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Real or Imaginary Space-Time?
Reality or Relativity?

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ABSTRACT The real space-time of Newtonian mechanics and the ether concept is contrasted with the imaginary space-time of the non-ether concept and relativity. In real space-time $(x, y, z, \bar{c}t)$ characteristic theory shows that Maxwell's equations and sound waves in any uniform fluid at rest have identical wave surfaces. Moreover, without charge or current, Maxwell's equations reduce to the *same* standard wave equation which governs such sound waves. This is not a general and invariant equation but it becomes so by Galilean transformation to any other reference-frame. So also do Maxwell's equations which are, likewise, not general but unique to one reference-frame. The mistake of believing that Maxwell's equations were invariant led to the Lorentz transformation and to relativity; and to the misinterpretation of the differential equation for the wave cone through any point as the quadratic differential form of a Riemannian metric in imaginary space-time $(x, y, z, i\bar{c}t)$. Mathematics is then required to tolerate the *same* equation being transformed in different ways for different applications. Otherwise, relativity is untenable and recourse must then be made to real space-time, normal Galilean transformation and an ether with Maxwellian statistics and Planck's energy distribution.

INTRODUCTION

Relativists and cosmologists regularly refer to space-time without specifying precisely what they mean by this term. Here the two different forms of space-time, real and imaginary, are introduced and contrasted.

It is shown that, in real space-time $(x, y, z, \bar{c}t)$, Maxwell's equations have the same wave surfaces as those for sound waves in any uniform fluid at rest, and thus that Maxwell's equations are not general and invariant but, like the standard wave equation, only hold in one unique frame of reference. In other words, Maxwell's equations only apply to electromagnetic waves in a uniform ether at rest. But both Maxwell's equations and the standard wave equation, and their identical wave surfaces, transform quite properly, by Galilean transformation, into a general invariant form which applies to waves in any uniform medium moving at any constant velocity relative to the reference-frame.

It was the mistaken idea, that Maxwell's equations and the standard wave equation should be invariant, which led, by a mathematical freak, to the Lorentz transform (which demands the non-ether concept and a universally constant wave-speed) and to special relativity. The mistake was further compounded by misinterpreting the differential equation for the wave hypercone through any point as the quadratic differential form of a Riemannian metric in imaginary space-time $(x, y, z, i\bar{c}t)$. Further complications ensued when this imaginary space-time was generalised to encompass gravitation in general relativity.

The outcome of these errors makes it necessary for mathematics to condone the *same* equation being transformed by different transformations for applications to different kinds of waves; and, presumably, being transformable by either or both of these transformations when not applied at all but considered purely as a mathematical equation. Otherwise, the non-ether concept, the Lorentz transform and the entire fabric of twentieth century non-Newtonian relativity all become mathematically untenable and must be discarded.

All is not lost, however, for recourse may then be made to the fact that Maxwell's equations transform by Galilean transformation in precisely the way they should, and to the existence of an ethereal gas, first discovered in 1976, which both has Maxwellian statistics and satisfies Planck's energy distribution.

REAL SPACE-TIME

Real space-time is a four dimensional space consisting of three-dimensional space plus a fourth length dimension obtained by multiplying time by a constant speed. (This is usually taken as the constant wave-speed $\bar{c}$ of electromagnetic waves). If the four lengths, which define a four-dimensional metric $(x, y, z, i\bar{c}t)$, are thought of as measured in directions mutually at right-angles, then the quadratic differential form of this metric is

$$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 + \bar{c}^2\,(dt)^2 \qquad (1)$$

THEORY OF CHARACTERISTICS

For any physical phenomenon governed by a set of first order linear partial differential equations in the independent variables x, y, z and $\bar{c}t$, the theory of characteristics (see, for example, Thornhill 1952, 1985b) gives the wave surfaces; these are always real when the system of equations is hyperbolic. The equations of these wave surfaces are independent of the non-differential terms in the governing equations.

ELECTROMAGNETIC AND SOUND WAVES

When the non-differential terms are removed from Maxwell's equations, i.e. when there is no charge distribution or current density, it can easily be shown that the components (E_1, E_2, E_3) of the electrical field-strength and the components (H_1, H_2, H_3) of the magnetic field-strength all satisfy the standard wave equation

$$\nabla^2\phi = \left(\frac{1}{\bar{c}^2}\right)\frac{\partial^2\phi}{\partial t^2} \qquad (2)$$

with a constant electromagnetic wave-speed $\overline{c}$.

It has been shown (Thornhill 1993) that the equations which govern, to the first order, general small-amplitude wave motions, in the general unsteady flow of any general fluid, also reduce to the equation (2), with a constant thermodynamic wave-speed $\overline{c}$, in the particularly simple case of a uniform fluid at rest. Moreover, it has also been shown (loc. cit.) that equation (2) can consequently only hold in one unique reference-frame; that it is not, therefore, invariant under Galilean transformation; but that it transforms quite normally by Galilean transformation into a form which is invariant for all other frames of reference.

It follows immediately, therefore, that the wave surfaces of Maxwell's equations are exactly the same as those for sound waves in any uniform fluid at rest, and that Maxwell's equations can only hold in one unique reference-frame and should not remain invariant when transformed into any other reference-frame.

