singularity is a simple singularity whose canonical form map-germ is x^3 + y^5.

Flegmann's thesis also described the groups E8 and H4 as having:

Order:
192 x 10! = 696,729,600 for E8 and 14,400 for H4

No. of Reflections in each Conjugacy Class:
120 for E8 and 60 for H4

Degrees of Basic Invariants:
2, 8, 12, 14, 18, 20, 24, 30 for E8 and 2, 12, 20, 30 for H4

Note that 2, 12, 20, 30 are in both E8 and H4
and
that 8, 18 are 6 greater than 2, 12
and
that 14, 24 are 6 less than 20, 30

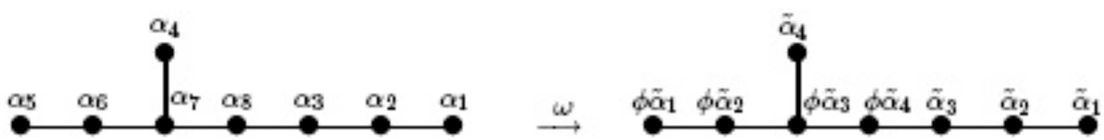
and
that
(1 + i sqrt(5)) (1 - i sqrt(5)) = 1+5 = 6 is related to 2 x Golden Ratio = (1 + sqrt(5))

Shcherbak worked with Arnol'd, and published
a 1983 paper "Singularities of a family of evolvents in the neighbourhood of a point of inflection of a curve, and the group H3 generated by reflections" (Funktsional'. Anal. i Prilozhen, 17:4, 70-2)
and
a 1984 paper "H4 in the problem of avoiding an obstacle" (Uspekhi Mat. Nauk. 39:5, 256).

Shcherbak's 1988 paper "Wavefronts and reflection groups" (Uspekhi Mat. Nauk. 43:3, 125-60) was posthumous, and was the basis for the papers by Fring and Korff at hep-th/0509152 and hep-th/0506226. John Baez, in his week 270 (11 October 2008), discussed H4 and E8 in the context of the later paper. In the earlier paper, entitled "Affine Toda field theories related to Coxeter groups of non-crystallographic type", Fring and Korff said:

"... E8 structure ...[exists]... in the form of a (minimal) E8-affine Toda field theory
...
there is an even more fundamental structure than E8 underlying this particular model,
the non-crystallographic Coxeter group H4.
We draw here on the observation made first by Sherbak in 1988 ...
namely that H4 can be embedded into E8 ...
Loosely speaking, one may regard the E8-theory as two copies of H4-theories.
We get a first glimpse of this structure from a more physical point of view when we bring the mass spectrum of minimal E8-affine Toda field theory ... into the form
m1 = 1
m2 = 2 cos(pi/30)
m3 = sqrt(sin(11pi/30) / sin(pi/30))
m4 = 2 PHI cos(7pi/30)
m5 = PHI m1
m6 = PHI m3
m7 = PHI m3
m8 = PHI m4
...

We observe here that there are four "fundamental" masses present in the theory, whereas the other ones can be obtained simply by a multiplication with the golden ratio
PHI = (1/2) (1 + sqrt(5)) = PHI^2 - 1 .
...
Note that higher powers of PHI can be reduced to that form ... A + PHI B with A,B in Q .. by a repeated use of ...
PHI^2 = 1 + PHI
PHI^3 = 1 + 2 PHI
PHI^4 = 2 + 3 PHI
PHI^5 = 3 + 5 PHI
...
PHI^n = f_(n-1) + PHI f_n
where f_n is then-th Fibonacci number obeying the recursive relation
f_(n+1) = f_n + f_(n-1)
...
It will turn out that each of the sets (m1,m2,m3,m4) and (m5,m6,m7,m8)
can be associated with an H4-[affine Toda field theory]
...[so that]...
H4 can be embedded into E8 ... such that the non-crystallographic structure is "visible" inside the theories related to crystallographic Coxeter groups.
...
[There is] a map w from a root system ... which is invariant under the action of a crystallographic Coxeter group ...[E8]... of rank ..[8]... into the union of two sets ... related to a non-crystallographic group ...[H4]... of rank ...[8/2 = 4]...
Introducing a ... labelling for the vertices on the Coxeter graphs, or equivalently the simple roots, we can always realize this map as ...[
aE8_i -> w(aE8_i) = aH4_i for i from 1 to 4
aE8_i -> w(aE8_i) = PHI aH4_(i-4) for i from 5 to 8
]...



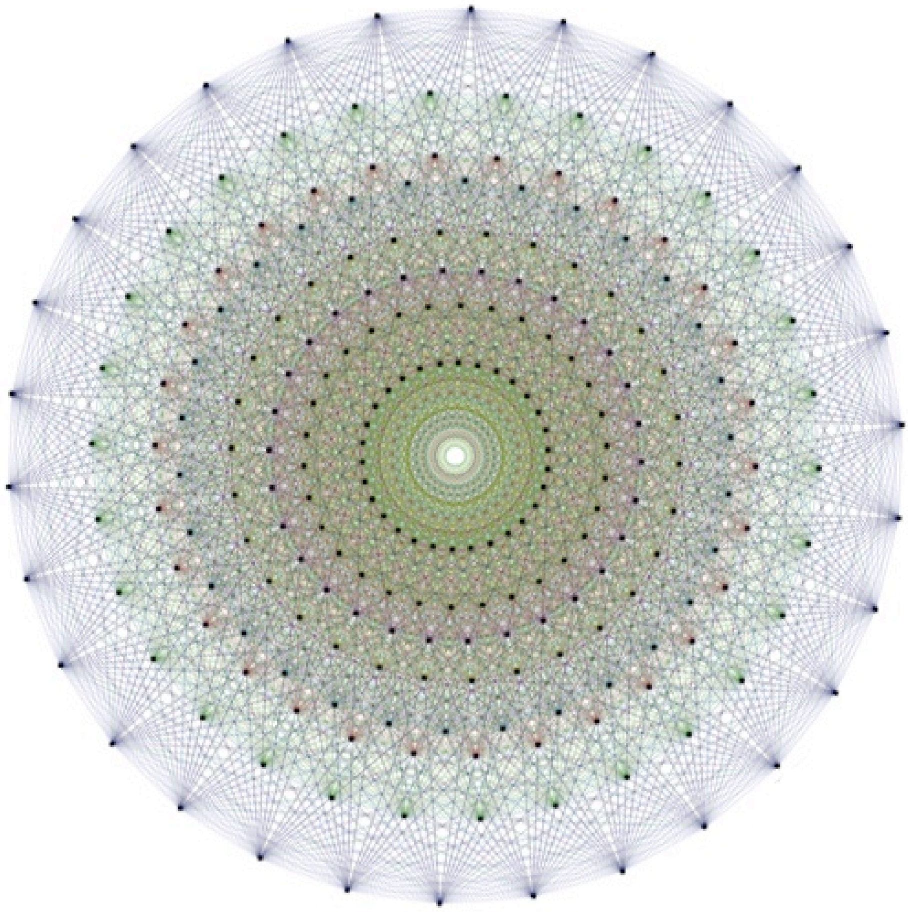
The corresponding Cartan matrix of E_8 together with its construction from inner products in $\tilde{\Delta}$ in agreement with (2.5) is

$$K = \begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & -1 & 0 & -1 & 2 & -1 \\ 0 & 0 & -1 & 0 & 0 & 0 & -1 & 2 \end{pmatrix} = R \begin{pmatrix} \tilde{K} & \phi\tilde{K} \\ \phi\tilde{K} & \phi^2\tilde{K} \end{pmatrix}. \tag{2.38}$$

The Cartan matrix of H_4 reads

$$\tilde{K} = \begin{pmatrix} 2 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 2 & -\phi \\ 0 & 0 & -\phi & 2 \end{pmatrix}. \tag{2.39}$$

...
the map w is an isometric isomorphism, such that we may compute inner products in the root system ...[E8 }... from inner products in ...[H4]...
A . B = R(w(A) . w(B))
Here the map R, called a rational form relative to PHI, extracts from a number of the form A + PHI B with A,B in Q the rational part of A
R(A + PHI B) = A
We normalize all our roots to have length 2 .. we may therefore compute the Cartan matrix related to ...[E8]... entirely from inner products in ...[H4]...
[and]... compute inner products in ...[H4]... from those in ...[E8]...
...
the entire root system ...[E8]... can be separated into ...[8]... orbits W_i ...[i from 1 to 8]... each containing ...[30]... roots ... [as shown in this image from a video by Garrett Lisi for FQXi07 on his web site deferentialgeometry.org



]... here ...[30]... is the order of ... the Coxeter number ...[of E8]...

The vertices ... of the Coxeter graph ... separate into two disjoint sets V+ and V-
...
The Coxeter numbers ... [for E8 and H4] are 30 and [with respect to the map w]
the set of exponents separate ... into
{ 1, 7, 11, 13, 17, 19, 23, 29 } = { 1, 11, 19, 29 } u { 7, 13, 17, 23 }

Note that 1, 11, 19, 29 are in both E8 and H4
and
that 7, 17 are 6 greater than 1, 11

and
that 13, 23 are 6 less than 19, 29
and
that
(1 + i sqrt(5)) (1 - i sqrt(5)) = 1+5 = 6 is related to 2 x Golden Ratio = (1 + sqrt(5)) ...".

E8 polytope has 240 vertices,
the sum of two H4 polytopes, each with 120 vertices, related by Golden Ratio,
so
E8 contains two copies of H4 related by Golden Ratio

In more detail:

248-dim E8 = 120-dim D8 + 128-dim half-spinorD8

240 E8 root vector vertices = 112 D8 root vector vertices + 128 half-spinor vertices

D8 contains two copies of D4
Each D4 has 24 root vector vertices forming a 24-cell

Golden Ratio points can be chosen (in two different ways) on each of the 96 edges of a 24-cell so that the 96 Golden Ratio points plus the 24 vertices form the 96 + 24 = 120 vertices of a 600-cell.

So, the two D4 of D8 give two 600-cells.
Each 600-cell acts as a 120-vertex symmetry polytope for a copy of H4.

In E8 physics, 8-dimensional spacetime is, at our low energies, reduced to
4-dimensional physical spacetime (denoted here 4)
plus
4-dimensional internal symmetry space (denoted here by 4*).

Each of the two D4 have 4-dimensional Cartan subalgebra spaces:
The Cartan space of one D4, denoted by D4, corresponds to 4-dim physical spacetime, denoted by 4. That D4 produces Gravity.
The Cartan space of the other D4, denoted by D4*, corresponds to 4-dim internal symmetry space, denoted by 4*. That D4 produces the Standard Model.

E8 = D8 + half-spinorD8 = (D4 + D4* + 64) + (64 + 64)

The 64 = 8x8 in (D4 + D4* + 64) represents 8 Dirac gamma components of the dimensions of 8-dim spacetime that is reduced to 4 + 4*.
E8 physics dimensional reduction of 8-dim spacetime to 4 + 4* also reduces the 8 Dirac gammas to 4 + 4*
so
that 64 = 8x8 = (4 + 4*) x (4 + 4*) = (4 + 4*) x 4 + (4 + 4*) x 4* = 32 + 32*
so that the 32 corresponds to the D4 of physical spacetime
and the 32* corresponds to the D4* of internal symmetry space.

Note that the 32 of D4 has some connection to internal symmetry space, as might be expected from the detailed structure of the SU(3) sector of Batakis M4xCP2 Kaluza-Klein theory, in that the 32 includes the 4 physical spacetime Dirac gamma components of the four dimensions of 4* internal symmetry space.

Each of the 64 = 8x8 in (64 + 64) represent 8 Dirac gamma components of either 8 fermion particles or 8 fermion antiparticles,
so
each of those 64 = 8x8 = 8x(4+4*) = 32 + 32*
where the 32 represents components with respect to 4-dim physical spacetime
and the 32* represents components with respect to 4-dim internal symmetry space
and
E8 = D8 + half-spinorD8 = (D4 + D4* + 32 + 32*) + (32 + 32* + 32 + 32*)

E8 = [(D4 + 32) + (32 + 32)] + [(D4* + 32*) + (32* + 32*)]

E8 = [24 + 96] + [24* + 96*] = 120 + 120* = H4 + H4*

where
H4 is a copy of H4 that corresponds to D4, Gravity, and 4-dim physical spacetime
and
H4* is a copy of H4 that corresponds to D4*, the Standard Model, and 4-dim internal symmetry space.

The following 4 pages from Appendix B of hep-th/0506226 by Fring and Korff show "… explicit computations of various orbits of … roots related to non-crystallographic and crystallographic Coxeter groups and exhibit how they can be embedded into one another …":

B. The orbits of H_4 and E_8

Successive action of $\sigma = \sigma_1\sigma_5\sigma_3\sigma_7\sigma_2\sigma_6\sigma_4\sigma_8$ and $\tilde{\sigma} = \tilde{\sigma}_1\tilde{\sigma}_3\tilde{\sigma}_2\tilde{\sigma}_4$ yields

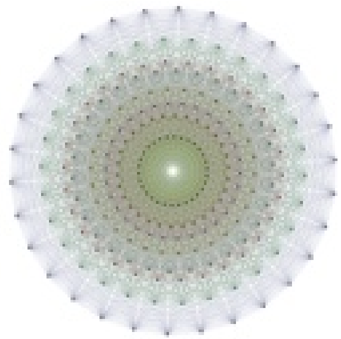
	Ω_1	$\omega(\Omega_1) = \tilde{\Omega}_1$	Ω_5
σ^0	α_1	$\tilde{\alpha}_1$	α_5
σ^1	$\alpha_2 + \alpha_3$	$\tilde{\alpha}_2 + \tilde{\alpha}_3$	$\alpha_6 + \alpha_7$
σ^2	$\alpha_7 + \alpha_8$	$\phi(\tilde{\alpha}_3 + \tilde{\alpha}_4)$	$\alpha_3 + \alpha_4 + \alpha_7 + \alpha_8$
σ^3	$\alpha_4 + \alpha_5 + \alpha_7 + \alpha_8$	$\phi(\tilde{\alpha}_1 + \tilde{\alpha}_2 + \tilde{\alpha}_3) + \tilde{\alpha}_4$	$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_5$ $+\alpha_6 + \alpha_7 + \alpha_8$
σ^4	$\alpha_3 + \alpha_6 + \alpha_7 + \alpha_8$	$\phi\tilde{\alpha}_2 + \phi^2\tilde{\alpha}_3 + \phi\tilde{\alpha}_4$	$\alpha_2 + \alpha_3 + \alpha_4 + \alpha_6$ $+2\alpha_7 + \alpha_8$
σ^5	$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4 + \alpha_7 + \alpha_8$	$\tilde{\alpha}_1 + \tilde{\alpha}_2 + \phi^2(\tilde{\alpha}_3 + \tilde{\alpha}_4)$	$\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6$ $+2\alpha_7 + \alpha_8$
σ^6	$\alpha_2 + \alpha_3 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8$	$\phi(\tilde{\alpha}_1 + \tilde{\alpha}_4) + \phi^2(\tilde{\alpha}_2 + \tilde{\alpha}_3)$	$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$ $+\alpha_5 + 2\alpha_6 + 2\alpha_7 + \alpha_8$
σ^7	$\alpha_4 + \alpha_6 + 2\alpha_7 + \alpha_8$	$\phi\tilde{\alpha}_2 + 2\phi\tilde{\alpha}_3 + \phi^2\tilde{\alpha}_4$	$\alpha_2 + 2\alpha_3 + \alpha_4 + \alpha_5$ $+\alpha_6 + 2\alpha_7 + 2\alpha_8$
σ^8	$\alpha_3 + \alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8$	$\phi(\tilde{\alpha}_1 + \tilde{\alpha}_2) + \phi^2(\tilde{\alpha}_3 + \tilde{\alpha}_4)$	$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$ $+\alpha_5 + \alpha_6 + 2\alpha_7 + 2\alpha_8$
σ^9	$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_6 + \alpha_7 + \alpha_8$	$\tilde{\alpha}_1 + \phi\tilde{\alpha}_4 + \phi^2(\tilde{\alpha}_2 + \tilde{\alpha}_3)$	$\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$ $+2\alpha_6 + 2\alpha_7 + \alpha_8$
σ^{10}	$\alpha_2 + \alpha_3 + \alpha_4 + \alpha_6 + \alpha_7 + \alpha_8$	$\tilde{\alpha}_2 + \phi^2(\tilde{\alpha}_3 + \tilde{\alpha}_4)$	$\alpha_3 + \alpha_4 + \alpha_6 + 2\alpha_7$ $+2\alpha_8$
σ^{11}	$\alpha_5 + \alpha_6 + \alpha_7 + \alpha_8$	$\phi(\tilde{\alpha}_1 + \tilde{\alpha}_2 + \tilde{\alpha}_3 + \tilde{\alpha}_4)$	$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_4$ $+\alpha_5 + \alpha_6 + \alpha_7 + \alpha_8$
σ^{12}	$\alpha_4 + \alpha_6 + \alpha_7$	$\phi(\tilde{\alpha}_2 + \tilde{\alpha}_3) + \tilde{\alpha}_4$	$\alpha_2 + \alpha_3 + \alpha_6 + \alpha_7$ $+\alpha_8$
σ^{13}	$\alpha_3 + \alpha_8$	$\tilde{\alpha}_3 + \phi\tilde{\alpha}_4$	$\alpha_4 + \alpha_7 + \alpha_8$
σ^{14}	$\alpha_1 + \alpha_2$	$\tilde{\alpha}_1 + \tilde{\alpha}_2$	$\alpha_5 + \alpha_6$
σ^{15}	$-\alpha_1$	$-\tilde{\alpha}_1$	$-\alpha_5$

	Ω_2	$\omega(\Omega_2) = \tilde{\Omega}_2$	Ω_6
σ^0	$-\alpha_2$	$\tilde{\alpha}_2$	$-\alpha_6$
σ^1	$\alpha_1 + \alpha_2 + \alpha_3$	$\tilde{\alpha}_1 + \tilde{\alpha}_2 + \tilde{\alpha}_3$	$\alpha_5 + \alpha_6 + \alpha_7$
σ^2	$\alpha_2 + \alpha_3 + \alpha_7 + \alpha_8$	$\tilde{\alpha}_2 + \phi^2 \tilde{\alpha}_3 + \phi\tilde{\alpha}_4$	$\alpha_3 + \alpha_4 + \alpha_6 + 2\,\alpha_7 + \alpha_8$
σ^3	$\alpha_4 + \alpha_5 + \alpha_6 + 2\,\alpha_7 + \alpha_8$	$\phi(\tilde{\alpha}_1 + \tilde{\alpha}_2 + 2\tilde{\alpha}_3 + \phi\tilde{\alpha}_4)$	$\alpha_1 + \alpha_2 + 2\,\alpha_3 + \alpha_4$ $+\alpha_5 + \alpha_6 + 2\,\alpha_7 + 2\,\alpha_8$
σ^4	$\alpha_3 + \alpha_4 + \alpha_5$ $+2(\alpha_6 + \alpha_7) + \alpha_8$	$\phi(\tilde{\alpha}_1 + 2\,\tilde{\alpha}_2) + \phi^3\tilde{\alpha}_3 + \phi^2\tilde{\alpha}_4$	$\alpha_1 + 2\,\alpha_2 + 2\,\alpha_3 + \alpha_4$ $+\alpha_5 + 2\,\alpha_6 + 3\,\alpha_7 + 2\,\alpha_8$
σ^5	$\alpha_1 + \alpha_2 + \alpha_4 + \alpha_6$ $+2(\alpha_3 + \alpha_7 + \alpha_8)$	$\tilde{\alpha}_1 + \phi^2(\tilde{\alpha}_2 + 2\tilde{\alpha}_3) + \phi^3\tilde{\alpha}_4$	$\alpha_2 + 2\,\alpha_3 + 2\,\alpha_4 + \alpha_5$ $+2\,\alpha_6 + 4\,\alpha_7 + 3\,\alpha_8$
σ^6	$\alpha_1 + \alpha_4 + \alpha_5 + \alpha_6$ $+2(\alpha_2 + \alpha_3 + \alpha_7 + \alpha_8)$	$\phi^2(\tilde{\alpha}_1 + 2\tilde{\alpha}_3 + \phi\tilde{\alpha}_4) + (2+\phi)\tilde{\alpha}_2$	$\alpha_1 + \alpha_2 + 2\alpha_3 + 2\alpha_4$ $+2\alpha_5 + 3\alpha_6 + 4\alpha_7 + 3\alpha_8$
σ^7	$\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$ $+3\,\alpha_7 + 2(\alpha_6 + \alpha_8)$	$\phi\,\tilde{\alpha}_1 + (\phi^4 - 1)\tilde{\alpha}_3 + \phi^3(\tilde{\alpha}_2 + \tilde{\alpha}_4)$	$\alpha_1 + 2\,\alpha_2 + 3\,\alpha_3 + 2\,\alpha_4$ $+\alpha_5 + 3\,\alpha_6 + 4\,\alpha_7 + 3\,\alpha_8$
σ^8	$\alpha_3 + \alpha_5 + 3\,\alpha_7$ $+2(\alpha_4 + \alpha_6 + \alpha_8)$	$\phi\,(\tilde{\alpha}_1 + \tilde{\alpha}_2 + 2\phi\tilde{\alpha}_4) + (\phi^4 - 1)\,\tilde{\alpha}_3$	$\alpha_1 + 2\,\alpha_2 + 3\,\alpha_3 + 2\,\alpha_4$ $+\alpha_5 + 2\,\alpha_6 + 4\,\alpha_7 + 4\,\alpha_8$
σ^9	$\alpha_1 + \alpha_2 + \alpha_4 + \alpha_5$ $+2(\alpha_3 + \alpha_6 + \alpha_7 + \alpha_8)$	$\phi^2\,(\tilde{\alpha}_1 + 2\tilde{\alpha}_3) + \phi^3\,(\tilde{\alpha}_2 + \tilde{\alpha}_4)$	$\alpha_1 + 2\,\alpha_2 + 2\,\alpha_3 + 2\,\alpha_4$ $+2\,\alpha_5 + 3\,\alpha_6 + 4\,\alpha_7 + 3\,\alpha_8$
σ^{10}	$\alpha_1 + 2(\alpha_2 + \alpha_3) + \alpha_4$ $+\alpha_6 + 2(\alpha_7 + \alpha_8)$	$\tilde{\alpha}_1 + (2 + \phi)\,\tilde{\alpha}_2 + \phi^2(2\tilde{\alpha}_3 + \phi\tilde{\alpha}_4)$	$\alpha_2 + 2\,\alpha_3 + 2\,\alpha_4 + \alpha_5$ $+3\,\alpha_6 + 4\,\alpha_7 + 3\,\alpha_8$
σ^{11}	$\alpha_2 + \alpha_3 + \alpha_4 + \alpha_5$ $+\alpha_6 + 2\,\alpha_7 + 2\,\alpha_8$	$\phi\,\tilde{\alpha}_1 + \phi^2\,\tilde{\alpha}_2 + \phi^3\,(\tilde{\alpha}_3 + \tilde{\alpha}_4)$	$\alpha_1 + \alpha_2 + 2\,\alpha_3 + 2\,\alpha_4$ $+\alpha_5 + 2\,\alpha_6 + 3\,\alpha_7 + 3\,\alpha_8$
σ^{12}	$\alpha_4 + \alpha_5 + 2(\alpha_6 + \alpha_7) + \alpha_8$	$\phi\tilde{\alpha}_1 + 2\,\phi\,(\tilde{\alpha}_2 + \tilde{\alpha}_3) + \phi^2\tilde{\alpha}_4$	$\alpha_1 + 2\,\alpha_2 + 2\,\alpha_3 + \alpha_4$ $+\alpha_5 + 2\,\alpha_6 + 2\,\alpha_7 + 2\,\alpha_8$
σ^{13}	$\alpha_3 + \alpha_4 + \alpha_6 + \alpha_7 + \alpha_8$	$\phi\tilde{\alpha}_2 + \phi^2\,(\tilde{\alpha}_3 + \tilde{\alpha}_4)$	$\alpha_2 + \alpha_3 + \alpha_4 + \alpha_6 + 2(\alpha_7 + \alpha_8)$
σ^{14}	$\alpha_1 + \alpha_2 + \alpha_3 + \alpha_8$	$\tilde{\alpha}_1 + \tilde{\alpha}_2 + \tilde{\alpha}_3 + \phi\tilde{\alpha}_4$	$\alpha_4 + \alpha_5 + \alpha_6 + \alpha_7 + \alpha_8$
σ^{15}	α_2	$\tilde{\alpha}_2$	α_6

	Ω_3	$\omega(\Omega_3)=\tilde{\Omega}_3$	Ω_7
σ^0	α_3	$\tilde{\alpha}_3$	α_7
σ^1	$\alpha_1+\alpha_2+\alpha_3+\alpha_7+\alpha_8$	$\tilde{\alpha}_1+\tilde{\alpha}_2+\phi^2\tilde{\alpha}_3+\phi\tilde{\alpha}_4$	$\alpha_3+\alpha_4+\alpha_5$ $+\alpha_6+2\,\alpha_7+\alpha_8$
σ^2	$\alpha_2+\alpha_3+\alpha_4+\alpha_5$ $+\alpha_6+2\,\alpha_7+\alpha_8$	$\phi\tilde{\alpha}_1+\phi^2\tilde{\alpha}_2+\phi^3\tilde{\alpha}_3+\phi^2\tilde{\alpha}_4$	$\alpha_1+\alpha_2+\alpha_4+\alpha_5$ $+3\,\alpha_7+2(\alpha_3+\alpha_6+\alpha_8)$
σ^3	$\alpha_3+\alpha_4+\alpha_5$ $+3\,\alpha_7+2(\alpha_6+\alpha_8)$	$\phi(\tilde{\alpha}_1+2\tilde{\alpha}_2)+(\phi^4-1)\tilde{\alpha}_3+\phi^3\tilde{\alpha}_4$	$\alpha_1+2(\alpha_2+\alpha_4+\alpha_6)$ $+\alpha_5+4\alpha_7+3(\alpha_3+\alpha_8)$
σ^4	$\alpha_1+\alpha_2+2(\alpha_3+\alpha_4)$ $+\alpha_5+3\,\alpha_7+2(\alpha_6+\alpha_8)$	$\phi^2(\tilde{\alpha}_1+\phi\tilde{\alpha}_2+\phi^2\tilde{\alpha}_3+2\tilde{\alpha}_4)$	$\alpha_1+2(\alpha_2+\alpha_4+\alpha_5)$ $+3(\alpha_3+\alpha_6)+5\,\alpha_7+4\,\alpha_8$
σ^5	$\alpha_1+\alpha_4+2(\alpha_2+\alpha_6)$ $+\alpha_5+3(\alpha_3+\alpha_7+\alpha_8)$	$\phi^2(\tilde{\alpha}_1+2\,\tilde{\alpha}_2+3\,\tilde{\alpha}_3)$ $+(\phi^4-1)\,\tilde{\alpha}_4$	$\alpha_1+2\,\alpha_2+3(\alpha_3+\alpha_4)$ $+2\,\alpha_5+6\,\alpha_7+4(\alpha_6+\alpha_8)$
σ^6	$2(\alpha_2+\alpha_3+\alpha_4+\alpha_6)$ $+\alpha_1+\alpha_5+4\,\alpha_7+3\,\alpha_8$	$\phi^2(\tilde{\alpha}_1+2\tilde{\alpha}_2+2\phi\tilde{\alpha}_3+\phi^2\tilde{\alpha}_4)$	$\alpha_1+2(\alpha_2+\alpha_5)+3\,\alpha_4$ $+4(\alpha_3+\alpha_6)+6\,\alpha_7+5\,\alpha_8$
σ^7	$\alpha_2+2(\alpha_3+\alpha_4+\alpha_5)$ $+4\,\alpha_7+3(\alpha_6+\alpha_8)$	$\phi(\tilde{\alpha}_1+2\phi^2\tilde{\alpha}_3+\phi^3\tilde{\alpha}_4)$ $+(\phi^4-1)\,\tilde{\alpha}_2$	$2(\alpha_1+\alpha_5)+3(\alpha_2+\alpha_4)$ $+4(\alpha_3+\alpha_6)+6\,\alpha_7+5\,\alpha_8$
σ^8	$\alpha_1+\alpha_2+2(\alpha_3+\alpha_4)$ $+\alpha_5+4\,\alpha_7+3(\alpha_6+\alpha_8)$	$\phi^2(\tilde{\alpha}_1+2\phi\tilde{\alpha}_3+\phi^2\tilde{\alpha}_4)$ $+(\phi^4-1)\,\tilde{\alpha}_2$	$\alpha_1+3(\alpha_2+\alpha_4)+2\,\alpha_5$ $+4(\alpha_3+\alpha_6)+6\,\alpha_7+5\,\alpha_8$
σ^9	$\alpha_1+2(\alpha_2+\alpha_4+\alpha_6)$ $+\alpha_5+3(\alpha_3+\alpha_7+\alpha_8)$	$\phi^2\,(\tilde{\alpha}_1+2\,\tilde{\alpha}_2+3\,\tilde{\alpha}_3)+\phi^4\tilde{\alpha}_4$	$\alpha_1+2(\alpha_2+\alpha_5)+4\,\alpha_6$ $+3(\alpha_3+\alpha_4)+6\,\alpha_7+5\,\alpha_8$
σ^{10}	$\alpha_1+2(\alpha_2+\alpha_3+\alpha_6)$ $+\alpha_4+\alpha_5+3(\alpha_7+\alpha_8)$	$\phi^2(\tilde{\alpha}_1+2\tilde{\alpha}_2+\phi^2\tilde{\alpha}_3)$ $+(\phi^4-1)\,\tilde{\alpha}_4$	$\alpha_1+2\,(\alpha_2+\alpha_5)+4\,\alpha_6$ $+3(\alpha_3+\alpha_4)+5\,\alpha_7+4\,\alpha_8$
σ^{11}	$\alpha_2+\alpha_3+\alpha_5+3\alpha_7$ $+2(\alpha_4+\alpha_6+\alpha_8)$	$\phi\,\tilde{\alpha}_1+\phi^3\tilde{\alpha}_2+(\phi^4-1)\tilde{\alpha}_3+2\phi^2\tilde{\alpha}_4$	$\alpha_1+2(\alpha_2+\alpha_4)+\alpha_5$ $+3(\alpha_3+\alpha_6)+4(\alpha_7+\alpha_8)$
σ^{12}	$\alpha_3+\alpha_4+\alpha_5$ $+2(\alpha_6+\alpha_7+\alpha_8)$	$\phi\tilde{\alpha}_1+2\phi\tilde{\alpha}_2+\phi^3\,(\tilde{\alpha}_3+\tilde{\alpha}_4)$	$\alpha_1+2(\alpha_2+\alpha_3+\alpha_4)$ $+\alpha_5+2\,\alpha_6+3(\alpha_7+\alpha_8)$
σ^{13}	$\alpha_1+\alpha_2+\alpha_3+\alpha_4$ $+\alpha_6+\alpha_7+\alpha_8$	$\tilde{\alpha}_1+\phi^2\,(\tilde{\alpha}_2+\tilde{\alpha}_3+\tilde{\alpha}_4)$	$\alpha_2+\alpha_3+\alpha_4+\alpha_5$ $+2(\alpha_6+\alpha_7+\alpha_8)$
σ^{14}	$\alpha_2+\alpha_3+\alpha_8$	$\tilde{\alpha}_2+\tilde{\alpha}_3+\phi\tilde{\alpha}_4$	$\alpha_4+\alpha_6+\alpha_7+\alpha_8$
σ^{15}	$-\alpha_3$	$-\tilde{\alpha}_3$	$-\alpha_7$

