Absorption of a circularly polarized beam in a dielectric, *etc.*

Radi I. Khrapko
Moscow Aviation Institute, 125993, Moscow, Russia

We calculate absorption of a circularly polarized light beam without an azimuth phase structure in a dielectric in the frame of the standard electrodynamics. A transfer of energy, momentum, and angular momentum from the beam to the dielectric is calculated. The calculation shows, in particular, that the angular momentum flux in the beam equals to two power of the beam divided by frequency. This result contradicts another part of the electrodynamics, which predicts the flux equals to power of the beam divided by frequency. In addition we show that this part of the electrodynamics contradicts the classical Beth's experiment. Our inference is: the electrodynamics is incomplete. To correct the electrodynamics, we introduce a spin tensor into the electrodynamics. The corrected electrodynamics is in accordance with our calculation and with the Beth's experiment.

Results of R. Loudon (PRA, 68, 013806) and A. Bishop et al. (PRL, 92, 198104) are mentioned.
PACS numbers: 42.25.Bs; 42.25.Ja; 42.87.-d

Keywords**:** Electrodynamics torque, angular momentum, spin tensor

1. Problem statement

Physicists recognize that circularly polarized light carries an angular momentum since the 19th century [1]. The angular momentum is absorbed if an absorbent absorbs the light no matter what nature of the absorbent. R. Feynman explained popularly a mechanism of the absorption using a circularly polarized plane wave as an example [2]. He proved that the ratio of the angular momentum flux density to the power density in such a wave was $1/\omega$. He recognized this angular momentum as spin of the wave.

Now physicists can calculate the power density in an electromagnetic field. It is a component of the Maxwell energy-momentum tensor or of the Poynting vector $\mathbf{E}\times\mathbf{B}$, $E^2/2+B^2/2$ (we set $\varepsilon_0=\mu_0=1$). However, they cannot calculate the angular momentum flux density because the modern electrodynamics does not know a spin tensor. And what is more, because of this inability, physicists claim that the angular momentum flux density in a circularly polarized plane wave is zero in direct contradiction to the quantum theory and to the Feynman's reasoning.

This claim is a corollary of a standard expression for a total angular momentum **J** [3]. In Maxwell's theory the Poynting vector $\mathbf{E}\times\mathbf{B}$ is interpreted as the density of momentum of the field. This is considered to be a reason that a total angular momentum relative to a point *O* or to an axis is defined as

$$\mathbf{J}=\int_V \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV\,, \qquad (1.1)$$

where **r** is the distance from *O*, and *V* is the volume of a slice of the beam.

According to this definition, only a beam can carry an angular momentum because only in a wave of finite transverse extent, the **E** and **B** fields have a component parallel to the wave vector (the field lines are closed loops) and the energy flow has components perpendicular to the wave vector. This circulating energy flow in the wave implies the existence of angular momentum, whose direction is along the direction of propagation, and

$$J_z=U/\omega \qquad (1.2)$$

in the case of a circularly polarized beam without an azimuth phase structure (here *U* is the energy of the slice of the beam) [3, 4]. This angular momentum is recognized as spin of the beam [5].

If the light gets through a doubly refracting object, which reverses the circularly polarization of the light in handedness, the object absorbs the double angular momentum of the incident light. So, a circularly polarized light passing through an isotropic absorbent or a doubly refracting medium exerts a moment of force, i.e. a torque τ on the medium. This torque arises due to the conservation law of angular momentum and should be equal to the angular momentum flux into the medium. So, according to (1.2),

$$\tau = \mathsf{P} / \omega \tag{1.3}$$

where P is the power of the absorbed beam.

Nevertheless we have reasons to doubt Eqns. (1.1) - (1.3) and the recognition that (1.1) is spin. To verify the statements (1.1), (1.2) we consider absorption of the beam by semi-infinite dielectric. We find that the dielectric absorbs twice as much as (1.1), (1.2), i.e.

$$\tau = 2\mathsf{P} / \omega, \quad J_z = 2U / \omega . \tag{1.4}$$

Thus we find that a circularly polarized light beam without an azimuth phase structure carries the double angular momentum compared with the prediction of the standard electrodynamics according to eqn. (1.1).

We argue that when an electromagnetic wave passes through a dielectric, the electric field polarizes the dielectric. The polarization and time derivative of the polarization, i.e. the displacement current, are

$$\mathbf{P} = (\varepsilon - 1)\mathbf{E}, \quad \mathbf{j} = \partial_t \mathbf{P} . \tag{1.5}$$

These allow us to calculate the torque on the absorbing dielectric using a standard formula [see, for example, [6] eqns. (5.1) and (7.18)]

$$\tau = \int [\mathbf{r} \times (\mathbf{P} \cdot \nabla)\mathbf{E} + \mathbf{r} \times (\mathbf{j} \times \mathbf{B}) + \mathbf{P} \times \mathbf{E}] dV . \tag{1.6}$$

