Complex dynamics as source of anomalous behavior in particle physics

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Abstract

Despite its remarkable predictive power, the Standard Model for particle physics (SM) leaves out many open questions. Two representative examples are the issue of CP violation and the anomalous magnetic moment of leptons (AMM). Our work develops from the premise that the postulate of unitary evolution no longer holds near the scale of electroweak interaction or near the "new physics" sector of SM. Results suggest that CP violation in kaon physics and the AMM problem are direct manifestations of fractional dynamics. Numerical predictions are found to be in close agreement with experimental data.

Introduction It is well known that symmetries are key ingredients of relativistic quantum field theory (*QFT*). In general, symmetry in *QFT* is associated with the invariance of field equations to certain transformations of its observables. In particular, invariance of field equations to the combined action of parity P (space reflection) and charge conjugation C (swapping particles with their antiparticles) is referred to as the CP *symmetry*. One puzzling aspect of *SM* is that CP symmetry is violated in weak interactions, that is, in quantum processes involving the exchange of massive vector bosons. Likewise, we define *dynamic anomalies* as deviations from a particular behavior predicted by field theory. For example, the *AMM* problem is defined as the departure of the measured magnetic moment of leptons g_l^{meas} from its nominal value of $g_l^{SM} = 2$ inferred from Dirac equation [1-3]. To simplify the terminology and presentation, symmetry violation and dynamic anomalies are treated hereafter under the generic name of *anomalies*. Using this ansatz, we focus on two important anomalies of particle physics: a) the violation of CP symmetry in neutral K-meson sector, b) the *AMM* problem. It is believed that a satisfactory resolution of these challenges may shed light on the poorly understood non-perturbative sector of *SM* [4-5]. The discussion is limited to these anomalies because, unlike other open questions surrounding *SM*, CP violation in kaon physics and the *AMM* problem have a well tested and reliable experimental basis. It is important to recall that, although *SM* is able to properly describe these anomalies, it cannot explain them. For example, CP violation effects are embodied in *SM* through a complex phase entering the quark mixing matrix. This phase is the single source of *CP* violation in flavor-changing transitions. However, its contribution fails to account for the magnitude of the observed matter-antimatter imbalance in the Universe [6-7].

The letter is organized according to the following plan: next section surveys the motivation for framing the dynamics of anomalous phenomena in the language of fractal operators. The following sections list the underlying definitions and hypotheses and discuss chiral properties of these operators. Analysis of anomalies is developed afterwards. Summary of results and future extensions are listed in the final section.

We acknowledge from the outset the introductory nature of our study. Independent research is required to validate, develop or falsify these preliminary predictions.

Onset of complex dynamics near the Fermi scale The electroweak interaction is transmitted by photon (γ) over large distances and by the triplet of massive vector bosons $W^{\pm}, Z^0$ over the short-distance region set by the Fermi constant ($G_F^{-1/2} = 293\, GeV$) [8-9]. The *Fermi scale* represents about the largest energy region probed with the current accelerator technology. We start with the observation that *P, CP* and dynamic anomalies are linked to either the electroweak interaction or the "new physics" (*NP*) sector of *SM* [6-9]. This observation suggests that unitary evolution postulated by *QFT* no longer holds near these large energy scales. Here, quantum processes evolve in a highly unstable, "noisy" environment and are prone to migrate outside equilibrium. An immediate question is then: How does this regime impact the dynamics of interacting fields? In searching for an answer to this question, it is sensible to follow the Renormalization Group ideas (*RG*) describing the approach to critical behavior in field theory. *RG* asserts that, near criticality, a) fast "noise" fluctuations can be averaged out and absorbed into a redefinition of observables, b) slow and fast fluctuations decouple [10-11]. This entails that the scale of "noise" and the scale on which quantum fields evolve are at least one order of magnitude different. There is, however, at least one *crossover region* where this ansatz fails and critical fluctuations can only be partially suppressed [11]. These considerations suggest that any attempt to extend the dynamic framework of field theory beyond *QFT* needs to comply with the following requirements:

a) must describe *non-unitary processes* where the scale of "noise" and scale of quantum fields overlap.

b) must have a *built-in asymmetry* to the reversal of space and time coordinates. [12]

c) has to *asymptotically approach* conventional *QFT*, which is by construction a theory of unitary phenomena.