	Ω_4	$\omega(\Omega_4)=\tilde{\Omega}_4$	Ω_8
σ^0	$-\alpha_4$	$-\tilde{\alpha}_4$	$-\alpha_8$
σ^1	$\alpha_4+\alpha_7$	$\phi\tilde{\alpha}_3+\tilde{\alpha}_4$	$\alpha_3+\alpha_7+\alpha_8$
σ^2	$\alpha_3+\alpha_5+\alpha_6+\alpha_7+\alpha_8$	$\phi(\tilde{\alpha}_1+\tilde{\alpha}_2+\phi\tilde{\alpha}_3+\tilde{\alpha}_4)$	$\alpha_1+\alpha_2+\alpha_3+\alpha_4$ $+\alpha_5+\alpha_6+2\,\alpha_7+\alpha_8$
σ^3	$\alpha_1+\alpha_2+\alpha_3+\alpha_4$ $+\alpha_6+2\,\alpha_7+\alpha_8$	$\tilde{\alpha}_1+\phi^2(\tilde{\alpha}_2+\tilde{\alpha}_4)+\phi^3\tilde{\alpha}_3$	$\alpha_2+\alpha_4+\alpha_5+3\,\alpha_7$ $+2(\alpha_3+\alpha_6+\alpha_8)$
σ^4	$\alpha_2+\alpha_4+\alpha_5+\alpha_6$ $+2(\alpha_3+\alpha_7+\alpha_8)$	$\phi\tilde{\alpha}_1+\phi^2\,(\tilde{\alpha}_2+2\,\tilde{\alpha}_3+\phi\tilde{\alpha}_4)$	$\alpha_1+\alpha_2+2(\alpha_3+\alpha_4+\alpha_6)$ $+\alpha_5+4\,\alpha_7+3\,\alpha_8$
σ^5	$\alpha_1+\alpha_2+\alpha_3+\alpha_4$ $+\alpha_5+3\alpha_7+2(\alpha_6+\alpha_8)$	$\phi^2(\tilde{\alpha}_1+\phi\,\tilde{\alpha}_2+\phi\tilde{\alpha}_4)$ $+(\phi^4-1)\tilde{\alpha}_3$	$\alpha_1+2(\alpha_2+\alpha_4+\alpha_5)$ $+3(\alpha_3+\alpha_6+\alpha_8)+4\,\alpha_7$
σ^6	$\alpha_2+2(\alpha_3+\alpha_4)+\alpha_5$ $+3\,\alpha_7+2(\alpha_6+\alpha_8)$	$\phi(\tilde{\alpha}_1+\phi^2\tilde{\alpha}_2+\phi^3\tilde{\alpha}_3+\phi\tilde{\alpha}_4)$	$\alpha_1+2(\alpha_2+\alpha_4)+\alpha_5$ $+3(\alpha_3+\alpha_6)+5\,\alpha_7+4\,\alpha_8$
σ^7	$\alpha_1+\alpha_2+2(\alpha_3+\alpha_6)$ $+\alpha_4+\alpha_5+3(\alpha_7+\alpha_8)$	$\phi^2(\tilde{\alpha}_1+\phi\tilde{\alpha}_2+\phi^2\tilde{\alpha}_3)$ $+(\phi^4-1)\tilde{\alpha}_4$	$\alpha_1+2(\alpha_2+\alpha_5)+5\alpha_7$ $+3(\alpha_3+\alpha_4+\alpha_6)+4\,\alpha_8$
σ^8	$\alpha_1+2(\alpha_2+\alpha_3+\alpha_4)$ $+\alpha_5+2(\alpha_6+\alpha_8)+3\alpha_7$	$\phi^2(\tilde{\alpha}_1+2\tilde{\alpha}_2+\phi^2\tilde{\alpha}_3+2\tilde{\alpha}_4)$	$\alpha_1+2(\alpha_2+\alpha_4+\alpha_5)$ $+3\,\alpha_3+5\,\alpha_7+4(\alpha_6+\alpha_8)$
σ^9	$\alpha_2+\alpha_4+\alpha_5$ $+2(\alpha_6+\alpha_3)+3(\alpha_7+\alpha_8)$	$\phi\tilde{\alpha}_1+\phi^3\tilde{\alpha}_2+\phi^4\tilde{\alpha}_3$ $+(\phi^4-1)\tilde{\alpha}_4$	$\alpha_1+2\,\alpha_2+\alpha_5+5\alpha_7$ $+3(\alpha_3+\alpha_4+\alpha_6)+4\,\alpha_8$
σ^{10}	$\alpha_1+\alpha_2+\alpha_3+\alpha_5$ $+2(\alpha_4+\alpha_6+\alpha_8)+3\alpha_7$	$\phi^2(\tilde{\alpha}_1+2\tilde{\alpha}_4)+\phi^3\tilde{\alpha}_2$ $+(\phi^4-1)\,\tilde{\alpha}_3$	$\alpha_1+2(\alpha_2+\alpha_4+\alpha_5)$ $+3(\alpha_3+\alpha_6)+4(\alpha_7+\alpha_8)$
σ^{11}	$\alpha_2+\alpha_4+\alpha_5$ $+2(\alpha_3+\alpha_6+\alpha_7+\alpha_8)$	$\phi\,\tilde{\alpha}_1+\phi^3(\tilde{\alpha}_2+\tilde{\alpha}_4)$ $+2\phi^2\,\tilde{\alpha}_3$	$\alpha_1+2(\alpha_2+\alpha_3+\alpha_4)$ $+\alpha_5+4\,\alpha_7+3(\alpha_6+\alpha_8)$
σ^{12}	$\alpha_1+\alpha_2+\alpha_3+\alpha_4$ $+\alpha_6+2(\alpha_7+\alpha_8)$	$\tilde{\alpha}_1+\phi^2\,\tilde{\alpha}_2+\phi^3\,(\tilde{\alpha}_3+\tilde{\alpha}_4)$	$\alpha_2+2(\alpha_3+\alpha_4+\alpha_6)$ $+\alpha_5+3(\alpha_7+\alpha_8)$
σ^{13}	$\alpha_2+\alpha_3+\alpha_4+\alpha_5$ $+\alpha_6+\alpha_7+\alpha_8$	$\phi\,\tilde{\alpha}_1+\phi^2\,(\tilde{\alpha}_2+\tilde{\alpha}_3+\tilde{\alpha}_4)$	$\alpha_1+\alpha_2+\alpha_3+\alpha_4$ $+\alpha_5+2(\alpha_6+\alpha_7+\alpha_8)$
σ^{14}	$\alpha_6+\alpha_7+\alpha_8$	$\phi(\tilde{\alpha}_2+\tilde{\alpha}_3+\tilde{\alpha}_4)$	$\alpha_2+\alpha_3+\alpha_4+\alpha_6+\alpha_7+\alpha_8$
σ^{15}	α_4	$\tilde{\alpha}_4$	α_8

The projection shown above from Garrett Lisi's FQXi07 video



of the 240 E8 root vectors from 8-dim to 2-dim with the form of 8 circles of 30 vertices each is described by H. S. M. Coxeter, in section 3.8 of his paper "Regular and Semi-Regular Polytopes III"(Math. Zeit. 200 (1988) 3-45, reprinted in "Kaleidoscopes: Selected Writings of H. S. M. Coxeter" (Wiley 1995)):
"…Du Val ... discovered ... ten-dimensional coordinates ...[u_1 ... u_10]... for the ... lattice 5_21 ... In fact, the vertices of a 5_21 of edge 5 sqrt(2) are all the points in Euclidean 10-space whose coordinates satisfy
the equations
x_1 + x_2 + x_3 + x_4 + x_5 = x_6 + x_7 + x_8 + x_9 + x_10 = 0
and the congruences
x_1 = x_2 = x_3 = x_4 = x_5 = 2 x_6 = 2 x_7 = 2 x_8 = 2 x_9 = 2 x_10 (mod 5)

In this lattice, the points at distance 5 sqrt(2) from the origin are, of course, the 240 vertices of a 4_21 . … In the accompanying table …[of]… new coordinates
u_v = (x_v + t x_(v+5))/sqrt(5) …[for]… v = 1,2,3,4,5
u_v = (t x_(v-5) – x_v)/sqrt(5) …[for]… v = 6,7,8,9,10
where t = (1/2)(sqrt(5) + 1) …

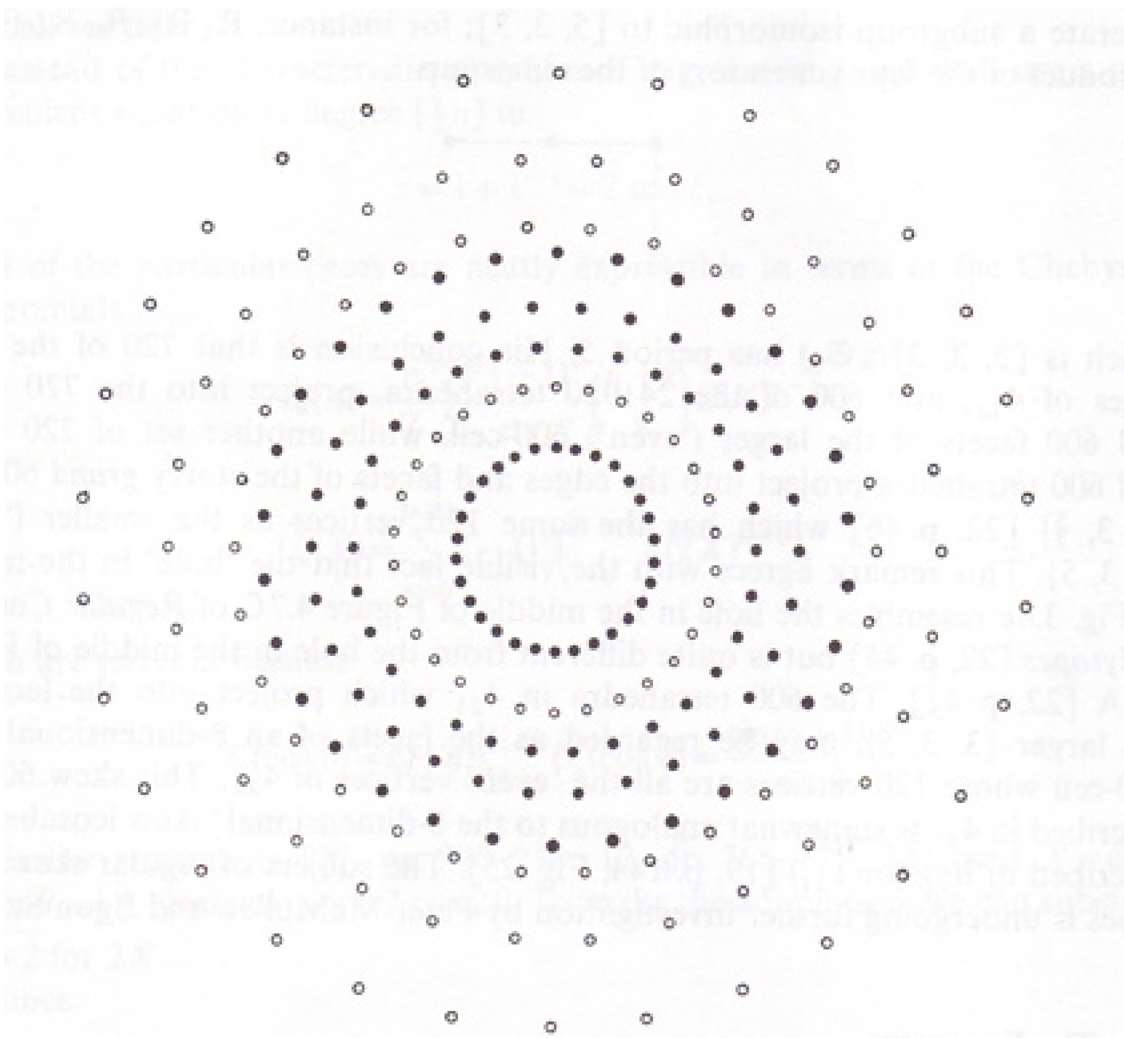
$$
\begin{array}{l}
(\sqrt{5}, 0, 0, 0, -\sqrt{5}; \tau\sqrt{5}, 0, 0, 0, -\tau\sqrt{5}) \\
(2\tau, 1, 1, -\tau^2, -\tau^2; 2, -\tau, -\tau, \tau^{-1}, \tau^{-1}) \\
(-\tau^{-2}, -\tau^2, 1, 1, 1; \tau^3, \tau^{-1}, -\tau, -\tau, -\tau) \\
(\tau^3, \tau^{-1}, -\tau, -\tau, -\tau; \tau^{-2}, \tau^2, -1, -1, -1) \\
(\tau^{-1}, \tau^{-1}, -\tau, -\tau, 2; \tau^2, \tau^2, -1, -1, -2\tau) \\
(\tau, \tau, \tau, -\tau^{-1}, -\tau^3; 1, 1, 1, -\tau^2, -\tau^{-2}) \\
(-2, \tau, \tau, -\tau^{-1}, -\tau^{-1}; 2\tau, 1, 1, -\tau^2, -\tau^2) \\
(\tau^2, \tau^2, -1, -1, -2\tau; -\tau^{-1}, -\tau^{-1}, \tau, \tau, -2) \\
(-1, -1, -1, \tau^2, \tau^{-2}; \tau, \tau, \tau, -\tau^{-1}, -\tau^3) \\
(\tau\sqrt{5}, 0, 0, 0, -\tau\sqrt{5}; -\sqrt{5}, 0, 0, 0, \sqrt{5})
\end{array}
$$

... By picking out alternate rows of the right-hand column of the table, we distinguish two sets of 120 vertices of 4_{21} ...

one set satisfying $u_1^2 + \dots + u_5^2 = 10$ [and] $u_6^2 + \dots u_{10}^2 = 10 t^2$

and the other satisfying $u_1^2 + \dots + u_5^2 = 10 t^2$ [and] $u_6^2 + \dots u_{10}^2 = 10$

Let us call these 'odd' and 'even' vertices, respectively. In Fig. 3.8 d …



… they appear as black and white dots. ... When we project
onto the 5-space $u_6 = \dots = u_{10} = 0$ by ignoring the last five coordinates,
we obtain the 120 + 120 vertices of two homothetic 600-cells ...
one having the coordinates …[of]… the other … multiplied by t

...
These 240 points are the vertices of the 8-dimensional uniform polytope 4_21
…[they]… represent the 240 lattice points at distance 2 from the origin:
the 16 permutations of (+/-2,0,0,0,0,0,0,0)
and the 112 + 112 cyclic permutations of (the last 7 coordinates in)

(+/-1 ; 0,0,0,+/-1,+/-1,0,+/-1)   (0 ; +/-1,+/-1,+/-1,0,0,+/-1,0)

..." using octonionic basis { 1, i, j, k, e, ie, je, ke }.

Notice that the structure can be seen as
16 + 112 + 112 = 128 + 112 = half-spinor D8 + adjoint root vectors D8.

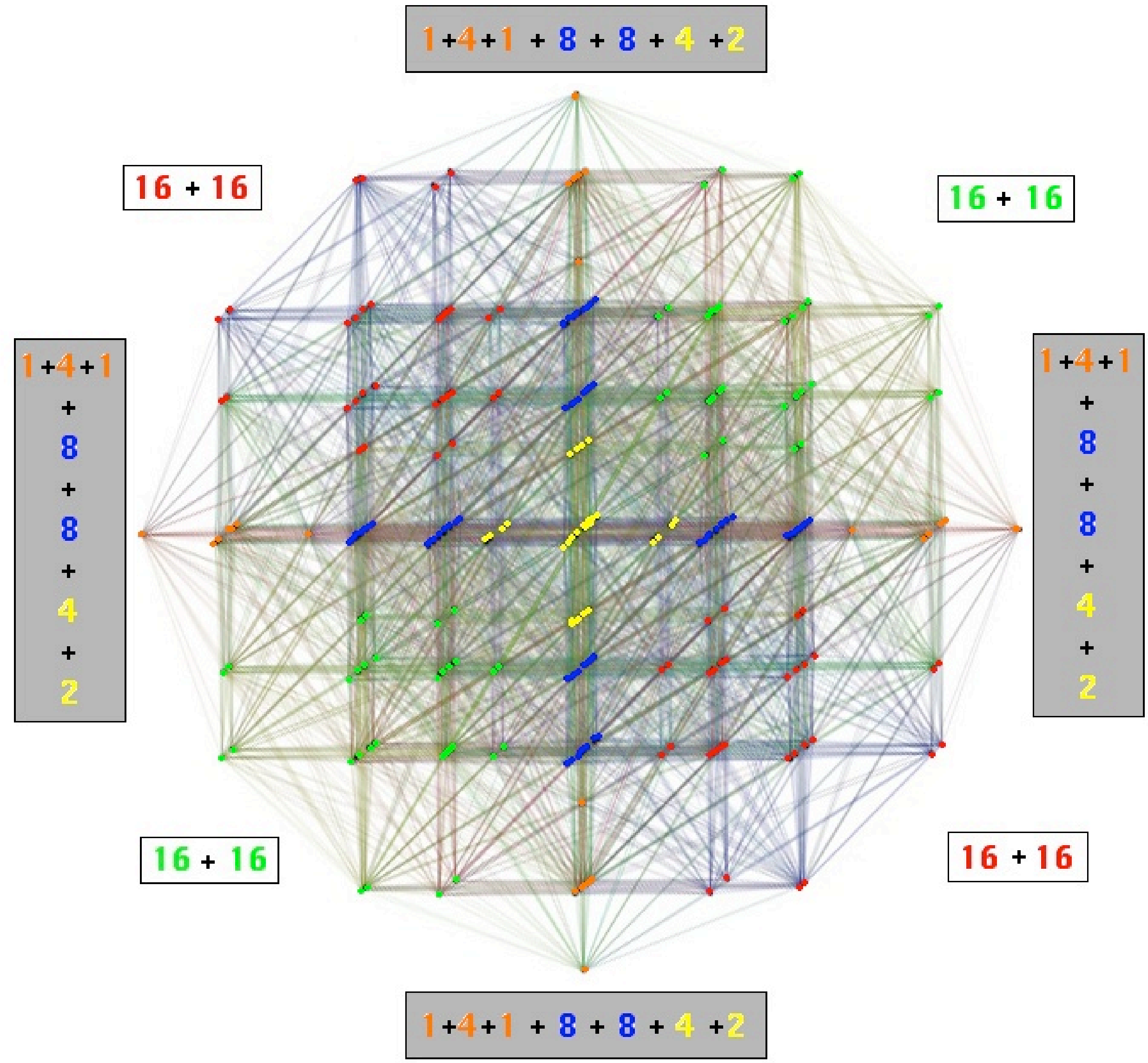
The E8 = 120 + 120 = (4+4)x30 = 8x30 decomposition does not directly correspond to the E8 = 112 + 128 decomposion because:

Each set of 120 vertices of each of the two 600-cells is made up of
120 = 24 vertices of a D4 + 3x32 where the three 32 are related by triality
so that each 600-cell contains one of the two D4 in D8, whose 112 vertices
are D8 = D4 + D4 + 8x8 = 24 + 24 + 64 = 112;

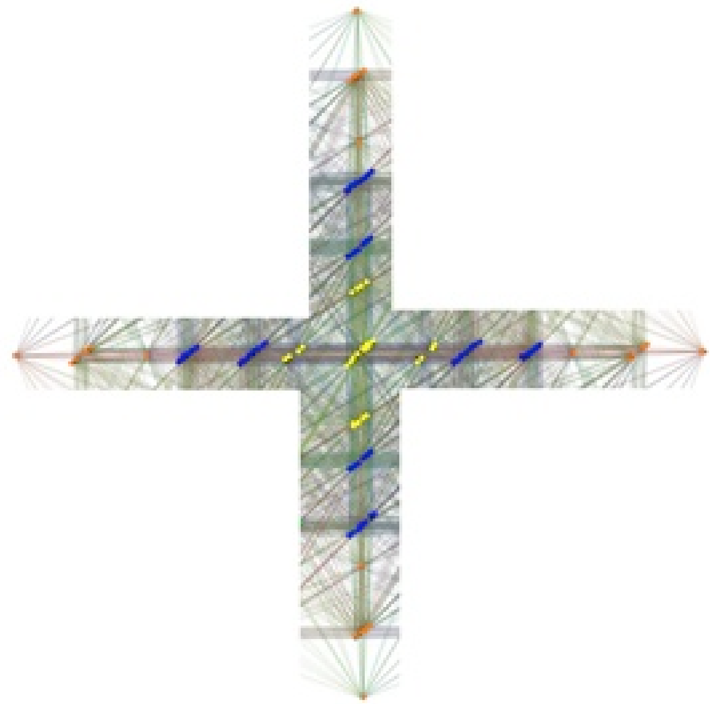
and, further, you cannot put all 128 of the D8 half-spinor into the 120 vertices of one 600-cell.

In his FQXi07 video, Garrett Lisi showed that the 8-Circle E8 projection can be rotated to see the 120 + 120 = 112 + 128 = 240 root vectors of E8 from another perspective, which I will call the H4+H4 Square projection.
I have put an m4v movie of the relevant part of Garrett Lisi's video on the web at
tony5m17h.net/8x30circletosquare.m4v
so that you can see how the rotation transition works.

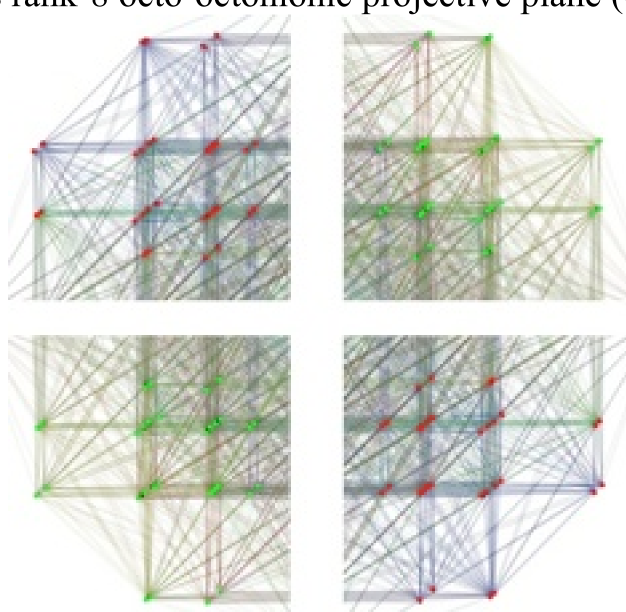
In the H4+H4 Square projection the 240 vertices are color-coded:



64 red for 8 components of 8 fermion particles,
60 of which are in the outer 120 of the 240
and 4 of which are in the inner 120 near the center;
64 green for 8 components of 8 fermion antiparticles,
60 of which are in the outer 120 of the 240
and 4 of which are in the inner 120 near the center;
64 blue for 8 components of 8 Kaluza-Klein spacetime dimensions
in the inner 120 – 4 – 4 = 112 of D8 in E8;
24 bright yellow for a D4 producing MacDowell-Mansouri Conformal Gravity;
24 orange for a D4* producing the Standard Model gauge bosons.
The 64 blue plus 24 bright yellow D4 plus 24 orange D4* make up the
64+24+24 = 112 root vectors of the D8 in E8:

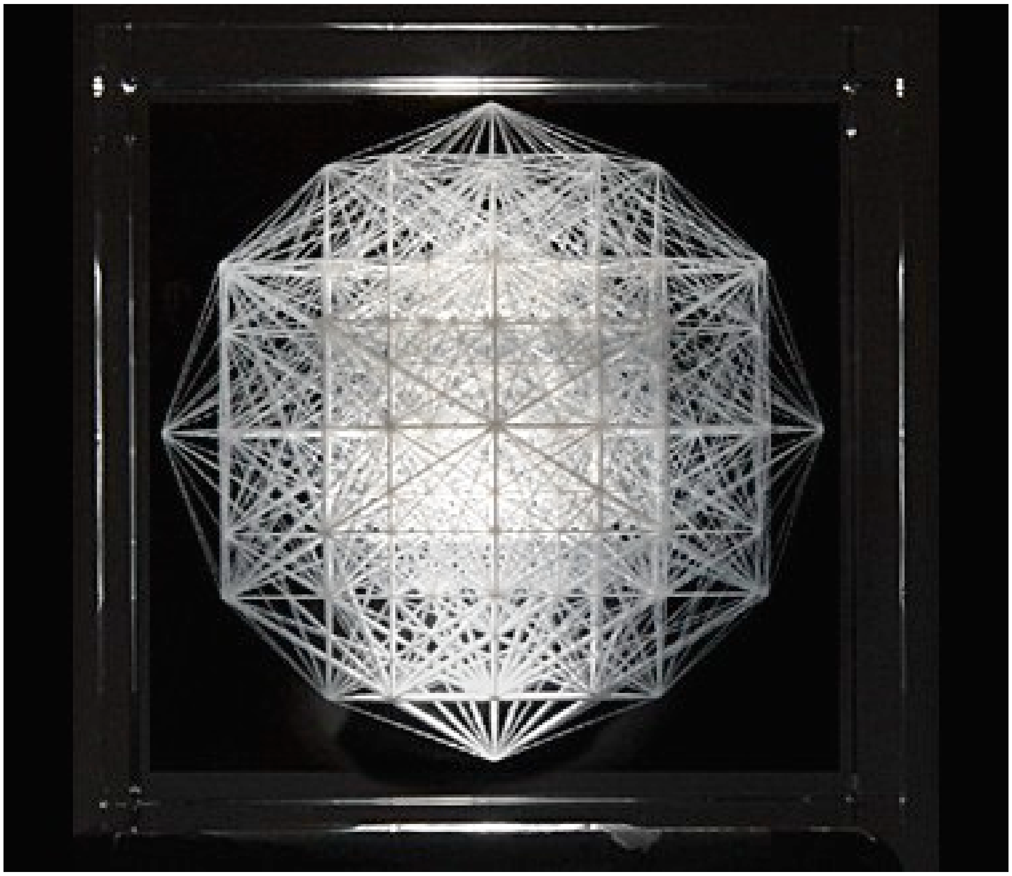


The 64 red plus 64 green make up the 64+64 = 128 root vectors that correspond to the +half-spinors of the D8 in E8, and to the 128-dim space E8 / D8 of Boris Rosenfeld's rank-8 octo-octonionic projective plane (OxO)P2.



Note that E8 contains +half-spinors of D8 representing one generation of fermions, and that E8 does NOT contain the –half-spinors of D8 representing one ANTI-generation of fermions, which enables (along with other natural math structures) my E8 physics model to be a realistic chiral model.

