A Note on Holographic Dark Energy

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The unknown constant in the holographic dark energy model is determined in terms of a recent conjecture. We find $d \approx \sqrt{\pi}/2$. The result is consistent with the present cosmological observations. The holographic dark energy is re-explained by considering the quantum uncertainty and discreteness of space-time. We also predict that there may exist more dark energy between two adjacent black holes.

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Recently one kind of holographic dark energy model is used to explain the observed dark energy [1-4]. According to the model, the dark energy density is

$$\rho_V = \frac{3d^2c^4}{8\pi G L_H{}^2} \tag{1}$$

where c is the speed of light, G is the Newton gravitational constant, L_H is the size of the event horizon of our universe, d is an undetermined constant. By comparing with the observations, it was found that the holographic model is a viable one in describing dark energy [5-9]. Ref [7] obtained $d \approx 0.85$ for the flat universe by using the supernova Ia (SN Ia) data and the shift parameter. In addition, the value of d smaller than one or the phantom-like holographic dark energy is also favored by the analysis results of the angular scale of the acoustic oscillation from the BOOMERANG and WMAP data on the cosmic microwave background (CMB) [9]. In this paper, we will theoretically determine the constant d in the holographic dark energy model in terms of a recent conjecture [10]. The holographic dark energy will be re-explained from a different point of views. We will also predict a new quantum effect of black holes.

According to a recent conjecture on the origin of dark energy [10], the dark energy may originate from the quantum fluctuations of space-time limited in our universe. By using the uncertainty principle in quantum theory, the quantum fluctuation energy of space-time of one degree of freedom limited in our universe is

$$\varepsilon \approx \frac{\hbar/2}{2L_H}c = \frac{\hbar c}{4L_H} \tag{2}$$

where is L_H the event horizon of our universe. Since the quantum fluctuations of space-time are essentially nonlocal, and one degree of freedom corresponds to two Planck area units in the two ends of the event horizon, the whole number of degrees of freedom of such fluctuations in our universe is

$$N = \frac{1}{2}\frac{A}{4L_P^{\ 2}} = \frac{\pi L_H^{\ 2}}{2L_P^{\ 2}} \tag{3}$$

where A is the area of event horizon, L_P is the Planck length. Then the energy density of the quantum fluctuations of space-time in our universe is

$$\rho_\Lambda \approx \frac{N\varepsilon}{4\pi L_H^{\ 3}/3} = \frac{3c^4}{32GL_H^{\ 2}} \tag{4}$$

This formula results from the quantum uncertainty and discreteness of space-time. In comparison with the formula (1), we can get

$$d \approx \sqrt{\pi}/2 \approx 0.886 \tag{5}$$

This value is in excellent agreement with the analysis result of observational data (see, for example, [7]). By inputting the current fraction value $\Omega_\Lambda \approx 0.73$, we can work out the equation of state:

$$w_\Lambda(z) = -\frac{1}{3}(1+\frac{2}{d}\sqrt{\Omega_V}) \approx -0.98 + 0.25z + O(z^2) \tag{6}$$

In addition, we can also determine the current event horizon of our universe:

$$L_H \approx \frac{\sqrt{\pi}}{2\sqrt{\Omega_V}} H_0^{-1}c \approx 1.04 H_0^{-1}c \tag{7}$$

This means that the current event horizon approximately satisfies the Schwarzschild relation $L_H = 2GM/c^2$, where $M = \rho_c 4\pi L_H^{\ 3}/3$.

It is noted that $d < 1$ will lead to dark energy behaving as phantom, and seems to violate the second law of thermodynamics during the evolution phase when the event horizon shrinks. However, the universe inside the event horizon is not an isolated system in case of the existence of the quantum process such as the Hawking radiation. The event horizon of a black hole can shrink due to the Hawking radiation, the event horizon of our universe can also do. Thus $d < 1$ does not violate the second law of thermodynamics when considering the whole universe system. The universe inside the event horizon and that outside the event horizon will inevitably exchange energy and information due to the quantum process. This may also explain the non-conservation of dark energy inside the event horizon of our universe.