A framework that naturally fits this description is based on fractional dynamics. Fractional dynamics operates with derivatives of non-integer order called *fractal* *operators* and is suitable for analyzing complex processes with long-range interactions [12-14]. Building on the current understanding of fractal operators, we take the dimensional parameter of the regularization program $\varepsilon = 4 - d$ to represent the order of fractional differentiation in physical space-time (alternatively, $\varepsilon = 1 - d$ in one-dimensional space) [13, 15-16]. According to this viewpoint, anomalous behavior starts to develop as soon as ε departs from zero. Full scale invariance and mean-field theory are asymptotically recovered as $\varepsilon \to 0$ [11].

Definitions and assumptions In this letter we use the Riemann-Liouville definition for the one-dimensional left and right fractal operators [16]. Taking the time coordinate to be the representative variable, one writes

$$ {}_0D_L^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dt} \int_0^t (t-\tau)^{-\alpha} f(\tau)\, d\tau \tag{1a} $$

$$ {}_0D_R^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} (-\frac{d}{dt}) \int_t^0 (\tau - t)^{-\alpha} f(\tau)\, d\tau \tag{1b} $$

Here, fractional dimension $0 < \alpha < 1$ denotes the order of fractional differentiation. We further specialize (1) to the case $\alpha = \varepsilon \ll 1$. Hence,

$$ {}_0D_L^{\varepsilon} f(t) \approx \frac{d}{dt} \int_0^t (t-\tau)^{-\varepsilon} f(\tau)\, d\tau \tag{2a} $$

$$ {}_0D_R^{\varepsilon} f(t) \approx (-\frac{d}{dt}) \int_t^0 (\tau - t)^{-\varepsilon} f(\tau)\, d\tau \tag{2b} $$

We also assume that analytic continuation is fully applicable to (1) and (2). As a result, substitution of the original time variable with its complex analogue $\tilde{t} = -it$ does not alter the physical content of the theory [17].

Chiral and mixing properties of fractal operators Left and right fractal operators *(L/R)* are natural analogues of chiral components associated with the structure of quantum fields [13]. It is important to note that there is an inherent mixing of *(L/R)* operators as described below. An equivalent representation of (1a) is given by

$$ {}_0D_L^{\alpha} f(t) = \frac{1}{\Gamma(1-\alpha)} (-\frac{d}{dt}) \int_t^0 [-(\tau - t)]^{-\alpha} f(\tau) d\tau $$

or,

$$ {}_0D_L^{\alpha} f(t) = \frac{(-1)^{-\alpha}}{\Gamma(1-\alpha)} (-\frac{d}{dt}) \int_t^0 (\tau - t)^{-\alpha} f(\tau) d\tau = (-1)^{-\alpha} {}_0D_R^{\alpha} f(t) $$

$$ {}_0D_R^{\alpha} = (-1)^{\alpha} {}_0D_L^{\alpha} = \exp(i\pi\alpha) {}_0D_L^{\alpha} \tag{3a} $$

Starting from (1b) instead, we find

$$ {}_0D_L^{\alpha} = (-1)^{\alpha} {}_0D_R^{\alpha} = \exp(i\pi\alpha) {}_0D_R^{\alpha} \tag{3b} $$

Let us iterate (3) a finite number of times ($n \geq 1$) under the assumption that $n\alpha = n\varepsilon \ll 1$. It follows that the fractal operator of any infinitesimal order may be only defined up to an arbitrary phase factor $\exp(i\pi n\varepsilon) \approx 1 + (i\pi n\varepsilon) = 1 - i\varepsilon$, that is,

$$ {}_0D_{L,R}^{\varepsilon} f(t) \approx [{}_0D_{L,R}^{0} - i\varepsilon] f(t) $$

or

$$ i\, {}_0D_{L,R}^{\varepsilon} f(t) = [i\, {}_0D_{L,R}^{0} + \varepsilon] f(t) \tag{4a} $$

where

$$ \lim_{\varepsilon \to 0} D_{L,R}^{\varepsilon} f(t) = f(t) \tag{4b} $$

Finally, the postulated property of analytic continuation transforms (4) into

$$ {}_0D_{L,R}^{\varepsilon} f(\tilde{t}) = [{}_0D_{L,R}^{0} + \varepsilon] f(\tilde{t}) \tag{5} $$

There is a fundamental distinction between (5) and the gauge transformation of conventional *QFT*. A gauge transformation is an operation that preserves unitarity, whilst (5) is built from fractal time operators that break time-reversal invariance [12]. As stated at the beginning of this section, these relations show that fractional dynamics induces a *topological mixing* among all quantum states. This phenomenon does not have a counterpart in conventional *QFT*.