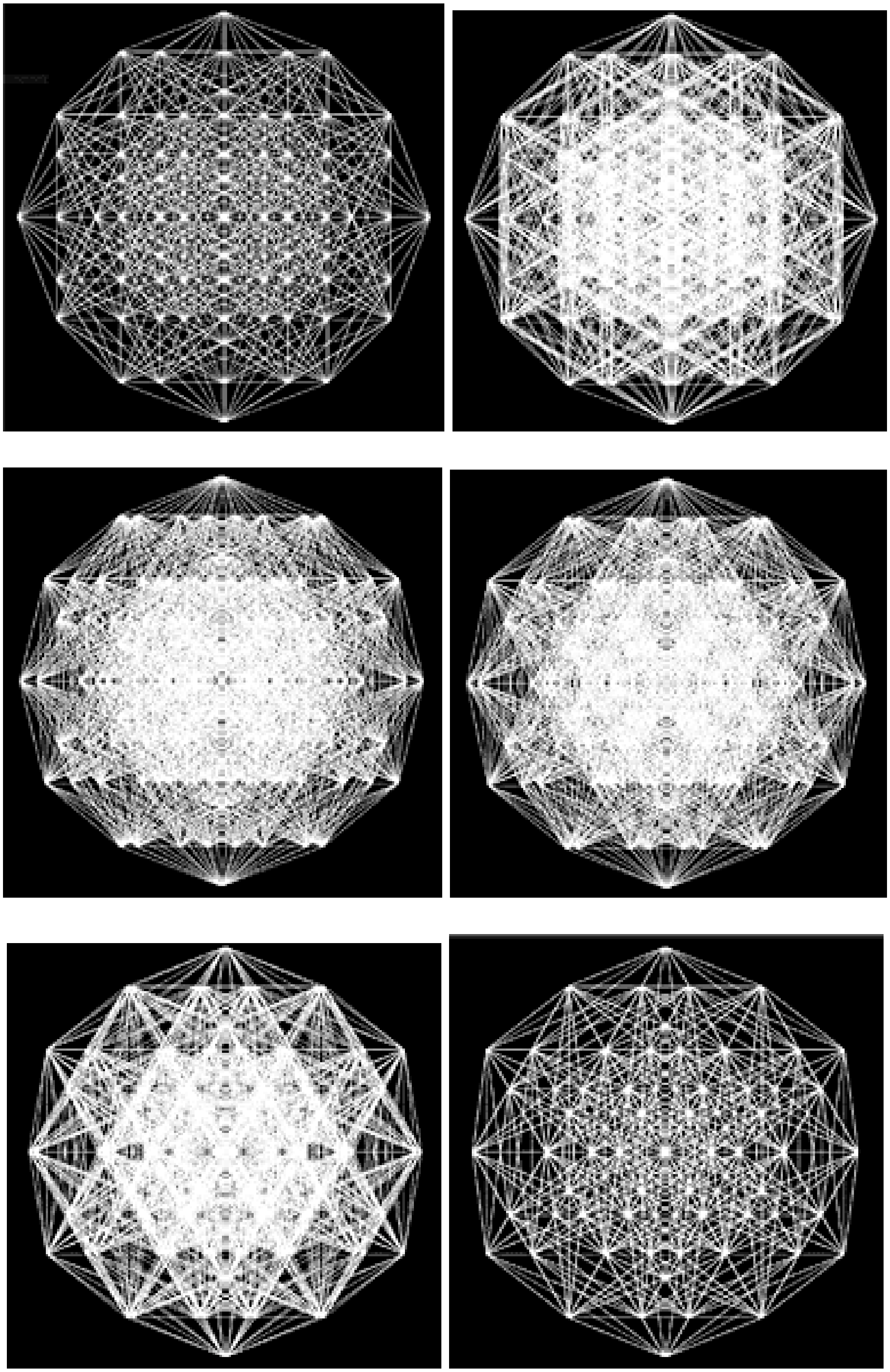
The H4+H4 Square projection has been used by Bathsheba Grossman to make an 8 cm glass model (see www.bathsheba.com/crystal/e8/) of the E8 root vector system:



As Bathsheba Grossman says at www.bathsheba.com/crystal/process/
"… The points are tiny (.1mm) fractures created by a focused laser beam. The conical beam, with a focal length of about 3", shines into the glass without damaging it except at the focal point. At that one point, concentrated energy heats the glass to the cracking point, causing a microfracture. To draw more points, the laser is pulsed on and off. To make the beam move between points, it's reflected from a mirror that is repositioned between pulses. The mirror is moved by computer-controlled motors, so many points can be drawn with great speed and accuracy. A typical design might use several hundred thousand points, or half a million isn't unusual in a large block, each placed with .001" accuracy. … I'm currently using … a Nd:YAG laser …".

In an animation at www.bathsheba.com/crystal/e8/ rotation of the Bathsheba E8

Glass shows that the H4+H4 Square projection can be transformed into



an E6 Hexagon projection.

Garrett Lisi said at FQXi07 (July 2007)
(see deferentialgeometry.org/talks/FQXi07/FQXi2007text.txt)
that he is
"…pretty sure … this hexagonal pattern … relates to E6 as a subgroup of E8. …".

To see how the E6 in E8 works,
look at the 240 = 72 + 168 decomposition that is natural in 9-dimensional coordinates, described by Coxeter in "Integral Cayley Numbers"
(Duke Math. J. vol. 13 no. 4, Dec. 1946,
reprinted in Coxeter's book "The Beauty of Geometry - Twelve Essays")
where Coxeter says (I am changing "l" to "z" for typographical reasons):

"… In terms of …

$z_1 = (1/2)(1 + e)$
$z_2 = (1/2)(1 - e)$
$z_3 = (1/2)(i + ie)$
$z_4 = (1/2)(i - ie)$
$z_5 = (1/2)(j + je)$
$z_6 = (1/2)(j - je)$
$z_7 = (1/2)(k + ke)$
$z_8 = (1/2)(k - ke)$

…
the 240 vertices of 4_{21} … consist of
112 like $\pm z_1 \pm z_2$
and
128 like $(1/2)(- z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8)$
(with an odd number of minus signs)
…

> [The 112 correspond to the 112 root vectors of D8,
> and the 128 to the -half-spinors of D8.
> Changing basis by changing sign of each of the basis elements
> { 1, i, j, k, e, ie, je, ke } gives 8 different lattices, 7 of which are independent.
> The 128 with even number of minus signs that are not in E8 correspond to the +half-spinors of D8 (physically, to an anti-generation of fermions).]

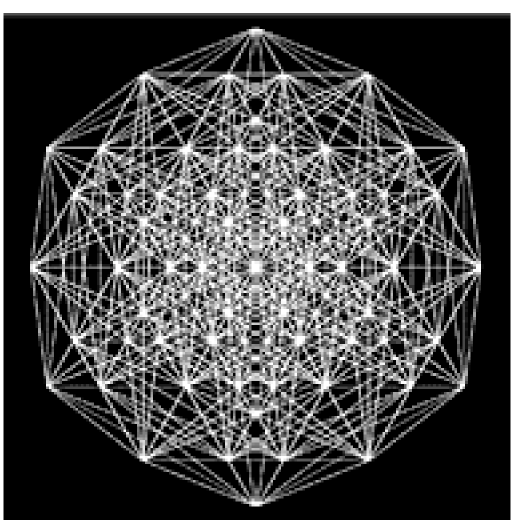
…

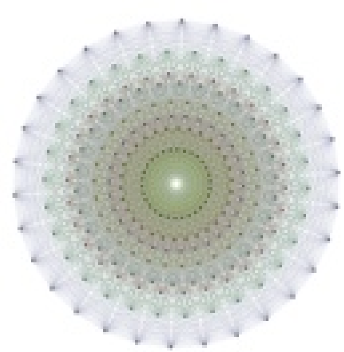
A … convenient notation, for some purposes, is obtained by defining …
$z_0 = (1/2)(1+i+j+k) = (1/2)(z_1 + z_2 + z_3 + z_4 + z_5 + z_6 + z_7 + z_8)$
… perhaps the greatest advantage of this notation is that enables us to replace the nine Cayley numbers …[corresponding to the 9 vertices of the extended E8 Dynkin diagram of the E8 lattice 5_{21}]… by

$z_8 – z_7$
$z_7 – z_6$
$z_6 – z_5$
$z_5 – z_4$
$z_4 – z_3$
$z_3 – z_2$
$z_2 – z_1$
$z_1 – z_0$
1
… the 240 integral Cayley numbers of norm 1 consist of …
72 like … $z_r – z_s$
… and 168 like … $+/-(z_r + z_s + z_t - z_0)$
where r,s,t take any three distinct values among 0,1,2,3,4,5,6,7,8
…”.

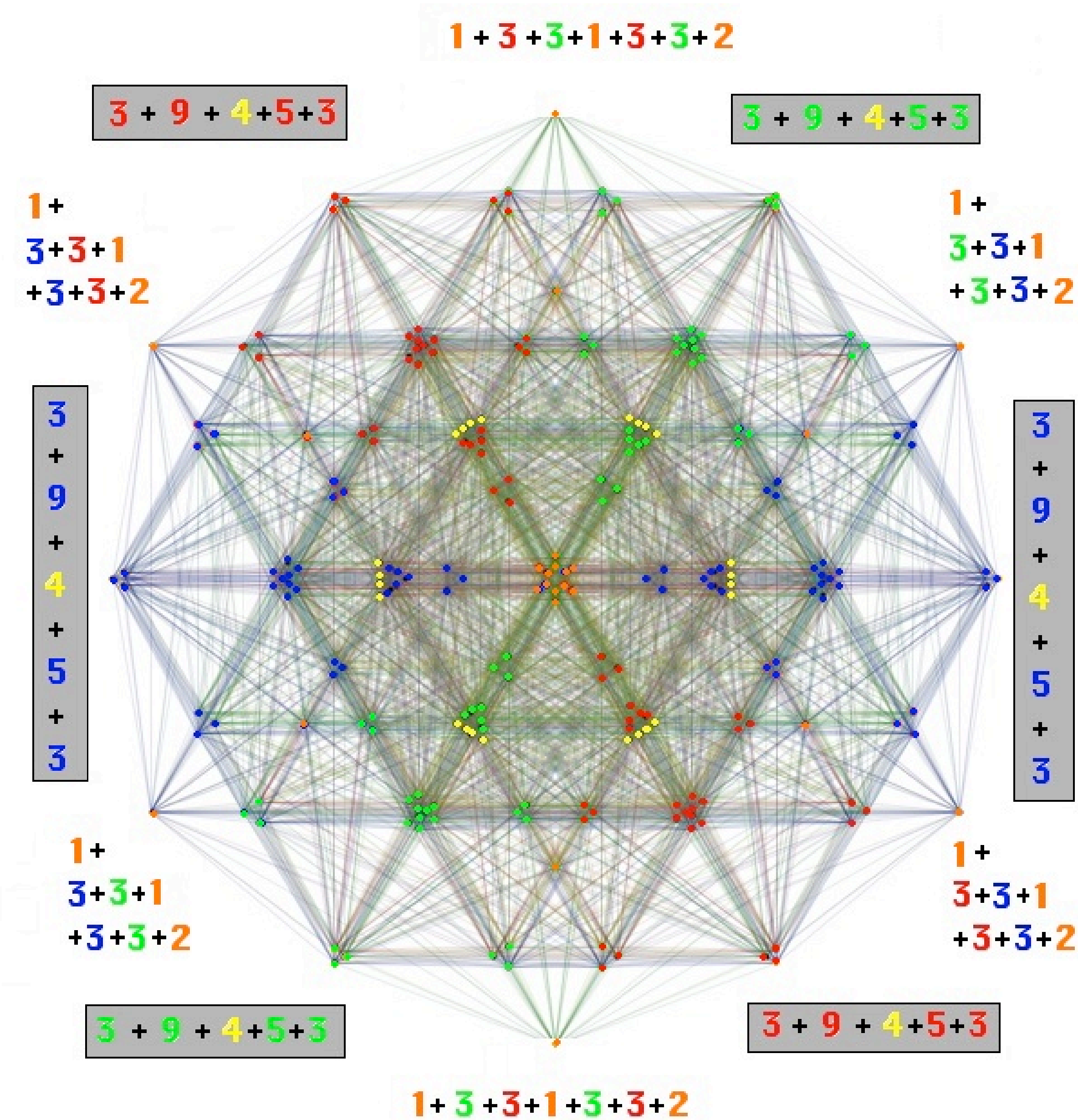
The decomposition 240 = 72 + 168
is related to the 72 root vectors of the 78-dimensional E6 subgroup
of 248-dimensional E8.

The E6 Hexagon projection can as shown in Garrett Lisi’s video for FQXi07 also be seen as a transformation of the 8-Circle E8 projection



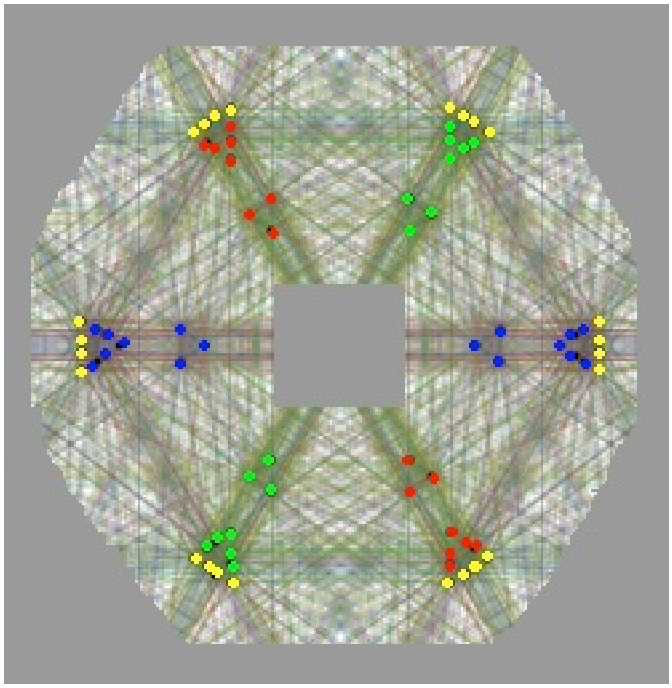


I have put an m4v movie of the relevant part of Garrett Lisi's video on the web at tony5m17h.net/8x30circletohex.m4v showing the transformation from the 8-Circle projection to the E6 Hexagon projection. Here is a more detailed image of the E6 Hexagon projection, adapted from Garrett Lisi's video:



In the E6 Hexagon projection the 240 vertices are color-coded:
64 red for 8 components of 8 fermion particles;
64 green for 8 components of 8 fermion antiparticles;
64 blue for 8 components of 8 Kaluza-Klein spacetime dimensions;
24 bright yellow for a D4 producing MacDowell-Mansouri Conformal Gravity;
24 orange for a D4* producing the Standard Model gauge bosons.

The 72 vertices that are E6 root vectors are



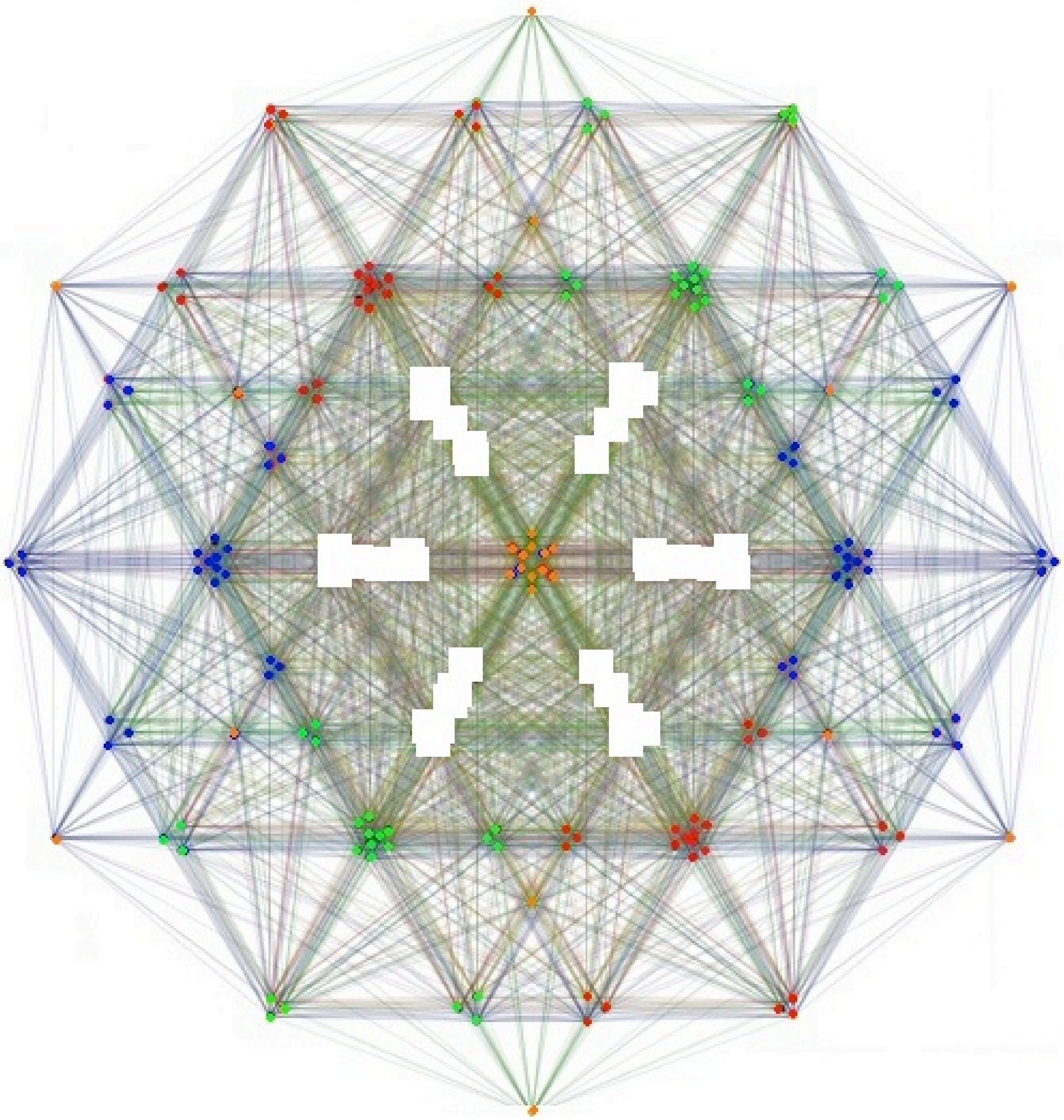
The E6 vertices are:
16 red for 2 components ({1,i} complex) of 8 fermion particles;
16 green for 2 components ({1,i} complex) of 8 fermion antiparticles;
16 blue for 2 components ({1,i} complex of 8-dim Kaluza-Klein spacetime;
24 bright yellow for a D4 producing MacDowell-Mansouri Conformal Gravity.

The 168 vertices in E8 outside E6 are:
48 red for 6 ({i,j,k,ie,je,ke} octonion) components of 8 fermion particles;
48 green for 6 ({i,j,k,ie,je,ke} octonion) components of 8 fermion antiparticles;
48 blue for 6 ({i,j,k,ie,je,ke} octonion) components of 8-dim K-K spacetime;
24 = 8+8+8 orange for a D4* producing the Standard Model gauge bosons.

The 48 red plus 8 orange give a red 48+8 = 56-dim Freudenthal algebra Fr(3,O).
The 48 green plus 8 orange give a green 48+8 = 56-dim Fr(3,O).
The 48 blue plus 8 orange give a blue 48+8 = 56-dim Fr(3,O).

E6 is the automorphism group of each of the three 56-dim Fr(3,O)
and the three 56-dim Fr(3,O) are related by Triality.

The 168 = 56+56+56 vertices of the three Fr(3,O) Freudenthal algebras are



The E6 itself corresponds to my E6 Bosonic Strings-as-World-Lines physics model with fermionic structure coming from orbifolding-see CERN CDS EXT-2004-031

If you regard the orange 24 vertices, not as root vectors of second D4, but
as the 8+8+8 = 8v vectors + 8s+ +half-spinors + 8s- –half-spinors of the D4 in E6,
then the 8v and 8s+ and 8 s- are related by triality.
Since the 48 blue and 48 red and 48 green are also related to vector spacetime,
+half-spinor fermion particles, and –half-spinor fermion antiparticles,
they can be considered as 48v and 48s+ and 48s- which are also related by triality.
Combining the 8v and 8s+ and 8s- with the 48v and 48s+ and 48s-
gives 56v and 56s+ and 56s- which are related by triality,
so that
the 168 = 56v + 56s+ + 56s-
where
the three 56 are related by the triality of the D4 in E6
and
each 56 corresponds to a Freudenthal algebra Fr(3,O)
of which E6 is the automorphism group.

The 56-dimensional Freudenthal algebra Fr(3,O) is 2x2 Zorn-type vector-matrices

$$\begin{matrix} a & X \\ Y & b \end{matrix}$$

where a and b are real numbers and X and Y are elements of the 27-dimensional Jordan Algebra J(3,O) of 3x3 Hermitian Octonionic matrices

$$\begin{matrix} d & S+ & V \\ S+^* & e & S- \\ V^* & S-^* & f \end{matrix}$$

where d, e, and f are real numbers; S+, V, and S- are Octonions;
and * denotes conjugation.

Fr(3,O) includes a complexification of J(3,O), so that each Half-Spinor Fermion Representation Space has 8 Complex Dimensions and a corresponding Bounded Complex Domain with 8-real-dimensional Shilov Boundary S7 x RP1 ,
as does the Vector SpaceTime representation space.

Introduction of a preferred Quaternionic structure at low energies gives the Vector SpaceTime an M4xCP2 8-dim Kaluza-Klein structure and a Higgs mechanism by the work of Meinhard Mayer, and produces the second and third generations of Fermion Particles and AntiParticles.

Each of the triality-related 56v and 56s+ and 56s- Freudenthal Fr(3,O) algebras has the structure of a 2x2 Zorn matrix as describned by Boris Rosenfeld in his book “Geometry of Lie Groups” (Kluwer 1997) (see particularly pages 56ff and 91ff):

1	1 8 8 * 1 8 * * 1
1 8 8 * 1 8 * * 1	1

The physical interpretation with respect to the E6 in E8 root vector decomposition is similar for all three of the triality-related 56v and 56s+ and 56s- Freudenthal Fr(3,O) algebras,
so the following description of physical interpretation will be in terms of one of them, the 56s+ red vertices corresponding to 48 of the 64 = 8x8 components in 8 dimensions of the 8 fundamental first-generation fermion particles.

The descriptions of the 56v blue root vectors corresponding to 48 of the 64 = 8x8 components in 8 dimensions of the 8 Dirac gammas and
the 56s- green vertices corresponding to 48 of the 64= 8x8 components in 8 dimensions of the 8 fundamental first-generation fermion antiparticles
are, as indicated by triality, similar.

In the Zorn matrix for the 56s+ red vertices corresponding to 48 of the 64 = 8x8 components in 8 dimensions of the 8 fundamental first-generation fermion particles,
the 6 entries labeled 8 are 6 octonions, corresponding to 6x8 = 48 root vectors,

representing the { i, j, k, ie, je, ke } components of each of the 8 fermion particles.

When combined with the 2x8 = 16 root vectors inside E6 representing
the { 1, e } components of each of the 8 fermion particles
the result is
6x8 + 2x8 = 48 + 16 = 64 = 8x8 root vectors representing
all 8 { 1, i, j, k, ie, je, ke, e } components, in 8 dimensions,
of each of all 8 fundamental first-generation fermion particles,
which
correspond to 64 of the 128 D8 half-spinor root vectors of the 240 of E8.
As to the remaining Zorn matrix real-number entries labeled 1, corresponding to 8 of
the 24 = 8+8+8 orange vertices outside the E6 part of E8

```
               1
  1             1
                  1



1
  1             1
    1
```

the two sets of 3 diagonal real numbers correspond to 3-vectors.

Gunaydin and Gursey, in J. Math.Phys. 14 (1973) 1651-1667, said:

"… the Zorn's vector matrices …[of]... scalars … and 3-vectors …[are a]… realization of the split octonion algebra …

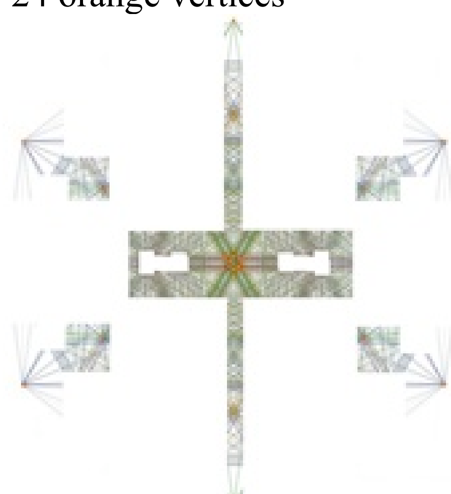
… the split octonion algebra contains divisors of zero and hence is not a division algebra …
We can represent the action of SO(8) on the split octonion basis [s]
… [s] = [u u*] … u = [u_1 , u_2 , u_3 , u_0] …
…[and]… we will construct the LSO(8) matrices that are in local triality with each other … given an element T^L in LSO(8) action on the octonions there exist unique T^R and T^P in LSO(8) such that …
(T^L x) y + x (T^R y) = T^P (xy) for all x,y in O
… Since the group SO(8) is the "Lie multiplication group" of octonions (i.e., that every action of SO(8) on O can be represented by octonion multiplication) …

Lie multiplication algebra of the octonions is defined as the Lie algebra with the elements: LMO = Der(O) (+) L_O_0 (+) R_O_0
where L_O_0 and R_O_0 correspond to multiplication from the left and the right by traceless (or imaginary) octonion units … the derivation (Lie) algebra of octonions is isomorphic to the Lie algebra of G2 … the automorphism group of …[the octonions]…

… one can reformulate the … principle of global triality … as follows:

Given d^1 in SO(8) and d^2, d^3 in SO(8)
(d^1 x) (d^2 y) = (d^3 (xy)*)*
where …[*]… denotes octonion conjugation …
we have cyclic symmetry between d^1, d^2 and d^3 …
Since given d^1, d^2, and d^3 are determioned uniquely up to a sign, the subgroup of SO(8) x SO(8) x SO(8) consisting of elements which are in triality will form a twofold covering group of SO(8), i.e., it will be isomorphic to Spin(8). …".

Physically,
the Zorn matrices of the three Fr(3,O) Freudenthal 56v, 56s+, and 56s- algebras correspond to the 8+8+8 = 24 orange vertices



and they produce three split octonions
and three elements T^L, T^ R, and T^P of LSO(8) related by triality,
so that
taken together they produce a Spin(8)
whose D4 Lie algebra is a second D4 in the 168 of the E6 in E8 root vector decomposition 240 = 72 + 168.

Therefore,
the physics of the E6 in E8 decomposition 240 = 72 + 168
is equivalent
to the physics of the E8 Physics model decomposition 240 = 112 + 128
They both have effectively:

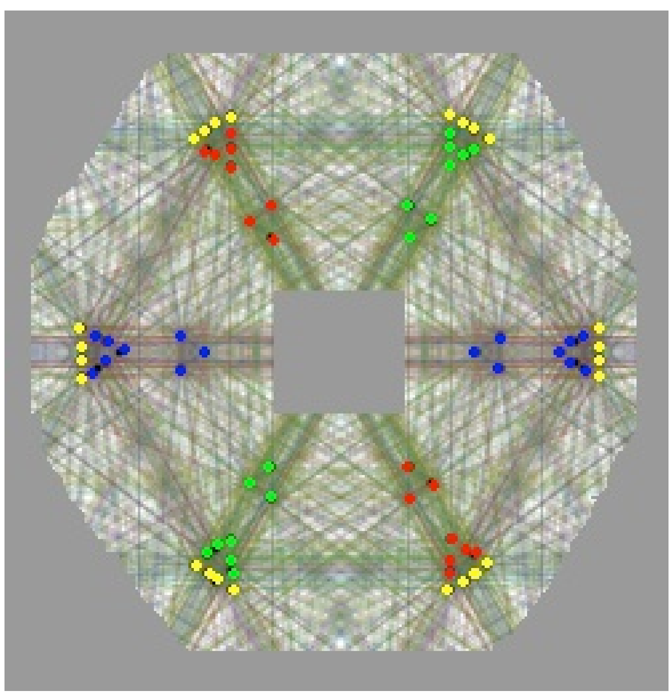
two (bright and orange/dark yellow vertices) 24-vertex D4 root vectors,
whose 4+4 = 8 Cartan subalgebra dimensions correspond
to the 8 E8 Cartan subalgebra dimensions and
to the 6 E6 plus 2 G2 Cartan subalgebra dimensions, with the G2 being the
automorphism group of the split octonions formed by the Zorn matrices

64 = 8x8 components (blue vertices) in 8 dimensions of 8 Dirac gammas

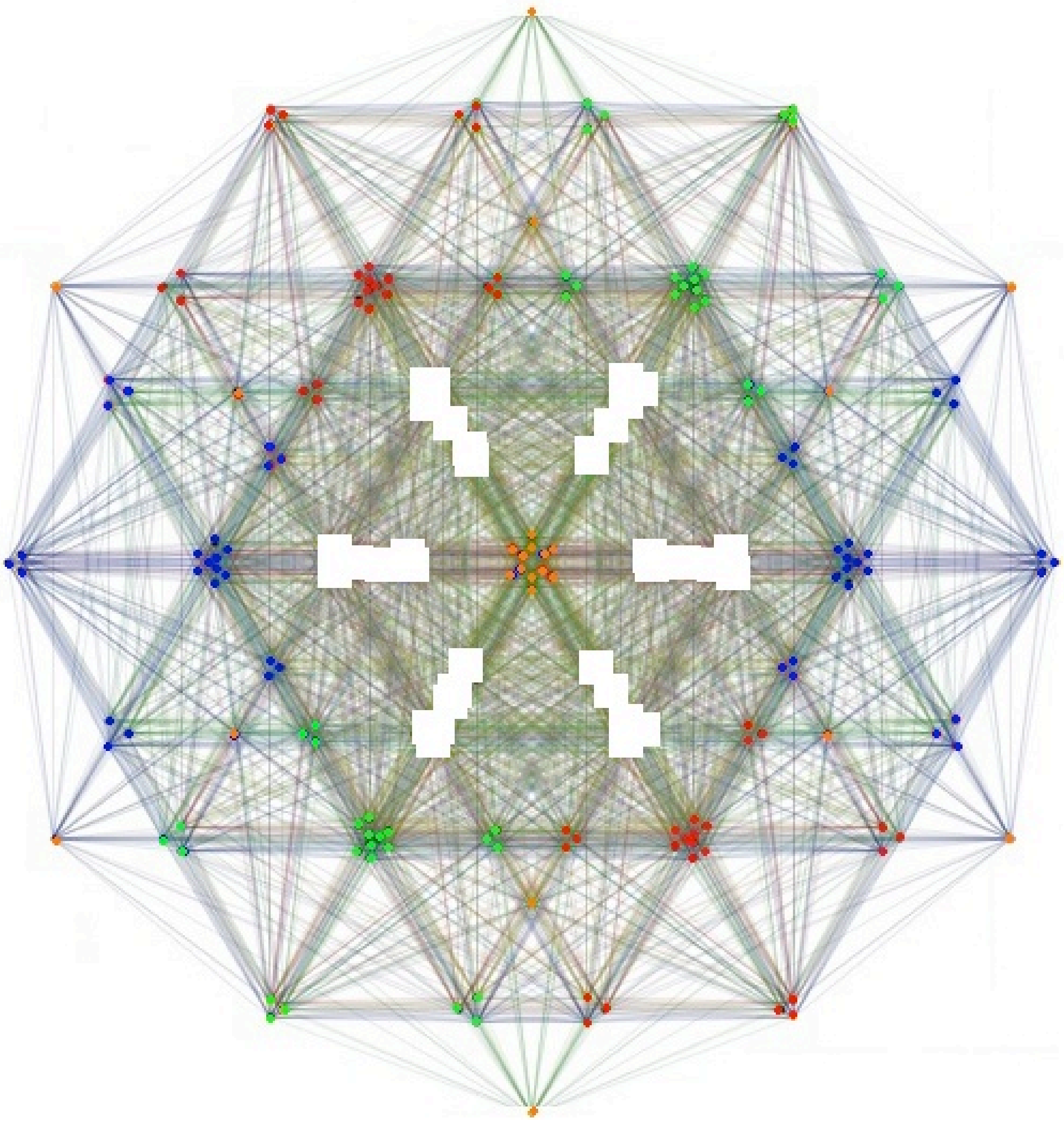
64 = 8x8 components (red vertices) in 8 dimensions of 8 fundamental
first-generation fermion particles

64 = 8x8 components (green vertices) in 8 dimensions of 8 fundamental
first-generation fermion antiparticles.

