Doppler shift and aberration for spherical electromagnetic waves

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Spherical wave vs. plane wave approximation to the nature of the electromagnetic waves in regard to the Doppler shift and aberration is considered. The first approach is free from the blue shift – red shift transition paradox innate for the second one. For spherical electromagnetic waves, in contrast with the plane ones, we have to assume that not only the magnitude, but also the direction of the light velocity ($\vec{c}$) is the same in any inertial frame, which leads to the accepted expression for time dilation. The rest frame of the source of electromagnetic waves is unique among all inertial frames (in it the angles of emission and reception are the same, and there is no shift in wavelength in all directions). The spherical approximation to electromagnetic waves preserves this uniqueness without violating the principle of relativity of the uniform motion, while the planar approximation ignores of the source completely. Both approaches give the same Doppler shift in the directions of the relative motion of the frames, but the differences at the angles close to the normal to those directions may be dramatic, which makes the validity of the Lorentz transformation questionable.

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Currently accepted mathematical expressions of the Doppler shift and aberration for electromagnetic waves was first derived by A. Einstein using the Lorentz transformation and assuming planar character of those waves [1].

If a source of plane electromagnetic wave is moving with constant velocity v in some frame, then the length of the emitted wave in this frame in the direction making angle θ with the direction of motion of the source (the angle of reception in the stationary frame), λ_θ, is connected with the length of the same wave in the rest frame of the source, λ_0, through the equation:

$$\lambda_\theta = \lambda_0 \frac{(1-\beta\cos\theta)}{\sqrt{1-\beta^2}}, \tag{1}$$

$\beta = v/c$, where c is the speed of electromagnetic waves in the free space.

The direction of propagation of the same wave in the rest frame of its source, angle θ', is connected with angle θ through the equation of aberration:

$$\cos\theta' = \frac{\cos\theta - \beta}{1-\beta\cos\theta}. \tag{2}$$

Angle θ' in the moving frame is seen as angle α from the stationary frame (the angle of emission in the stationary frame). The connection between those angles is:

$$\tan\theta' = \sqrt{1-\beta^2}\,\tan\alpha. \tag{3}$$

Spherical electromagnetic waves do not fit into the picture of the Lorentz transformation. The phase of an electromagnetic wave at any point of space (which is closely related with the number of waves passed trough that point) must be an invariant. Under the Lorentz transformation the phase of a plane electromagnetic wave is the same in any inertial frame and that of a spherical one is not.

That the plane wave approximation to the electromagnetic waves is controversial, we can see from the following example.

Let us consider the case in which the observer is moving relative to the source of light. In that case

$$\lambda_{\theta'} = \lambda_0 \frac{\sqrt{1-\beta^2}}{(1+\beta\cos\theta')}; \tag{4}$$

θ' is the direction of propagation of light in the moving frame, i.e. the angle of reception for the moving observer in his rest frame.

According to Eq. (4), when $\frac{\sqrt{1-\beta^2}-1}{\beta} < \cos\theta' < 0$, then $\lambda_{\theta'} < \lambda_0$. Thus, despite the fact that the direction of reception makes an obtuse angle with the direction of motion of the source, i.e. the source of light is moving away from the observer, the latter gets a blue shift instead of a red shift, which is inconsistent with the constancy of the speed of light in the free space. Indeed, if we consider instead of a wave of light a sequence of short light pulses issued with the frequency of the wave, then each later pulse will have to travel faster than former one in order to give a blue shift.

(The strange transition from blue shift to red shift and vice versa for a given angle of observation by changing only the intensity of the relative velocity of the source and observer, which cannot be accounted for in the framework of the currently accepted theory is considered in [2].)

In a correct equation of the Doppler shift $\cos\theta'$ (as well as $\cos\alpha$), must be changing sign when passing the angle where $\lambda_{\theta'} = \lambda_0$ (zero shift angle), i.e. that angle must be $\theta' = \alpha = \pm\pi/2$, or, which is the same, $\theta = \pm \mathrm{acos}\beta$.

We can get the expression of the Doppler shift for spherical electromagnetic waves using some obvious properties of waves.

It is clear that in any frame all waves (wavelengths) emitted by a source of electromagnetic wave in the free space are contained between the source and the (outer) wavefront: at any instant of time the last wave, or a part of it, has just left the source and the wave first emitted has just arrived at the location of the wavefront. In any direction the distance between the source and the front of the emitted wave is equal to the sum of wavelengths contained between them.