Here $(\mathbf{P} \cdot \nabla)\mathbf{E} + \mathbf{j} \times \mathbf{B}$ is the total Lorentz force per unit volume (see, for example, [7]), and $\mathbf{P} \times \mathbf{E}$ is the torque on electric dipoles per unit volume [8]. Note that the volume density of torque, i.e. the integrand

$$d\tau / dV = \mathbf{r} \times (\mathbf{P} \cdot \nabla)\mathbf{E} + \mathbf{r} \times (\mathbf{j} \times \mathbf{B}) + \mathbf{P} \times \mathbf{E} , \tag{1.7}$$

consists of two part:

(i) Two first terms in (1.7),

$$\mathbf{r} \times (\mathbf{P} \cdot \nabla)\mathbf{E} + \mathbf{r} \times (\mathbf{j} \times \mathbf{B}) , \tag{1.8}$$

equal to zero in the case of a plane wave and near the axis of a beam. In the case of a wide beam, they are presented only near the wall of the beam. Simmonds and Guttmann wrote: "The skin region is the only one in which the *z*-component of angular momentum does not vanish" [9]. We name these terms wall terms,

$$\tau_w = \int [\mathbf{r} \times (\mathbf{P} \cdot \nabla)\mathbf{E} + \mathbf{r} \times (\mathbf{j} \times \mathbf{B})] dV . \tag{1.9}$$

The electric contribution to the total Lorentz force, i.e. the Coulomb force acting on dipoles, $(\mathbf{P} \cdot \nabla)\mathbf{E}$, is not a volume force [see. (2.11)]. In reality, this force acts only on a surface bound charge σ of the dielectric because the average macroscopic volume bound charge density is zero (as it always is for neutral linear dielectrics). So, we name the first term in (1.6) a surface torque,

$$\tau_s = \int \mathbf{r} \times (\mathbf{P} \cdot \nabla)\mathbf{E} dV , \tag{1.10}$$

and we name the second term in (1.6) a magnetic torque,

$$\tau_B = \int \mathbf{r} \times (\mathbf{j} \times \mathbf{B}) dV . \tag{1.11}$$

So,

$$\tau_w = \tau_s + \tau_B \tag{1.12}$$

(ii) The last term in (1.7),

$$\mathbf{P} \times \mathbf{E} , \tag{1.13}$$

presents a volume torque inside the beam or in the plain wave. We name this term an electric torque,

$$\tau_E = \int \mathbf{P} \times \mathbf{E} dV . \tag{1.14}$$

So, the total torque,

$$\tau = \tau_w + \tau_E = \tau_s + \tau_B + \tau_E . \tag{1.15}$$

Now we can present our calculation.

2. Torque

We use paraxial beams of light [4, 10, 11],

$$\breve{\mathbf{E}} = \exp[i(\breve{k}z - t)][\mathbf{x} + i\mathbf{y} + \mathbf{z}\frac{1}{\breve{k}}(i\partial_x - \partial_y)]u(\rho), \quad \breve{\mathbf{B}} = -i\breve{k}\breve{\mathbf{E}}, \quad \rho^2 = x^2 + y^2, \quad \breve{k}^2 = \breve{\varepsilon}, \quad \breve{k} = \eta + i\kappa. \tag{2.1}$$

The symbol 'breve' marks complex vectors and numbers excepting i. $\breve{k}$ is the complex wave number, and $\breve{\varepsilon}$ is the permittivity. For short, we set speed of light in vacuum, $c = 1$, and the frequency, $\omega = 1$.

A profile of the beam (2.1) may be Gaussian [11],

$$u = \exp\{-\rho^2 / w^2\}\sqrt{2/\pi}/w, \quad kw >> 1, \tag{2.2}$$

but it doesn't matter. We use the cylindrical coordinates ρ, ϕ, z

$$x = \rho\cos\phi, \quad y = \rho\sin\phi, \quad \rho = \sqrt{x^2 + y^2} \tag{2.3}$$

with the metric

$$dl^2 = d\rho^2 + \rho^2 d\phi^2 + dz^2, \quad g_{\rho\rho} = 1, \quad g_{\phi\phi} = \rho^2, \quad g_{zz} = 1, \quad \sqrt{g}_{\wedge} = \rho, \quad g^{\phi\phi} = 1/\rho^2 \tag{2.4}$$

Square root of determinant of the metric tensor is a scalar density of weight $+1$. Gothic symbols are usually applied to denote tensor densities. We shall, instead, mark the density with the symbol 'wedge' at the level of bottom indices for a density of weight $+1$ and at the level of top indices for a density of weight -1. A volume element and a surface element are densities of weight -1, $dV^{\wedge} = d\rho d\phi dz$, $da^{\wedge} = d\rho d\phi$, as well as the absolute antisymmetric density $e^{\wedge}_{ijk}$, which equals to ± 1, or 0.