The E6 in E8 decomposition 240 = 72 + 168 gives insight into the physical interpretation of the two D4 (dark yellow and bright yellow vertices):

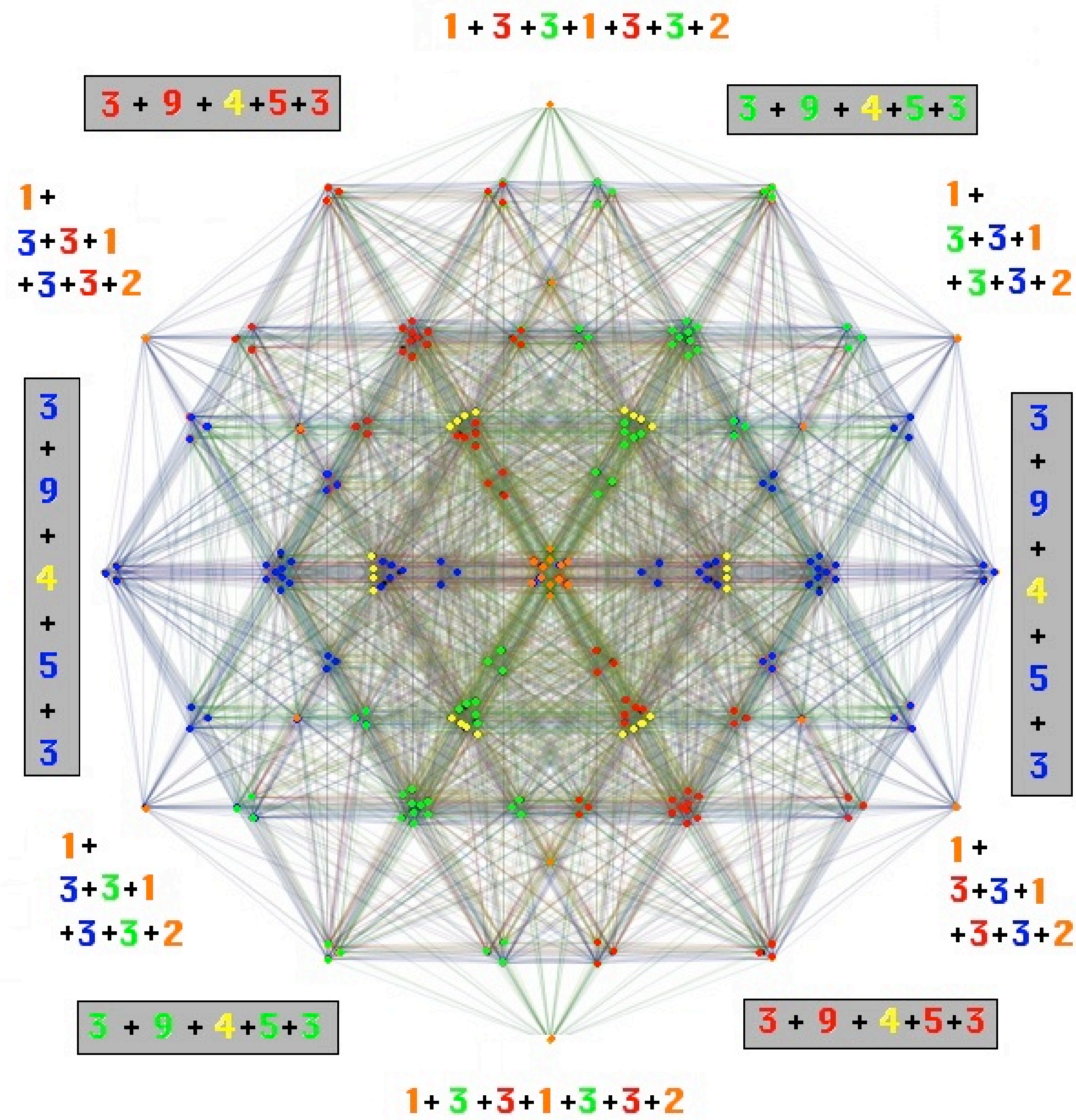


the 24 bright yellow D4 vertices share the 72 E6 vertices with { 1, e } components of the fermions and Dirac gammas and those components (through the physical spacetime being the Shilov boundary related to complex geometry related to the complex number interpretation of { 1, e }) naturally correspond to Gravity and the geometry of the mass-producing Higgs mechanism;



the 24 orange D4 vertices share the 168 vertices outside E6 with the { i, j, k, ie, je, ke } components of the fermions and Dirac gammas, and those components correspond in the octonion identification of fundamental fermions to the red, blue, green up quarks and red, blue, green down quarks and so to the color SU(3) of the Standard Model, so it is natural for the D4 of the 24 bright yellow vertices to correspond to the gauge groups of the Standard Model. Further, the 24 orange D4 vertices are in the 168 = 56v + 56s+ + 56s- which contain the three Zorn matrices of split octonions, and, as Gunaydin and Gursey said in J. Math.Phys. 14 (1973) 1651-1667,

"… Under the SU(3) subgroup of split G2 leaving u_0 and u_0* invariant, the three split octonions (u_1, u_2, u_3) transform like a unitary triplet (quarks) …", so that it is natural for the D4 of the 24 orange vertices to correspond to the Standard Model including its color SU(3).



The relationship between E8 root vector decompositions and coordinate dimension

8-dim coordinates give a natural E8 Physics model 240 = 112 + 128
9-dim coordinates give a natural E6 in E8 240 = 72+ 168
10-dim coordinates give a natural E8 = H4 + H4 240 = 120 + 120

seems similar what happens for the 120 vertices of the 600-cell

4-dim coordinates give a natural 120 = 24 + 96
5-dim coordinates give a natural 120 = 20 + 40 + 60 = 60 + 60

Since H4 is the symmetry group of the 600-cell and the 120 vertices of the 600-cell are half of the 240 E8 root vectors, such a similarity may give further insight into the structure of E8.
The 600-cell structures are described by H. S. M. Coxeter, in his paper "Regular and Semi-Regular Polytopes II" (Math. Zeit. 188 (1985) 555-591, reprinted in "Kaleidoscopes: Selected Writings of H. S. M. Coxeter" (Wiley 1995)) where he uses 4-dim Golden ratio t coordinates (his x-coordinates) to describe the 120 vertices as the permutations of

(t, t, t, t^(-2)) with even number of minus signs - there are 4x16/2 = 32 of these

(t^2, t^(-1), t^(-1), t^(-1)) with even number of minus signs - there are 4x16/2 = 32 of these

(sqrt(5), 1, 1, 1) with an odd number of minus signs - there are 4x16/2 = 32 of these
and
(+/-2, +/-2, 0, 0) there are 24 of these

so that the natural decompositon is 120 = 32+32+32+24 = 96 + 24
which corresponds to the 96 edges plus 24 vertices of a 24-cell.

Then to transform to 5-dim coordinates (his u-coordinates) he starts with x-coordinates and adds fifth coordinate zero $(x_1, x_2, x_3, x_4, 0)$ and makes the transformation

$$2u_1 = - x_1 + t^2 x_2 + t^{-2} x_3$$
$$2u_2 = t^{-2} x_1 - x_2 + t^2 x_3$$
$$2u_3 = t^2 x_1 + t^{-2} x_2 - x_3$$
$$2u_4 = - x_1 - x_2 - x_3 + \sqrt{5}\, x_4$$
$$2u_5 = - x_1 -x_2 - x_3 - \sqrt{5}\, x_4)$$

In terms of the u-coordinates, Coxeter gets a natural 120 = 20 + 40 + 60 = 60 + 60 decomposition, saying:
"… the 120 vertices … are the permutations of

$(\sqrt{5}, 0, 0, 0, -\sqrt{5})$ [20 of these]…

$(t^2, t^{-2}, -1, -1, -1)$ [20 of these]…
$(1, 1, 1, -t^{-2}, -t^2)$ [20 of these]…

$(2, t^{-1}, t^{-1}, -t, -t)$ [30 of these]…
$(t, t, -t^{-1}, -t^{-1}, -2)$ [30 of these]…".

More insight into these structures can be gained by considering what Conway and Sloane, in Chapter 8 of their book "Sphere Packings, Lattices and Groups" (Springer, 3rd ed 1999), said:
"... The icosian group is a multiplicative group of order 120 consisting of the quaternions … where (a, b, c, d) means a + bi + cj + dk …

$(1/2)(\pm 2, 0, 0, 0)$

$(1/2)(\pm 1, \pm 1, \pm 1, \pm 1)$

$(1/2)(0, \pm 1, \pm s, \pm t)$

... all even permutations of the coordinates are permitted ...

$s = (1/2)(1 - \sqrt{5})$, $t = (1/2)(1 + \sqrt{5})$

... The 240 minimal vectors of ... E8 ... consist of the elements q and s q where q is any element of the icosian group …

> [The three sets of icosians have 8, 16, and 96 elements,
> and the first two sets form the 8+16 = 24 vertices of a 24-cell
> and the third set forms the 96 Golden Ratio points on the 96 edges of that 24-cell,
> corresponding to the 4-dim decomposition of the 120 vertices of the 600-cell
> as described above.]

… We use the particular names
w = (1/2)(-1, 1, 1, 1)
i_H = (1/2)(0, 1, s, t)
There is a homomorphism from this [icosian] group to the Alternating group A5 on five letters {G,H,I,J,K} defined by
i = (1/2)(0, 2, 0, 0) -> (H,I)(J,K)
j = (1/2)(0, 0, 2, 0) -> (H,J)(K,I)
k = (1/2)(0, 0, 0, 2) -> (H,K)(I,J)
w = (1/2)(-1, 1, 1, 1) -> (I,J,K)
i_H = (1/2)(0, 1, s, t) -> (G,I)(J,K)
in which the kernel is {+/-1}. Table 8.1 below gives much more information about this homomorphism. Abstractly the [120-element] icosian group is the perfect double cover 2.A5 of [60-element] A5, and is sometimes called the binary icosahedral group.
The icosian ring I is the set of all finite sums q_1 + … q_n where each q_i is in the icosian group. Elements of the icosian ring are … called icosians.
The typical icosian q has the form q = a + bi + cj + dk where the coordinates a,b,c,d belong to the golden field Q(t) and so have the form x + y sqrt(5) where x,y are in Q [the rational numbers].
The conjugate icosian … is qbar = a - bi - cj - dk
and q qbar = a^2 + b^2 + c^2 + d^2
Two vectors … v = (q_1, q2, …) … and w = (r_1, r_2, …) have a quaternionic inner product
(v,w) = q_1 r_1bar + q_2 r_2bar + …
We shall use two different norms for such vectors, the quaternionic norm
QN(v) = (v,v)
which is a number of the form a + b sqrt(5) with a,b in Q
and the Euclidean norm
EN(v) = a + b

The icosians of quaternionic norm 1 are the elements of the icosian group.
With respect to the quaternionic norm the icosians belong to a four-dimensional space over Q(t);
with the Euclidean norm they lie in an eight-dimensional space.
In fact under the Euclidean norm the icosian ring I is isomorphic to an E8 lattice in this space. …
Table 8.1 has 60 entries, one for each pair of elements +/-q of the icosian group. …

Table 8.1. Icosians and corresponding elements of A_5 and E_8.

$1 \to$ identity	$i = i_G \to (HI)(JK)$	$j = j_G \to (HJ)(KI)$	$k = k_G \to (HK)(IJ)$
2 2 2 − +	0 0 0 − +	0 0 0 − +	0 0 0 − +
0 0 0 + −	2 2 2 − +	0 0 0 − +	0 0 0 + −
0 0 0 + −	0 0 0 + −	2 2 2 − +	0 0 0 − +
0 0 0 + −	0 0 0 − +	0 0 0 + −	2 2 2 − +

$\omega = \omega_{GH} \to (IJK)$	$\omega^i = \omega_{GI} \to (HKJ)$	$\omega^j = \omega_{GJ} \to (HIK)$	$\omega^k = \omega_{GK} \to (HJI)$
−1 − − − +	−1 − − + −	−1 − − + −	−1 − − + −
1 + + − +	1 + + − +	−1 − − − +	−1 − − + −
1 + + − +	−1 − − + −	1 + + − +	−1 − − − +
1 + + − +	−1 − − − +	−1 − − + −	1 + + − +

$\bar{\omega} = \omega_{HG} \to (IKJ)$	$\bar{\omega}^i = \omega_{IG} \to (HJK)$	$\bar{\omega}^j = \omega_{JG} \to (HKI)$	$\bar{\omega}^k = \omega_{KG} \to (HIJ)$
−1 − − 2 −2	−1 − − 0 0	−1 − − 0 0	−1 − − 0 0
−1 − − 0 0	−1 − − 0 0	1 + + 0 0	1 + + −2 2
−1 − − 0 0	1 + + −2 2	−1 − − 0 0	1 + + 0 0
−1 − − 0 0	1 + + 0 0	1 + + −2 2	−1 − − 0 0

$i_H \to (GI)(JK)$	$j_H \to (GJ)(KI)$	$k_H \to (GK)(IJ)$	$i_I \to (HG)(JK)$	$j_J \to (HG)(KI)$	$k_K \to (HG)(IJ)$
0 0 0 − +	0 0 0 − +	0 0 0 − +	0 0 0 0 0	0 0 0 0 0	0 0 0 0 0
1 0 2 − +	τ 2 0 − −	σ 0 0 + +	1 2 0 0 0	−τ −2 0 0 2	−σ 0 0 0 −2
σ 0 0 + +	1 0 2 − ∓	τ 2 0 − −	−σ 0 0 0 −2	1 2 0 0 0	−τ −2 0 0 2
τ 2 0 − −	σ 0 0 + +	1 0 2 − +	−τ −2 0 0 2	−σ 0 0 0 −2	1 2 0 0 0

$i_J \to (HI)(GK)$	$j_K \to (HJ)(GI)$	$k_I \to (HK)(GJ)$	$i_K \to (HI)(JG)$	$j_I \to (HJ)(KG)$	$k_J \to (HK)(IG)$
0 + − 0 0	0 + − 0 0	0 + − 0 0	0 − + − +	0 − + − +	0 − + − +
1 + + 0 0	τ + + −2 0	−σ + − 0 −2	1 + + − +	−τ − − + +	σ − + + +
−σ + − 0 −2	1 + + 0 0	τ + + −2 0	σ − + + +	1 + + − +	−τ − − + +
τ + + −2 0	−σ + − 0 −2	1 + + 0 0	−τ − − + +	σ − + + +	1 + + − +

$\omega_{HI} \to (GJK)$	$\omega_{HJ} \to (GKI)$	$\omega_{HK} \to (GIJ)$	$\omega_{JK} \to (GHI)$	$\omega_{KI} \to (GHJ)$	$\omega_{IJ} \to (GHK)$
−1 − − 0 0	−1 − − 0 0	−1 − − 0 0	−1 0 −2 + −	−1 0 −2 + −	−1 0 −2 + −
0 + − 0 0	σ − + 0 2	τ + + −2 0	0 0 0 − +	−σ 0 0 − −	τ 2 0 − −
τ + + −2 0	0 + − 0 0	σ − + 0 2	τ 2 0 − −	0 0 0 − +	−σ 0 0 − −
σ − + 0 2	τ + + −2 0	0 + − 0 0	−σ 0 0 − −	τ 2 0 − −	0 0 0 − +

$\omega_{IH}\to(GKJ)$	$\omega_{JH}\to(GIK)$	$\omega_{KH}\to(GJI)$	$\omega_{KJ}\to(GIH)$	$\omega_{IK}\to(GJH)$	$\omega_{JI}\to(GKH)$
$1\ \ -\ +\ -$	$-1\ -\ -\ +\ -$	$-1\ -\ -\ +\ -$	$-1\ -2\ 0\ 0\ 0$	$-1\ -2\ 0\ 0\ 0$	$-1\ -2\ 0\ 0\ 0$
$0\ \ +\ -\ +$	$-\sigma\ +\ -\ -\ -$	$-\tau\ -\ -\ +\ +$	$0\ 0\ 0\ 0\ 0$	$\sigma\ 0\ 0\ 0\ 2$	$-\tau\ -2\ 0\ 0\ 2$
$\tau\ \ -\ +\ +$	$0\ -\ +\ -\ +$	$-\sigma\ +\ -\ -\ -$	$-\tau\ -2\ 0\ 0\ 2$	$0\ 0\ 0\ 0\ 0$	$\sigma\ 0\ 0\ 0\ 2$
$\sigma\ +\ -\ -\ -$	$-\tau\ -\ -\ +\ +$	$0\ -\ +\ -\ +$	$\sigma\ 0\ 0\ 0\ 2$	$-\tau\ -2\ 0\ 0\ 2$	$0\ 0\ 0\ 0\ 0$

$t_{HI}\to(HGIJK)$	$t_{HJ}\to(HGJKI)$	$t_{HK}\to(HGKIJ)$	$t_{JK}\to(JGKHI)$	$t_{KI}\to(KGIHJ)$	$t_{IJ}\to(IGJHK)$
$\tau\ 2\ 0\ 0\ 2$	$-\tau\ -2\ 0\ 0\ 2$	$-\tau\ -2\ 0\ 0\ 2$	$-\tau\ -2\ 0\ +\ +$	$-\tau\ -2\ 0\ +\ +$	$-\tau\ -2\ 0\ +\ +$
$0\ 0\ 0\ 0\ 0$	$1\ 2\ 0\ 0\ 0$	$\sigma\ 0\ 0\ 0\ 2$	$0\ 0\ 0\ -\ +$	$-1\ 0\ -2\ +\ -$	$\sigma\ 0\ 0\ +\ +$
$\sigma\ 0\ 0\ 0\ 2$	$0\ 0\ 0\ 0\ 0$	$1\ 2\ 0\ 0\ 0$	$\sigma\ 0\ 0\ +\ +$	$0\ 0\ 0\ -\ +$	$-1\ 0\ -2\ +\ -$
$1\ 2\ 0\ 0\ 0$	$\sigma\ 0\ 0\ 0\ 2$	$0\ 0\ 0\ 0\ 0$	$-1\ 0\ -2\ +\ -$	$\sigma\ 0\ 0\ +\ +$	$0\ 0\ 0\ -\ +$

$t_{IH}\to(IGHKJ)$	$t_{JH}\to(JGHIK)$	$t_{KH}\to(KGHJI)$	$t_{KJ}\to(KGJIH)$	$t_{IK}\to(IGKJH)$	$t_{JI}\to(JGIKH)$
$\tau\ -\ -\ 2\ 0$	$-\tau\ -\ -\ 2\ 0$	$-\tau\ -\ -\ 2\ 0$	$-\tau\ -\ -\ +\ +$	$-\tau\ -\ -\ +\ +$	$-\tau\ -\ -\ +\ +$
$0\ +\ -\ 0\ 0$	$-1\ -\ -\ 0\ 0$	$-\sigma\ +\ -\ 0\ -2$	$0\ +\ \ +\ -$	$1\ +\ +\ -\ +$	$-\sigma\ +\ -\ -\ -$
$\sigma\ +\ -\ 0\ -2$	$0\ +\ -\ 0\ 0$	$-1\ -\ -\ 0\ 0$	$-\sigma\ +\ \ -\ -$	$0\ +\ -\ +\ -$	$1\ +\ +\ -\ +$
$1\ -\ -\ 0\ 0$	$-\sigma\ +\ -\ 0\ -2$	$0\ +\ -\ 0\ 0$	$1\ +\ +\ -\ +$	$-\sigma\ +\ -\ -\ -$	$0\ +\ -\ +\ -$

$s_{JK}\to(KGJHI)$	$s_{KI}\to(IGKHJ)$	$s_{IJ}\to(JGIHK)$	$s_{HI}\to(IGHJK)$	$s_{HJ}\to(JGHKI)$	$s_{HK}\to(KGHIJ)$
$\sigma\ 0\ 0\ -\ -$	$-\sigma\ 0\ 0\ -\ -$	$-\sigma\ 0\ 0\ -\ -$	$-\sigma\ +\ -\ -\ -$	$-\sigma\ +\ -\ -\ -$	$-\sigma\ +\ -\ -\ -$
$0\ 0\ 0\ -\ +$	$-\tau\ -2\ 0\ +\ +$	$1\ 0\ 2\ -\ +$	$0\ -\ +\ -\ +$	$\tau\ +\ +\ -\ -$	$1\ +\ +\ -\ +$
$1\ 0\ 2\ -\ +$	$0\ 0\ 0\ -\ +$	$-\tau\ -2\ 0\ +\ +$	$1\ +\ +\ -\ +$	$0\ -\ +\ -\ +$	$\tau\ +\ +\ -\ -$
$\tau\ -2\ 0\ +\ +$	$1\ 0\ 2\ -\ +$	$0\ 0\ 0\ -\ +$	$\tau\ +\ +\ -\ -$	$1\ +\ +\ -\ +$	$0\ -\ +\ -\ +$

$s_{KJ}\to(JGKIH)$	$s_{IK}\to(KGIHJ)$	$s_{JI}\to(IGJKH)$	$s_{IH}\to(HGIKJ)$	$s_{JH}\to(HGJIK)$	$s_{KH}\to(HGKJI)$
$\sigma\ +\ -\ 0\ -2$	$-\sigma\ +\ -\ 0\ -2$	$-\sigma\ +\ -\ 0\ -2$	$-\sigma\ 0\ 0\ 0\ -2$	$-\sigma\ 0\ 0\ 0\ -2$	$-\sigma\ 0\ 0\ 0\ -2$
$0\ -\ +\ 0\ 0$	$\tau\ +\ +\ -2\ 0$	$-1\ -\ -\ 0\ 0$	$0\ 0\ 0\ 0\ 0$	$-\tau\ -2\ 0\ 0\ 2$	$-1\ -2\ 0\ 0\ 0$
$1\ -\ -\ 0\ 0$	$0\ -\ +\ 0\ 0$	$\tau\ +\ +\ -2\ 0$	$-1\ -2\ 0\ 0\ 0$	$0\ 0\ 0\ 0\ 0$	$-\tau\ -2\ 0\ 0\ 2$
$\tau\ +\ +\ -2\ 0$	$-1\ -\ -\ 0\ 0$	$0\ -\ +\ 0\ 0$	$-\tau\ -2\ 0\ 0\ 2$	$-1\ -2\ 0\ 0\ 0$	$0\ 0\ 0\ 0\ 0$

[some pen and ink modifications were made by me in accord with errata statements in the book by Conway and Sloane]…

The typical entry in this table: wbar^I = w_IG -> (HJK) … should be read as follows. The top line gives name(s) for q (in this case
Wbar^i = w_IG = (1/2)(-1 - i + j + k))
and indicates the corresponding even permutation of {G,H,I,J,K}.

The four quaternionic coordinates of 2q appear in the first column, followed by two columns giving the E8 vectors representing 2q and 2 s q

As usual, - stands for -1 and + for +1.

The formulae …

w^i = i^(-1) w i = j w = w k = wbar + i = (1/2)(-1 + i - j - k)

w^j = j^(-1) w j = k w = w i = wbar + j = (1/2)(-1 - i + j - k)

w^k = k^(-1) w k = i w = w j = wbar + k = (1/2)(-1 - i - j + k)

wbar^i = i^(-1) wbar i = - k wbar = - wbar j = w - i = (1/2)(-1 - i + j + k)

wbar^j = j^(-1) wbar j = - i wbar = - wbar k = w - j = (1/2)(-1 + i - j + k)

wbar^k = k^(-1) wbar k = - j wbar = - wbar i = w - k = (1/2)(-1 + i + j - k)

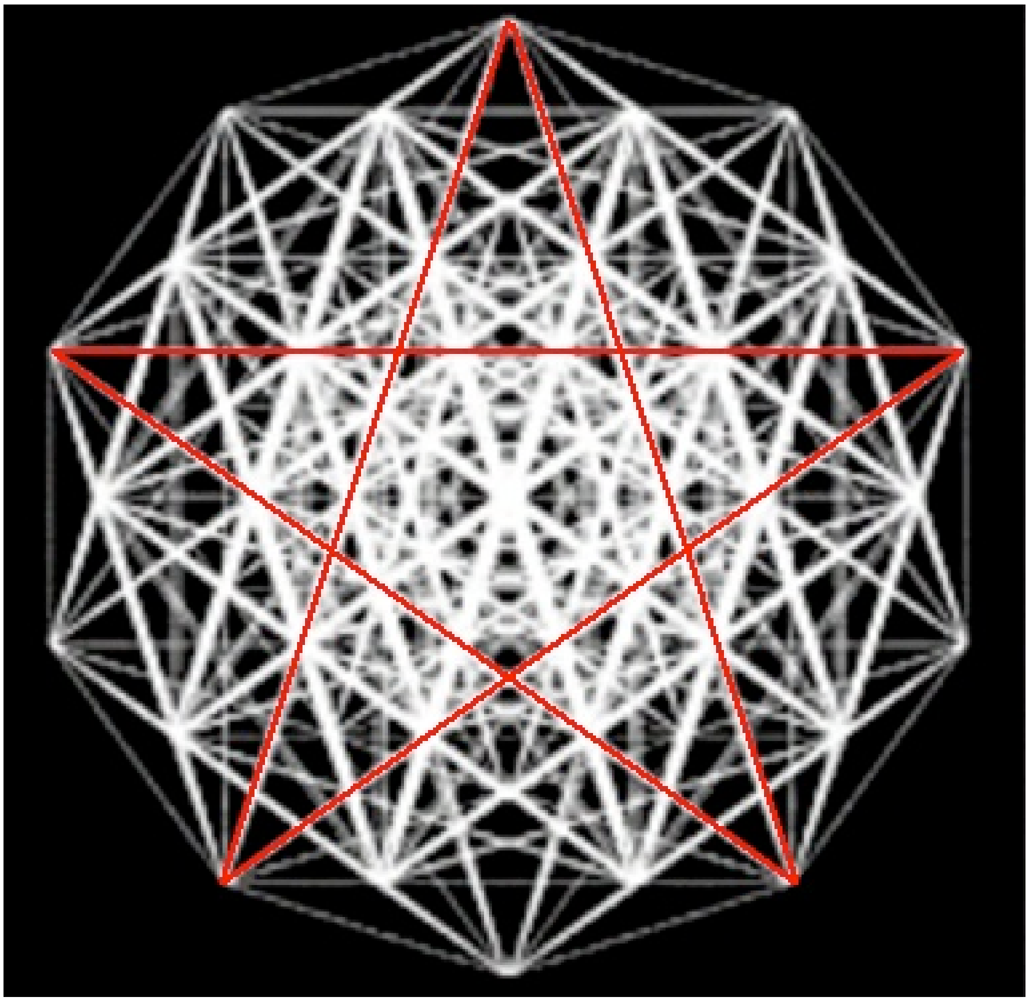
… are helpful for manipulating these quaternions.
The naming system for q = (1/2)(a, b, c, d) is as follows.
The letters i,j,k indicate that a = 0
w indicates a = -1 …[and]… s indicates a = -s ...[and]… t indicates a = -t
and the subscripts indicate the signs of a,b,c,d:

signs	subscript
0(+00)	G
-+++	GH
----	HG
-+--	GI
--++	IG
0+++	H
0+--	I
-0++	HI
-0--	IH
-0+-	JK
-0-+	KJ
…	…

For example w_XY corresponds to the permutation (Z,T,U) where X,Y,Z,T,U is an even permutation of G,H,I,J,K. The triple { i_X, j_X, k_X }where X is any of G,H,I,J,K forms a system of unit quaternions (with i_x^2 = -1, i_X j_X = k_X , etc).

The 240 minimal vectors of this version of E8 have Euclidean norm 1, and quaternionic norm either 1 or s^2. They consist of the elements q and s q , where q is any element of the icosian group. …”.