THE WAVE HYPERCONOID

In particular, the equation for the envelope of all wave surfaces which pass through any point at any time is, for equation (2), and therefore also for Maxwell's equations,

$$(dx)^2 + (dy)^2 + (dz)^2 = \overline{c}^2\,(dt)^2$$

or

$$\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2 = \overline{c}^2 \tag{3}$$

This is a differential equation and the first objective should be, therefore, to solve it and obtain its general integral.

It is by no means trivial, but it is, nevertheless, not very difficult to show, by elementary standard methods, that the general integral of the differential equation (3), which passes through (x_1, y_1, z_1) at time t_1, is the right spherical hypercone

$$(x - x_1)^2 + (y - y_1)^2 + (z - z_1)^2 = \overline{c}^2\,(t - t_1)^2 \tag{4}$$

In two space variables and time, equation (3) reduces to

$$(dx)^2 + (dy)^2 = \overline{c}^2\,(dt)^2 \tag{5}$$

of which the general integral, passing through (x_1, y_1) at time t_1, is the right circular cone

$$(x - x_1)^2 + (y - y_1)^2 = \overline{c}^2\,(t - t_1)^2 \tag{6}$$

Although the formal derivation of the cone (6), as the general integral of the differential equation (5), is never given, this cone is often used in books and papers, purporting to illustrate the special theory of relativity. The backward part (in time) of the cone (i.e. the 'domain of dependence' of characteristic theory), is labelled 'the past'; and the forward part (i.e. the 'region of influence' of characteristic theory) is labelled 'the future'.

THE LORENTZ TRANSFORM

If the mistake were made of wrongly thinking that equation (2) (and therefore also Maxwell's equations) should be invariant under transformation, then this could also be achieved by means of some alternative transformation to the Galilean. Unfortunately, by a quirk of mathematics, such a transformation exists and was probably first discovered by Woldemar Voigt (1887) although it is known as the Lorentz transform. Voigt accepted the existence of an ethereal fluid but, without reference to or derivation from Maxwell's equations, he thought that electromagnetic waves must be governed by equation (2) in the same way as some particularly simple waves in material fluids. He therefore believed that equation (2) should be invariant under transformation, even though such invariance was not required in fluid dynamics.

The Lorentz transform demands the assumption of a universally constant electromagnetic wave-speed $\overline{c}$ and is based on keeping the *quantity* $\left[(dx)^2 + (dy)^2 + (dz)^2 - \overline{c}^2\,(dt)^2\right]$ invariant under transformation, whether it is zero or non-zero, even though the essential requirement was only to keep the zero values invariant so that solutions of equation (3) (i.e. wave surfaces) may remain wave surfaces when transformed.

The entire purpose, however, of a transformation is simply to *refer*, to other metrics, equations which govern the variation of physical quantities; that is why these different metrics are called, quite simply, frames of *reference.* Thus the Galilean transformation changes Maxwell's equations in $\{E_i\}$ and $\{H_i\}(i=1,2,3)$ which are invariant in all other frames of reference. The Lorentz transform, on the contrary, does not succeed, as is claimed, in keeping Maxwell's equations in $\{E_i\}$ and $\{H_i\}$ invariant as equations in $\{E_i\}$ and $\{H_i\}$. The form of Maxwell's equations is only preserved by the Lorentz transform if the transformed equations are written not as equations in $\{E_i\}$ and $\{H_i\}$ but in $\{E_i'\}$ and $\{H_i'\}$ as given by

$$E_1' = E_1 \qquad\qquad H_1' = H_1$$

$$E_2' = \beta\left(E_2 - H_3\,\frac{\overline{u}}{\overline{c}}\right) \qquad\qquad H_2' = \beta\left(H_2 + E_3\,\frac{\overline{u}}{\overline{c}}\right)$$

$$E_3' = \beta\left(E_3 + H_2\,\frac{\overline{u}}{\overline{c}}\right) \qquad\qquad H_3' = \beta\left(H_3 - E_2\,\frac{\overline{u}}{\overline{c}}\right)$$

where the new frame of reference has constant velocity $(-\overline{u},0,0)$ relative to the original reference-frame and $\beta = \left(1-\overline{u}^2/\overline{c}^2\right)^{-\frac{1}{2}}$, (see, for example, Einstein 1905).

IMAGINARY SPACE-TIME

It was Minkowski (1908) who first failed to recognise the equation (3) as a differential equation. Instead, he wrote

$$(ds)^2 = (dx)^2 + (dy)^2 + (dz)^2 - \overline{c}^2\,(dt)^2 \tag{7}$$

so that he could interpret this as the quadratic differential form of an imaginary Riemannian metric $(x, y, z, i\overline{c}t)$, i.e. space-imaginary time. The null lines $ds = 0$, of this metric satisfy the differential equation (3) and these, he thought, were light rays. He announced with great emotion, "Henceforth space by itself and time by itself are doomed to fade away into mere shadows". The imaginary metric $(x, y, z, i\overline{c}t)$ is usually also referred to as space-time.