The coordinate transformation for the *covariant* components of the vectors **E**, **B** in (2.1) gives [12]

$$\underset{\rightarrow}{\breve{E}} = \exp[i(\breve{k}z - t + \phi)](\underset{\rightarrow}{\rho} + i\rho\underset{\rightarrow}{\phi} + \underset{\rightarrow}{z}\frac{i}{\breve{k}}\partial_\rho)u(\rho), \qquad \underset{\rightarrow}{\breve{B}} = -i\breve{k}\underset{\rightarrow}{\breve{E}}. \tag{2.5}$$

The arrow placed under a symbol means a covariant vector, or a covariant coordinate vector.

If a beam of the type (2.5) with $\breve{k} = 1$ for $z < 0$,

$$\underset{\rightarrow}{\breve{E}}_1 = \exp[i(z - t + \phi)](\underset{\rightarrow}{\rho} + i\rho\underset{\rightarrow}{\phi} + \underset{\rightarrow}{z}\, i\partial_\rho)u(\rho), \qquad \underset{\rightarrow}{\breve{B}}_1 = -i\underset{\rightarrow}{\breve{E}}_1. \tag{2.6}$$

impinges normally on a surface of a dielectric which is characterized by $\breve{k}$, the beam divides into a reflected part (for $z < 0$)

$$\underset{\rightarrow}{\breve{E}}_2 = \frac{1-\breve{k}}{1+\breve{k}}\exp[i(-z - t + \phi)](\underset{\rightarrow}{\rho} + i\rho\underset{\rightarrow}{\phi} - \underset{\rightarrow}{z}\, i\partial_\rho)u(\rho), \qquad \underset{\rightarrow}{\breve{B}}_2 = i\underset{\rightarrow}{\breve{E}}_2 \tag{2.7}$$

and a transmitted part (for $z > 0$)

$$\underset{\rightarrow}{\breve{E}}_3 = \frac{2}{1+\breve{k}}\exp[i(\breve{k}z - t + \phi)](\underset{\rightarrow}{\rho} + i\rho\underset{\rightarrow}{\phi} + \underset{\rightarrow}{z}\frac{i}{\breve{k}}\partial_\rho)u(\rho), \qquad \underset{\rightarrow}{\breve{B}}_3 = -i\breve{k}\underset{\rightarrow}{\breve{E}}_3 \tag{2.8}$$

in accordance with the reflected and the transmission coefficients $\breve{R} = \frac{1-\breve{k}}{1+\breve{k}}$, $\breve{T} = \frac{2}{1+\breve{k}}$. We set

$$\int u^2 2\pi\rho\, d\rho = 1 \tag{2.9}$$

(we do not write a dimension), so an average power that enters the dielectric is

$$\mathsf{P} = 1 - R^2 = \eta T^2 = \frac{4\eta}{1 + 2\eta + k^2}, \quad R = |\breve{R}|, \quad T = |\breve{T}|, \quad |\breve{k}| = k. \tag{2.10}$$

Now we transform Eqn. (1.10) of *z*-component of the surface torque,

$$\underset{s}{\tau}_z = \int \rho(\varepsilon - 1)E^i\partial_i E_\phi dV = \int(\varepsilon - 1)\partial_i(\rho E^i E_\phi)dV = \oint(\varepsilon - 1)\rho E^i E_\phi da_i = \int \rho\sigma E_\phi da_z \tag{2.11}$$

The surface charge density σ is obtained by the use of E_z-components from (2.6) – (2.8),

$$\breve{\sigma} = [\breve{E}_{3z} - \breve{E}_{1z} - \breve{E}_{2z}]_{z=0} = \frac{2i(1-\breve{\varepsilon})}{(1+\breve{k})\breve{k}} \exp[i(-t+\phi)]\partial_\rho u(\rho) . \qquad (2.12)$$

Substituting (2.12) and $E_{3\phi}$ from (2.8) into (2.11) yields the time average surface torque

$$\underset{s}{\tau} = \int_{z=0} \rho \Re\{\breve{\sigma}\overline{E}_{3\phi}\} d\rho d\phi / 2 = \frac{2\pi\Re\{(1-\breve{\varepsilon})\overline{k}\}}{k^2(1+2\eta+k^2)} \int \rho^2 \partial_\rho (u^2)\, d\rho = \frac{2\eta(k^2-1\}}{k^2(1+2\eta+k^2)} \qquad (2.13)$$

(the over lines mark complex conjugate complex numbers).

The time average magnetic torque (1.11) equals to the integral of

$$d\underset{B}{\tau}_z = \Re\{\rho(\breve{j}_{3z}\overline{B}_{3\rho} - \breve{j}_{3\rho}\overline{B}_{3z})\sqrt{g}_{\wedge}\} dV^{\wedge} / 2 \qquad (2.14)$$

over the dielectric $(z > 0)$. Substituting $\breve{E}_{3\rho}, \breve{E}_{3z}, \overline{B}_{3\rho} = i\overline{k}\overline{E}_{3\rho}, \overline{B}_{3z} = i\overline{k}\overline{E}_{3z}$ from (2.8) into (2.14) and integrating with respect to ϕ yields

$$\underset{B}{\tau}_z = \pi \int \rho^2 \Re\{(\breve{\varepsilon}-1)i\overline{k}(\partial_t \breve{E}_{3z}\overline{E}_{3\rho} - \partial_t \breve{E}_{3\rho}\overline{E}_{3z})\} d\rho dz = \frac{4\pi\eta\Re\{(\breve{\varepsilon}-1)i\overline{k}\}}{k^2(1+2\eta+k^2)} \int \rho^2 \partial_\rho (u^2) \exp(-2\kappa z)\, d\rho dz . \qquad (2.15)$$

