Gravitational redshift

May 9, 2008.

José Francisco García Juliá
C/ Dr. Marco Merenciano, 65, 5.
46025 Valencia (Spain)
E-mail: jose.garcia@dival.es

In this paper we obtain in an elemental way the basic equations that explain the observed redshift of the light emitted by the stars.

Key words: Light redshift, gravity.

1. Introduction

In this article we go to study in an elemental way how the gravity can change the frequency of the light emitted by the stars. We will show that this light is red shifted or blue shifted, depending on the values of the gravitational potential in the points of emission and observation. Then, if we only see the redshift it is because the gravitational field is lesser at the Earth than at the stars.

2. Gravitational redshift

For a particle of mass m in a gravitational field with potential ϕ we have that

$$E = m\,c^2 \qquad (2.1)$$

$$V = m\phi \qquad (2.2)$$

being E and V the relativistic and potential energies of the particle, respectively, and c the velocity of the light in the vacuum. Therefore, for a photon (assuming an "effective mass" [1] $m = \frac{E}{c^2}$)

$$E = h\nu \qquad (2.3)$$

$$V = \frac{h\nu}{c^2}\phi \qquad (2.4)$$

being h the constant of Planck. As the gravitational field is conservative, then

$$T + V = h\nu + \frac{h\nu}{c^2}\phi = const. \qquad (2.5)$$

$$h\nu_0 + \frac{h\nu_0}{c^2}\phi_0 = h\nu + \frac{h\nu}{c^2}\phi \tag{2.6}$$

$$\nu_0\left(1+\frac{\phi_0}{c^2}\right) = \nu\left(1+\frac{\phi}{c^2}\right)$$

$$\frac{\nu_0}{\nu} = \frac{c^2+\phi}{c^2+\phi_0} \tag{2.7}$$

being ν and ν_0 the light frequencies emitted and observed, respectively, of the photon, $T = h\nu$ its kinetic energy, and ϕ and ϕ_0 the potentials in the points of emission and observation, respectively. The gravitational potential ϕ varies with the inverse of the distance and always is $\phi < 0$, only $\phi(\infty) = 0$. If $|\phi_0| < |\phi|$, then $\nu_0 < \nu$, and the light is red shifted, and $h\nu_0 < h\nu$. As *V* increases (it is less negative) then *T* decreases. But, if $|\phi_0| > |\phi|$, then $\nu_0 > \nu$, and the light is blue shifted.

On the other side, from (2.5), and adapting the escape velocity concept [2] for a photon (with $m = \frac{E}{c^2} = \frac{h\nu}{c^2}$), we have

$$\frac{h\, v_{eph}}{\lambda} + \frac{h\, c}{\lambda\, c^2}\phi = 0$$

$$v_{eph} = -\frac{\phi}{c} = \frac{GM}{R\, c} \tag{2.8}$$

being *G* the gravitational constant of Newton and v_{eph} the escape velocity of a photon of wavelength λ emitted by a star of mass *M* and radius *R*.

Now, from (2.7)

$$\frac{\nu_0-\nu}{\nu} = \frac{\nu_0}{\nu} - 1 = \frac{c^2+\phi}{c^2+\phi_0} - 1 = \frac{\phi-\phi_0}{c^2+\phi_0} \tag{2.9}$$

For $|\phi_0| << c^2$

$$\frac{\nu_0-\nu}{\nu} = \frac{\phi-\phi_0}{c^2} \tag{2.10}$$

And, for $|\phi_0| << |\phi|$

$$\frac{\nu_0 - \nu}{\nu} = \frac{\phi}{c^2} = -\frac{GM}{R\,c^2} = -\frac{v_{eph}}{c} \tag{2.11}$$

From which, we have: $\nu = \dfrac{c}{\lambda}$ and $\nu_0 = \dfrac{c - v_{eph}}{\lambda}$, and the velocity of the observed photon would be $c - v_{eph}$.

3. Conclusion

As the gravity is an attractive force, the light would be red shifted if this force is greater in the point of emission than in the point of observation. This is what occurs with the light emitted by the stars when it is observed at the Earth.

References

[1] R. F. Evans and J. Dunning-Davies, The Gravitational Red-Shift, arXiv: gr-qc/0403082v1 (2004).

[2] John W. Norbury, From Newton's Laws to the Wheeler-DeWitt Equation, arXiv: physics/9806004v2 (1998).