Universal transition to equilibrium According to the so-called Feigenbaum paradigm, asymptotic transition to mean-field behavior and scale invariance is described by $\varepsilon \to 0$ and occurs through a universal sequence of period-doubling bifurcations [13, 18-19]. For $n = 2^p \gg 1$, the bifurcation set is represented by the geometric progression

$$\varepsilon_n - \varepsilon_\infty = C\bar{\delta}^{-n} \tag{6a}$$

in which *C*, $\bar{\delta}$ are scaling constants and $\varepsilon_\infty = 0 \; (d_\infty = 4)$. Dividing both sides by *C*, rescaling the dimensional parameter and dropping the tildes yields

$$\varepsilon_p = \bar{\delta}^{-2^p} \tag{6b}$$

The expectation is that, near the transition from classical to fractional dynamics, index p assumes a range of values that are close to unity ($p = 1, 2, 3...$). In addition, if there are N independent states involved in the anomalous process, it is reasonable to assume that the dimensional contribution “per state” is given by

$$\varepsilon_p^0 = (\frac{1}{N})\bar{\delta}^{-2^p} \tag{7}$$

In what follows we take $\bar{\delta} = 3.9$. This is the best fit value for recovering the spectra of particle masses and gauge couplings, as discussed in [13, 19].

***CP* violation in *K*-meson physics** The first application of (7) may be found in understanding *CP* violation in kaon physics. One way to describe the *CP* anomaly of *K*-mesons is to refer to the structure of the long-lived kaon $\left|K_L^0\right\rangle$ [6-7]. It differs slightly from the *CP*-odd eigenstate $\left|K_2\right\rangle \approx \left|K^0\right\rangle - \left|\overline{K^0}\right\rangle$ and is rather represented as

$$\left|K_L^0\right\rangle \approx (1+\varepsilon_K)\left|K^0\right\rangle - (1-\varepsilon_K)\left|\overline{K}^0\right\rangle \tag{8}$$

Here ε_K is a complex deviation parameter whose modulus is $\left|\varepsilon_K\right| = (2.254 \pm 0.083)\times 10^{-3}$ and which measures the amplitude of *CP*-violation in mixing of *K*-mesons [6-7]. On account of (4), (5), (7) and on the fact that $\left|\overline{K^0}\right\rangle$ is, by definition, a time-reversed replica of $\left|K^0\right\rangle$, we obtain

$$\begin{aligned} \left|K^0\right\rangle &\approx (1+\varepsilon_p^0)\left|K^0\right\rangle \\ \left|\overline{K}^0\right\rangle &\approx (1-\varepsilon_p^0)\left|\overline{K}^0\right\rangle \end{aligned} \tag{9}$$

Since there are two independent states involved in mixing ($\left|K^0\right\rangle$ and $\left|\overline{K^0}\right\rangle$) we take $N = 2$ and an overall contribution $n_1 n_2 = 2^{p_1} 2^{p_2} = 2^{p_1+p_2} = 2^p$. Considering the simplest possible scenario $(p_1 = p_2 = 1)$ leads to

$$\varepsilon_2^0 = \frac{1}{2}\overline{\delta}^{-4} = 2.162\times 10^{-3} \tag{10}$$

in close numerical agreement with $\left|\varepsilon_K\right|$.

Theory of *CP* violation introduces a second deviation parameter $\varepsilon_K^{'}$ in connection with the occurrence of direct decay channels [6-7, 8-9]. Experimental data shows that

$\left|\frac{\varepsilon_K^{'}}{\varepsilon_K}\right| = 1.72\times 10^{-3}$ or $\left|\varepsilon_K^{'}\right| = 3.878\times 10^{-6}$. We repeat the same reasoning for $\varepsilon_K^{'}$ and assume $N = 4$ since there are four independent states participating in the decay, i.e. $\left|K_L\right\rangle, \left|K_S\right\rangle, \left|\pi^+\pi^-\right\rangle, \left|\pi^0\pi^0\right\rangle$. Using $p = 3$ results in a value that matches well the magnitude of $\varepsilon_K^{'}$, that is,

$$\varepsilon_3^0 = \frac{1}{4}\overline{\delta}^{-8} = 4.67\times 10^{-6} \tag{11}$$