Integrating with respect to *z* and, by parts, with respect to ρ yields [12]

$$\underset{B}{\tau}_z = \frac{2\eta(k^2+1)}{k^2(1+2\eta+k^2)} . \qquad (2.16)$$

Calculating of the electric torque (1.14) yields:

$$\underset{E}{\tau}^z = \int \Re(\breve{P}_\rho \overline{E}_{3\phi} - \breve{P}_\phi \overline{E}_{3\rho}) e^{\rho\phi z}_{\wedge} (d\rho d\phi dz)^{\wedge} / 2 = \frac{4\eta}{1+2\eta+k^2} \qquad (2.17)$$

It is remarkable that the sum of the surface part (2.13) and the magnetic part (2.16) of the total torque τ equals to the electric part (2.17) of the torque

$$\underset{s}{\tau} + \underset{B}{\tau} = \frac{2\eta(k^2-1)}{k^2(1+2\eta+k^2)} + \frac{2\eta(k^2+1)}{k^2(1+2\eta+k^2)} = \frac{4\eta}{1+2\eta+k^2} = \underset{E}{\tau} . \qquad (2.18)$$

So, the total torque that is experienced by our dielectric equals to the double quantity

$$\tau = \underset{s}{\tau} + \underset{B}{\tau} + \underset{E}{\tau} = \frac{8\eta}{1+2\eta+k^2} , \qquad (34)$$

and eqn. (2.10) gives $\tau = 2\mathsf{P}$, i.e. we arrive to eqn. (1.4),

$$\tau = 2\mathsf{P} / \omega .$$

instead of (1.3).

Sorry, this result does not coincide with the value and the division of the torque in [6] and with the result of [13].

Hereby we show that eqns. (1.1) - (1.3), which are a part of the standard electrodynamics, contradict the calculation of angular momentum in the frame of the standard electrodynamics because the angular momentum flux according to (1.1) is half the torque acting on the absorber.

In view of this result, we undertake another verification of the statements (1.1) - (1.3). We apply Eq. (1.1) to the classical Beth's experiment [14] and immediately find that the statement (1.1) predicts zero result of the experiment [15]. The point is the circularly polarized beam, which exerts a torque on a doubly refracting plate in the Beth's experiment, passes through the plate there and back. Therefore the Poynting vector $\mathbf{E} \times \mathbf{B}$ is obviously zero in the experiment because the passed beam is added with the reflected one. So, Eq. (1.1) yields zero (see the appendix 1).

3. Pressure

In this section we use our technique to calculate pressure acting on the dielectric from the beam. The pressure can be readily obtained as a component of the Maxwell tensor (density), $\underset{\wedge}{T}^{zz}$, by the use of the summary field in vacuum

$$\underset{\to}{\breve{E}}_V = \underset{\to}{\breve{E}}_1 + \underset{\to}{\breve{E}}_2, \quad \underset{\to}{\breve{B}}_V = \underset{\to}{\breve{B}}_1 + \underset{\to}{\breve{B}}_2 : \tag{3.1}$$

$$< \underset{\wedge}{T}^{zz} >|_{z<0} = \sqrt{g}_{\wedge} (E_{V\rho}^2 + E_{V\varphi}^2 - E_{Vz}^2 + B_{V\rho}^2 + B_{V\varphi}^2 - B_{Vz}^2)/4 = \rho[u^2 - (\partial_\rho u)^2/2](1+R^2) . \tag{3.2}$$

Integration over the complete cross section with use of the Gaussian beam waist w gives:

$$F|_{z<0} = \int \rho[u^2 - (\partial_\rho u)^2/2](1+R^2) d\rho d\phi = (1 - 2/w^2)(1+R^2) = (1-2/w^2)\frac{2(\eta^2+1+\kappa^2)}{1+2\eta+k^2} . \tag{3.3}$$

For a wide beam, $w \to \infty$, this expression reduces to that for a plane wave and coincides with eqn. (7.1) from [6] in spite of the eqn. (7.1) is obtained for a beam of finite transverse extent. The force (3.3) is less than that for a plane wave because of z-components of the **E** and **B** fields (the field lines are closed loops). These components give rise to a negative pressure.