Let a source of spherical electromagnetic wave, O_1, be moving with a constant velocity $v = \beta c$ from the origin O along the axis X of a stationary frame (Fig. 1).

The total number of waves emitted in the period of time t in the stationary frame, n_0', (which is equal to the number of oscillations within the source of wave), is contained in the space between point O_1 and the sphere with centre in O and radius $R = ct$, which represents the front of those waves in the stationary frame. Assuming that the light velocity is the same in the stationary frame and the rest frame of the source, for some point A of the wavefront in the stationary frame we have:

$$|O_1A| = ct\sqrt{1-2\beta\cos\theta+\beta^2} = n_0\lambda_0\sqrt{1-2\beta\cos\theta+\beta^2} = n_0'\lambda_\theta, \tag{5}$$

where n_0 is the number of oscillations within the source of wave during the interval of time t in the rest frame of the source; λ_0 is the length of the emitted wave in the same frame; λ_θ is the length of the wave emitted in direction $\angle AOO_1 = \theta$ relative to the direction of motion of the source in the stationary frame.

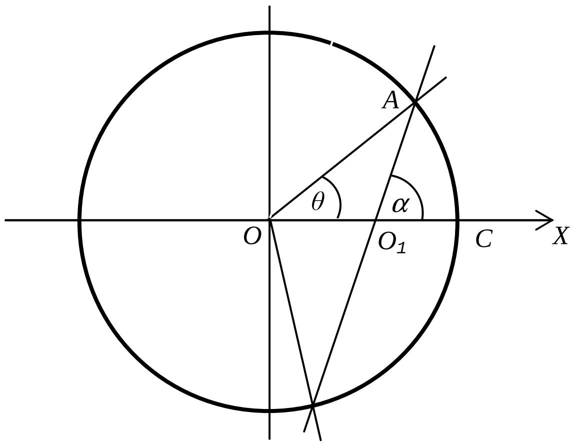

FIG. 1 The wavefront of a spherical electromagnetic wave emitted by a moving source O_1 with the angles of reception and emission in the stationary frame: θ and α respectively.

From Eq. (5):

$$\lambda_\theta = \frac{n_0 \lambda_0 \sqrt{1 - 2\beta\cos\theta + \beta^2}}{n_0'}. \tag{6}$$

It is sometimes convenient to use angle $\angle AO_1C = \alpha$ instead of angle θ. Angle α is the one at which, from the point of view of the stationary frame, the ray OA (or the point A of the wave front) propagates in the moving frame connected with the source. If we assume that the ray OA propagates through an imaginary moving tube of a very small diameter, then angle α is the angle of slope of the tube relative to the direction of the source's motion. From ΔAOO_1

$$\sin\alpha = \frac{\sin\theta}{\sqrt{1 - 2\beta\cos\theta + \beta^2}}.$$

So, instead of Eq. (6) we can use the equation:

$$\lambda_\alpha = \frac{n_0 \lambda_0 \left(\sqrt{1 - \beta^2 \sin^2\alpha} - \beta\cos\alpha\right)}{n_0'}.$$

Due to the constancy of the speed of electromagnetic waves in the free space, when $\lambda_\alpha = \lambda_0$, then reception angle $\alpha = \pm\pi/2$ and $n_0' = n_0\sqrt{1 - \beta^2}$, which gives:

$$\lambda_\theta = \frac{\lambda_0 \sqrt{1 - 2\beta\cos\theta + \beta^2}}{\sqrt{1 - \beta^2}}.$$

and

$$\lambda_\alpha = \frac{\lambda_0\left(\sqrt{1-\beta^2\sin^2\alpha}-\beta\cos\alpha\right)}{\sqrt{1-\beta^2}}.$$

Using vectors we can rewrite the above equations in compact forms:

$$\lambda = \lambda_0 \frac{|\vec{c}-\vec{v}|}{\sqrt{c^2-v^2}}, \tag{7}$$

$$\sin\alpha = \frac{c\cdot\sin\theta}{|\vec{c}-\vec{v}|}, \tag{8}$$

where λ is the wavelength in the considered direction of propagation of the electromagnetic wave, i.e. in the direction of $\vec{c}$.

At $\theta = 0$ and $\theta = \pm\pi$ Eqs. (1) and (7) coincide absolutely.

