The Exact Analytic Solution of Blasius Equation

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Abstract

We find Blasius function to satisfy the boundary condition $f(\infty)=1$ and obtain the exact analytic soultion of Blasius equation.

Keywords: Jiang functons; Blasius function; Blasius equation; Exact analytic solution.

1. Introduction. In 1967 using Jiang functions we defined Blasius function. Using it we obtained the exact analytic solution for Blasius eqaution [1]. Here we rewrite this paper.
Blasius equation [2] is

$$f'''(x)+f(x)f''(x)=0, \qquad (1)$$

with boundary conditions

$$f(0)=f'(0)=0, \quad f''(0)=1, \; f'(\infty)=1. \qquad (2)$$

Blasius gave a soultion in power series

$$f(x)=\sum_{k=0}^{\infty}(-1)^k\frac{A_k}{(3k+2)!}x^{3k+2} \qquad (3)$$

where

$$A_0=A_1=1, \quad A_k=\sum_{i=0}^{k-1}\binom{3k-1}{3i}A_iA_{k-i-1}, k\geq 2. \qquad (4)$$

From (3) and (4) we have

$$f(x)=\frac{1}{2'}x^2-\frac{1}{5!}x^5+\frac{11}{8!}x^8-\frac{375}{11!}x^{11}+\frac{27897}{14}x^{14}-\cdots \qquad (5)$$

To our knowledge, up to now, no one has given an exact analytic solution of Blasius equation.

2. We find Blasius function to satisfy the boundary condition $f(\infty)=1$. We define Jiang functions [3]

$$J_1(x)=\frac{1}{3}\left[e^x+2e^{-x/2}\cos(\frac{\sqrt{3}}{2}x)\right]=\sum_{i=0}^{\infty}\frac{x^{3i}}{(3i)!}, \qquad (6)$$

$$J_2(x)=\frac{1}{3}\left[e^x-2e^{-x/2}\cos(\frac{\sqrt{3}}{2}x-\frac{\pi}{3})\right]=\sum_{i=0}^{\infty}\frac{x^{3i+2}}{(3i+2)!}, \qquad (7)$$

$$J_3(x)=\frac{1}{3}\left[e^x+2e^{-x/2}\cos(\frac{\sqrt{3}}{2}x-\frac{2\pi}{3})\right]=\sum_{i=0}^{\infty}\frac{x^{3i+1}}{(3i+1)!}. \qquad (8)$$

From (6) and (8) we define Blasius function

$$F'(x)=\frac{J_3(x)}{J_1(x)}=\sum_{k=0}^{\infty}\frac{B_k}{(3k+1)!}x^{3k+1}, \qquad (9)$$

where $B_0=1, B_1=-3$,

$$B_k=-3k+\sum_{i=1}^{k-1}(-1)^{i+1}\binom{3k+1}{3(k-i)}|B_i|, \quad k\geq 2 \qquad (10)$$

From (9) we have

$$F(x)=\frac{1}{2!}x^2-\frac{3}{5!}x^5+\frac{99}{8!}x^8-\frac{11259}{11!}x^{11}+\cdots \qquad (11)$$

Blasius function $F(x)$ is the approximate analytic solution of the following Blasius equation

$$F'''(x)+F(x)F''(x)=0, \qquad (12)$$

with boundary conditions

$$F(0)=F'(0)=0, \quad F''(0)=1, \quad F'(\infty)=1. \qquad (13)$$

3. We assume $x=\frac{\eta}{\sqrt[3]{3}}$. Substituting it into (11) we have

$$F\left(\frac{\eta}{\sqrt[3]{3}}\right)=\frac{1}{\sqrt[3]{9}}\left[\frac{1}{2!}\eta^2-\frac{1}{5!}\eta^5+\frac{11}{8!}\eta^8-\frac{417}{11!}\eta^{11}+\cdots\right] \qquad (14)$$

From (14) we define Blasius function

$$\phi(\eta)=\frac{1}{2!}\eta^2-\frac{1}{5!}\eta^5+\frac{11}{8!}\eta^8-\frac{417}{11!}\eta^{11}+\cdots \qquad (15)$$

Blasius function $\phi(\eta)$ is the analytic solution of the following Blasius equation

$$\phi'''+\phi\phi''=0, \qquad (16)$$

with boundary conditions

$$\phi(0)=\phi'(0)=0, \quad \phi''(0)=1, \quad \phi'(\infty)=1. \qquad (17)$$

4. We define Blasius function

$$\Psi'_{(x)}=\frac{J_3(x)+D(x)}{J_1(x)} \qquad (18)$$

From (1) we assume

$$\Psi'(x) = f'(x) \tag{19}$$

Substituting (19) into (18) we have

$$D(x) = f'(x)J_1(x) - J_3(x) \tag{20}$$

From (20) we have

$$D(x) = \sum_{k=1}^{\infty} \frac{C_k}{(3k+1)!} x^{3k+1} \tag{21}$$

where

$$C_k = \left[\sum_{i=0}^{k} (-1)^{i+1} A_i \binom{3k+1}{3(k-i)} \right] - 1, \ k \geq 1. \tag{22}$$

From (22) we have

$$C_1 = 2, C_2 = -18, C_3 = 744, C_4 = -61180, C_5 = 11628918, \cdots \tag{23}$$

We may prove that

$$\lim_{x \to \infty} \frac{D(x)}{J_1(x)} = 0. \tag{24}$$

From (18) we have

$$\psi(x) = \int_0^x \frac{J_3(\eta) + D(\eta)}{J_1(\eta)} d\eta \tag{25}$$

Blasius function $\psi(x)$ is the exact analytic solution of the following Blasius equation

$$\psi''' + \psi\psi'' = 0 \tag{26}$$

with boundary conditions

$$\psi(0) = \psi'(0) = 0, \psi''(0) = 1, \psi'(\infty) = 1. \tag{27}$$

5. We assume that

$$H(x) = \frac{2}{4!}x^4 - \frac{18}{7!}x^7 + \frac{744}{10!}x^{10} - \frac{61180}{13!}x^{13} \tag{28}$$

Using (28) we define Blasius function

$$\phi'(x) = \frac{J_3(x) + H(x)}{J_1(x)} = x - \frac{1}{4!}x^4 + \frac{11}{7!}x^7 - \frac{375}{10!}x^{10} + \frac{27897}{13!}x^{13} - \frac{3817137}{16!}x^{16} + \cdots \tag{29}$$

Blasius finction $\phi(x)$ is the exact analytic solution of the following Blasius equation

$$\phi''' + \phi\phi'' = 0, \qquad (30)$$

with boundary conditions

$$\phi(0) = \phi'(0) = 0, \phi''(0) = 1, \phi'(\infty) = 1.$$

References

[1] Chun-Xuan, Jiang, Analystic Solution of Blasius Equation, Unpublished Notebook, (1967).

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[3] Chun-Xuan, Jiang, Foundations of Santilli's Isonumber Theory, with Applications to New Cryptograms, Fermat's Theorem and Goldbach's Conjecture Inter. Acad, Proess, America-Europe- Asia(2002), 263-268, http://www.i-b-r.org/docs/jiang.pdf.