Pressure acting on the dielectric *under* its surface can be readily obtained as a component of the Maxwell tensor by the use of (2.8) instead of (3.1):

$$\begin{aligned} < \underset{\wedge}{T}^{zz} >|_{z>0} &= \sqrt{g}_{\wedge} (E_{3\rho}^2 + E_{3\varphi}^2 - E_{3z}^2 + B_{3\rho}^2 + B_{3\varphi}^2 - B_{3z}^2)\exp(-2\kappa z)/4 \\ &= \rho[u^2 - (\partial_\rho u)^2/2k^2](1+R^2)\exp(-2\kappa z) . \end{aligned} \tag{3.4}$$

It is seen that the flux density of momentum (3.4) decreases with z. Accordingly, a force density acts on the dielectric,

$$\underset{\wedge}{f}^z = -\partial_z < \underset{\wedge}{T}^{zz} >|_{z>0} = 2\kappa\rho[u^2 - (\partial_\rho u)^2/2k^2]\frac{2(\eta^2+1+\kappa^2)}{1+2\eta+k^2}\exp(-2\kappa z) . \tag{3.5}$$

A resulting force

$$F|_{z>0} = \int \rho[u^2 - (\partial_\rho u)^2/2k^2](1+R^2) d\rho d\phi = (1-2/k^2w^2)(1+R^2) = (1-2/k^2w^2)\frac{2(\eta^2+1+\kappa^2)}{1+2\eta+k^2} \tag{3.6}$$

acting on the dielectric at $z > 0$ exceeds the force (3.3) acting from vacuum by a small surface force

$$F|_{z>0} - F|_{z<0} = \frac{k^2-1}{k^2w^2}2(1+R^2) = \frac{4(\eta^2-1+\kappa^2)(\eta^2+1+\kappa^2)}{k^2w^2(1+2\eta+k^2)} \tag{3.7}$$

directed against the z-direction and acting near the beam wall. If ignoring the wall effect, the surface force is zero because of continuity of **E** and **B** field at $z = 0$.

Thus the force (3.3) acting on the dielectric from vacuum is divided into the small surface part (3.7) and the bulk part (3.6):

$$F = (1-2/w^2)\frac{2(\eta^2+1+\kappa^2)}{(\eta+1)^2+\kappa^2} = \overbrace{-\frac{4(\eta^2-1+\kappa^2)(\eta^2+1+\kappa^2)}{k^2w^2[(\eta+1)^2+\kappa^2]}}^{surface} + \overbrace{(1-2/k^2w^2)\frac{2(\eta^2+1+\kappa^2)}{(\eta+1)^2+\kappa^2}}^{bulk} . \tag{3.8}$$

This division is not coincided with eqns (7.10), (8.1) from [6] even if $w \to \infty$:

$$\frac{2(\eta^2+1+\kappa^2)}{(\eta+1)^2+\kappa^2} = \overbrace{\frac{2(\eta^2-1+\kappa^2)}{(\eta+1)^2+\kappa^2}}^{surface} + \overbrace{\frac{4}{(\eta+1)^2+\kappa^2}}^{bulk} , \tag{7.10}$$

The surface force (3.7) can be readily obtained also by the use of the surface charge (2.12). The z-component of the time averaged surface force density $\underset{a}{F}_{\wedge}$ acts on this charge and directs against the z-direction:

$$F_{\underset{a}{}\wedge} = \sqrt{g}_{\wedge}(\varepsilon^2 - 1)E_{3z}^2/4 = \rho(k^4 - 1)T^2(\partial_\rho u)^2/4k^2 . \qquad (3.9)$$

Integration over the complete cross section with the use of $(1+k^2)T^2 = 2(1+R^2)$ gives eqn. (3.7).

4. Appendix 1. The Beth's experiment [15]

We show that the angular momentum flux into the Beth's doubly refracting plate is zero, according to (1.1).

In the Beth's experiment a circularly polarized light beam (power $P = 80$ mW, $\lambda = 1.2\ \mu$m, $\omega = 1.6\cdot 10^{15}\,\mathrm{s}^{-1}$) passes through the half-wave plate, then it is reflected and passes twice through the quarter-wave plate, and then returns through the half-wave plate. The torque exerting on the half-wave plate is 20 dyne cm. This result is in accordance with the formula

$$\tau = 4P/\omega . \qquad (4.1)$$

However, let us calculate the Poynting vector $\mathbf{E}\times\mathbf{B}$ in the Beth's experiment. We start from expression (2.6) for the Beth's circularly polarized beam.

When a mirror reflects the beam (2.6), signs preceding z and the sign in the formula for $\breve{B}$ are changed. But because of the quarter-wave plate a helicity of the beam is conserved, and so signs preceding ϕ is changed and the sign in the formula for $\breve{B}$ is changed once more. Thus the reflected beam in the Beth experiment is expressed by the formula (we use index 4 for the reflected beam)

$$\underset{\rightarrow}{\breve{E}}_4 = \exp[i(-z - t - \phi)](\underset{\rightarrow}{\rho} - i\rho\underset{\rightarrow}{\phi} - \underset{\rightarrow}{z}\, i\partial_\rho)u(\rho), \qquad \underset{\rightarrow}{\breve{B}}_4 = -i\underset{\rightarrow}{\breve{E}}_4 . \qquad (4.2)$$

Let us calculate the Maxwell energy-momentum tensor

$$T^{\lambda\mu} = -g^{\lambda\alpha}F_{\alpha\nu}F^{\mu\nu} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4 \qquad (4.3)$$

for the total field $\underset{\rightarrow}{\breve{E}} = \underset{\rightarrow}{\breve{E}}_1 + \underset{\rightarrow}{\breve{E}}_4$, $\underset{\rightarrow}{\breve{B}} = \underset{\rightarrow}{\breve{B}}_1 + \underset{\rightarrow}{\breve{B}}_4$.