For the spherical electromagnetic waves the transition to the rest frame of the source is trivial: we have only to alter the sign before the relative velocity in Eqs. (7) and (8):

$$\lambda = \lambda_0 \frac{|\vec{c}+\vec{v}|}{\sqrt{c^2-v^2}}, \tag{9}$$

$$\sin\alpha = \frac{c\cdot\sin\theta}{|\vec{c}+\vec{v}|}.$$

Comparing Eqs. (7) and (9) we can assume that vector $\vec{c} = \overrightarrow{OA}/t$ (the vector connecting the points of emission and reception of a pulse or ray of light at the instant of its emission divided by the time of its travel from one end to the other) is the same in the moving and stationary frames, i.e. for the spherical electromagnetic waves, the vector of the velocity of a electromagnetic wave is an invariant.

Eq. (8) is the equation of aberration for spherical electromagnetic waves. At the instant when a light pulse arrives at the observer, the angle of the slope of the line connecting the reception point with the source is angle α; at the instant when this pulse is emitted by the source that angle is θ. In the rest frame of the source those angles coincide, in any other frame they are different.

For spherical electromagnetic waves the difference between the angles θ and α is fundamental: angle α is always the direction of a ray in the frame connected with the source, and θ may be a direction in any frame, which makes the rest frame of the source unique. That, of course, does not violate the principle of relativity of uniform motion, because we can always single out the rest frame of the source from the other frames (e.g., in that frame, in contrast with any other frame, the angles of emission and reception are the same, and there is zero shift in all directions). In other words, we cannot ignore the source in the case of a spherical electromagnetic wave. In mathematical language this is tantamount to the statement that every sphere has its centre point.

For the plane electromagnetic waves the angles θ and θ' in Eq. (2) are equivalent ones, which means that we are ignoring the source of wave completely and are considering only a wave in two equivalent inertial frames. That is why when making transition from one frame to the other we have not only to alter the sign before the relative velocity of the frames, but also to swap the angles and wavelengths in Eqs. (1) and (2). Thus, under the Lorentz transformation the unique character of the rest frame of the source of electromagnetic wave gets lost, which, of course, is also a distortion of the physical reality.

Now let us get the equation of time dilation using as an axiom the constancy of the vector of the light velocity $\vec{c}$, instead of the information about the zero shift angle as above, in order to show that they mean essentially the same thing.

According to Eq. (6), for the case when $\lambda_\theta = \lambda_0$, we have:

$$\frac{n_0'}{n_0} = \frac{|\vec{c} - \vec{v}|}{c}.$$

In the other frame:

$$\frac{n_0'}{n_0} = \frac{|\vec{c} + \vec{v}|}{c}.$$

Since n_0' / n_0 is the ratio of the numbers of wavelengths, which are equal to the numbers of *internal oscillations* of the source of waves, its final expression cannot contain any evidence of direction (the "source" may be not emitting anything and the direction of the observer's motion cannot affect those numbers).

Besides, while both equations are essentially the same ($|\vec{c} - \vec{v}|$ in one frame is the same as $|\vec{c} + \vec{v}|$ in the other frame), their forms differ in the sign before the relative velocity. The principle of relativity of uniform motion requires that not only the essence of those equations, but also their forms be the same (otherwise an observer would be at a loss which form to use in his frame: in regard to internal oscillations all inertial frames are equivalent). Thus, both forms of those equations must give the same result for both observers, i.e. when $\lambda_\theta = \lambda_0$, then $|\vec{c} - \vec{v}| = |\vec{c} + \vec{v}|$ in both frames, which gives $n_0' / n_0 = \sqrt{1 - \beta^2}$ for the case when $(\vec{c} - \vec{v}) = (\vec{c} + \vec{v})$ (or $\cos\theta = \beta$ in Eq. (6)). (As for the other solution: $n_0' / n_0 = \sqrt{1 + \beta^2}$ (i.e. $\cos\theta = 0$ in Eq. (6)), we discard it because that would mean that a moving observer is be able to count more numbers of waves than emitted by the source.)

In conclusion it may be stated that the differences between the spherical and plane wave approaches to the electromagnetic waves cannot be ignored, which makes the validity of the Lorentz transformation questionable.

References:

[1] Einstein A. *"Zur Elektrodynamik bewegter Körper"*, *Annalen der Physik* **17**, 891-921.
[2] Pierseaux Yves, *"Einstein's relativistic Doppler formula"*, arXiv:physics/0509163v1 [physics.class-ph], 2008