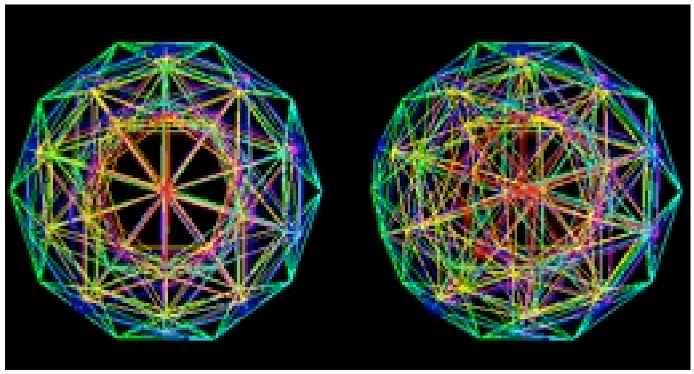
The 5-fold symmetry of the 120 Icosians and the 240 E8 Root Vectors that are formed by two sets of H4 Root Vectors (600-cell vertices), one set being the other dilated by the Golden Ratio,
is seen in this Decagon Projection of Bathsheba Grossman's E8 Glass model:



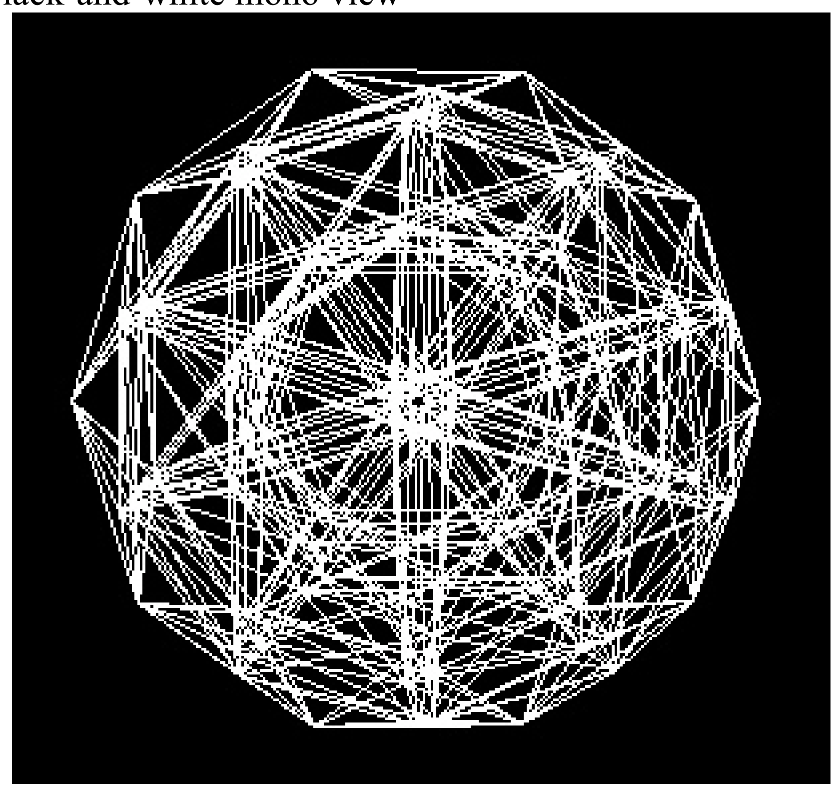
I added the red Pentagram that shows the Golden Ratio Dilated set of 120 vertices as being 10+30+10+60 outside the inner pentagon plus 10 at the center and the Undilated set of 120 vertices as being 10+30+10+60 inside the inner pentagon plus another 10 at the center.

More detail can be seen by looking at each set of 120 Icosian vertices by itself as the set of vertices of a 600-cell, and using these 600-cell images constructed by a java applet by Michael Gibbs:

a color stereo view



and a larger black-and-white mono view



The mathematical structure is related to a talk given in early 2008 at the University of California Riverside by Bertram Kostant. In his notes of the talk, John Baez said: "…The Cartan subalgebra … h … for the Lie algebra e8 … is 8-dimensional, and there are 240 roots, so the dimension of e8 is 248 … the 248-dimensional Lie

algebra e8 is a direct sum of 31 8-dimensional Cartan algebras. …
There's … a copy of (Z/5)^3 in E8. If we think of this as a 3-dimensional vector space containing "lines" (1-dimensional subspaces), then it contains

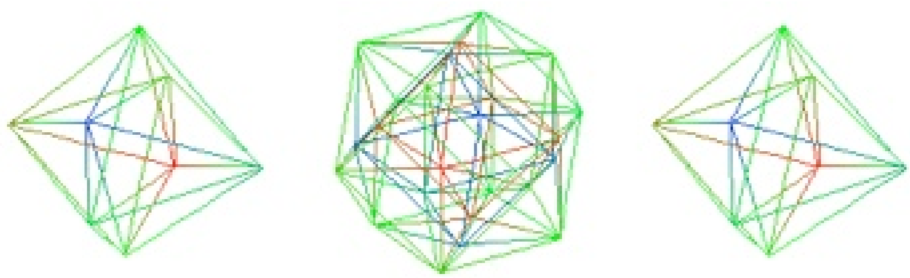
$$1 + 5 + 5^2 = 31$$

lines. The centralizer in E8 of any such line is

$$SU(5) \times SU(5)$$

This group is 48-dimensional. It has a 248-dimensional representation coming from the adjoint action on e8. This is the direct sum of
the 48-dimensional subrepresentation coming from su(5) (+) su(5) in e8
and a representation of dimension 248 - 48 = 200 …".

From the point of view of my E8 Physics model, the 200 breaks down into two copies of 100, and each 100 breaks down into 12 + 24+32+32,
and the 40 Root Vector Vertices assigned by John Baez to two copies of SU(5) are seen as the 40 Root Vector Vertices of 45-dim D5:



8+8+8 = 24 for a 4-dim 24-cell (the Root Vectors of a D4 in D5)
8 for a 4-dim HyperOctahedron above (in a 5 th dim) the 24-cell
8 for a 4-dim HyperOctahedron below (in 5 th dim) the 24-cell
for 8+8+8 + 8 + 8 = 5x8 = 40 Root Vector Vertices for D5.

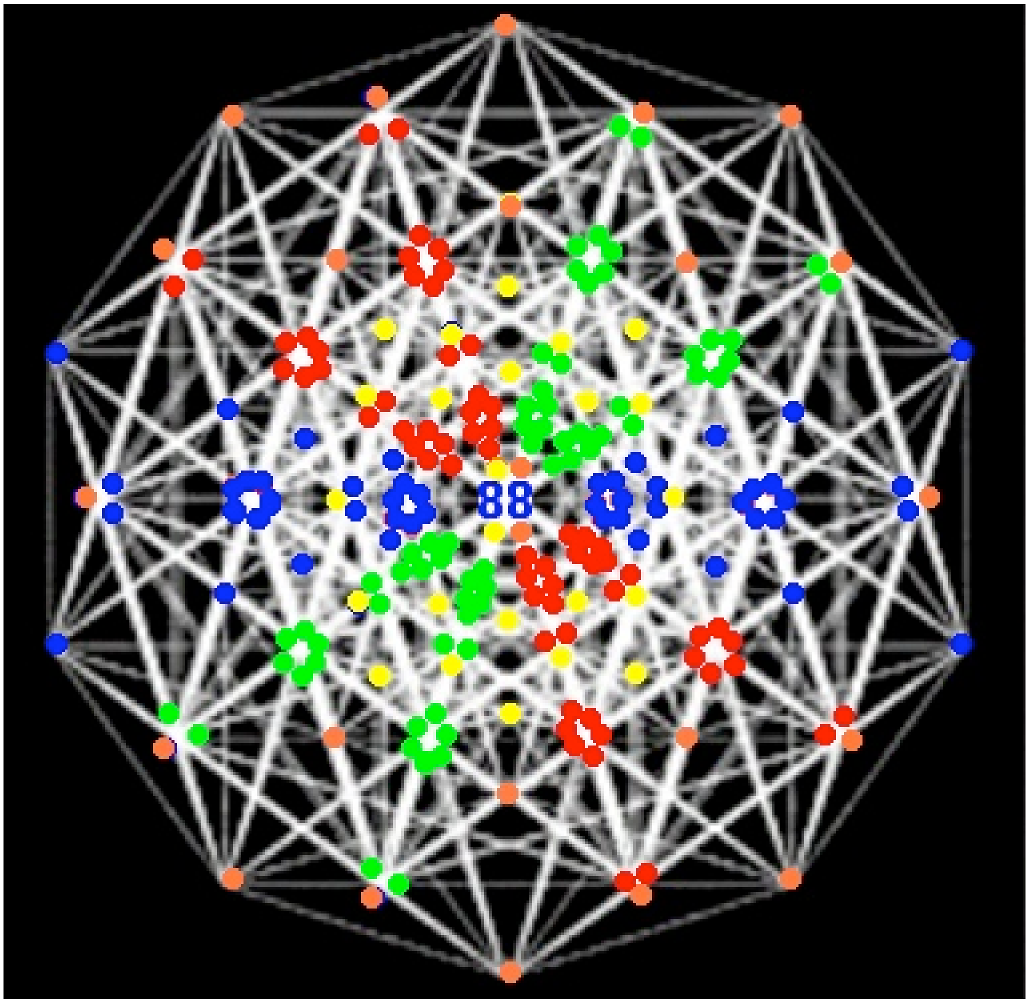
D5 is a Lie subalgebra of the E6 Lie subalgebra of the E8 Lie algebra,
with 24 of the 40 representing the Root Vectors of a D4 containing a
D3 Conformal Group Lie subalgebra for MacDowell-Mansouri Gravity
and the 40-24 = 16 representing a Complex 8-dim Kaluza-Klein Spacetime
whose 8-Real-dim Shilov Boundary is
a Kaluza-Klein Physical Spacetime plus Internal Symmetry Space.

The 24-12 = 12 Root Vectors of D4 not in the D3 Conformal Group
represent the 12 generators of S(U(3)xU(2)) as to which John Baez said in his notes on Bertram Kostant's U. C. Riverside 2008 lecture:
"…The gauge group of the Standard model is usually said to be
SU(3)xSU(2)xU(1), but this group has a Z/6 subgroup that acts trivially on all known particles. The quotient (SU(3)xSU(2)xU(1))/(Z/6) is isomorphic to

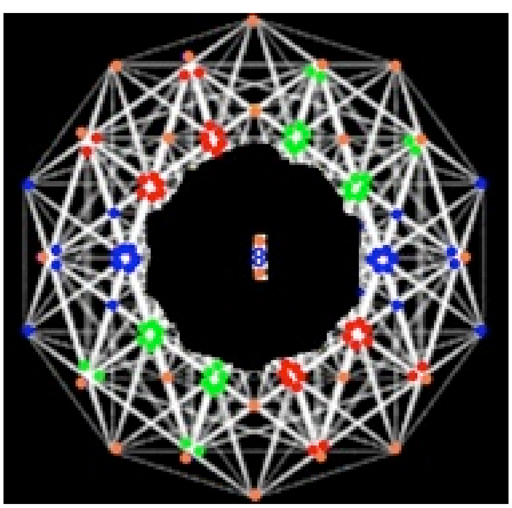
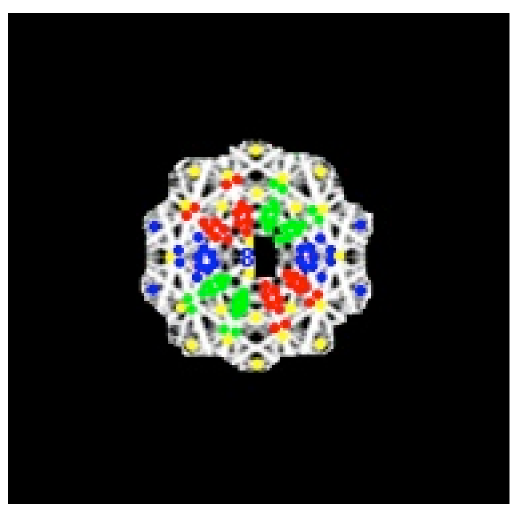
S(U(3)xU(2)) - that is, the subgroup of SU(5) consisting of block diagonal matrices with a 3x3 block and a 2x2 block.
So … S(U(3)xU(2)) …
could be called the "true" gauge group of the Standard Model. …".

Color-code the Decagon Projection vertices by:
Orange/Yellow for 24+24 = 48 for the two copies of D4 in D8 in E8,
each D4 giving Gauge Bosons for Gravity and the Standard Model,
and the Yellow D4 being inside D5 which has 24+16 = 40 Root Vectors;
Blue for 16 for Complex-8-dim-Spacetime of D5;
Blue for 6x8 = 48 for 8-dim Spacetime components with respect to the 3 colors and 3 anticolors of the SU(3) Color Force, which, combined with the two Complex components of the 16 Blue inside D5, give all 8 Octonionic components of 8-dim Spacetime;
Red for 8x8 = 64 for the 8 Octonionic components of the 8 First-Generation Fundamental Fermion Particles;
Green for 8x8 = 64 for the 8 Octonionic components of the 8 First-Generation Fundamental Fermion AntiParticles.

Then, the 48 + 200 breakdown 248-dim E8 described above by John Baez
gives a 40 + 200 breakdown of the 240 Root Vector Vertices of E8,
which in turn can be broken down into
an inner set of 20 + 100 = 120 = 10+(2+8) + 12 + 24+32+32
(the (2+8) being a Central 10 at the zero-radius center of the Decagon Projection,
the 10 and 2 and 12 being the 24 of a D4,
the 8 and 24 being 32 of 64 Octonionic components of 8 Spacetime Dimensions,
a 32 being 32 of 64 Octonionic components of 8 Fermion Particles,
the other 32 being 32 of 64 Octonionic components of 8 Fermion Particles)
and
an outer set of 20 + 100 = 120 = 10+(2+8) + 12 + 24+32+32
(the (2+8) being a Central 10 at the zero-radius center of the Decagon Projection,
the 10 and 2 and 12 being the 24 of a D4,
the 8 and 24 being 32 of 64 Octonionic components of 8 Spacetime Dimensions,
a 32 being 32 of 64 Octonionic components of 8 Fermion Particles,
the other 32 being 32 of 64 Octonionic components of 8 Fermion Particles)
with
the Outer 120 vertices being the Inner 120 dilated by the Golden Ratio,
and the mapping between the Inner 120 and the Outer 120 corresponding to
the {-1,+1} of the Electroweak U(2) Charges.



The Inner 120 and Outer 120 look like

 and 

E8 and Exotic 4-dim Physical Spacetime

Frank Dodd (Tony) Smith, Jr. 2008

E8 physics is based on the structure of the 248-dim E8 Lie algebra
which in turn is based on the 120-dim D8 bivector a 128-dim D8 half-spinor parts
of the Cl(16) real Clifford algebra, which is the tensor product Cl(8) (x) Cl(8)
of two 256-dim Cl(8) Clifford algebras.
Due to the 8-periodicity of real Clifford algebras,
any real Clifford algebra, no matter how large, is part of a tensor product
Cl(8) (x) ... (x) Cl(8) and therefore of Cl(16) (x) ... (x) Cl(16)
Taking the completion of the union of all such tensor products produces
a generalized hyperfinite II1 von Neumann algebra factor
with infinite Clifford algebra structure, denoted here by Cl(R^oo) = T ,
and it in turn gives an Algebraic Quantum Field Theory (AQFT) with E8 structure.
Here, the relation of E8 AQFT based on T = Cl(R^oo) to exotic structure on 4-dimensional physical spacetime is described.

Exotic R4 and Physics

Torsten Asselmeyer-Maluga and Helge Rose, in gr-qc/0511089 , said: "... there is only one differential structure of any manifold of dimension smaller than four. For all manifolds larger than four dimensions there is only a finite number of possible differential structures ...

n	1	2	3	4	5	6	7	8	9	10	11
#Diff_n	1	1	1	oo	1	1	28	2	8	6	992

In dimension four there is a countable number of differential structures on most compact four-manifolds and an uncountable number for most non-compact four-manifolds. ...".

Torsten Asselmeyer-Maluga and Carl H. Brans, in their book "Exotic Smoothness and Physics" (World 2007) said: "... Einstein's theory ... of general relativity ... requires a given differential structure on a 4-manifold to express the field equations describing the gravitational field.
From the beginning Einstein questioned the need to find a separate source for gravitation which ultimately turned out to be the stress-energy tensor of the system ... So we now look at the singularities associated with the change of differentiable structures as possible "sources" ...

Choose two homeomorphic 4-manifolds M and M' with different differential structures ...[and]... a Lorentzian structure ... S x [0,1] . We call the coordinaties of S space-like coordinates and the coordinate of [0,1] time-like coordinates ...[and]... assume that this splitting induces a foliation of the whole 4-manifold ...
the change of the differential structure of M (leading to M') is given by a local modification of the 3-manifold S ... this local modification is essentially defined on a 2-manifold ...[for which]... we obtain the new [2-dimensional] metric ...[and]... Extend ... to obtain the new [4-dimensional] metric ...[from which]... we obtain ... the Ricci tensor ...[and]... the scalar curvature ...[and]... the source of the gravitational field .. a singular energy-monentum tensor. The conservation law (with singularities) for this tensor follows from its construction as an Einstein tensor. ...
... there are uncountably many exotic smooth structures on R4 ... it is possible to divide the set of exotic R4's into two classes according to whether or not they can be embedded smoothly in compact subsets of standard R4:
large ... exotic ... non-existence or embedding ... into the standard smooth 4-sphere S4 (or into the standard R4) ... the exoticness is "located" at the end of the space which means in some sense the neighborhood of the infinity ...
and
small ... exotic ... ribbon R4's can be embedded into the standard smooth 4-sphere S4 (or into the standard R4) ... the exoticness is in some sense "localized" in the interior of the interior of R4 . Thus, this is a candidate of an exotic R4 which can be used as coordinate patch. ...".

Large Exotic Structures from E8 Intersection Form:

Torsten Asselmeyer-Maluga and Helge Rose, in gr-qc/0609004, said: "... In the case of topological, closed, and simply connected 4-manifolds,
every quadratic symmetric matrix may arise as an intersection form,
while in the case of differentiable manifolds, only two types of intersection forms can arise :
1) Q = +/- 1 (+) +/- 1 (+) ... (+) +/- 1
the intersection form is diagonal.
2) Q = nE8 (+) mH
with the 8x8 Matrix E8 (the Cartan matrix of the exceptional Lie algebra E8)
and the hyperbolic matrix H = Q(S2xS2).
Only the second case is non-trivial and topologically interesting. This is the case particularly for manifolds with the intersection form E8:
The topology of the corresponding 4-manifold is relatively complicated,
and the boundary of the latter is the Poincare sphere S3 / I* ...".

Alexandru Scorpan, in "The Wild World of 4-Manifolds" (AMS 2005), said: "... the E8-matrix [is]... the intersection form of [the 4-dimensional]... manifold PE8 ... PE8 contains eight spheres,
each with self-intersection -2 and intersecting the other spheres either 0 or +1
... the boundary of PE8 ... is the Poincare homology sphere ... sometimes called the dodecahedral space ...
Reversing orientation ... the Poincare homology sphere ... is still a homology sphere ...
[it]... must bound some contractible topological 4-manifold, a fake 4-ball D ...
Then we can glue PE8 and D along their common boundary ... and thus obtain a ... simply connected ... closed manifold ME8 = PE8 u D ...
known as the E8-manifold ... the E8-manifold, ME8 [is]... non-smoothable ...
Combining Donaldson's theorem ... Rokhlin's theorem ... Freedman's classification of topological manifolds ... and Serre's algebraic classification of forms yields
... Every smooth simply-connected 4-manifold is homeomorphic to either
m CP2 # n CP2bar or # +/- m ME8 # n S2xS2 .
However, since many of the # +/- m ME8 # n S2xS2 's are non-smoothable, this last statement is somewhat unsatisfactory ...
If ... the ... open ... 11/8 - Conjecture [that]... we must have ... at least three H ;s for every couple of E8 's in ... Q = (+) +/- m E8 (+) n H ... were true,
an immediate consequence would be:
Every smooth simply-connected 4-manifold is homeomorphic to either of
m CP2 # n CP2bar or # +/- m K3 # n S2xS2
...[where]... the intersection form of K3 is (+) (-2) E8 (+) 3 H ...".

Torsten Asselmeyer-Maluga and Carl H. Brans, in their book "Exotic Smoothness and Physics" (World 2007) said: "... topological manifolds corresponding to ... intersection form ... E8 cannot carry any smooth structure ... Because of the fact the PL = DIFF in dimension 4, this manifold also admits no combinatorial structure ...
Gompf ... produce[d] a 2-parameter family of uncountably many exotic R4...
the space R_0,0 is the standard R4 ...
Gompf conjectured .. that the space R_oo,oo is diffeomorphic to the universal space U ...[that]... contains every smoothing of R4 embedded within it. ...".

Small Exotic Structures and T = Cl(R^oo):

Torsten Asselmeyer-Maluga and Carl H. Brans, in their book "Exotic Smoothness and Physics" (World 2007) said: "... the information about the differential structure

can be localized into a contractible 4-dimensional submanifold, called an Akbulut cork.
The difference between two non-diffeomorphic differential structures is encoded in the non-trivial h-cobordism between these contractible submanifolds.
... the non-triviality of the h-cobordism is related to the existence of a Casson handle defining the h-cobordism.
The set of all Casson handles can be described by a binary tree, i.e., every path in that tree defines a specific Casson handle.
For the smoothness change, we need a pair of Casson handles or a pair of paths in the tree.
... a pair of paths has the structure of an algebra T as was pointed out by Ocneanu. ...
Algebraic methods can be used to study T. ...
In some sense we can say that the DIFF difference between the two manifolds M , M' is given by the non-canceling 2-/3-handle pair represented by a non-trivial Casson handle. ...
Every Casson handle is determined by its 6th [actually, 4th] Casson tower

> [Michael H. Freedman and Laurence R. Taylor, in "A Universal Smoothing of Four-Space" (J. Differential Geometry 24 (1986) 69-78), say:
> "... any Casson handles may always be imbedded in a ... 4-stage ... tower. The latter are determined by ...[eight]... positive countable integers (the number of +/- kinks at each stage) and are therefore countable.
> Thus, any Casson handle may be trimmed down to a fixed representative ..contained in its first ...[four]... stages. ...".]

and the two factors express the self-intersections in the tower.
... a Casson handle can be represented by a labeled finitely-branching tree Q with base point * , having all edge paths infinitely extendable away from * ...
this ... algebra T ... can be related to the Clifford algebra of the infinite Euclidean space ...[denoted by Cl(R^oo)]...
this ... algebra T ... is ... a factor II1 ... algebra with finite valued trace ... given by the Clifford algebra on R^oo ...[Cl(R^oo)]...".

Note that the 4-stage Casson towers are determined by 8 integers, corresponding to the rank of E8 and the dimensionality of the vector space of Cl(8).

Exotic R4 (Small and Large) and E8 AQFT:

The 4-stage Casson towers of Small Exotic R4 give T = Cl(R^oo), that by the 8-periodicity of real Clifford algebras, is the completion of the union of all tensor products Cl(8) (x) ...(x) Cl(8) and therefore includes all tensor products

Cl(16) (x) ...(x) Cl(16) with each Cl(16) containing an E8.
Therefore, Small Exotic R4 give a generalized hyperfinite II1 von Neumann algebra factor which in turn gives **an Algebraic Quantum Field Theory (AQFT) with E8 structure**.

The Large Exotic R4 of the form # m CP2 # n CP2bar
are related to the 8-dimensional M4 x CP2 Kaluza-Klein spacetime structure of E8 physics at energies low enough for a preferred quaternionic structure to freeze out.
The CP2 internal symmetry space structure is related to the exotic R4 structure of the M4 physical spacetime by the CP2 and/or CP2bar of the Large Exotic R4 of the form # m CP2 # n CP2bar

Since the K3 intersection form is (+) (-2) E8 (+) 3 H
an E8 intersection form lives inside each Large Exotic R4 of the form
 # +/- m K3 # n S2xS2

Torsten Asselmeyer-Maluga and Carl H. Brans, in their book "Exotic Smoothness and Physics" (World 2007) said: "... the smoothness properties of ... the base manifold, M , ... can be gleaned from a study of ... the moduli space of connections, M1 , ...[which]... is smoothly collared into ... M ...

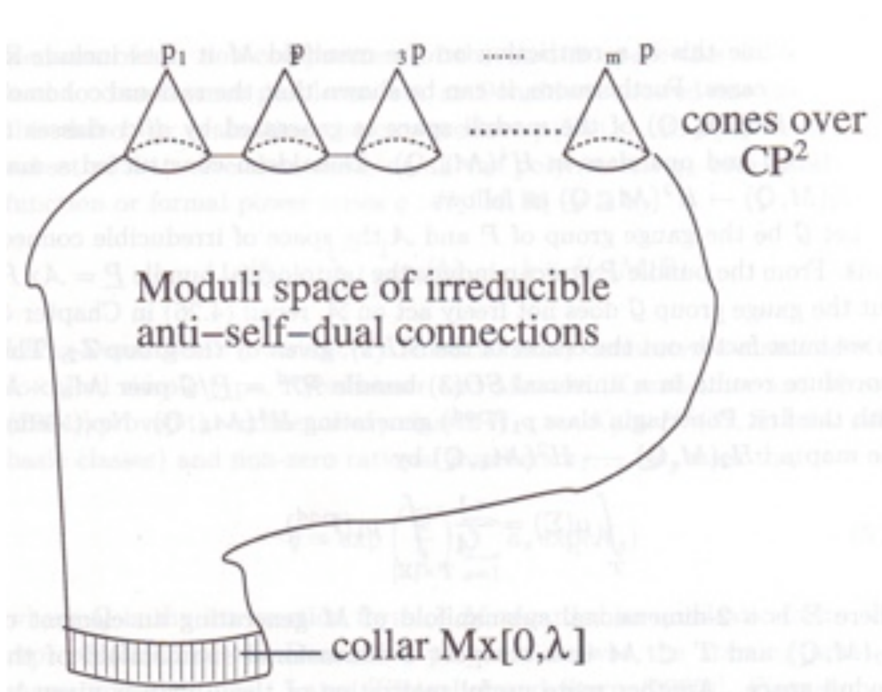


Fig. 5.1 Structure of the moduli space $\mathcal{M}_1$

...

... Donaldson's theorem ...[is based on]... the structure of the moduli space ... Formally ... moduli space = A / G ...[where]]... A is the set of structures and G is the group ...[and].. moduli space ...[is]... equivalence classes of such structures under ... group action ...
the space of connections A is an affffine space and thus contractible. By dividing out the gauge group G we obtain a topologically non-trivial space A / G , the moduli space. This moduli space can be seen as a base space of a principal bundle A -> A / G with structure group G .
Because of the contractibility of the space A , the moduli space is the classifying space BG = A / G of the gauge group G , i.e., any principal bundle over M with structure group G can be classified by the homotopy classes [M , BG] .
... Consider a Yang-Mills theory with respect to a compact Lie group G over a compact simply-connected 4-manifold M . Thus we are looking at connections on some G principal fiber bundle P over this manifold. The field strength of the Yang-Mills theory F is the curvature of P while the connection A is the gauge potential.
... for a large class of interesting groups ... and related Yang-Mills equations ... the solution space (moduli space) of the (anti)-self-dual equations ... has the structure of a smooth finite dimensional manifold, at least locally. ...
If we consider the parameterized space of all connections, the irreducible connections generate a smooth manifold part, with the reducible ones singularities where the dimension is lower ...
Atiyah, Hitchin and Singer ... showed that this moduli space Mk ... of irreducible, anti-self-dual connections with second Chern number k ... admits a smooth manifold structure. The dimension is given by the following table wiht respect to the compact group G , the compact 4-manifold M = S4 , the second Chern number k and the irreducibility condition.

G	dim Mk	Irreducibility condition
SU(N)	4Nk – N^2 – 1	k >= N/2
Spin(N)	4(N-2)k – N(N-1)/2	k >= N/4 , N >= 7
Sp(N)	4(N+1)k – N(2N+1)	k >= N
E6	48k – 78	k >= 3
E7	72k – 133	k >= 3
E8	120k – 248	k >= 3
F4	36k – 52	k >= 3
G2	16k – 14	k >= 2

... a result of Taubes ... leads to the extension of the formula to nearly all interesting smooth 4-manifolds. In particular, the results obtained for dim Mk using M = S4 agree for those for which M is an arbitrary 1-connected manifold as long as it has a positive definite intersection form ...".

Note that for k = 3 , the E8 moduli space has dimension 120x3 - 248 = 112.

What sort of E8 structures have dimensionality 112 ?

112 of the 240 root vectors of E8 correspond to the 112 root vectors of the D8 Lie algebra, with the remaining 240 - 112 = 128 corresponding to half-spinors of D8 and to the Type EVIII rank 8 symmetric space E8 / Spin(16) = (OxO)P2.
The set of (QxO)P2 in (OxO)P2 is the type EIX rank 4 symmetric space E8 / E7xSU(2) that has 248 - 133 - 3 = 112 dimensions.
The 112 = 4x28 dimensions correspond to a quaternification of a 28-dimensional Jordan algebra J4(Q).

J4(Q) contains the traceless 28 - 1 = 27 dimensional part J4(Q)o.
J4(Q)o has structure similar to that of the 27-dimensional exceptional Jordan algebra J3(O).
J3(O) contains a traceless 27 - 1 = 26-dimensional part J3(O)o.
J3(O)o that is the basis for E6 physics of strings-as-world-lines with the structure of 26-dimensional bosonic string theory, with orbifold structure for fermions.

Further: 27-dimensional J3(O) has as its automorphism group F4.