Likewise, semi-leptonic decay channels are characterized by four independent states ($N = 4$), that is, $\left|K^0\right\rangle, \left|\overline{K}^0\right\rangle, \left|\pi^-\right\rangle, \left|\pi^+\right\rangle$ [6-7][1]. Using $p = 2$ gives

$$\varepsilon_{2,SL}^0 = 1.081\times 10^{-3} \tag{12}$$

This result falls close to the laboratory value reported in the literature ($\mathrm{Re}\,\varepsilon_{K,SL}^{'} = 1.656\times 10^{-3}$) [6-7].

The lepton magnetic moment problem We turn now to the *AMM* issue. The predicted magnetic moment carried by a charged lepton $l = \{e, \mu, \tau\}$ is [2-4]

$$\overrightarrow{\mu_l} = g_l \frac{e\hbar}{2m_l}\frac{\vec{S}}{\hbar} \tag{13}$$

in which $S = 1/2$ represents the lepton spin, m_l the lepton mass and g_l its gyromagnetic ratio. The cumulative contribution of quantum fluctuations leads to a small deviation from the Dirac value $g_l = 2$ that is parameterized by the *anomalous magnetic moment*:

$$a_l \equiv \frac{1}{2}(g_l - 2) \tag{14}$$

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[1] Here, leptonic doublets $(e^+\nu_e)$ and $(e^-\overline{\nu}_e)$ are not included in the count on the assumption that their contribution to *CP* violation may be safely ignored to a first-order approximation.

This quantity can be accurately measured in experiments and, through the perturbative framework of *SM*, precisely determined. The difference between experiment and theory ($\Delta a_l = \left| a_l^{OBS} - a_l^{SM} \right|$) provides a stringent test of *SM* at its quantum loop level. The contribution a_l^{SM} is generally expected to scale linearly with $({}^{m_l}\!/_{\Lambda_{NP}})^2$, where Λ_{NP} stands for the scale above which the "new physics" sector of *SM* is likely to emerge [2-4]. It is thus apparent from these remarks that a_μ, a_τ are a much more sensitive signal for *NP* effects on or above the Λ_{NP} scale, since $({}^{m_\mu}\!/_{m_e})^2 = 4.275\times10^4$ and $({}^{m_\tau}\!/_{m_e})^2 = 1.209\times10^7$.

By definition, the magnetic moment of leptons depends on ratio $({}^{e}\!/_{m})$. As a result, departures from $g_l = 2$ are linked to quantum processes that distort the relative distribution of charge and mass. Since both charge and mass are scale-dependent, the gyromagnetic ratio depends also on the observation scale. From the standpoint of *RG*, they represent observables that "run" with the observation scale μ according to the flow equation

$$\mu \frac{da_l}{d\mu} \equiv \frac{da_l}{dt} = \beta_a(a_l) = d_1 + d_2 a_l + d_3 a_l^2 + ... \tag{15}$$

in which d_i $(i = 1, 2, 3...)$ denote expansion coefficients and $t \square \log({}^{\mu}\!/_{\mu_0})$ stands for the sliding scale. Analysis of the *RG* flow for couplings and masses in the presence of a generic control parameter reveals the onset of a scaling pattern near the chaotic attractor of the flow [13, 20]. This fact is consistent with the universal scenario for transition to chaos in unimodal maps [18, 20]. On this basis, we posit that $\Delta a = \left| a^{OBS} - a^{SM} \right| = a^{NP}$ obeys a scaling relationship similar to (7)

$$\Delta a_l \propto a_0 \overline{\delta}^{-2^{n(l)}} \tag{16}$$

where $n(l) > 1$ is a natural index associated with each lepton flavor $"l"$. By analogy with (7), the ratio of consecutive terms of the lepton family can be presented as

$$\frac{\Delta a_l}{\Delta a_{l+1}} = (\frac{1}{N})\overline{\delta}^{-(2^l)} \tag{17}$$

Here, $N = 2$ since there are two participating states, $|e,\mu\rangle$ and $|\mu,\tau\rangle$. Table 1 summarizes the wealth of current knowledge on lepton anomalous moments, along with their respective references. a^{OBS} denotes the 'observed' data, whereas a^{SM} represents the *SM* values computed according to [2-4, 21-23, 24-27].