It is generally believed that the holographic form of dark energy is obtained by setting the UV and IR cutoff to saturate the holographic bound set by formation of a black hole [1-3]. Thus the dark energy can still come from the usual vacuum zero-point energy in quantum field theory. However, a simple calculation shows that this may be not right. The lowest frequency of the vacuum zero-point energy limited in our universe is

$$E_1 = \frac{hc}{8L_H} \tag{8}$$

According to the holographic principle [13-15], the whole number of degrees of freedom in our universe is

$$N_H = \frac{A}{4{L_P}^2} = \frac{\pi {L_H}^2}{{L_P}^2} \tag{9}$$

Then the vacuum zero-point energy density should satisfy the following inequality:

$$\rho_{VZE} \geq N_H E_1 = \frac{3\pi c^4}{16G{L_H}^2} \tag{10}$$

This requires that $d \geq \sqrt{2}\pi / 2$ in the holographic form of dark energy. Since the total energy in a region of the size L should not exceed the mass of a black hole of the same size, there should exist a theoretical upper bound $d \leq 1$. In addition, the result $d \geq \sqrt{2}\pi / 2$ is also ruled out by the cosmological observations. This can be shown more directly from the equation of state:

$$w = -\frac{1}{3}(1 + \frac{2}{d}\sqrt{\Omega_V}) \geq -\frac{1}{3}(1 + \frac{2\sqrt{2}}{\pi}\sqrt{\Omega_V}) \tag{11}$$

By inputting the current fraction value $\Omega_V \approx 0.73$ we obtain $w_0 \geq -0.59$. This has been ruled out by the observational constraint $w_0 < -0.75$. Thus the observed dark energy may not come from the vacuum zero-point energy in quantum field theory. The analysis also implies that the usual vacuum zero-point energy may not exist [16-18]. Even a holographic number of modes with the lowest frequency will give more vacuum zero-point energy than the observed dark energy. By comparison, the quantum fluctuations of space-time, which energy density is described by the equation (4) and is consistent with the observations, may be the origin of dark energy. Since the quantum fluctuations of space-time may be also called quantum-gravitational vacuum fluctuations, the vacuum fluctuation energy still exists. It does not come from matter, but from space-time. This may have some deep implications for a complete theory of quantum gravity.

Lastly, we will predict a new quantum effect of black holes in terms of the above analysis of dark energy. If the quantum fluctuations of space-time limited in the event horizon of our universe do exist, then it will also exist between two black holes. This means that there will exist more quantum fluctuation energy or dark energy between two black holes. Consider two black holes with the same radius R. The distance between them is $L >> R$. The quantum fluctuation energy of space-time of one degree of freedom limited between them is $\varepsilon \approx \frac{\hbar c}{2L}$. The whole number of degrees of freedom of such fluctuations is $N \approx \frac{\pi R^2}{2{L_P}^2}$. Then the whole quantum fluctuation energy of space-time between the black holes is

$$E_{BHV} = N\varepsilon \approx \frac{\pi \hbar c R^2}{4{L_P}^2 L} = \frac{\pi R}{2L} E_{BH} \tag{12}$$

where E_{BH} is the energy of black hole. The energy density is

$$\rho_{BHV} \approx \frac{N\varepsilon}{\pi r^2 L} \approx \frac{\hbar c}{4{L_p}^2 L^2} \tag{13}$$

It is evident that the density of the quantum fluctuation energy between the black holes is much larger than that of the observed dark energy. Such energy can be detected in the local part of the universe such as the center of Milky Way. For example, the repulsive acceleration of an object near one black hole is $a \approx \frac{2\pi c^2}{3L^2} r$, where r is the distance between the object and the black hole. The repulsive force equalizes the gravitational force of the black hole when $r \approx (RL^2)^{1/3}$.

In conclusion, the unknown constant in the holographic dark energy model is determined in theory. We find $d = \sqrt{\pi}/2 \approx 0.886$. This value is perfectly consistent with the observational data. The holographic dark energy is re-explained by considering the quantum fluctuations of space-time. We also predict a new quantum effect of black holes. There may exist more quantum fluctuation energy or dark energy between two black holes.

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