E6 is the automorphism group of the 56-dimensional Freudenthal algebra Fr3(O). From Boris Rosenfeld's book "Geometry of Lie Groups" (Kluwer (1997) at pages 91 and 56), it seems to me that 56-dim Fr3(O) with automorphisms E6 should be written as a 2x2 Zorn-type array:

$$\begin{array}{cc} 1 & \begin{array}{ccc} 1 & 8 & 8 \\ * & 1 & 8 \\ * & * & 1 \end{array} \\ \\ \begin{array}{ccc} 1 & 8 & 8 \\ * & 1 & 8 \\ * & * & 1 \end{array} & 1 \end{array}$$

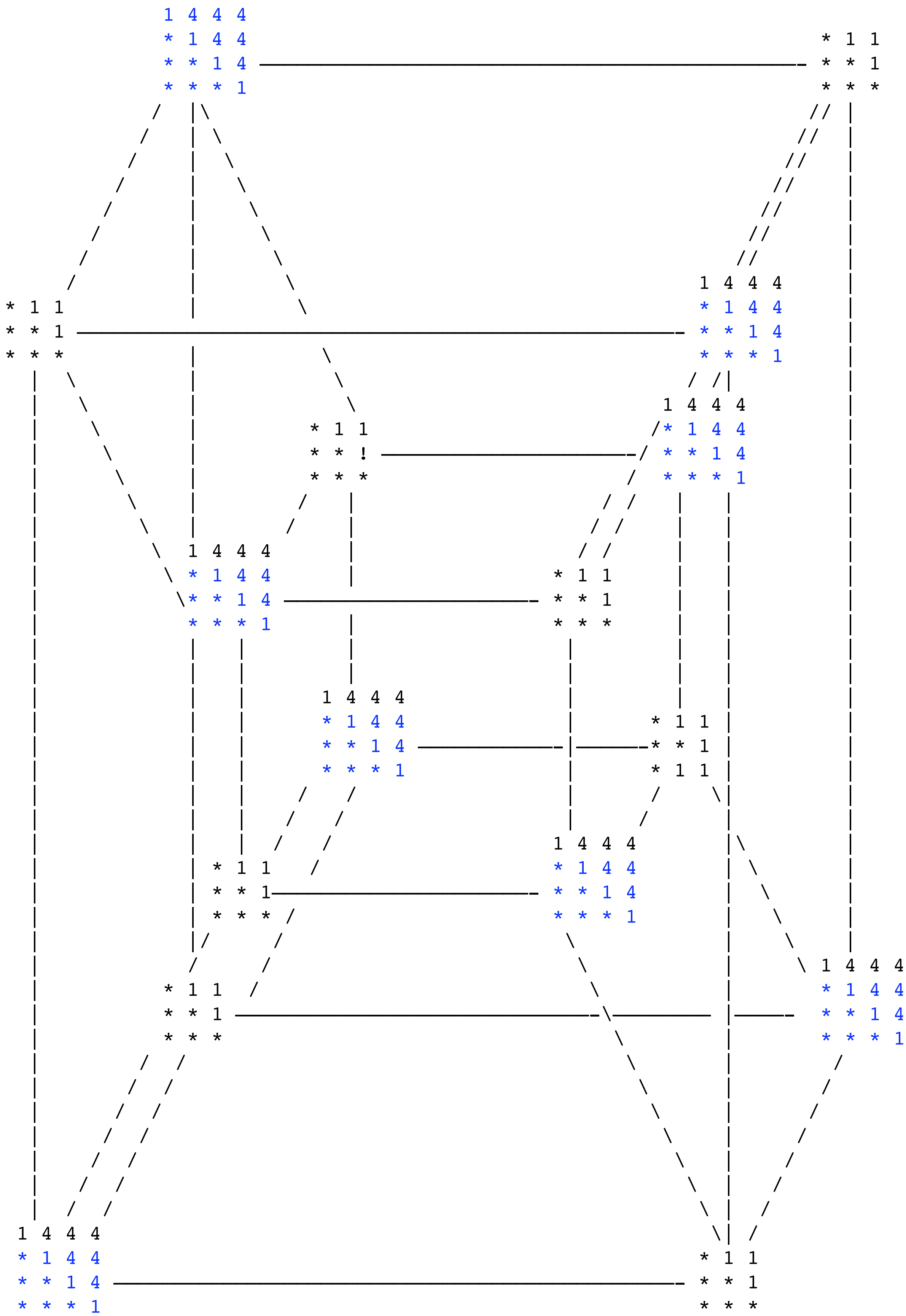
where the 1 are real numbers and the 8 are octonions.

E7 is the automorphism group of the 112-dimensional Brown algebra Br3(O) which has twice the dimensionality of the 56-dimensional Fr3(O). Br3(O) is not a binary algebra, but is a ternary algebra. If you try to “think like a Vegan”,you might see that E7 and E8 might be represented as higher-dim arrays, such as 2x2x2 , instead of the 2x2 Zorn array of Fr3(O). When you go to a 3-dim 2x2x2 array for the 112-dim Brown "algebra-like thing" corresponding to E7, you get a picture like this:

```
                          1 8 8
          1 ------------- * 1 8
        /  |              * * 1
       /   |            /   |
      /    |           /    |
     /     |          /     |
1 8 8      |         /      |
* 1 8 ------------ 1        |
* * 1      |                |
  |        |       |        |
  |        |       |        |
  |        |       |        |
  |      1 8 8     |
  |      * 1 8 ----|------- 1
  |      * * 1     |       /
  |     /          |      /
  |    /           |     /
  |   /            |    /
    /            1 8 8
  1 ------------ * 1 8
                 * * 1
```

133-dimensional E7 plus 112-dimensional Br3(O) plus a Quaternionic 3-dimensional SU(2) combine to form 133+112+3 = 248-dimensional E8.

E8 has its adjoint 248-dimenional representation as its lowest dimensional non-trivial representation, so E8 is self-automorphic, and is the final step in the E series of Lie algebras. Since E8 / D8 = 248 - 120 = 128-dim (OxO)P2
you might say E8 = D8 + (OxO)P2 and also that
248-dim E8 = 8 x 31 = 8 x 28 + 8 x 3 = 8 x J4(Q) + 8 x SU(2) looks like a tesseract


1 4 4 4
* 1 4 4
* * 1 4
* * * 1
* 1 1
* * 1
* * *
1 4 4 4
* 1 4 4
* * 1 4
* * * 1
* 1 1
* * 1
* * *
1 4 4 4
* 1 4 4
* * 1 4
* * * 1
* 1 1
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* * *
1 4 4 4
* 1 4 4
* * 1 4
* * * 1
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1 4 4 4
* 1 4 4
* * 1 4
* * * 1
* 1 1
* * 1
* * *

Note that half of the J4(Q) correspond to 4 x 28 = 112 dimensions
while
the other half plus 7 of the SU(2) correspond to 4 x 28 + 7 x 3 = 133 dimensions
and
the remaining SU(2 corresponds to 3 dimensions
in E8 / E7xSU(2) = (QxO)P2 with 248-133-3 = 112 dimensions.

If the full E8 = 8 x J4(Q) + 8 x SU(2) = D8 + (OxO)P2 array is seen as an 8-dimensional E8 lattice then the high-order Quantum Many-Worlds look like a 1+7 dimensional Feynman Checkerboard.
Since there are 7 independent E8 lattices, each corresponding to an imaginary Octonion, the 1+7 dimensional Feynman Checkerboard should be regarded as a superpositiion of all 7 of them. Such superpositions have been described, using the term “brocade”, by Robert P. C. de Marrais in his papers such as arxiv 0804.3416 in which he said:
“... a brocade ...[is]... a 7-in-1 representation ...[of]... 7 Sedenion box-kites ... seen as collected on the frame of just one ...”.

Murat Gunaydin, in hep-th/0008063, describes “... quasiconformal nonlinear realization of E8 on a space of 57 dimensions. This space may be viewed as the quotient of E8 by its maximal parabolic subgroup; there is no Jordan algebra directly associated with it, but it can be related to a certain Freudenthal triple system which itself is associated with the "split" exceptional Jordan algebra J3(OS) where OS denote the split real form of the octonions O .It furthermore admits an E7 invariant norm form N4 , which gets multiplied by a (coordinate dependent) factor under the nonlinearly realized "special conformal" transformations. Therefore the light cone, defined by the condition N4 = 0, is actually invariant under the full E8, which thus plays the role of a generalized conformal group. ... results are based on the following five graded decomposition of E8 with respect to its E7 x D subgroup ... with the one-dimensional group D consisting of dilatations ...

```
g(-2)    g(-1)    g(0)    g(1)    g(2)
  1        56     133+1     56      1
```

... D itself is part of an SL(2; R) group, and the above decomposition thus corresponds to the decomposition ... of E8 under its subgroup E7 x SL(2;R) ...".

To see how E8 intersection forms of those Large Exotic R4 are related to the E8 structure of E8 AQFT physics,
look at the **E8 Cartan Matrix** Structure
and then also look at the structure of **hyperfinite II1 von Neumann factor** algebra used for AQFT in E8 physics.

E8 Cartan Matrix Structure:

E8 has an 8x8 Cartan matrix

$$
\begin{matrix}
2 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\
-1 & 0 & 2 & -1 & 0 & 0 & 0 & 0 \\
0 & -1 & -1 & 2 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 2 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & -1 & 2 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & 2 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 2
\end{matrix}
$$

and Coxeter-Dynkin diagram

```
*-*-*-*-*-*-*
        |
        *
```

John Baez, in his week 164, said "... make a model of this [E8 Coxeter-Dynkin] diagram by linking together 8 rings:

```
   /\   /\   /\   /\   /\   /\   /\
  /  \ /  \ /  \ /  \ /  \ /  \ /  \
 /    \    \    \    \    \    \    \
/    / \  / \  / \  / \  / \  / \    \
\    \ /  \ /  \ /  \ /  \ /  \ /    /
 \    \    \    \    \ /\ \    \    /
  \  / \  / \  / \  / \  \ \  / \  /
   \/   \/   \/   \/ / \/ \ \/   \/
                    /      \
                    \      /
                     \    /
                      \  /
                       \/
```

Imagine this model as living in S3.
Next, hollow out all these rings: actually delete the portion of space that lies inside them! We now have a 3-manifold M whose boundary dM consists of 8 connected components, each a torus. Of course, a solid torus also has a torus as its boundary. So attach solid tori to each of these 8 components of dM, but do it via this attaching map:

(x,y) -> (y,-x+2y)

where x and y are the obvious coordinates on the torus, numbers between 0 and 2pi, and we do the arithmetic mod 2pi. We now have a new 3-manifold without boundary... and this is the Poincare homology sphere.
... The Poincare homology sphere is actually the boundary of a 4-manifold, and it's not hard to say what this 4-manifold is. I just gave you a recipe for cutting out 8 solid tori from the 3-sphere and gluing them back in with a twist. Suppose we think of 3-sphere as the boundary of the 4-disk D4, and think of each solid torus as part of the boundary of a copy of D2 x D2, using the fact that

d(D2 x D2) = S1 x D2 + D2 x S1.

Then the same recipe can be seen as instructions for gluing 8 copies of D2 x D2 to the 4-ball along part of their boundary, getting a new 4-manifold with boundary. If you ponder it, you'll see that the boundary of this 4-manifold is the Poincare homology 3-sphere. ...

the whole story generalizes to higher dimensions! ... start with an analogous pattern of 8 n-spheres linked in the (2n+1)-sphere. Do all the same stuff, boosting the dimensions appropriately... and you'll get an interesting (2n+1)-dimensional manifold dM which is the boundary of a (2n+2)-dimensional manifold M.

When n is *odd* and greater than 1, this manifold dM is actually an "exotic sphere". In other words, it's homeomorphic but not diffeomorphic to the usual sphere of dimension 2n+1. ...
For example: In dimension 7 ... there are 28 exotic spheres ... (up to orientation-preserving diffeomorphism), and they are all connected sums of the exotic 7-sphere dM formed by the above construction. ...”.

I think that is somewhat analogous to

the 28 differential-exotic structures of the 7-sphere S7
(in E8 physics, S7 represents the spatial part of 8-dim high-energy unified spacetime)
corresponding to
the 28 generators of the D4 Lie algebra, two copies of which live inside E8
(in E8 physics, the D4 give Gravity and the Standard Model).

The E8 diagram/matrix also describes an intersection form, as Alexandru Scorpan said in his book "The Wild World of 4-Manifolds" (AMS 2005): "... the E8-matrix ...[is]... the intersection form of ... [the 4-dimensional]... manifold PE8 ... PE8 contains eight spheres,
each with self-intersection -2 and intersecting the other spheres either 0 or +1 ...".
Let the 8 spheres of PE8 correspond to the 8 basis elements of the E8 Cartan matrix. Since the entire E8 Lie algebra can be constructed from its Cartan matrix
(see for example section 21.3 of "Representation Theory"
by William Fulton and Joe Harris (Springer-Verlag 1991)
the 4-manifold PE8 contains the Lie algebra E8 (and therefore the Lie group E8).

Roughly, I visualize it as:
the intersections of the 8 spheres of PE8
correspond to
the reflections in the 8-dim E8 root vector space.

hyperfinite II1 von Neumann factor algebra

Vaughan F. R. Jones, in his review of the book Quantum symmetries on operator algebras, by D. Evans and Y. Kawahigashi, Oxford Univ. Press, New York, 1998, Bull. (N.S.) Am. Math. Soc., Volume 38, Number 3, Pages 369-377, said:
"... The "algebraic quantum field theory" of Haag, Kastler and others ... is an attempt to approach quantum field theory by seeing what constraints are imposed on the underlying operator algebras by general physical principles such as relativistic invariance and positivity of the energy. A von Neumann algebra of "localised observables" is postulated for each bounded region of space-time. Causality implies that these von Neumann algebras commute with each other if no physical signal can travel between the regions in which they are localised. The algebras act simultaneously on some Hilbert space which carries a unitary representation of the Poincare (=Lorentz plus 4-d translations) group. The amount of structure that can be deduced from this

data is quite remarkable. ... Just as remarkably, more than one type II1 factor (up to isomorphism) was constructed ... and ... uncountably many were shown to exist and the classification of factors is not at all straightforward. That is the bad news.
Now the good news. A von Neumannn algebra is called hyperfinite if it contains an increasing dense sequence of finite dimensional *-subalgebras ... it was shown that there is a unique hyperfinite II1 factor. (It can be realised as U(G) where G is the group of all finite permutations of [the natural numbers] N .) ...".

Irving Segal, in his review (Bull. AMS 33 (1996) 459-465) of the book Noncommutative Geometry (Academic Press 1994) by Alain Connes, said: "... The W*-algebras ... investigated by Murray and von Neumann (henceforth, MvN) ... and ... C*-algebras ... investigated by the Gelfand school were formally similar and in fact essentially identical in the finite-dimensional case. But they differ in topology in the infinite-dimensional case, which ... in fact ... rather fundamentally changes the character of the theory. The algebras of MvN were closed in the weak operator topology, while those of Gelfand were closed in the uniform (operator bound norm) topology. ... **von Neumann ... was particularly fond of the "approximately finite" factor of type II1, which in fact plays a basic role in the representation of fermion fields**. ... **the Clifford algebra over a Hilbert space ... is the simplest of the type II W*-algebras** ... the Clifford algebra is a central simple algebra that is altogether different from the algebra of all bounded operators on Hilbert space ... this algebra plays a fundamental role in the analysis of free fermionic quantum fields. ... it is the algebra generated by the canonical fermionic Q's ... The Clifford algebra is the simplest of the factors that are direct limits of matrix algebras ...".

The E8 physics model uses a Cl(8) generalization of the conventional hyperfinite II1 von Neumann algebra factor, which should describe (to paraphrase Vaughan F. R. Jones) all finite permutations of Clifford Algebra State Patterns.
By taking the limit as n goes to infinity of the real-Clifford-periodicity tensor factorization of order 8
Cl(8n,R) = Cl(8,R) x ...(n times tensor)... x Cl(8,R)
the generalized hyperfinite II1 von Neumann algebra R can be denoted as the real Clifford algebra Cl(infinity,R) whose half-spinors are sqrt(2^(infinity))-dimensional. In other words, since the halfspinors of Cl(2n,R) are 2^(n-1)-dimensional, the dimension of the full spinors grows exponentially with the dimension of the vector space of the Clifford algebra.

It seems to me likely that the subfactor structure of the generalized hyperfinite II1 von Neumann factor algebra based on real Clifford algebra 8-periodicity and used for an AQFT for E8 physics
might have substantial similarities to that of the standard hyperfinite II1 von Neumann factor algebra based on complex Clifford algebra 2-periodicity.

Adrian Ocneanu, in his article Quantized Groups, String Algebras and Galois Theory for Algebras, at pages 119-172 in Operator Algebras and Applications, Volume 2, edited by David E. Evans and Masamichi Takesaki (Cambridge 1988), said:
"... We introduce a Galois type invariant for the position of s subalgebra inside an algebra, called a paragroup, which has a group-like structure. Paragroups are the natural quantization of (finite) groups. ... harmonic analysis for the paragroup corresponding to the group Z2 is done in the Ising model ...

In paragroups the underlying set of a group is replaced by a graph, the group elements are substituted by strings on the graph and a geometrical connection stands for the composition law ... we can ... use as invariant the coupling system, which is similar to the duality coupling between an abelian group and its dual. ...

The algebra ... R, or the hyperfinite II1 factor ... also called ... the elementary von Neumann algebra ... is the weak closure of the Clifford algebra of the real separable Hilbert space, [and] is a factor ... which has very many symmetries ...

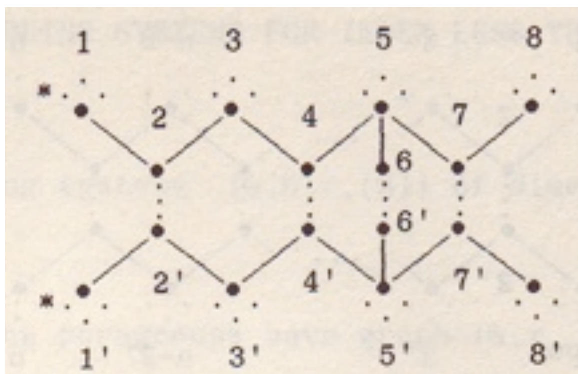
A ... theorem of Connes implies that any closed subalgebra of R which is a factor ... is isomorphic either to Matn(C) or to R itself. Thus any finite index subfactor N of R is isomorphic to R, and all the information in the inclusion N in R comes from the relative position of N in R and not from the structure of N. ... in our context this guarantees that the closure of all finite dimensional constructions done below will us back to R. ... for subfactors of finite Jones index, finite depth and scalar centralizer of ... R ...”.

Marta Asaeda, in math/0605318, said: “... For a subfactor N ⊂ M, the index value [M : N] belongs to the set

{ 4 cos^2 (pi / n) | n = 3, 4, 5 ... } u [4 , oo]

... subfactors with index less than 4 are completely classified by the Dynkin diagrams An , D2n , E6 , and E8 ... Popa ... gave a classification of subfactors ... of the hyperfinite II1 factor ... with index equal to 4 ...[in which case]... the dual principal graph of a subfactor is the same as the principal graph ...”.

Adrian Ocneanu, in his article in Operator Algebras and Applications, Volume 2 cited above, said: “... In index less than 4 ... the conjugacy classes are rigid: ... axioms eliminate one connection for each Dn and the pair of connections on E7 [because they are not geometrically flat]. Thus there is one subfactor for each diagram An, one for each diagram D2n, and a pair of opposite conjugate but nonconjugate subfactors for each diagram E6 and E8. ... there are two nonisomorphic coupling systems for ... the graph... E8 ... A vertex and its contragredient are joined by a dotted line ...



...".

...

Fractal Spacetime, Feynman Checkerboards, and Pure Spinors

by Frank D. (Tony) Smith, Jr. - 2009

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1 - Fractal Spacetime - page 440
The material on Fractal Spacetime is closely related to similar work by Carlos Castro

- a - Each individual Spatial dimension - page 440

- b - 8 Octonionic Spacetime dimensions - page 447

2 – Feynman Checkerboards – page 453

3 - Pure Spinors, and Lucas and Fibonacci Numbers - page 460
Pure Spinors are related to my E8 Physics model
(see www.tony5m17h.net/E8physicsbook.pdf)
which makes extensive use of Clifford Algebras in ways related to the work of Carlos Castro. The Lucas and Fibonacci Numbers are based on the Golden Ratio which (in line with the work of Carlos Castro) connects them to Fractal Spacetime.

Note – I wrote this while reading what little I could find for free on the web by M. S. El Naschie, because some web material written by John Baez et al called his work to my attention. Therefore, some of the topics are also topics about which M. S. El Naschie has written. However, my overall feeling is that his approaches to those topics may be substantially different from my work.

1 – Fractal Spacetime

a - Each individual Spatial dimension

Michael F. Barnsley, J. S. Geronimo, and A. N. Harrington, in an early draft of Comm. Math. Phys. 88 (1983) 479-501 that included some material omitted from the published version), said:

"… In this paper we consider the Julia set B_L for the mapping

T_L(z) = (z - L)^2

… of the complex plane into itself, where L is a parameter which may be real or complex …

With the notation T_L^0(z) = z and T_L^(n+1)(z) = T_L(T_L^n(z)) for n in {1,2,3, … }
B_L can be defined to be ... the closure of all repulsive k-cycles, k in {1,2,3, … }

…

notice … the critical values of L, at which occur such phenomena as the first appearance of k-cycles and the onset of ergodic behavior …

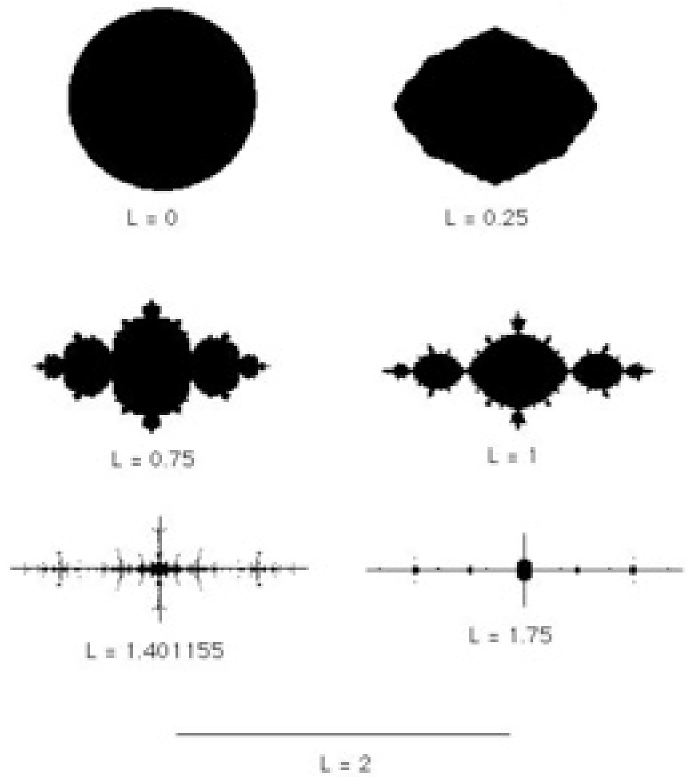
Our approach is to consider the set B*_L of formal objects which we call L-chains { L +/- sqrt(L +/- sqrt(L +/- … } where all half-infinite sequences of plus and minus signs are included, and where the branch cut is fixed, for example, on the negative real axis.

For L > 2 we … work out a one-to-one correspondence between the elements of B*_L and the points of B_L.

For $-1/4 \leqq L \leqq 2$ the correspondence is exhibited via the Bottcher equation and conformal mapping, and for some values of L we show only that almost all L-chains correspond to individual points in B_L.

… Mandlebrot, who … views B_L as an example of a fractal set, and also as an attractor for an appropriately defined (generalized) discrete dynamical system, based upon inverse mappings … has produces some beautiful pictures of B_L …".

I generated the following pictures:



Michael Barnsley in his book “Fractals Everywhere” (Academic 1988) said (in slightly different notation and context):
“… B_L is connected for all L in [0,2],
and totally disconnected when L > 2.
In the latter case B_L may be described as a “Cantor-like” set …

… or as a “dust”.

Further,
Barnsley, Geronimo, and Harrington said:
“… One reason why we first became interested in B_L was because it arose for L = 3 in the context of the Diophantine Moment Problem (D.M.P.) ...[which]... appeared in an attempt to predict the critical indices for Ising model lattice gases ... The ...[D.M.P.]... was completely solved for L < 4 and largely solved for L = 4 (leading, incidentally, to a novel resolution for a one-dimensional Ising model) …”.

Since the one-dimensional Ising model has been shown to by H. A. Gersch (Int. J. Theor. Phys. 20 (1981) 491) to be equivalent to the (1+1)-dimensional Feynman Checkerboard model of Quantum Physics, the Cantorian Fractal Structures of Barnsley, Geronimo,and Harrington usefully describe the Many-Worlds-Sum-Over-Histories Quantum Structure for each individual dimension of Spacetime.

As will be seen in more detail in section 1b of this paper,
the Cantorian Fractal Structure of each individual Spacetime dimension naturally extends to all 8 dimensions of Octonionic Spacetime and its Quaternionic (4+4)=8-dimensional Kaluza-Klein Spacetime with (1+3)=4-dimensional Physical Spacetime in which physics is realistically described by the (1+3)-dimensional HyperDiamond Feynman Checkerboard described in my paper at
CERN-CDS-EXT-2004-030
(also at www.tony5m17h.net/FckbUSGR.pdf)

With respect to more details for the case of one individual Spacetime dimension, Barnsley, Geronimo, and Harrington said:
"… Throughout this section we assume L in [2,oo) .. for which it is known that B_L lies entirely upon the real axis …
the operation of T_L on B_L …[is]… equivalent to that of the right-shift operator upon the set W of half-infinite Bernoulli sequences …
Lebesgue measure … of B_L … is zero when L > 2
… we construct the measure upon B_L, which is invariant and mixing under T_L, by means of a special sequence of approximating measures.
This measure is singular with respect to Lebesgue measure and has no purely atomic component. …
The approximating measures are related to a set of monic polynomials, orthogonal respect to the invariant measure. …
The polynomials are none other than the Tschebycheff polynomials when L = 2, and … they generalize the latter in a nontrivial way when L > 2.
an isomorphism of systems which relates the invariant to the uniform measure upon W … shows that the action of T_L upon B_L has entropy equal to ln2. …

we will sometimes use the "L-chain" notation

s(w) = L + e1 sqrt(L + e2 sqrt(L + e3 sqrt(…)))

… the Julia set for T_L(z) = (z – L)^2 , L in [2,oo) is precisely

B_L = { s(w) | w in W }

This follows from the fact that the Julia set is the set of all limit points of all finite order preimages of any point in the plane, with at most two exceptions.
…
the Hausdorff dimension of B_L is bounded above … for L = 5 …[by]… 0.564 …[which]… is good enough … to distinguish B_L from the classical ternary set of Cantor, whose Hausdorff dimension is ln2 / ln3 = 0.631
…
One way of characterizing the invariant measure S when
2 ≦ L < oo is by means of the associated set of monic polynomials.
We denote this set by { P_n(x)}_(n=-1)^oo
where P_(-1)(x) = 0.
For n ≧ 0, P_n(x) has degree n and the coefficient of x^n is unity.
The polynomials obey

INTEGRAL_I P_n(x) P_m(x) dS(x) = 0 for n =/= m

These polynomials provide an interesting generalizaton of the Tchebycheff polynomials { TCH_n(x) = cos(n cos^(-1)(x) }
to which … they must be related by

P_n(x) = 2 TCH_n((1/2) x - 1) when L = 2

…
the zeroes of TCH_2^n((1/2)x - 1) are precisely the set of numbers

2 +/- sqrt(2 +/- sqrt(2 +/- … +/- sqrt(2)…)
…[sqrt n times]…".

If you take the square root of the L-chain structure for L = 2 you get

sqrt(2 +/- sqrt(2 +/- sqrt(2 +/- sqrt(2 +/- …

which is closely related to the chain

G = sqrt(1 +/- sqrt(1 +/- sqrt(1 +/- sqrt(1 +/- …

If you square both sides of that equation you get

G^2 = 1 +/- sqrt(1 +/- sqrt(1 +/- sqrt(1 +/- …

which is equivalent to

G^2 = 1 + G or G^2 - G - 1 = 0

which is a quadratic equation with solutions

G = (1/2x1) (-(-1) +/- sqrt ((-1)^2 – 4x1x(-1)) =
= (1/2)(1 +/- sqrt(5)

Let G denote the + solution G = (1/2)(1 + sqrt(5)) = Golden Ratio
and
let g denote the - solution g = (1/2)(1 - sqrt(5))
so that –g, the inverse of G, is the Golden Mean 0.681… = G - 1.