Lepton flavor	a^{OBS}	a^{SM}	$\Delta a = a^{OBS} - a^{SM}$
e	$1159652188.3 \times 10^{-12}$ [21]	0.0011596521859 [22]	5.05×10^{-12}
μ	$116592080 \times 10^{-11}$ [23]	$116591858 \times 10^{-11}$ [4]	22×10^{-10}
τ	$-0.052 < a_\tau^{OBS} < 0.013$ [24]	$117721(5) \times 10^{-8}$ [25]	unknown
	$\langle a_\tau^{OBS} \rangle = -0.018(17)$ [26]	$117721(5) \times 10^{-8}$ [25]	unknown
	$-0.007 < a_\tau^{OBS} < 0.005$ [27]	$117721(5) \times 10^{-8}$ [25]	unknown

Tab. 1: Observed and computed spectrum of lepton magnetic moments

The set of values for Δa_e and Δa_μ are known to a great degree of precision [2-4, 21-23]. Hence taking $l(e,\mu) = 2$ and applying (17) to the first two components of the charged lepton triplet yields

$$\frac{\Delta a_e}{\Delta a_\mu} = (\frac{1}{2})\overline{\delta}^{-2^{l(e,\mu)}}$$

It follows from these arguments that $l(\mu,\tau)=l(e,\mu)+1=3$ and the estimate of the τ -lepton magnetic moment is given by

$$\Delta a_{\tau}=(\frac{1}{2})\frac{\Delta a_{\mu}}{\overline{\delta}^{-8}}\Rightarrow a_{\tau}^{OBS}=2.179917\times 10^{-3} \tag{19}$$

A quick glance at Tab. 1 shows that this prediction complies with the range of experimental data recently reported in [24-27]. Narrowing the range Δa_{τ} through new rounds of high-precision tests on the τ -lepton will either confirm or refute the validity of (19).

Concluding remarks In summary, our work is based on the premise that the postulate of unitary evolution is likely to break down either near the Fermi scale or the “new physics” (*NP*) scale. Anomalous behavior emerges as a direct manifestation of this far-from-equilibrium setting whose adequate description requires the use of fractal operators. Fractional dynamics suggests a straightforward explanation of *CP* violation in the kaon sector and of the *AMM* problem. Invoking *RG* equations leads to the conclusion that the spectrum of lepton magnetic moments follows a scaling pattern. The predicted moment of the τ - lepton is found to fall in line with current experimental data. Main results are tabulated below.

We caution that our model can be best qualified as “work in progress”. Parallel efforts are needed to either disprove or expand these preliminary findings. In particular, it is interesting to evaluate if a similar approach may explain the long-standing puzzle known as the “strong *CP* problem” of particle physics [28]. Can one cancel the θ-vacuum angle of quantum chromo-dynamics (*QCD*) through the combined use of (5), (6) and of fractal operators? It is also instructive to determine if our model can generate predictions that comply with recent laboratory data on *B*-meson decays and their *CP* violating effects. We plan on reporting these developments in a sequel study.

Name	Parameter	Behavior	Predicted	Actual
K-meson mixing	$\lvert\varepsilon_K\rvert$	$(1/2)(\bar{\delta})^{-4}$	2.162×10^{-3}	2.280×10^{-3}
K-meson decay	$\lvert\varepsilon'_K\rvert$	$(1/4)(\bar{\delta})^{-8}$	4.670×10^{-6}	3.878×10^{-6}
Semileptonic K-meson decay	$\mathrm{Re}\,\varepsilon'_{K,SL}$	$(1/4)(\bar{\delta})^{-4}$	1.081×10^{-3}	1.656×10^{-3}
Magnetic moment	$\frac{\Delta a_e}{\Delta a_\mu}$	$(1/2)(\bar{\delta})^{-4}$	2.295×10^{-3}	2.162×10^{-3}
Magnetic moment	$\frac{\Delta a_\mu}{\Delta a_\tau}$	$(1/2)(\bar{\delta})^{-8}$	$-0.007 < a_\tau^{EXP} < 0.005$	$a_\tau^{PR} = 2.180\times10^{-3}$

Tab 2: Summary of predicted versus actual parameters

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