Inertial Mass In A Machian Framework

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We revisite the law of proportionality between gravitational mass and inertial mass within a framework consistent with the Principle of Mach as recently implemented by Assis..

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Relativity In Newtonian And Post Newtonian Mechanics

The law of force of Newton's theory of gravitation, written in obvious notation,

$$\vec{F}_{21}^{N} = -\left(\frac{m_{g1}\, m_{g2}}{r^2} \right) \hat{r} \qquad (1)$$

is the first relativistic universal law which appeared in the development of science. The above since in its formulation only enters the involved *gravitational masses*, m_{gk}, and its instantaneous mutual distance, $r = r_{12} = |\vec{r}_1 - \vec{r}_2|$. $\vec{F}_{21}$ is the force exerted by the point mass 2 on the point mass 1. Note that the masses involved in equ .(1) has nothing to do, a *priori*, with the *inertial mass* appearing in the Newton' s second law, $f = m_i\, \boldsymbol{a}$ .The force law, plus the proportionality law[1,2], valid for any material particle k,

$$m_{gk} = \sqrt{G}\; m_{ik} \qquad (2)$$

between gravitational mass and inertial mass, m_{ik} , suffice to explain most of the observed gravitational facts. Here $G = 6.67\ 10^{-11}\, m^3\, kg^{-1} s^{-2}$ means the universal constant of gravitation. The mutual gravitational energy (potential energy) coherent with equ.(1) is worth,

$$U^{N} = -\left(\frac{m_{g1}\, m_{g2}}{r} \right) \qquad (3)$$

from which follows equ.(1) when performing the usual procedure; $\vec{F} = -\nabla U$.

Despite being one of the best verified laws of physics, with a relative uncertainty below 10 $^{-11}$, equ.(2) only appears as a fortuitous coincidence in classical mechanics (CM). This fact intrigued Mach along his life who therefore envisaged the idea that distant matter should regulate, inertially, local interactions. When referring to the well known Newton's bucket, he said[3] : *"Try to fix Newton's bucket and rotate the heaven of fixed stars and then prove the absence of centrifugal forces"*.

In 1925 Schrödinger tried to seek for the origin of inertia by modifying equ.(3) in a suitable manner[4,5]. Guided for heuristic arguments he wrote, for two interacting point masses:

$$U^S = U^N\left(1 - \frac{\varepsilon \dot{r}^2}{c^2}\right) \tag{4}$$

wherein $\dot{r} \equiv dr/dt$; c is the velocity of light in a vacuum; and e is a dimensionless parameter that becomes 3 in order to fit the observed planetary precession.

Schrödinger emphasized the fact that any interaction energy should depend only on the separation and relative velocity between the particles, in order to follow Mach's views.

With the aid of his modified energy, Schrödinger calculates the energy of interaction for a spherical shell (gravitational mass M_g, radius R) interacting with an internal point mass m_g , moving in the neighborhood of its center. Thus, he obtains:

$$U = -\left(\frac{M_g m_g}{R}\right)\left(1 - \frac{v^2}{c^2}\right) \tag{5}$$

Schrödinger identified the component of this potential energy which depends on the velocity with the kinetic energy of the particle, $K = m_i\, v^2/2$. That is, $M_g m_g\, v^2/R c^2 = m_i\, v^2/2$. It then follows:

$$m_i = \left(\frac{2\, M_g}{R\, c^2}\right) m_g = \left(\frac{8\, \pi\, \sigma\, R}{c^2}\right) mg \qquad (6)$$

wherein σ labels the (assumed constant) surface density of gravitational mass and v is the velocity of the moving particle, referred to the rest frame centered in the sphere. Later on, Schrödinger integrates the result of the spherical shell for a "world" of radius R_o, supposing a constant mass density. He concludes that taking the radius and the mass density of our own galaxy, then we would obtain a value of G some 10^{11} times smaller than what is really measured. Therefore, the inertia of particles in the solar system must be mainly due to matter farther away from our galaxy.

Relational Mechanics: A Recent Implementation Of Mach's Principle

The pioneer work of Schrödinger was recently improved by Assis [6,7,8,9], who was able to implement Mach's ideas in a rigorous, entirely general, way. Taking departure with Schrödinger, the startpoint of Assis formulation is a Weber-like law of force which reads, in obvious notation,

$$\vec{F}_{21} = H_g\ \vec{F}_{21}^{\,N} \left(1 - \frac{\xi\, \dot{r}^2}{2\, c^2} + \frac{\xi\, r\, \ddot{r}}{c^2}\right) \qquad (7)$$

wherein $\ddot{r} \equiv d^2 r/dt^2$; and H_g and ξ are constants. The outstanding mathematical property of equ.(7) is that it is *invariant* (frame independent), which means that each term in the Weber-Assis's force has the same value to *all* observers, even for the non inertial ones[8,9].

