# Interpretation of Solution of Radial Biquaternion Klein-Gordon Equation and Comparison with EQPET/TSC Model

**V. Christianto* and F. Smarandache****


## Abstract

In a previous publication,[1] we argued that the biquaternionic extension of the Klein-Gordon equation has numerical solution with sinusoidal form, which differs appreciably from conventional Yukawa potential. In the present article we interpret and compare this result from the viewpoint of the EQPET/TSC (Electronic Quasi-Particle Expansion Theory/Tetrahedral Symmetric Condensate) model described by Takahashi.[2] Further observation is of course recommended in order to refute or verify this proposition.


## Introduction

In the preceding article[1] we argued that biquaternionic extension of the radial Klein-Gordon equation (radialBQKGE) has numerical solution with sinusoidal form, which differs appreciably from conventional Yukawa potential—and may be interpreted as plausible implication of "local potential" in the Yang-Mills field.[3] We also argued that this biquaternionic extension of KGE may be useful in particular to explore new effects in the context of low-energy nuclear reaction (LENR).[5]

Interestingly, Takahashi[2] has discussed key experimental results in condensed matter nuclear effects in light of EQPET/TSC. We argue here that the potential model used in his paper, STTBA (Sudden Tall Thin Barrier Approximation), may be comparable to our derived sinusoidal potential from radial biquaternion KGE.[1] While we don't yet offer numerical prediction, our qualitative comparison may be useful in verifying further experiments.

## Solution of Radial Biquaternionic KGE (radial BQKGE)

In our previous paper,[1] we argued that it is possible to write the biquaternionic extension of the Klein-Gordon equation as follows:

$$\left(\Diamond\Diamond + m^2\right) \varphi(x,t) = 0 \quad (1)$$

Provided we use this definition:[1,3]

$$\Diamond = \nabla^q + i\nabla^q = \left( -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z} \right) + i \left( -i \frac{\partial}{\partial T} + e_1 \frac{\partial}{\partial X} + e_2 \frac{\partial}{\partial Y} + e_3 \frac{\partial}{\partial Z} \right) \quad (2)$$

where $e_1$, $e_2$, $e_3$ are *quaternion imaginary units* obeying (with ordinary quaternion symbols $e_1=i$, $e_2=j$, $e_3=k$):[3,4]

$$i^2 = j^2 = k^2 = -1,\ ij = -ji = k,\quad jk = -kj = i,\ ki = -ik = j. \quad (3)$$

And quaternion Nabla operator is defined as:[2]

$$\nabla^q = -i \frac{\partial}{\partial t} + e_1 \frac{\partial}{\partial x} + e_2 \frac{\partial}{\partial y} + e_3 \frac{\partial}{\partial z} \quad (4)$$

By using polar coordinates transformation,[1,6] we get this for the one-dimensional situation:

$$\left( \frac{\partial}{\partial r} \left( \frac{\partial}{\partial r} \right) - i \frac{\partial}{\partial r} \left( \frac{\partial}{\partial r} \right) + m^2 \right) \varphi(x,t) = 0 \quad (5)$$

The solution is given by:[1]

$$y = k_1 \cdot \sin\left( \frac{|m|r}{\sqrt{-i-1}} \right) + k_2 \cdot \cos\left( \frac{|m|r}{\sqrt{-i-1}} \right) \quad (6)$$

Therefore, we may conclude that numerical solution of radial biquaternionic extension of Klein-Gordon equation yields different potential compared to the well-known Yukawa potential:[1]

$$u(r) = -\frac{g^2}{r} e^{-mr} \quad (7)$$

In the next section we will discuss an interpretation of this new potential (6) compared to the findings discussed by Takahashi[2] from condensed matter nuclear experiments.

## Comparison with Takahashi's EQPET/TSC/STTBA model

Takahashi[2] reported some findings from condensed matter nuclear experiments, including intense production of helium-4 ($^4$He) atoms by electrolysis and laser irradiation experiments.

Takahashi analyzed those experimental results using EQPET formation of TSC were modelled with numerical estimations by STTBA. This STTBA model includes strong interaction with negative potential near the center (where $r \to 0$). See Figure 1.

Takahashi described that Gamow integral of STTBA is given by:

$$\Gamma_n = 0.218\,(\mu^{1/2}) \int_{r_0}^{b} (V_b - E_d)^{1/2}\, dr \quad (8)$$