A signature of the metric tensor $g^{\lambda\alpha}$ in Eq. (4.3) is $(+---)$. $F^{\mu\nu} = -F^{\nu\mu}$, $F_{\mu\nu} = F^{\alpha\beta}g_{\mu\alpha}g_{\nu\beta}$ is the field strength tensor. The sense of its components is

$$F^{ti} = -E^i, \quad F_{ti} = E_i, \quad F^{ij} = -B^{ij}, \quad F_{ij} = -B_{ij}, \quad B_k = B^{ij}_{\wedge}e_{\underset{\wedge}{ijk}}, \quad B^k = B_{\underset{\wedge}{ij}}e^{ijk}_{\wedge}, \quad i,\ j,... = \rho,\ \phi,\ z . \qquad (4.4)$$

For example,

$$F^{\phi t} = -g^{\phi\phi}F_{\phi t} = E^{\phi} = g^{\phi\phi}E_{\phi}, \quad F^{\rho\phi} = g^{\phi\phi}F_{\rho\phi} = -B^{\rho\phi} = -g^{\phi\phi}B_{\rho\phi} . \qquad (4.5)$$

The component $T^{t\phi}_{\wedge}$ is the density of an orbital mass-energy flux, i.e. the ϕ-component of the Poynting vector; infinitesimal time averaged mass

$$dp^t = dm = <T^{t\phi}_{\wedge}> da_{\underset{\phi}{\wedge}}dt = <T^{t\phi}_{\wedge}> dzd\rho dt \qquad (4.6)$$

passes through the surface element $da_{\underset{\phi}{\wedge}} = dzd\rho$ during dt. The component $T^{\phi t}_{\wedge} = T^{t\phi}_{\wedge}$ is the volume density of an orbital momentum; infinitesimal time averaged momentum

$$dp^{\phi} = <T^{\phi t}_{\wedge}> d\rho d\phi dz \qquad (4.7)$$

is contained in the volume element $d\rho d\phi dz$. Using (4.3) yields zero:

$$<T^{\phi t}_{\wedge}> = <T^{t\phi}_{\wedge}> = \Re[(\breve{E}_{1z} + \breve{E}_{4z})(\overline{B}_{1\rho} + \overline{B}_{4\rho}) - (\breve{E}_{1\rho} + \breve{E}_{4\rho})(\overline{B}_{1z} + \overline{B}_{4z})]/2 = 0 . \qquad (4.8)$$

This means that no mass rotates in the Beth experiment.

The component $T^{\phi z}_{\wedge}$ is the flux density of an orbital momentum; infinitesimal time averaged momentum

$$dp^{\phi} = <T^{\phi z}_{\wedge}> da_{\underset{z}{\wedge}}dt = <T^{\phi z}_{\wedge}> d\rho d\phi dt \qquad (4.9)$$

passes through the surface element $da_{\underset{z}{\wedge}} = d\rho d\phi$ during dt. This mean that an infinitesimal torque

$$d\tau_z = dL_z/dt = e_{\underset{\wedge}{z\rho\phi}}dL^{\rho\phi}_{\wedge}/dt = e_{\underset{\wedge}{z\rho\phi}}dL^{\rho\phi}\sqrt{g}_{\wedge}/dt = \rho dp^{\phi}\sqrt{g}_{\wedge}/dt = <T^{\phi z}_{\wedge}> \rho^2 d\rho d\phi \qquad (4.10)$$

acts on the surface element $da_z^{\wedge} = d\rho d\phi$. But

$$< T^{\phi z}_{\wedge} >= -\Re[(\breve{E}_{1\phi} + \breve{E}_{4\phi})(\overline{E}_{1z} + \overline{E}_{4z}) + (\breve{B}_{1\phi} + \breve{B}_{4\phi})(\overline{B}_{1z} + \overline{B}_{4z})]/2\rho = 0 . \tag{4.11}$$

So, no torque acts on the Beth plate according to the standard electrodynamics. Why then the plate experiences the torque (4.1)?

5. Appendix 2. Spin tensor [15, 16]

We see that the Maxwell electrodynamics provides the deficit of angular momentum in the absorbed beam and the lack of the angular momentum flux into the Beth's plate. So, the electrodynamics is not complete. We introduce a spin tensor in the electrodynamics.