(see The Golden Ratio (Broadway Books 2002) by Mario Livio)
Still further, Barnsley, Geronimo, and Harrington said:
"… We will say that a set of points is a Cantor set it it is compact, non-denumerable, and contains no intervals. A set is perfect if every element is a limit point of other members of the set and the set contains all of its limit points. … For 2 < L < oo,
B_L is a Cantor set with Lebesgue measure for zero.
For 2 ≦ L < oo, B_L is compact and perfect.
…
We define … distributions … by …
S_n(x) = (1/2^n) [No. of members of K_n which are less than x]
…[where]…
K_n = T_L^(-n)(L) , which consists of the 2^n real points
L +/- sqrt(L +/- sqrt(L +/- … +/- sqrt(L)…)
where there are n plus - or - minus signs …
For 2 ≦ L < oo the sequence { S_n(x) }_1^oo converges to a continuous distribution S(x), uniformly for x in R …
There is a unique Borel measure, which for economy of notation we denote by S, such that
S(c,d] = S(d) - S(c)
for all c < d in R. we denote the corresponding Borel measurable subsets of R by B so that (R,B,S) is a measure space. …
For L ≧ 2 the measure S in invariant under T_L.
…
When L = 2 , T_2(z) = (z - 2 ^2, and we have

dS(x) = 0 for x ≦ 0
= (1/pi)(dx / (sqrt(x (4 – x)))) for 0 < x < 4
= 0 for x > 4

[Note that S(x) for 0 < x < 4 is the arccosine measure.
To see that in standard form,
translate [0,4] to [-2,+2] changing x to z+2 to change dx / (sqrt(x (4 - x))) to
dz / (sqrt((2+z)(2-z))) = dz / (sqrt(4 - z^2))
and then contract [-2,+2] to [-1,+1] changing z to 2y
which changes dx / (sqrt(x (4 – x)))) and dz / (sqrt(4 - z^2)) to
2dy / (sqrt(4 - 4y^2)) = dy / (sqrt(1 - y^2))]

Let F denote the set of all Borel measurable subsets of B_L. Then {B_L,F,S,T_L) is a system … isomorphic to the system formed by the left-shift on W with the usual uniform measure. Consequently (B_L,F,S,T_L) is mixing with entropy ln2.
The system is also isomorphic to the one formed by z -> z^2 on the unit circle in C, with circular Lebesgue measure …[showing]… the connection between the system which exists when L ≧ 2 and that which exists when L = 0.
… since (B_L,F,S,T_L) is a mixing system, so is (B_L,F,S,T_L^n) for n in {1,2,3, …}.
Hence P_2^n(x) + L provides a mixing transformation on B_L with respect to S.
Shifting B_L to the left by subtracting L, and correspondingly adjusting the measure, this shows that each of the polynomials
P_2^n(x + L) provides a mixing transformation upon the shifted system. …".

So,
we see that the intervals containing the zeroes of Tchebycheff polynomials of degree 2^n form nth order Borel sets for B_2.
The corresponding Borel measure is the singular measure concentrated at the zeroes of the Tchebycheff polynomials of degree 2 taking the value 2^(-n) at each zero.
Note that T_2^n maps each nth order Borel set densely onto the whole set B_2 = [0,4].
T_2 acts as a Bernoulli shift operator for theTchebycheff measure system on B_2 and, as n becomes large, the Tchebycheff measure goes to the arccosine measure.

Define M_n as the lattice constructed from B_2 by identifying the nth order Borel sets of B_2, each with its Tchebycheff measure, as vertices of the lattice.
M_n is a 1-dimensional lattice with structure of the Tchebycheff measures on B2.
The fineness of the lattice M_n Is determined by the order n of the
Borel sets.

As the nth order Tchebycheff measure is a singular measure concentrated at zeroes of Tchebycheff polynomials of degree 2^n,
the lattice M_n has a natural singular measure that converges as n becomes large, or as lattice spacing becomes small, to the Tchebycheff measure that is isomorphic as a Bernoulli scheme to Lebesgue measure on the closed interval [0,4].

This lattice structure,
and its relation to the aforementioned “novel resolution for a one-dimensional Ising model”,
and the showing by H. A. Gersch (Int. J. Theor. Phys. 20 (1981) 491) that the one-dimensional Ising model is equivalent to the (1+1)-dimensional Feynman Checkerboard model of Quantum Physics,
show that
the Cantorian Fractal Structure of Spacetime naturally represents (1+1)-dimensional Feynman Checkerboard Quantum Physics for one dimension of Spacetime.

Note that Bernoulli Shifts can be seen as coming from Binary Decision Trees, similar to those that produce Markov Processes and Surreal Numbers, and that
Surreal Numbers can be a basis for the Feynman Checkerboard representation of the Many-Worlds of Sum-Over-Histories Quantum Field Theory.

b - 8 Octonionic Spacetime dimensions

The Cantorian Fractal Structure of each individual Spacetime dimension, which represents (1+1)-dimensional Feynman Checkerboard Quantum Physics, naturally extends to all 8 dimensions of Octonionic Spacetime and its Quaternionic (4+4)=8-dimensional Kaluza-Klein Spacetime with (1+3)=4-dimensional Physical Spacetime in which physics is realistically described by the (1+3)-dimensional HyperDiamond Feynman Checkerboard described in section 2 hereof and in my paper at CERN-CDS-EXT-2004-030
(also at www.tony5m17h.net/FckbUSGR.pdf)

Consider my E8 Physics model and the root vector decomposition E8 = H4 + H4, with each H4 containing a D4.

The gauge bosons of Gravity plus the Standard Model come from
the D4 + D4 inside E8 = H4 + H4.

There are 24 root vectors is each D4,
and since each D4 is rank 4, each D4 is 24+4 = 28-dimensional.

Each D4 produces a transformation group acting on the 8-dimensional Octonionic Spacetime (and, after freezing out of a preferred Quaternionic substructure, the (4+4)-dimensional Kaluza-Klein Spacetime) of my E8 Physics model,
so
the fundamental structure of E8 Physics and its (4+4)=8-dimensional Spacetime can be studied by looking at each D4.

The 28 generators of each D4 can be generated from
the 7 imaginary Octonions {i,j,k,e,ie,je,ke} that correspond to a basis for the unit Octonion sphere S7 in 8-dim Octonion space.

S7 is parallelizable, but due to the Non-Associativity of Octonions its 7 generators do not close to form a Lie algebra.

When you form Lie bracket operations with them, the 7 generators of S7 expand by producing 14 generators of the Lie algebra of the Octonion automorphism group G2 plus 7 generators of another S7, thus producing the 7+14+7 = 28-dimensional D4 Lie algebra.
Let { Oi | i = 1, … 7 } denote basis elements {i,j,k,e,ie,je,ke} of the Imaginary Octonions.

Since the full 28-dim D4 Lie algebra is generated from the original 7 generators of a parallelizable S7, you can describe E8 Physics Spacetime
by looking at each of the 7 Imaginary Octonion generators of S7
and, for each of them, at the (1+1)-dimensional spaces spanned by { 1, Oi }
where
{ Oi | i = 1, … 7 } denote basis elements {i,j,k,e,ie,je,ke} of the Imaginary Octonions.

Define iS7, for i = l, ..., 7, as the intersection of S7 with the
subspace of the octonions spanned by { 1, Oi } .
Then each of the subspaces iS7 = { 1, Oi } can be regarded as a one-dimensional Spacetime as discussed in section 1a .

Consider the map TL: O -> O defined on the Octonions by T_L(o) = (o - L)^2,
where L is a real number in [0,2] or in L > 2 including values arbitrarily close to 2 .

If L = 0, then the unit Octonions S7 are invariant under TL and also under n iterations denoted by T_L^n for any n.

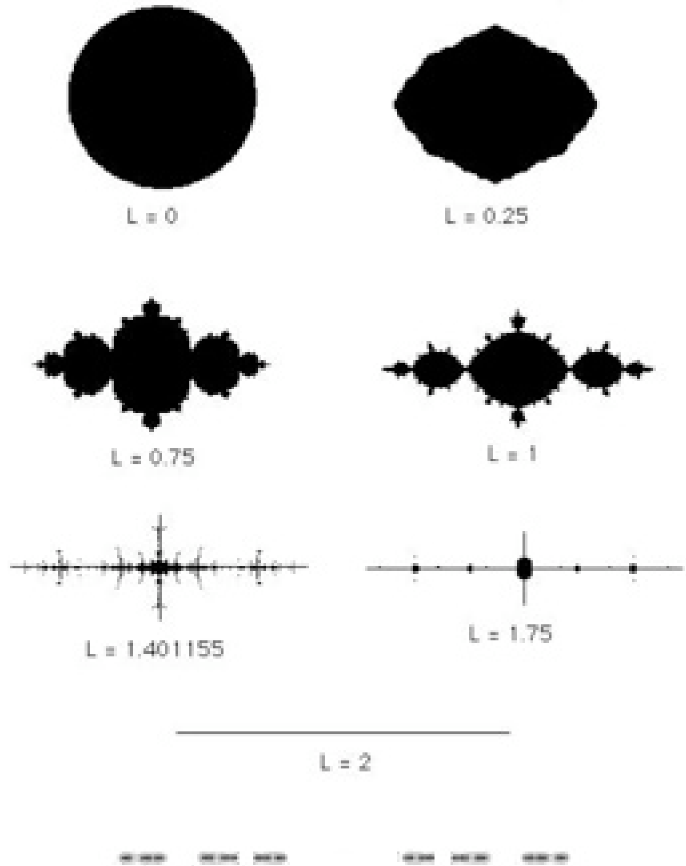
S7 and its interior is the subset of O that remains bounded under the iterated map T_L^n as n becomes arbitrarily large.

Denote by D_L the subset of O that remains bounded under the iterated map T_L^n as n becomes large.

Call the boundary of D_L the Julia set for L, denoted by B_L.

For octonions,the intersections of D_L and B_L with any 2-dimensional plane containing the real axis is just D_L and the Julia set B_L defined for the complex plane as described in section 1a for the case of one Spacetime dimension, and in the work of Barnsley, Geronimo, and Harrington that is cited there.

Consider the images of Julia sets for one Spacetime dimension shown in section 1a :



L > 2

The Julia sets B_L range from B_0 = unit sphere S7
through a number of complicated shapes to B_2 = [0,4] on the Octonionic real axis
to a disconnected Cantor dust on the Octonionic real axis for B_L for L > 2.
Since B_0 = S7, the iS7 described above can be regarded as iB_0. To generalize beyond L = 0, define iB_L, for i = l, ..., 7, as the intersection of B_L with the subspace of the octonions spanned by { 1, Oi } .
Since all 7 of the iB_L share the same Real Axis of the Octonions,
the real interval [0,4] inherits 7 different Tchebycheff measure structures, one for each imaginary octonion Oi,
and each denoted by iS for i = 1, … , 7
in addition to the natural Real Axis Tchebycheff measure structure which is here denoted by 0S.
Therefore, we have a set of 8 Tchebycheff measures

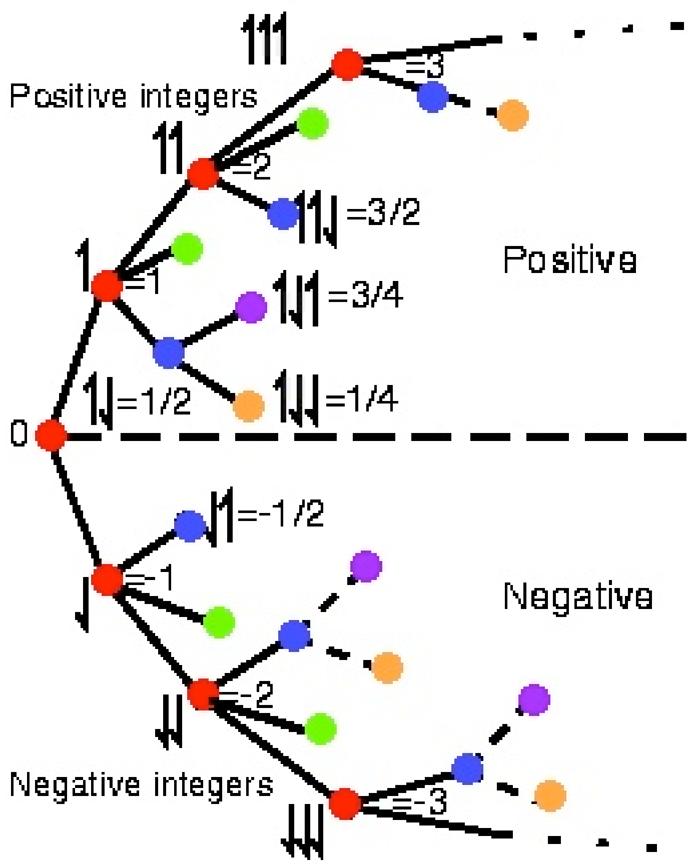
{ 0S ; 1S, 2S, 3S, 4S, 5S, 6S, 7S }

that are each the limit as n becomes large of measures based on the Tchebycheff polynomials of degree 2^n. The intervals containing the zeroes of each set of Tchebycheff polynomials of degree 2^n form nth order Borel sets for B_2.
Each of the corresponding set of 8 Borel measures is the singular measure concentrated at the zeroes of the Tchebycheff polynomials of degree 2 taking the value 2^(-n) at each zero.
Note that T_2^n maps each nth order Borel set densely onto the whole set B_2 = [0,4].
T_2 acts as a Bernoulli shift operator for each Tchebycheff measure system on B_2 and, as n becomes large, each Tchebycheff measure goes to the arccosine measure.

Consider each of the set of 8 Tchebycheff measures as the measures on 8 Real Line segments that form 8 orthonormal basis vectors for an 8-dimensional Spacetime.
Define M_n as a Lattice on that 8-dimensional Spacetime by identifying the zeroes of each set of Tchebycheff polynomials of degree 2^n as Lattice Vertices, so that the fineness of the lattice M_n Is determined by the order n of the Borel sets.
To complete the M_n Lattice, vertices on the basis axes combine by Octonion and Quaternion multiplication to produce full HyperDiamond lattices in 8 and 4 dimensions.
Since the M_n Lattice for (1+7)=8-dimensional Spacetime generalizes the construction in section 1a for (1+1)-dimensional Spacetime that produced a solution of the one-dimensional Ising model which has been shown to by H. A. Gersch (Int. J. Theor. Phys. 20 (1981) 491) to be equivalent to the (1+1)-dimensional Feynman Checkerboard model of Quantum Physics,
the M_n Lattice in 8 dimensions of Octonionic Spacetime and its counterparts in the two components of Quaternionic (4+4)=8-dimensional Kaluza-Klein Spacetime with (1+3)=4-dimensional Physical Spacetime allows physics to realistically described by the (1+3)-dimensional HyperDiamond Feynman Checkerboard.
For detailed structure of the Lattice for the (1+3)-dimensional HyperDiamond Feynman Checkerboard, see section 2 hereof and my paper at CERN-CDS-EXT-2004-030
(also at www.tony5m17h.net/FckbUSGR.pdf)

Note that Bernoulli Shifts of the Tchebycheff measures can be seen as coming from Binary Decision Trees, similar to those that produce Markov Processes and Surreal Numbers, and that
Surreal Numbers can be a basis for the Feynman Checkerboard representation of the Many-Worlds of Sum-Over-Histories Quantum Field Theory. To see how this works, look at the lattice spacetime of a Feynman Checkerboard, and call it the lattice spacetime of "our" universe in the ManyWorlds.
For simplicity, look at one of the space-time dimensions.
Represent its vertices by the red dots - the integers of the Surreal Numbers.
Then consider the "set" of all "other" lattice spacetimes of the other universes in the

ManyWorlds. Let the blue dots represent one of the "nearest neighbor" lattice spacetimes, and let the green dots represent the other "nearest neighbor" lattice spacetime. Then go one step further, and let the purple and gold dots represent the two "next nearest neighbor" lattice spacetimes that are "accessible through" the blue spacetime, and also (not shown on figure) go to the two "next nearest neighbor" lattice spacetimes that are "accessible through" the green spacetime.
Continuing to fill out the Surreal Number Binary Tree,
you have a representation of all the universes of the ManyWorlds
by the "set" of all Surreal Rationals with finite expansions.



As Onar Aam has noted, the Surreals have natural mirrorhouse structure (see my web page at www.tony5m17h.net/miroct.html).

The two "nearest neighbors" of the origin in this 1-dimensional Surreal "spacetime", Sur^1, correspond to the set {+1, -1} (represented in the diagram by {blue, green}) which are reflected into each other through the origin (represented by red). The reflection group is the Weyl group of the rank-1 Lie group Spin(3) = SU(2) = Sp(1) = S3.

If we look at k-dimensional Surreal spacetimes Sur^k, we see that the "nearest neighbors" of the origin correspond to the root vectors of the Weyl reflection groups for the largest rank-k Lie group that contains as a subgroup the rank-k Lie group Spin(2k).
Here I am using the term "nearest neighbors" to include both nearest and next-nearest neighbors in the case of Weyl groups whose root vectors are not all of the same length, as, for example, G2, F4, and Spin(2k+1) for k greater than 1. For instance:

The origin of Sur^2 has 4+8 = 12 "nearest neighbors", corresponding to (12+2)-dim G2 containing (4+2)-dim Spin(4). The "nearest neighbors" are in a Star-of-David pattern, with 30-degree angle between adjacent root vectors. Since 2x30 = 60, 3x30 = 90, and 4x30 =120, the G2 lattice of all "neighbors" combines both the square Gaussian lattice and the triangular Eisenstein lattice. Sur^2 has complex structure.

The origin of Sur^4 has 24+24 = 48 "nearest neighbors", corresponding to (48+4)-dim F4 containing (24+4)-dim Spin(8). The "nearest neighbors" are in a double 24-cell pattern, and all "neighbors" form a double D4 lattice. Sur^4 has quaternionic structure.

The origin of Sur^8 has 112+128 = 240 "nearest neighbors", corresponding to (240+8)-dim E8 containing (112+8)-dim Spin(16). The "nearest neighbors" are in a Witting polytope pattern, and all "neighbors" form an E8 lattice. If the other 6 of the 7 E8 lattices are included, then there are 480 “nearest neighbors”Sur^8 has octonionic structure, and can therefore represent my E8 Physics model.

The "neighbors" of the origin of Sur^16 form a /\16 Barnes-Wall lattice that is related to the Fermionic Orbifold Structure of E6 Bosonic String Theory described at CERN-CDS-EXT-2004-031 and at www.tony5m17h.net/E6StringBraneStdModelAR.pdf

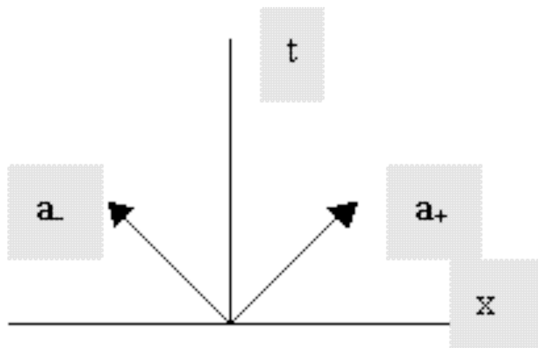
The "neighbors" of the origin of Sur^24 form a /\24 Leech lattice that is related to the Monster Group of Lattice Bosonic String Theory that describes one cell that, in the continuum limit, produces the Hyperfinite II1 von Neumann factor Algebraic Quantum Field Theory of my E8 Physics model
(see my web page at www.tony5m17h.net/LatBStrMonster.html)

2 – Feynman Checkerboards

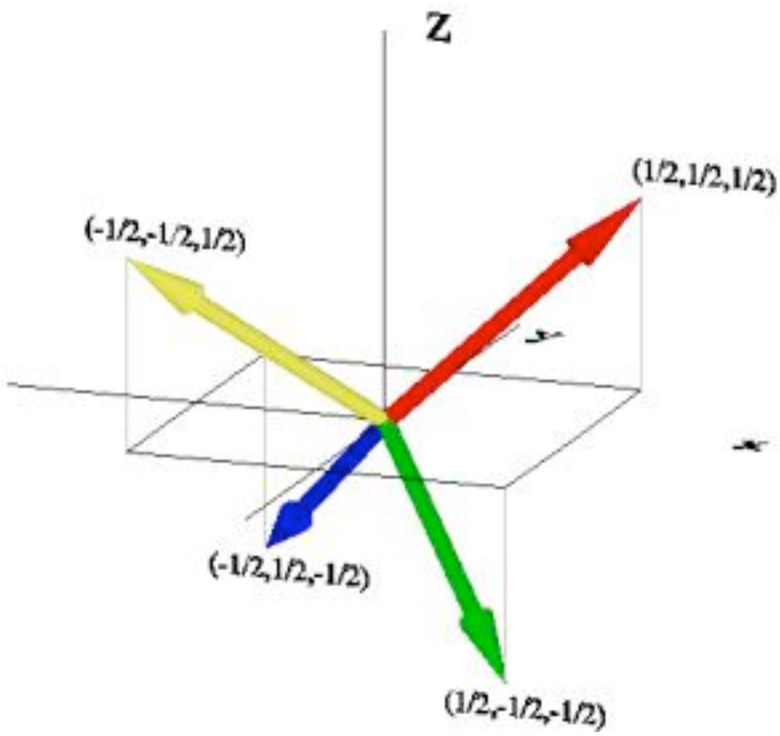
The HyperDiamond Feynman Checkerboard in 1+3 dimensions reproduces the correct Dirac equation.

Urs Schreiber has done the work necessary for the proof, after reading the work of George Raetz presented on his web site.

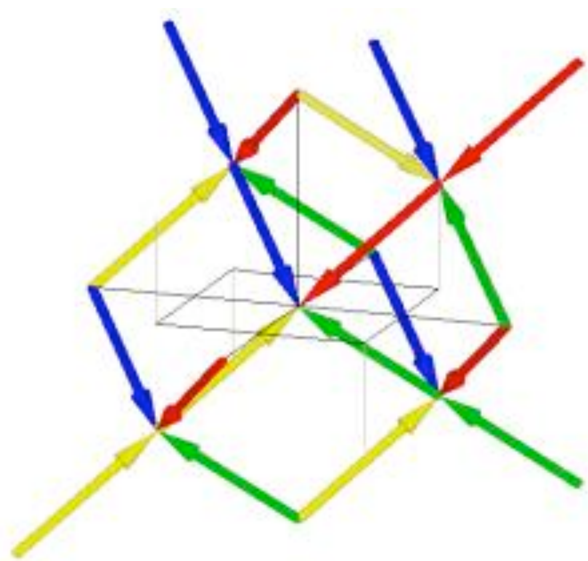
A very nice feature of the George Raetz web site is its illustrations, which include an image of a vertex of a 1+1 dimensional Feynman Checkerboard



and an image of a projection into three dimensions of a vertex of a 1+3 dimensional Feynman Checkerboard



and an image of flow contributions to a vertex in a HyperDiamond Random Walk from the four nearest neighbors in its past



Urs Schreiber wrote on the subject: Re: Physically understanding the Dirac equation and 4D in the newsgroup sci.physics.research on 2002-04-03 19:44:31 PST (including an appended forwarded copy of an earlier post) and again on 2002-04-10 19:03:09 PST.

Here are some excerpts from those posts:
"... I know ... the lanl paper [quant-ph/9503015]... and I know that Tony Smith does give a generalization of Feynman's summing prescription from 1+1 to 1+3 dimensions. But I have to say that I fail to see that this generalization reproduces the Dirac propagator in 1+3 dimensions, and that I did not find any proof that it does. Actually, I seem to have convinced myself that it does not, but I may of course be quite wrong. I therefore take this opportunity to state my understanding of these matters. First, I very briefly summarize (my understanding of) Tony Smith's construction:

The starting point is the observation that the left |-> and right |+> going states of the 1+1 dim checkerboard model can be labeled by complex numbers
|-> ---> (1 + i)
|+> ---> (1 - i)
(up to a factor) so that multiplication by the negative imaginary unit swaps components:
(-i) (1 + i)/2 = (1 - i)/2
(-i) (1 - i)/2 = (1 + i)/2 .

Since the path-sum of the 1+1 dim model reads phi = sum over all possible paths of (-i eps m)^(number of bends of path) = sum over all possible paths of product over all steps of one path of -i eps m (if change of direction after this step generated by i) 1 (otherwise)

this makes it look very natural to identify the imaginary unit appearing in the sum over paths with the "generator" of kinks in the path. To generalize this to higher dimensions, more square roots of -1 are added, which gives the quaternion algebra in 1+3 dimensions. The two states |+> and |-> from above, which were identified with complex numbers, are now generalized to four states identified with the following quaternions (which can be identified with vectors in M^4 indicating the direction in which a given path is heading at one instant of time):
(1 + i + j + k)
(1 + i - j - k)
(1 - i + j - k)
(1 - i - j + k) ,
which again constitute a (minimal) left ideal of the algebra (meaning that applying i,j, or k from the left on any linear combination of these four states gives another linear combination of these four states).
Hence, now i,j,k are considered as "generators" of kinks in three spatial dimensions and the above summing prescription naturally generalizes to
phi = sum over all possible paths of product over all steps of one path of
-i eps m (if change of direction after this step generated by i)
-j eps m (if change of direction after this step generated by j)
-k eps m (if change of direction after this step generated by k)
1 (otherwise)

The physical amplitude is taken to be A * e^(i alpha) where A is the norm of phi and alpha the angle it makes with the x0 axis.

As I said, this is merely my paraphrase of Tony Smith's proposal as I understand it.
I fully appreciate that the above construction is a nice (very "natural") generalization of the summing prescription of the 1+1 dim checkerboard model. But if it is to describe real fermions propagating in physical spacetime, this generalized path-sum has to reproduce the propagator obtained from the Dirac equation in 1+3 dimensions, which we know to correctly describe these fermions.
Does it do that? ...

Hence I have taken a look at the material [that] ... George Raetz ... present[s] ... titled "The HyperDiamond Random Walk" … which is mostly new to me.... I am posting this in order to make a suggestion for a more radical modification ...

[The]... equation ... DQ = (iE)Q ... is not covariant. That is because of that quaternion E sitting on the left of the spinor Q in the rhs of [the] equation ... . The Dirac operator D is covariant, but the unit quaternion E on the rhs refers to a specific frame.
Under a Lorentz transformation L one finds
L DQ = iE LQ = L E' Q <=> DQ = E'Q now with E' = L~ E L instead of E.

This problem disappears when the unit quaternion E is brought to the *right* of the spinor Q. What we would want is an equation of the form DQ = Q(iE) .
In fact, demanding that the spinor Q be an element of the minimal left ideal generated by the primitive projector P = (1+y0)(1+E)/4 , so that Q = Q' P , one sees that
DQ = Q(iE) almost looks like the the *Dirac-Lanczos equation*. (See hep-ph/0112317, equation (5) or ... equation (9.36) [of]... W. Baylis, Clifford (Geometric) Algebras, Birkhaeuser (1996) ...).
To be equivalent to the Dirac-Lanczos equation, and hence to be correct,
we need to require that D = y0 @0 + y1 @1 + y2 @2 + y3 @3
instead of ... = @0 + e1 @1 + e2 @2 + e3 @3 .

All this amounts to sorting out in which particular representation we are actually working here. In an attempt to address these issues, I now redo the steps … with some suitable modifications to arrive at the correct Dirac-Lanczos equation (this is supposed to be a suggestion subjected to discussion): So consider a lattice in Minkoswki space generated by a unit cell spanned by the four (Clifford) vectors
r = (y0 + y1 + y2 + y3)/2
g = (y0 + y1 - y2 - y3)/2
b = (y0 - y1 + y2 - y3)/2
y = (y0 - y1 - y2 + y3)/2 .
(yi are the generators of the Dirac algebra {yi,yj} = diag(+1,-1,-1,-1)_ij.)
This is Tony Smith's "hyper diamond".
(Note that I use Clifford vectors instead of quaternions.)
Now consider a "Clifford algebra-weighted" random walk along the edges of this lattice, which is described by four Clifford valued "amplitudes":
Kr, Kg, Kb, Ky
and such that
@r Kr = k (Kg y2 y3 + Kb y3 y1 + Ky y1 y2)
@b Kb = k (Ky y2 y3 + Kr y3 y1 + Kg y1 y2)
@g Kg = k (Kr y2 y3 + Ky y3 y1 + Kb y1 y2)
@y Ky = k (Kb y2 y3 + Kg y3 y1 + Kr y1 y2) .
(This is geometrically motivated.
The generators on the rhs are those that rotate the unit vectors corresponding to the amplitudes into each other. "k" is some constant.)

Note that I multiply the amplitudes from the *right* by the generators of rotation, instead of multiplying them from the left.
Next, assume that this coupled system of differential equations is solved by a spinor Q
 Q = Q' (1+y0)(1+iE)/4
 E = (y2 y3 + y3 y1 + y1 y2)/sqrt(3)
with
 Kr = r Q
 Kg = g Q
 Kb = b Q
 Ky = y Q .

This ansatz for solving the above system by means of a single spinor Q is, as I understand it, the central idea. But note that I have here modified it on the technical side:
Q is explicitly an algebraic Clifford spinor in a definite minial left ideal, E squares to -1, not to +1, and the Ki are obtained from Q by premultiplying with the Clifford basis vectors defined above.
Substituting this ansatz into the above coupled system of differential equations one can form one covariant expression by summing up all four equations:
 (r @r + g @g + b @b + y @y) Q = k sqrt(3) Q E
The left hand side is immediate.
To see that the right hand side comes out as indicated simply note that r + g + b + y = y0 and that Q y0 = Q by construction.
The above equation is the Dirac-Lanczos-Hestenes-Guersey equation, the algebraic version of the equation describing the free relativistic electron. The left hand side is the flat Dirac operator r @r + g @g + b @b + y @y = ym @m and the right hand side, with k = mc / (hbar sqrt(3)) , is equal to the mass term i mc / hbar Q.
As usual, there are a multitude of ways to rewrite this.
If one wants to emphasize biquaternions then premultiplying everything with y0 and splitting off the projector P on the right of Q to express everything in terms of the, then also biquaternionic, Q' (compare the definitions given above) gives Lanczos' version (also used by Baylis and others).
I think this presentation improves a little on that given on George Raetz's web site:
The factor E on the right hand side of the equation is no longer a nuisance but a necessity.
Everything is manifestly covariant (if one recalls that algebraic spinors are manifestly covariant when nothing non-covariand stands on their *left* side). The role of the quaternionic structure is clarified, the construction itself does not depend on it. Also, it is obvious how to generalize to arbitrary dimensions. In fact, one may easily check that for 1+1 dimensions the above scheme reproduces the Feynman model.

While I enjoy this, there is still some scepticism in order as long as a central questions remains to be clarified: How much of the Ansatz K(r,g,b,y) = (r,g,b,y) Q is wishful thinking?
For sure,
every Q that solves the system of coupled differential equations that describe the amplitude of the random walk on the hyper diamond lattice also solves the Dirac equation. But what about the other way round?

Does every Q that solves the Dirac equation also describe such a random walk. ...".