Using $b$=5.6 fm and $r_0$=5 fm, he obtained:

$$P_{4d} = 0.77 \quad (9)$$

and

$$V_B = 0.257 \text{ MeV} \quad (10)$$

While his EQPET model gave significant underestimation

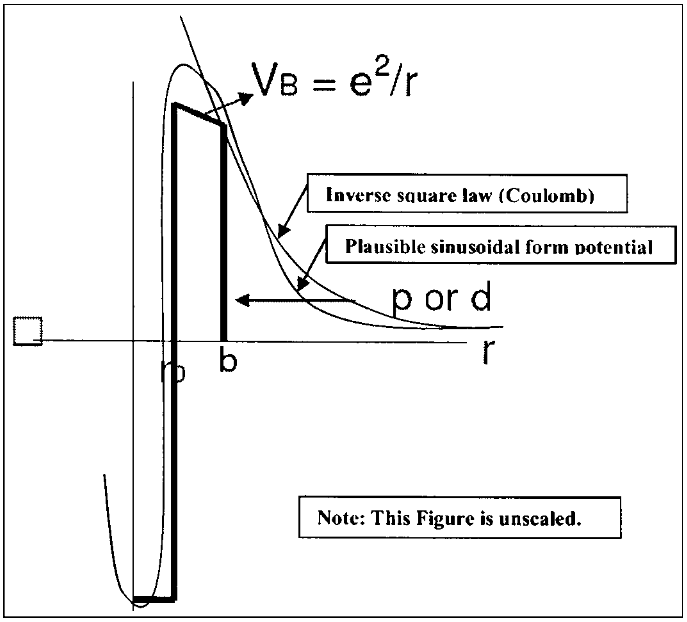


Figure 1. Potential for Coulomb barrier reversal for STTBA, reprinted with permissionfrom Takahashi.[2]

for 4D fusion rate when rigid constraint of motion in 3D space was attained, introducing different values of $\lambda_{4d}$ can improve the result.[2] Therefore we may conclude that STTBA can offer good approximation of condensed matter nuclear reactions.[5]

Interestingly, the STTBA lacks sufficient theoretical basis, therefore one can expect that a sinusoidal form (or combined sinusoidal waves such as in Fourier method) may offer better result which agrees with experiments. This will be pursued in a later paper.

Nonetheless, we recommend further observation in order to refute or verify this proposition of a new type of potential derived from the biquaternion radial Klein-Gordon equation.

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## References

**1.** Christianto, V. and Smarandache, F. 2008. "Numerical Solution of Radial Biquaternion Klein-Gordon Equation," *Progress in Physics*, 4, 1, January; Smarandache, F. and Christianto, V., eds. 2008. *Hadron Models and Related New Energy Issues*, InfoLearnQuest, January (www.gallup-unm.edu/~smarandache/NewEnergy.pdf).

**2.** Takahashi, A. 2005. "A Theoretical Summary of Condensed Matter Nuclear Effects," Siena Workshop on Anomalies in Metal-D/H Systems, Siena, May (http://lenr-canr.org); Takahashi, A. 2007. "A Theoretical Summary of Condensed Matter Nuclear Effects," *J. Condensed Matter Nucl. Sci.*, 1, p. 129 (www.iscmns.org/CMNS/JCMNS-Vol1.pdf).

**3.** Yefremov, A., Smarandache, F., and Christianto, V. 2007. "Yang-Mills Field from Quaternion Space Geometry, and Its Klein-Gordon Representation," *Progress in Physics*, 3, 3, www.ptep-online.com.

**4.** Christianto, V. 2006. "A New Wave Quantum Relativistic Equation from Quaternionic Representation of Maxwell-Dirac Equation as an Alternative to Barut-Dirac Equation," *Electronic Journal of Theoretical Physics*, 3, 12, www.ejtp.com.

**5.** Storms, E. http://lenr-canr.org.

**6.** Nishikawa, M. 2004. "A Derivation of Electroweak Unified and Quantum Gravity Theory without Assuming Higgs Particle," arXiv:hep-th/0407057, p.15.

**7.** Maxima from http://maxima.sourceforge.net. (Using Lisp GNU Common Lisp).

**About the Authors**

V. Christianto is www.sciprint.org administrator and an independent researcher. His research interests include quantization in astrophysics; use of quaternion and biquaternion number in various aspects of theoretical physics; and computational methods in science. He has published more than 14 scientific papers (independently and with F. Smarandache and others); some of them can be found at AFLB, EJTP, and *Progress in Physics*, http://www.ptep-online.com. He also co-authored some books (independently and with F. Smarandache and others), including: *Multivalued Logic, Neutrosophy, and Schrödinger Equation* (Hexis-Phoenix, 2006); *Unfolding the Labyrinth: Open Problems in Physics, Mathematics, Astrophysics, and Other Areas of Science* (Hexis-Phoenix, 2006); *Quantization in Astrophysics, Brownian Motion, and Supersymmetry* (MathTiger, Chenai, India, 2007).

Dr. Smarandache was born in Romania and received a Ph.D. in mathematics from the State University of Kishinev in 1997. After emigration to the U.S., he continued postdoctoral studies at various universities. Smarandache worked as a software engineer for Honeywell (1990-1995) and was an adjunct professor at Pima Community College (1995-1997). In 1997 he became an assistant professor at the University of New Mexico (Gallup) and was promoted to associate professor of mathematics in 2003. Smarandache is the author of many books and has published in many journals and proceedings.

*Email: admin@sciprint.org ; website: www.sciprint.org
**Department of Mathematics, University of New Mexico, Gallup, NM ; Email: fsmarandache@yahoo.com