The standard classical electrodynamics starts from the free field canonical Lagrangian,

$$\underset{c}{L} = -F_{\mu\nu}F^{\mu\nu}/4, \quad F_{\mu\nu} = 2\partial_{[\mu}A_{\nu]} . \tag{5.1}$$

Using this Lagrangian, by the Lagrange formalism physicists obtain the canonical energy-momentum tensor

$$\underset{c}{T}^{\lambda\mu} = \partial^{\lambda}A_{\alpha}\frac{\partial \underset{c}{L}}{\partial(\partial_{\mu}A_{\alpha})} - g^{\lambda\mu}\underset{c}{L} = -\partial^{\lambda}A_{\alpha}F^{\mu\alpha} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4, \tag{5.2}$$

and the canonical total angular momentum tensor

$$\underset{c}{J}^{\lambda\mu\nu} = 2x^{[\lambda}\underset{c}{T}^{\mu]\nu} + \underset{c}{Y}^{\lambda\mu\nu} \tag{5.3}$$

where

$$\underset{c}{Y}^{\lambda\mu\nu} = -2A^{[\lambda}\delta^{\mu]}_{\alpha}\frac{\partial \underset{c}{L}}{\partial(\partial_{\nu}A_{\alpha})} = -2A^{[\lambda}F^{\mu]\nu}, \tag{5.4}$$

is the canonical spin tensor.

Then physicists accomplish a Belinfante-Rosenfeld procedure [17, 18]. They add specific terms to the canonical tensors and arrive to the standard energy-momentum tensor $\Theta^{\lambda\mu}$, the standard total angular momentum tensor $\underset{st}{J}^{\lambda\mu\nu}$, and the standard spin tensor $\underset{st}{Y}^{\lambda\mu\nu}$, which is zero,

$$\Theta^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} - \partial_{\nu}\underset{c}{\tilde{Y}}^{\lambda\mu\nu}/2 = -\partial^{\lambda}A_{\nu}F^{\mu\nu} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4 + \partial_{\nu}(A^{\lambda}F^{\mu\nu}),$$

$$\underset{c}{\tilde{Y}}^{\lambda\mu\nu} \overset{def}{=} \underset{c}{Y}^{\lambda\mu\nu} - \underset{c}{Y}^{\mu\nu\lambda} + \underset{c}{Y}^{\nu\lambda\mu} = -2A^{\lambda}F^{\mu\nu}, \tag{5.5}$$

$$\underset{st}{J}^{\lambda\mu\nu} = \underset{c}{J}^{\lambda\mu\nu} - \partial_{\kappa}(x^{[\lambda}\underset{c}{\tilde{Y}}^{\mu]\nu\kappa}), \tag{5.6}$$

$$\underset{st}{Y}^{\lambda\mu\nu} = \underset{st}{J}^{\lambda\mu\nu} - 2x^{[\lambda}\Theta^{\mu]\nu} = \underset{c}{Y}^{\lambda\mu\nu} - \underset{c}{\tilde{Y}}^{[\lambda\mu]\nu} = 0 . \tag{5.7}$$

This means that standard addends $\underset{st}{t}^{\lambda\mu}, \underset{st}{s}^{\lambda\mu\nu}$ are added to the canonical energy-momentum and spin tensors:

$$\Theta^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} + \underset{st}{t}^{\lambda\mu}, \qquad \underset{st}{t}^{\lambda\mu} = -\partial_{\nu}\underset{c}{\tilde{Y}}^{\lambda\mu\nu}/2 = \partial_{\nu}(A^{\lambda}F^{\mu\nu}), \tag{5.8}$$

$$\underset{st}{Y}^{\lambda\mu\nu} = \underset{c}{Y}^{\lambda\mu\nu} + \underset{st}{s}^{\lambda\mu\nu} = 0, \qquad \underset{st}{s}^{\lambda\mu\nu} = -\underset{c}{Y}^{\lambda\mu\nu} = 2A^{[\lambda}F^{\mu]\nu} . \tag{5.9}$$

We use another addends; our addends,

$$t^{\lambda\mu} = \partial_{\nu}A^{\lambda}F^{\mu\nu}, \qquad s^{\lambda\mu\nu} = 2A^{[\lambda}\partial^{\mu]}A^{\nu}, \tag{5.10}$$

satisfy the equation

$$2t^{[\lambda\mu]} = \partial_{\nu}s^{\lambda\mu\nu} \tag{5.11}$$

and lead to the Maxwell energy-momentum tensor

$$T^{\lambda\mu} = \underset{c}{T}^{\lambda\mu} + t^{\lambda\mu} = -F^{\lambda}{}_{\nu}F^{\mu\nu} + g^{\lambda\mu}F_{\alpha\beta}F^{\alpha\beta}/4 \tag{5.12}$$

and a tensor, which is doubled electric part $\underset{e}{\Upsilon}^{\lambda\mu\nu}$ of the electrodynamics spin tensor $\Upsilon^{\lambda\mu\nu}$,

$$2\underset{e}{\Upsilon}^{\lambda\mu\nu} = \underset{c}{\Upsilon}^{\lambda\mu\nu} + s^{\lambda\mu\nu} = 2A^{[\lambda}\partial^{|\nu|}A^{\mu]}. \tag{5.13}$$

This result was submitted to "JETP Letters" on May 12, 1998.