[The question ends this quotation of Urs Schreiber.]

My proposal to answer the question raised by Urs Schreiber uses symmetry.
The hyperdiamond random walk transformations include the transformations of the Conformal Group:
rotations and boosts (to the accuracy of lattice spacing);
translations (to the accuracy of lattice spacing);
scale dilatations (to the accuracy of lattice spacing): and
special conformal transformations (to the accuracy of lattice spacing).

Therefore, to the accuracy of lattice spacing, the hyperdiamond random walks give you all the conformal group Dirac solutions, and since the full symmetry group of the Dirac equation is the conformal group,
the answer to the question is "Yes".

So. thanks to the work of Urs Schreiber:

The HyperDiamond Feynman Checkerboard in 1+3 dimensions does reproduce the correct Dirac equation.

Here are some references to the conformal symmetry of the Dirac equation:
R. S. Krausshar and John Ryan in their paper Some Conformally Flat Spin Manifolds, Dirac Operators and Automorphic Forms at math.AP/0212086 say:
"... In this paper we study Clifford and harmonic analysis on some conformal flat spin manifolds. ... manifolds treated here include RPn and S1 x S(n-1). Special kinds of Clifford-analytic automorphic forms associated to the different choices of are used to construct Cauchy kernels, Cauchy Integral formulas, Green's kernels and formulas together with Hardy spaces and Plemelj projection operators for Lp spaces of hypersurfaces lying in these manifolds. ... Solutions to the Dirac equation are called

Clifford holomorphic functions or monogenic functions. Such functions are covariant under ... conformal or .... Mobius transformations acting over Rn u {oo}. ...".

Barut and Raczka, in their book Theory of Group Representations and Applications (World 1986), say, in section 21.3.E, at pages 616-617:
"... E. The Dynamical Group Interpretation of Wave Equations.
... Example 1. Let G = O(4,2) .Take U to be the 4-dimensional non-unitary representation in which the generators of G are given in terms of the 16 elements of the algebra of Dirac matrices as in exercise 13.6.4.1. Because (1/2)L_56 = gamma_0 has eigenvalues n = +/-1, taking the simplest mass relation mn = K, we can write (m gamma_0 - K) PSI(dotp) = 0, where K is a fixed constant.
Transforming this equation with the Lorentz transformation of parameter E
PSI(p) = exp(i E N) PSI(p)
N = (1/2) gamma_0 gamma
gives
(gamma^u p_u - K) PSI(p) = 0 which is the Dirac equation ...".

P. A. M. Dirac, in his paper Wave Equations in Conformal Space, Ann. Math. 37 (1936) 429-442, reprinted in The Collected Works of P. A. M. Dirac: Volume 1: 1924-1948, by P. A. M. Dirac (author), Richard Henry Dalitz (editor), Cambridge University Press (1995), at pages 823-836, said:
"... by passing to a four-dimensional conformal space ... a ... greater symmetry of ... equations of physics ... is shown up, and their invariance under a wider group is demonstrated. ... The spin wave equation ... seems to be the only simple conformally invariant wave equation involving the spin matrices. ... This equation is equivalent to the usual wave equation for the electron, except ...[that it is multiplied by]... the factor (1 + alpha_5) , which introduces a degeneracy. ...".

3 – Pure Spinors and Lucas and Fibonacci numbers

Reese Harvey (in his book Spinors and Calibrations (Academic 1990)) says
"... the set Cpx(n) of orthogonal complex structures on R^2n has two connected components Cpx+(n) and Cpx-(n), with
Cpx(n) = O(2n)/U(n)
Cpx+/-(n) = SO(2n)/U(n)
…".
Harvey calls O(2n)/U(n) "the twistor space (at a point on a manifold), i.e., the twistor fiber", and says(I am using "n" instead of Harvey's "p" here):
"... Let PURE(n) .. denote the set of all pure spinors ...
consider the complex case ...
PURE_C / C* = Cpx(2n) = O(2n)/U(n)
... The square of a pure spinor represents the associated null plane in /\R(n,n) ...".

For examples:

In the case of n=4:
O(8)/U(4) which has 28 - 16 = 12 real dimensions is the Twistor Space,
and it has two components (for the two mirror image half-spinors of Spin(8)) that each are SO(8)/U(4) with 28-16 = 12 real dimensions.
Since PURE_C /C* = O(8)/U(5) has 28-16 = 12 real dimensions,
it has 6 complex dimensions.
Since the C* of complex scalars is 1-complex-dimensional,
PURE_C has 6+1 = 7 complex dimensions.
Since Spin(8) half-spinors are 8-dimensional, 7 of the 8 dimensions of Spin(8) half-spinors are PURE (real structure).

In the case of n=5:
O(10)/U(5) which has 45 - 25 = 20 real dimensions is the twistor space, and it has two components (for the two mirror image half-spinors of Spin(10)) that each are SO(10)/U(5) with 45-25 = 20 real dimensions.
Since PURE_C /C* = O(10)/U(5) has 45-25 = 20 real dimensions, it has 10 complex dimensions.
Since the C* of complex scalars is 1-complex-dimensional,
PURE_C has 10+1 = 11 complex dimensions.
Since Spin(10) half-spinors are 16-dimensional, 11 of the 16 dimensions of Spin(10) half-spinors are PURE (real structure).

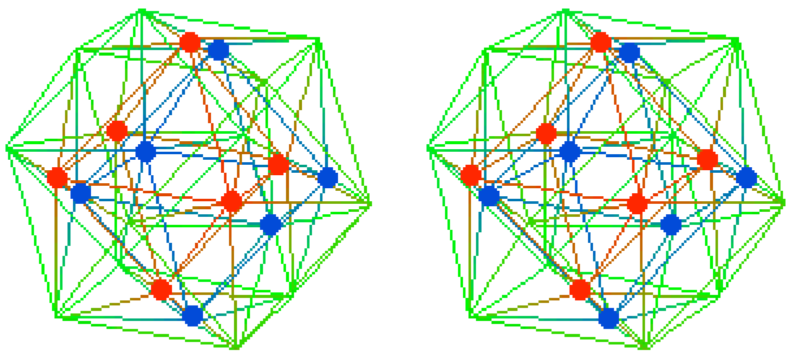
In the case of n=7:
O(16)/U(8) which has 120 - 64 = 56 real dimensions is the Twistor Space,
and it has two components (for the two mirror image half-spinors of Spin(8)) that each are SO(16)/U(8) with 120-64 = 56 real dimensions.
Since PURE_C /C* = O(16)/U(8) has 129-64 = 56 real dimensions, it has 28 complex dimensions.
Since the C* of complex scalars is 1-complex-dimensional,
PURE_C has 28+1 = 29 complex dimensions.
Since Spin(16) half-spinors are 128-dimensional, 29 of the 128 dimensions of Spin(16) half-spinors are PURE (real structure).

Note that the 120 dimensions of Spin(16) plus the 128 dimensions of Spin(16) half-spinors form 248-dimensional E8
and that 240 – 72 = 168 = 3x56 root vectors of E8 not in its E6 subalgebra form 3 copies of the Fr(3,O) Freudental algebra of which E6 is automorphism group
and that each of the 3 copies of Fr(3,O) correspond to the Twistor Space of the D8 Lie algebra of O(16) and Spin(16) from which E8 is constructed,
so that E8 contains 3 copies of Fr(3,O) Twistor Space, all related to each other by Triality.

That is all consistent with the table in vol. 2 of the book Spinors and Spacetime by Penrose and Rindler (Cambridge 1986) that lists in table B.65 on page 453 (extended by me to Dimensions 15 and 16) (the Half in (Half)-Spinors is for the even Dimensions):

Dimensions	Pure Spinors	(Half)-Spinors
1,2	1	1
3,4	2	2
5,6	4	4
7,8	7	8
9,10	11	16
11,12	16	32
13,14	22	64
15,16	29	128

Fibonacci Numbers and Lucas Numbers are defined in terms of the Golden Mean which I denote by g = (1/2)(sqrt(5) - 1)
and its inverse which I denote by G = (1/2)(sqrt(5) + 1)
g is related to icosahedral symmetry in general
and in particular to the 120 vertices of the 4-dim 600-cell polytope,
which vertices can be obtained from the 24 vertices of a 24-cell



plus a Golden Ratio point on each of the 96 1-dim edges of the 24-cell, giving the 24+96 = 120 vertices of the 600-cell,
which can be seen as a 4-dim HyperIcosahedron.

Those 120 vertices form the Root Vector Polytope of H4, the full group of symmetries of the 4-dim HyperIcosahedron,
and
two sets of 120 form the 240 vertices of the E8 Root Vector Polytope,
so that it can be said that E8 = H4 + H4.

The Golden Ratio G is not only inherent in the construction of each of the two H4 each with 120 Root Vectors,
but also in the construction E8 = H4 + H4,
where the lengths of the 120 Root Vectors of one of the H4
are the lengths of the 120 Root Vectors of the other dilated by G.

H. S. M. Coxeter gave details in section 3.8 of his paper
"Regular and Semi-Regular Polytopes III"(Math. Zeit. 200 (1988) 3-45, reprinted in "Kaleidoscopes: Selected Writings of H. S. M. Coxeter" (Wiley 1995)) where he said (using notation “t” for “G”):
“…Du Val ... discovered ... ten-dimensional coordinates ...[u_1 ... u_10]... for the ... lattice 5_21 ... In fact, the vertices of a 5_21 of edge 5 sqrt(2) are all the points in Euclidean 10-space whose coordinates satisfy
the equations

x_1 + x_2 + x_3 + x_4 + x_5 = x_6 + x_7 + x_8 + x_9 + x_10 = 0
and the congruences
x_1 = x_2 = x_3 = x_4 = x_5 = 2 x_6 = 2 x_7 = 2 x_8 = 2 x_9 = 2 x_10 (mod 5)

In this lattice, the points at distance 5 sqrt(2) from the origin are, of course, the 240 vertices of a 4_21 . … In the accompanying table …[of]… new coordinates
u_v = (x_v + t x_(v+5))/sqrt(5) …[for]… v = 1,2,3,4,5
u_v = (t x_(v-5) – x_v)/sqrt(5) …[for]… v = 6,7,8,9,10
where t = (1/2)(sqrt(5) + 1) …

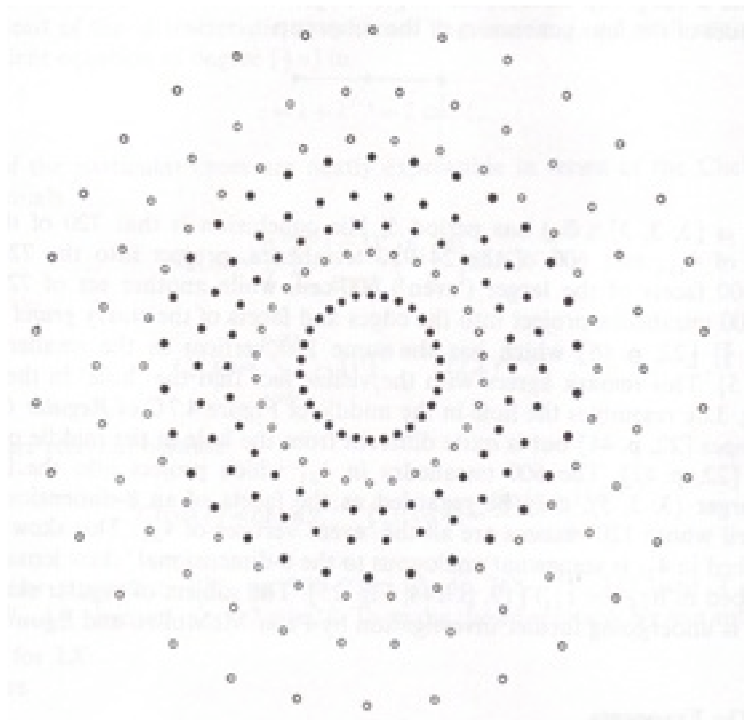
$$
\begin{array}{l}
(\sqrt{5}, 0, 0, 0, -\sqrt{5} ; \tau\sqrt{5}, 0, 0, 0, -\tau\sqrt{5}) \\
(2\tau, 1, 1, -\tau^{2}, -\tau^{2} ; 2, -\tau, -\tau, \tau^{-1}, \tau^{-1}) \\
(-\tau^{-2}, -\tau^{2}, 1, 1, 1 ; \tau^{3}, \tau^{-1}, -\tau, -\tau, -\tau) \\
(\tau^{3}, \tau^{-1}, -\tau, -\tau, -\tau ; \tau^{-2}, \tau^{2}, -1, -1, -1) \\
(\tau^{-1}, \tau^{-1}, -\tau, -\tau, 2 ; \tau^{2}, \tau^{2}, -1, -1, -2\tau) \\
(\tau, \tau, \tau, -\tau^{-1}, -\tau^{3} ; 1, 1, 1, -\tau^{2}, -\tau^{-2}) \\
(-2, \tau, \tau, -\tau^{-1}, -\tau^{-1} ; 2\tau, 1, 1, -\tau^{2}, -\tau^{2}) \\
(\tau^{2}, \tau^{2}, -1, -1, -2\tau ; -\tau^{-1}, -\tau^{-1}, \tau, \tau, -2) \\
(-1, -1, -1, \tau^{2}, \tau^{-2} ; \tau, \tau, \tau, -\tau^{-1}, -\tau^{3}) \\
(\tau\sqrt{5}, 0, 0, 0, -\tau\sqrt{5} ; -\sqrt{5}, 0, 0, 0, \sqrt{5})
\end{array}
$$

... By picking out alternate rows of the right-hand column of the table, we distinguish two sets of 120 vertices of 4_21 ...

one set satisfying u_1^2 + ... + u_5^2 = 10 [and] u_6^2 + ... u_10^2 = 10 t^2

and the other satisfying u_1^2 + ... + u_5^2 = 10 t^2 [and] u_6^2 + ... u_10^2 = 10

Let us call these 'odd' and 'even' vertices, respectively. In Fig. 3.8 d …



… they appear as black and white dots. ... When we project
onto the 5-space u_6 = ... = u_10 = 0 by ignoring the last five coordinates,
we obtain the 120 + 120 vertices of two homothetic 600-cells ...
one having the coordinates …[of]… the other … multiplied by t
...
These 240 points are the vertices of the 8-dimensional uniform polytope 4_21 …[they]…
represent the 240 lattice points at distance 2 from the origin:
the 16 permutations of (+/-2,0,0,0,0,0,0,0)
and the 112 + 112 cyclic permutations of (the last 7 coordinates in)

(+/-1 ; 0,0,0,+/-1,+/-1,0,+/-1)   (0 ; +/-1,+/-1,+/-1,0,0,+/-1,0)

[using octonionic basis { 1, i, j, k, e, ie, je, ke }] …".

Projected into two dimensions, the 240 vertices are in 4+4 = 8 circles of 30 vertices each.
The two sets of 120 vertices decompose physically this way:

Inner 120:
- 24 of one of the two D4 in E8
- 32 of the 8x8 = 64 components of 8-dim Kaluza-Klein Spacetime
- 32 of the 8x8 = 64 components of 8 Fermion Particles
- 32 of the 8x8 = 64 components of 8 Fermion AntiParticles

Outer 120 (dilated by G):

24 of the other of the two D4 in E8
the other 32 of the 64 components of 8-dim Kaluza-Klein Spacetime
the other 32 of the 64 components of 8 Fermion Particles
the other 32 of the 64 components of 8 Fermion AntiParticles

Since the decomposition 240 E8 = 120 H4 + 120 H4
is of rank 8 E8 into one rank 4 H4 plus another rank 4 H4
that decomposition corresponds to
the phase transition at low (relative to Planck) energies
from a full Octonionic 8-dim Spacetime
to a (4+4)-dim Kaluza-Klein Spacetime produced by freezing out a preferred Quaternionic substructure.

The Golden Mean g is a transcendental number and in some sense the most irrational of the real numbers. As Mario Livio notes in his book The Golden Ratio (Broadway Books 2002):

G = sqrt(1 + sqrt(1 + sqrt(1 + sqrt(1 + …

Therefore G has fundamental structure similar to Fractal Spacetime,
which uses representations in terms of numbers of the form
L +/- sqrt(L +/- sqrt(L +/- sqrt(L +/- ...
which in turn are zeroes of (generalized) Tchebycheff polynomials of degree 2^n.

Acccording to a web site at
www.mcs.surrey.ac.uk/Personal/R.Knott/Fibonacci/lucasNbs.html
"… A. W. W. J. M. van Loon noticed that, since G - g = 1
and G + g = sqrt(5), there is a particularly nice way of writing the Lucas numbers forumula that shows a closer relationship with the Fibonacci numbers forumula:
F(n) = (G^n - (-g)^n) / (G - (-g))
L(n) = (G^n + (-g)^n) / (G + (-g)) …" so we have:

n	F(n)	L(n)	F(n)+L(n)
0	0	2	2
1	1	1	2
2	1	3	4
3	2	4	6
4	3	7	10
5	5	11	16
6	8	18	26
7	13	29	42
8	21	47	68
9	34	76	110
10	55	123	178
11	89	199	288

Note that the Lucas numbers for n = 3, 4, 5, and 7 correspond to Pure Spinors.
Here are some comments on F(n), L(n) and F(n)+L(n):

F(2)+L(2) = 4 is the dimensionality of physical spacetime and of the Spinors of D3 and of the Full Spinors of D2
with F(2) = 1 as the dimensionality of physical time
and L(2) = 3 as the dimensionality of physical space

F(3)+L(3) = 6 is the dimensionality of conformal spacetime
with F(3) = 2 as the dimensionality of conformal time
and L(3) = 4 as the dimensionality of conformal space and of the Pure Spinors of D3 = A3 = Conformal Group

F(4)+L(4) = 10 is the dimensionality of conformal octonions and of conformal space over 8-dimensional octonionic spacetime and the Half-Spinors of D4
with F(4) = 3 as the dimensionality of imaginary quaternions
and L(4) = 7 as the dimensionality of imaginary octonions and of the Pure Spinors of D4

F(5)+L(5) = 16 = (4+4) + (1+7) as the dimensionality of (4+4)-dim Kaluza-Klein Spacetime plus (1+7) Fermions and of the Half-Spinors of D5
with F(5) = 5 = 4+1 as the dimensionality of 4-dim Physical Spacetime plus 1 Neutrino-Type Fermion and of the Non-Pure Spinors of D5
and L(5) = 11 = 4+7 as the dimensionality of 4-dim Internal Symmetry Space plus 7 Tree-Level-Massive Fermions and of the Pure Spinors of D5

F(6)+L(6) = 26 is the dimensionality of bosonic string theory
with F(6) = 8 as the dimensionality of octonionic spacetime and of the Half-Spinors of D4
and L(6) = 18 as the dimensionality of conformal space over the 16-dimensional Half-Spinors of D5 and Full Spinors of D4

F(7)+L(7) = 42 is Deep Thought's Final Answer
With L(7) = 29 as Pure Spinors of D8 and related to E8

F(11)+L(11) = 288 is the number of vertices in the norm = 22 layer of the D4+ lattice of the (1+3)-dim HyperDiamond Feynman Checkerboard described in my paper at CERN-CDS-EXT-2004-030
(also at www.tony5m17h.net/FckbUSGR.pdf)

Here are the numbers of vertices in some of the layers of the D4+ lattice. Note the occurrence patterns of interesting numbers. The even-numbered layers correspond to the even D4 sublattice:

```
 norm        number of vertices
   0           1
   1           8   =    1 x 8
   2          24   =    1 x 24
   3          32   =  ( 1 + 3 ) x 8
   4          24   =    1 x 24
   5          48   =  ( 1 + 5 ) x 8
   6          96   =  ( 1 + 3 ) x 24
   7          64   =  ( 1 + 7 ) x 8
   8          24   =    1 x 24
   9         104   =  ( 1 + 3 + 9 ) x 8
  10         144   =  ( 1 + 5 ) x 24
  11          96   =  ( 1 + 11 ) x 8
  12          96   =  ( 1 + 3 ) x 24
  13         112   =  ( 1 + 13 ) x 8
  14         192   =  ( 1 + 7 ) x 24
  15         192   =  ( 1 + 3 + 5 + 15 ) x 8
  16          24   =    1 x 24
  17         144   =  ( 1 + 17 ) x 8
  18         312   =  ( 1 + 3 + 9 ) x 24
  19         160   =  ( 1 + 19 ) x 8
  20         144   =  ( 1 + 5 ) x 24
  21         256   =  ( 1 + 3 + 7 + 21 ) x 8
  22         288   =  ( 1 + 11 ) x 24
  23         192   =  ( 1 + 23 ) x 8
  24          96   =  ( 1 + 3 ) x 24
  25         248   =  ( 1 + 5 + 25 ) x 8
  26         336   =  ( 1 + 13 ) x 24
  27         320   =  ( 1 + 3 + 9 + 27 ) x 8
  28         192   =  ( 1 + 7 ) x 24
  29         240   =  ( 1 + 29 ) x 8
  30         576   =  ( 1 + 3 + 5 + 15 ) x 24
  31         256   =  ( 1 + 31 ) x 8
  32          24   =    1 x 24
  33         384   =  ( 1 + 3 + 11 + 33 ) x 8
  34         432   =  ( 1 + 17) x 24
  35         384   =  ( 1 + 5 + 7 + 35 ) x 8
```

```
 36        312  =  ( 1 + 3 + 9 ) x 24
 37        304  =  ( 1 + 37 ) x 8
 38        480  =  ( 1 + 19 ) x 24
 39        448  =  ( 1 + 3 + 13 + 39 ) x 8
 40        144  =  ( 1 + 5 ) x 24
 41        336  =  ( 1 + 41 ) x 8
 42        768  =  ( 1 + 3 + 7 + 21 ) x 24
 43        352  =  ( 1 + 43 ) x 8
 44        288  =  ( 1 + 11) x 24
 ...           ...

2x127                    3,072 = 128 x 24

2x128                       24

2x65,536 = 2^17             24

2x65,537             1,572,912 = 65,538 x 24

2x2,147,483,647        51,539,607,552 =
                     = 2,147,483,648 x 24

2x2,147,483,648 = 2^32      24
```

Comet Holmes

While Garrett Lisi was writing his paper 0711.0770, just before the Full Moon before Halloween (according to a Tommaso Dorigo blog post on 25 October 2007) "... Comet 17P/Holmes has experienced a huge outburst, brightening ... in the matter of hours ... 400,000 times ...[around]... 13:40UT on October 24th ..[according to]...Seiichi Yoshida ...".

On Guy Fawkes Day, 5 November 2007, the day before Garrett Lisi posted 0711.0770, the Astronomy Picture of the Day was the above image, and the antwrp.gsfc.nasa.gov web site said:

> "... Comet Holmes continues to be an impressive sight to the unaided eye. The comet has diminished in brightness only slightly, and now clearly appears to have a larger angular extent than stars and planets. Astrophotographers have also noted a distinctly green appearance to the comet's coma over the past week. Pictured above [by Vicent Peris and Jose Luis Lamadrid] over Spain in three digitally combined exposures, Comet 17P/Holmes now clearly sports a tail. The blue ion tail is created by the solar wind impacting ions in the coma of Comet Holmes and pushing them away from the Sun. Comet Holmes underwent an unexpected and dramatic increase in brightness starting only two weeks ago. The detail visible in Comet Holmes' tail indicates that the explosion of dust and gas that created this dramatic brightness increase is in an ongoing and complex event. ...".

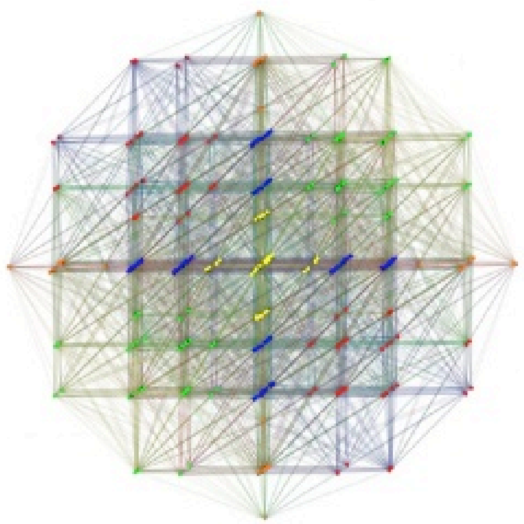
Maybe Comet Holmes should be nicknamed "Comet E8", since it shares with Garrett Lisi's model "... dramatic brightness ... in an ongoing and complex event ...".

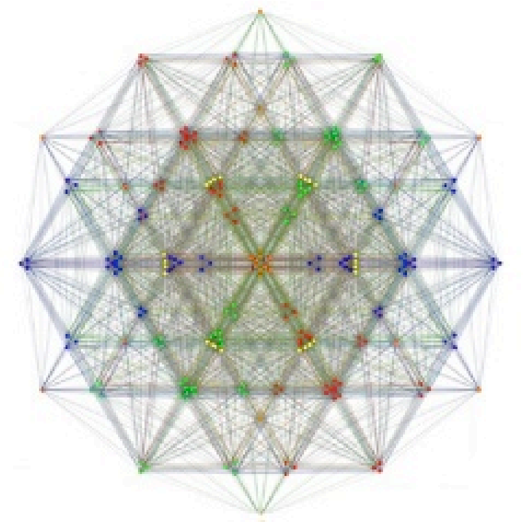
Puzzles

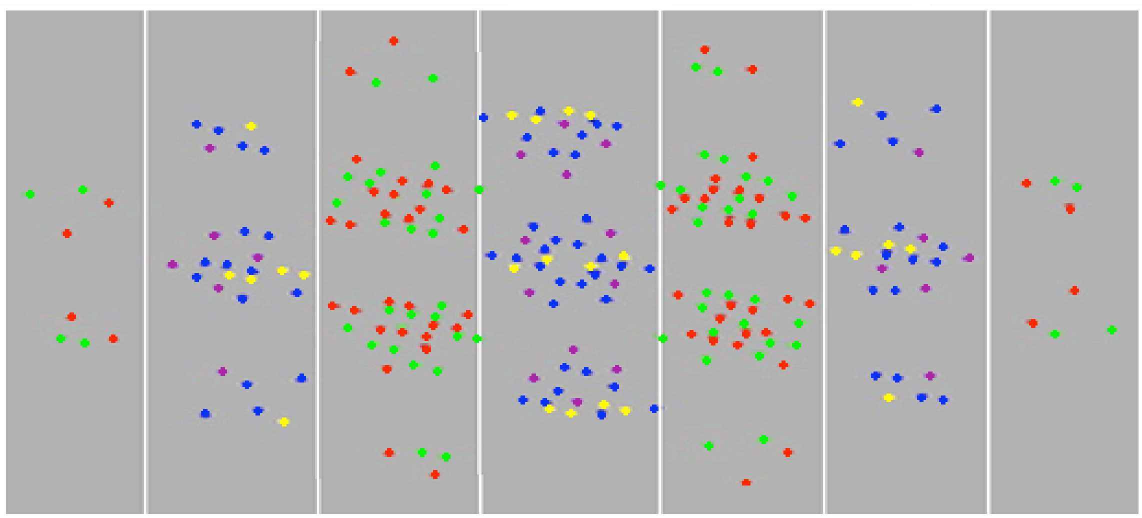
The reaction of the North American Physics establishment to E8 Physics has predominantly ranged from hostility to indifference (see page 5). For example, the poster

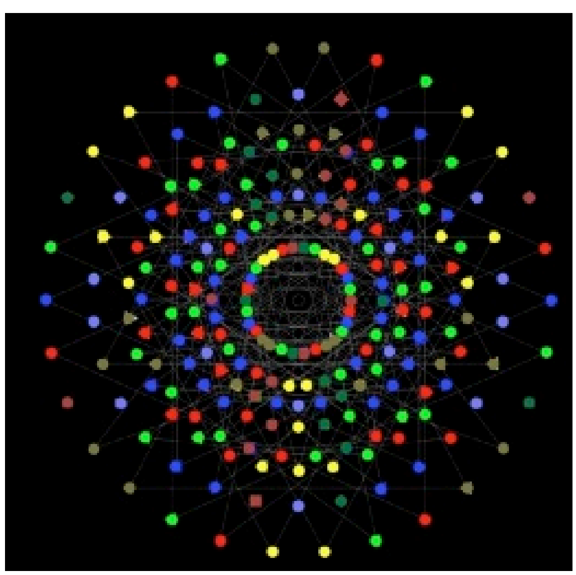
for the PASCOS (PArticles, Strings, and COSmology) meeting at the Perimeter Institute in June 2008 depicted Physics as a jigsaw puzzle whose pieces were either unconnected or forced to fit where they don't belong, in marked contrast to the January 2008 cover of European Science et Vie

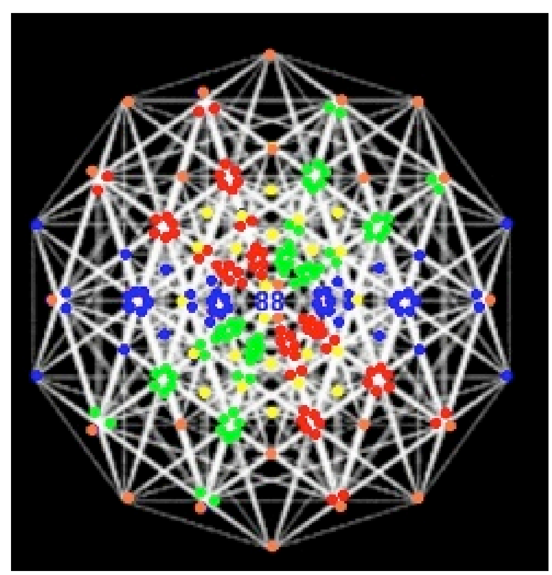
which celebrated E8 Physics as the missing piece that, having been found, solved the puzzle.







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