DUAL TRANSFORMATION

The question that now arises is how, in general, the equation (2) should be transformed. Is it permissible to ask, before transforming this equation, whether it is to be applied to sound waves in a stationary fluid, or to electromagnetic waves, or only to be considered purely as a mathematical equation? Does mathematics allow equation (2) to conform to Galilean transformation when it is applied to sound waves, to Lorentz transformation when it is applied to electromagnetic waves, and to either or both of these transformations when it is considered purely as a mathematical equation; or does mathematics insist that the Galilean transformation is unique and must apply equally to all equations so that the *same* equation must always be transformed by the *same* Galilean transformation, no matter to what it is applied, or whether it is applied to anything at all? In the latter case, the non-ether concept, the Lorentz transform and non-Newtonian relativity all become mathematically untenable and must be abandoned; and recourse must then be made to the fact that equation (2) and Maxwell's equations transform quite normally by Galilean transformation in exactly the way they should.

GALILEAN TRANSFORMATION OF ELECTROMAGNETIC EQUATIONS

In the derivation of Maxwell's equations, for the electromagnetic field with a constant wave-speed $\overline{c}$, no account whatsoever is taken of the motion, relative to the frame of reference, of any possible ethereal medium in which electromagnetic waves may propagate. It follows, therefore, that Maxwell's equations can only be valid either (a) if there is no ethereal medium, or (b) if the validity of the equations is confined to a uniform ether with constant wave-speed $\overline{c}$, and to the one unique reference-frame relative to which this uniform ether is at rest.

If then, the non-ether concept is mathematically untenable, it follows that Maxwell's equations can only apply to electromagnetic waves in a uniform ether at rest. When transformed by Galilean transformation to a reference-frame, moving with constant relative velocity $(-\overline{u}, -\overline{v}, -\overline{w})$, the operator $\partial/\partial t$ in Maxwell's equations and in equation (2) is changed into Euler's total time-derivative moving with the fluid, namely

$$\frac{D}{Dt} \equiv \frac{\partial}{\partial t} + \overline{u}\frac{\partial}{\partial x} + \overline{v}\frac{\partial}{\partial y} + \overline{w}\frac{\partial}{\partial z}. \tag{8}$$

The resulting progressive equations are then invariant and apply to electromagnetic waves in a uniform ether moving with any constant velocity $(\overline{u}, \overline{v}, \overline{w})$

relative to the reference-frame. When Maxwell's equations are generalised to give equations for the electromagnetic field in an ether in general unsteady motion (Thornhill 1993), they are again invariant, and so also is the differential equation for the characteristic wave-hyperconoid through any point at any time, namely

$$(dx - udt)^2 + (dy - vdt)^2 + (dz - wdt)^2 = c^2\,(dt)^2$$

or

$$\left(\frac{dx}{dt} - u\right)^2 + \left(\frac{dy}{dt} - v\right)^2 + \left(\frac{dz}{dt} - w\right)^2 = c^2 \tag{9}$$

where (u, v, w) is the local ethereal velocity and c the local wave-speed at any time.

Equation (9) states quite simply that, in general unsteady flow, waves travel in all directions with the local wave-speed c relative to the local fluid velocity (u, v, w); in the special and particularly simple case, when the flow is uniform and at rest, equation (3), by contrast, states merely that waves travel in all directions with the constant wave-speed $\overline{c}$.

MOVING WITH THE LOCAL ETHER

It follows that, at the origins of all reference-frames that move with the local ether, the general wave-hyperconoid (9) reduces locally, when acceleration effects are negligible, to the simple form

$$(dx)^2 + (dy)^2 + (dz)^2 = c^2\,(dt)^2 \tag{10}$$

in which the wave-speed c is not constant but varies with position and time in the general unsteady ethereal flow. This simple wave-hyperconoid (10) will thus appear to be invariant for Galilean transformations between all observers travelling with the local ether; and all such observers will obtain locally the same null result as Michelson and Morley.

This would imply that the Michelson-Morley experiment demonstrates only that the ether has viscosity, as would be expected of a fluid. The relative velocity between the ether and the Earth's surface would then decline across the viscous ethereal boundary layer surrounding the Earth, becoming zero at the Earth's surface.

The apparent invariance of the hyperconoid (10), (with a variable wave-speed c), achieved by the Galilean transformation for all observers travelling with the local ether, contrasts remarkably with the spurious invariance, achieved by the Lorentz transform, of the hypercone (3), which is limited to an assumed universally constant wave-speed $\overline{c}$ and the denial of an ether.

RETURN TO REALITY

The abandonment of relativity and a return to Newtonian mechanics, with real space-time and an ether, would create a backlog of problems still requiring conventional solutions, e.g. stellar aberration, the speed of light in a moving medium, the construction of an ethereal cosmology, etc. It would also require a suitable ether.

Such problems do eventually yield, however, to the methods of unsteady gas-dynamics and the theory of characteristics, and some of them have already been solved.

The thermodynamics of an ether, first discovered in 1976, that has Maxwellian statistics and satisfies Planck's energy distribution is given by Thornhill (1985a).

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