The expression (5.13) was obtained heuristically. It is not final one. The tensor (5.13) is obvious not symmetric in the sense of electric - magnetic symmetry. It represents only the electric field, $\mathbf{E}$, $\mathbf{A} = -\int \mathbf{E}dt$. A true spin tensor of electromagnetic waves must depend symmetrically on the magnetic vector potential A_α and on an electric vector potential

$$\Pi_\alpha = e_{\alpha\lambda\mu\nu}\Pi^{\lambda\mu\nu}, \quad \partial_\nu \Pi^{\lambda\mu\nu} = F^{\lambda\mu}. \tag{5.14}$$

So the spin tensor of electromagnetic waves has the form

$$\Upsilon^{\lambda\mu\nu} = \underset{e}{\Upsilon}^{\lambda\mu\nu} + \underset{m}{\Upsilon}^{\lambda\mu\nu} = A^{[\lambda}\partial^{|\nu|}A^{\mu]} + \Pi^{[\lambda}\partial^{|\nu|}\Pi^{\mu]}, \tag{5.15}$$

and the total angular momentum has the form

$$J^{\lambda\mu} = \int (2x^{[\lambda}T^{\mu]\nu} + \Upsilon^{\lambda\mu\nu})dV_\nu, \text{ or } \quad \mathbf{J} = \int \mathbf{r}\times(\mathbf{E}\times\mathbf{B})dV + \int \Upsilon^{ij0}dV, \tag{5.16}$$

instead of (1.1), and the angular momentum (1.1) is an orbital angular momentum rather than spin.

Conclusions, Notes, and Acknowledgements

This paper conveys new physics. We review existing works concerning electrodynamics spin and indicate that existing theory is insufficient to solve spin problems because spin tensor of the modern electrodynamics is zero. Then we show how a change of the Belinfante-Rosenfeld procedure resolves the difficulty by introducing a true electrodynamics spin tensor. Our spin tensor, in particular, doubles a predicted angular momentum of a circularly polarized light beam without an azimuth phase structure and explains the Beth experiment.

Unfortunately, materials of this paper were rejected more than 350 times by scientific journals. For example (I show an approximate number of the rejections in parentheses): JETP Lett. (8), JETP (13), TMP (10), UFN (9), RPJ (75), AJP (16), EJP (4), EPL (5), IJTP (1), JOSAB (1), PRA (5), PRD (4), PRE (2), PRL (2), APP (5), FP (6), PLA (9), OC (5), JPA (4), JPB (1), JMP (6), JOPA (4), JMO (2), CJP (1), OL (4), NJP (5), MPEJ (3), arXiv (75).

I am deeply grateful to Professor Robert H. Romer for publishing my question [19] (was submitted on Oct. 7, 1999) and to Professor Timo Nieminen for valuable discussions (Newsgroups: sci.physics.electromag). Unfortunately, Jan Tobochnik, the present-day Editor of AJP, rejected my papers more than 20 times.

References

1. A. Sadowsky, Acta et Commentationes Imp. Universitatis Jurievensis **7**, No 1-3 (1899).
2. R. P. Feynman et al., *The Feynman Lectures on Physics* (Addison–Wesley, London, 1965) Vol. 3, p. 17–10.
3. W. Heitler, *The Quantum Theory of Radiation*, (Clarendon, Oxford, 1954), p. 401.
4. J. D. Jackson, *Classical Electrodynamics*, (John Wiley, 1999), p. 350.
5. H. C. Ohanian, Amer. J. Phys. **54**, 500 (1986).
6. R. Loudon, Phys. Rev. A68, 013806 (2003)
7. J. P. Gordon, Phys. Rev. A **8**, 14 (1973).
8. W. K. H. Panofsky and M. Phillips, *Classical Electricity and Magnetism* (Addison-Wesley, Reading, MA, 1955).
9. J. W. Simmonds, M. J. Guttmann, *States, Waves and Photons* (Addison-Wesley, Reading, MA, 1970)
10. L. Allen et al., in *Progress in Optics XXXIX* (Elsevier, Amsterdam, 1999) p.298

11. H. Kogelnik, T Li, Appl. Opt. **5**, 1550 (1966)
12. R. I. Khrapko, “Discrepancy between theory and experiment”, www.sciprint.org (2005)
13. A. I. Bishop et al., Phys. Rev. Lett. **92**, 19, 198104 (2004).
14. R. A. Beth, Phys. Rev. **48**, 471 (1935); Phys. Rev. **50**, 115 (1936).
15. R. I. Khrapko, Measurement Techniques, **46**, 317 (2003)
16. R. I. Khrapko, Gravitation & Cosmology, **10**, 91 (2004)
17. F. J. Belinfante, Physica **6,** 887 (1939).
18. L. Rosenfeld, Memoires de l'Academie Royale des Sciences de Belgiques **8** No 6 (1940).
19. R. I. Khrapko, Amer. J. Phys., **69**, 405 (2001)