FIFTEEN CONSECUTIVE INTEGERS WITH EXACTLY k PRIME FACTORS

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Abstract

In this paper using the arithmetic function $J_2(w)$ we prove that there exist infinitely many integers n such that each of consecutive integers $n, n+1, \cdots, n+14$ is exactly k prime factors.

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We have proved that there exist infinitely many integers n such that each of $n, n+1, n+2$ is product of k distinct primes [1]. In this paper using the arithmetic function $J_2(w)$ we prove that there exist infinitely many integers n such that each of consecutive integers $n, n+1, \cdots, n+14$ is exactly k prime factors.

Theorem 1 There exist infinitely many integers n such that each of consecutive integers $n, n+1, \cdots, n+14$ is exactly five prime factors.

Proof. The first number is

$$m_1 = 48899543056776531 7569$$

and for each of the fifteen integers $m_1 + i, i = 0,1,2,\cdots,14$ as can be seen from the factorizations [2]:

$$m_1 = 3 \cdot 3 \cdot 3 \cdot 1811094187288019 6947$$
$$m_2 = m_1 + 1 = 2 \cdot 5 \cdot 11 \cdot 4445413005161502887$$
$$m_3 = m_1 + 2 = 6917 \cdot 19973 \cdot 130843 \cdot 27051617$$
$$m_4 = m_1 + 3 = 2 \cdot 2 \cdot 3 \cdot 4074961921398044 3131$$
$$m_5 = m_1 + 4 = 13 \cdot 17 \cdot 283 \cdot 7818547728247211$$
$$m_6 = m_1 + 5 = 2 \cdot 7 \cdot 7 \cdot 498974929150780 9363$$
$$m_7 = m_1 + 6 = 3 \cdot 5 \cdot 5 \cdot 651993907423687 0901$$
$$m_8 = m_1 + 7 = 2 \cdot 2 \cdot 2 \cdot 6112442882097066 4697$$
$$m_9 = m_1 + 8 = 149 \cdot 28229 \cdot 4622647 \cdot 25149671$$
$$m_{10} = m_1 + 9 = 2 \cdot 3 \cdot 3 \cdot 2716641280932029 5421$$
$$m_{11} = m_1 + 10 = 31 \cdot 2963 \cdot 34871 \cdot 152667661633$$
$$m_{12} = m_1 + 11 = 2 \cdot 2 \cdot 5 \cdot 2444977152838826 5879$$
$$m_{13} = m_1 + 12 = 3 \cdot 7 \cdot 11 \cdot 2116863335791191851$$
$$m_{14} = m_1 = 13 = 2 \cdot 37 \cdot 922213309 \cdot 7165420727$$
$$m_{15} = m_1 + 14 = 19 \cdot 29 \cdot 60607 \cdot 14643011879719$$

Suppose that $m = \prod_{i=1}^{15} m_i$. We define the prime equations

$$P_i = \frac{m}{m_i} x + 1 \quad (1)$$

where $i = 1, 2, \cdots, 15$.

We have the arithmetic function [3-14]

$$J_2(w) = \prod_{3 \le P} (P - 16 - c(P)) \ne 0, \qquad (2)$$

where $c(P) = -15$ if $P = 3,5,7,11$; $c(P) = -14$ if $P \mid m$, but $P \ne 3,5,7,11$; $c(P) = 0$ otherwise, $w = \prod_{2 \le P} P$.

Since $J_2(w) \to \infty$ as $w \to \infty$, there exist infinitely many integers x such that $P_1, P_2, \cdots, P_{15}$ are all primes.

We have the asymptotic formula of the number of integers $x \le N$ [3-14]

$$p_{16}(N,2) \sim \frac{J_2(w) w^{15}}{f^{16}(w)} \frac{N}{\log^{16} N}, \qquad (3)$$

where $f(w) = \prod_{2 \le P} (P - 1)$.

From (1) we have $n = m_1 P_1 = mx + m_1$, $n + 1 = m_1 P_1 + 1 = mx + m_1 + 1 = mx + m_2 = m_2 \left(\frac{m}{m_2} x + 1 \right) = m_2 P_2, \cdots,$ $n + 14 = mx + m_1 + 14 = mx + m_{15} = m_{15} \left(\frac{m}{m_{15}} x + 1 \right) = m_{15} P_{15}$.

If $P_1, P_2, \cdots, P_{15}$ are all primes, then each of consecutive integers $n, n+1, \cdots, n+14$ is exactly five prime factors.

Theorem 2 There exist infinitely many integers n such that each of consecutive integers $n, n+1, \cdots, n+14$ is exactly k prime factors.

Proof. From theorem 1 we have that each of consecutive integers $m_1, m_2 = m_1 + 1, \cdots, m_{15} = m_1 + 14$ is exactly $k - 1$ prime factors.

Suppose that $m = \prod_{i=1}^{15} m_i$. We define the prime equations

$$P_i = \frac{m}{m_i} x + 1, \qquad (4)$$

where $i = 1, 2, \cdots, 15$.

We have the arithmetic function [3-14]

$$J_2(w) = \prod_{3\le P} (P - 16 - c(P)) \ne 0, \tag{5}$$

where $c(P) = -15$ is $P = 3,5,7,11$; $c(P) = -14$ if $P \mid m$, but $P \ne 3,$

$5,7,11$; $c(P) = 0$ otherwise..

Since $J_2(w) \to \infty$ as $w \to \infty$, there exist infinitely many integers x such that $P_1, P_2, \cdots, P_{15}$ are all primes.

We have the asymptotic formula of the number of integers $x \le N$ [3-14]

$$p_{16}(N,2) \sim \frac{J_2(w)w^{15}}{f^{16}(w)} \frac{N}{\log^{16} N}.$$

From (4) we have $n = m_1 P_1 = mx + m_1$, $n+1 = m_1 P_1 + 1 = mx + m_1 + 1 = mx$

$+ m_2 = m_2 \left(\frac{m}{m_2} x + 1 \right) = m_2 P_2, \cdots,$ $n + 14 = mx + m_1 + 14 = mx + m_{15}$

$= m_{15} \left(\frac{m}{m_{15}} x + 1 \right) = m_{15} P_{15}$

If $P_1, P_2, \cdots, P_{15}$ are all primes, then each of consecutive integers $n, n+1, \cdots,$ $n+14$ is exactly k prime factors.

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