With the aid of equ.(7) Assis was able to explain the origin of inertia and the *reality* of the so called *fictitious* forces of inertia ($-m\vec{a}$, centrifugal, Coriolis, etc.). The above forces are due, in a Machian scenario, to the gravitational interaction between any accelerated particle and the whole universe[10,11,12,13,14]

In short, Assis was able to develop a *true* relativistic mechanics which, besides comply with Mach's requirements, can be considered as a genuine extension of CM. Assis coined the name *Relational Mechanics* (RM) when referring to his model.

Epistemological And Dimensional Considerations

Recently we have revisited Assis's formulation of RM by stressing some dimensional ambiguities concerning the net distinction between gravitational mass and inertial mass[1,2,13]. In fact, we performed a critical revision of RM based upon the physical and dimensional *hierarchy* of the involved magnitudes[1,2].

First at all, we consider the gravitational mass as being a *primary* magnitude[1,2,13], similar in this sense to the electric charge and to the spin. A primary magnitude cannot be derived, *up to now*, from other previously known properties.

Some authors prefer to write equ.(1) with a multiplicative constant η, $F_{21}^{N} = \eta \left(m_{g1}\, m_{g2}\right)/r^2$. Now we will show that the above constant is *superfluous.*

In the first place, gravitational force doesn´t depend upon the medium in which the particles are immersed. There is no *gravitational permittivity*. This is a very important difference when comparing with the Priestley-Coulomb law for material media. Thus,

η being a number *independent of the medium*, for the *sake of symmetry*, it will affect in the same way each point mass.

Thereby, $F_{21}^{N} = \left[\left(\sqrt{\eta}\, m_{g1}\right)\left(\sqrt{\eta}\, m_{g2}\right)\right]/r^2$. In such a case, we *define* $m'_{gj} = \sqrt{\eta}\, m_{gj}$ as being the gravitational mass of the point mass j. QED.

***"We must avoid including superfluous elements in the description of physical phenomena".* Newton, Principia**.

By inserting equ.(2) in equ.(1) we get the familiar force law, written in terms of *inertial masses*, m_{ik}. Equation (2) allows us to grasp the "size" of the standards of gravitational mass in terms of the most familiar standards of inertial mass. Thus, in the cgs system, a body having 1 *Unit* of gravitational mass has an inertial mass amounting to $1/\sqrt{G} \approx 4\ 10^3\ g$, i.e. some 4 *kg*. From equ.(1) and $[F] = L^1\ M^1\ T^{-2}$ we deduce the *dimensional formula* for gravitational mass[15].

$$[m_g] = L^{3/2}\ M^{1/2}\ T^{-1} \tag{8}$$

wherein, as shown by Palacios[15], the bracket means the ratio of the standards employed to measure gravitational mass in two *coherent* systems of units (such as the cgs and the MKS), . Thus, $[m_g] = U'_{gm}/U_{gm}$ = *a real number*. The same meaning have the symbols $L \equiv U'_L/U_L$ for length, and M and T, for inertial mass and time, respectively[15].

On account of equ.(8) we get $U'_{gm} = U_{gm}\,(100/1)^{\frac{3}{2}}(1000/1)^{\frac{1}{2}}(1/1)^{-1}$ = $3.162\ 10^4$, being U'_{gm} and U_{gm} the MKS and cgs standards of gravitational mass, respectively. Thus, 1 *MKS unit* of gravitational mass is 31,620 times greater than the *cgs Unit* of gravitational mass, despite being 1 kg = 1,000 g.

Completing Relational Mechanics

The customarily adopted, very strong, constraint $m_g = m_i$ precludes the rigorous implementation of the Mach's Principle [1,2] since for such purpose it must be $m_i/m_g = f\left(\rho_g, H_0\right)$, being ρ_{go} the average matter density of the distant universe (galaxies) and H_o the Hubble's constant. For the above reasons, we adopt, as a startpoint for calculations, equ.(7) with $H_g=1$, dimensionless, as done by Schrodinger in 1925.

The force exerted by the whole isotropic universe on an accelerated test particle k (gravitational mass m_{gk}) is worth[1,2] $-m_{gk}\,\Phi_G\,\vec{a}$. Here, $\Phi_G = \left(2\pi\xi\rho_g/3H_0^2\right)$, wherein ρ_g means the mean density of gravitational mass in the universe and H_o labels the Hubble's constant. If $\vec{f}$ is the local force responsible for the acceleration, then we get $\vec{f} = m_{gk}\,\Phi_G\,\vec{a} \equiv m_{ik}\,\vec{a}$, equation in which we have *defined* the inertial mass, m_{ik}, of the test particle, in order to recover CM:

$$m_{ik} \equiv m_{gk}\,\Phi_G \tag{9}$$

A New Theorem Of Relational Mechanics

Theorem

An increase in the number of galaxies contained in the universe also increases the density of inertial mass as the square of the density of gravitational mass[16]:

$$\rho_I = \Phi_G\,\rho_g \propto \rho_g^2 \tag{10}$$

Proof

Adding equ.(9) for the N particles contained in an arbitrary volume V we get $\sum_{k=1}^{N} m_{ik} = \Phi_{GV} \sum_{1}^{N} m_{gk} \propto \rho_g \sum_k m_{gk}$. If, remaining the volume unchanged, N changes to $N+dN$, it will be $d\left(\sum_k m_{ik}\right) \propto d(\rho_g)\sum_k m_{gk} + \rho_g\, d\left(\sum_k m_{gk}\right)$. Bearing in mind that $\rho_g \equiv (1/V)\sum_k m_{gk}$, the above relation becomes $d\left(\sum_k m_{ik}\right) \propto V\,(2\,\rho_g\; d\rho_g) = V\, d(\rho_g^2)$ which means that $(1/V)\sum_k m_{ik} \equiv \rho_i \propto \rho_g^2$, in accordance with equ.(10). QED.

On account of eqs.(2, 9, 10) we find $G = \left(3H_0^2/2\pi\,\xi\rho_i\right)$ in agreement with the relation advanced, as far as in 1938, by Dirac[17].

Equation (10) has a clear physical meaning: an increase in the density of inertial mass arises from two different causes:

- An increase in the number of galaxies in the universe also increases ρ_g, and consequently ρ_i ("cumulative effect" taken into account in CM, equ.(2)).
- The increase in the density of gravitational mass also increases the *individual* inertial mass of each particle (here is the core of Mach's Principle).

The above theorem becomes ambiguous in Assis's formulation since in his algorism it results $\Phi_A \equiv \left(2\pi\xi\rho_g\, G/3H_0^2\right)$ and, in order to recover CM, we are compelled to take $\Phi_A \equiv 1$, dimensionless [6,8,9].

Related Considerations

Our above considerations enhance the role of Dimensional Analysis in the formulation of straightforward algorisms able to describe physical facts without ambiguities. The concerned algorisms must preserve the neat distinction which really do exist between two related cualitatively different magnitudes.

Thermodynamics provides us another interesting example: After Carnot we know that $Q_1/T_1 = Q_2/T_2$, wherein Q_1 and Q_2 mean, respectively, the input and output heat in an ideal cyclic machine working between the absolute temperatures T_1, T_2. The above ratios can be expressed in *cal / abs.degree, J/ ºK, etc.*

As far as we know, no author has never adopted an *ad hoc* system of standards in order to get the meaningless equation $Q = T$. As everybody know, the core of thermodynamics is anchored to the largely ignored distinction between heat and temperature. The above crucial differentiation only comes after the lasting works of Black, Davy, Rumford, Mayer, Joule, Thomsom, Helmholtz, and others.

Statistical Mechanics provides us another interesting example, when dealing with the connection between mechanical energy per degree of freedom and absolute temperature[1,2]. Here, the link between the two above magnitudes is one half of the Boltzmann's constant, $\langle E \rangle = (k/2)\,T$; $k = 1.38\ 10^{-16}$ *erg/ K*..

The equation (1) resembles the Priestley-Coulomb law, when expressed in terms of the *electrostatic unit of charge*. We cannot avoid to quote Maxwell when referring to the above law of force [18`]: " *We may now write the general law of electrical action in the simple form* $F = e\,e'\,r^{-2}$ .... *If* [Q] *is the concrete electrostatic unit of quantity itself, and e,e´ the numerical values of particular quantities, if* [L] *is the unit of length, ..., then the equation becomes*

$$[Q]=[L^{3/2}\ T^{-1}\ M^{1/2}] \quad (11)$$

Other units may be employed for practical purposes, and in other departments of electrical science, but in the equations of electrostatics, quantities of electricity are understood to be estimated in electrostatic units, just as in physical astronomy we employ a unit of mass which is founded on the phenomena of gravitation, and which differs from the units of mass in common use."

As we saw, the view advocated by Maxwell was embodied by Schrodinger when dealing with gravitational mutual energy.

Palacios was able to develop a sound and rigorous vectorial theory of Dimensional Analysis based upon the ideas of Fourier [15,18]. In his theory, the squared brackets means the *ratio* of two *coherent* units (i.e. *real numbers*), instead of the units itself, as claimed by Maxwell. As far as we know Maxwell was the first man who wrote squared brackets when referring to units.

The ideas of Maxwell concerning dimensional analysis, when properly updated, are entirely consistent with our actual views. Translating equ.(11) to modern symbolism [19,20,21] we get, according to Maxwell (Ref.17, chapter 1): $U_Q=(U_L)^{3/2}(U_T)^{-1}(U_M)^{1/2}$, a *symbolic*, *operationally undefined*, relation between *coherent* units, say the cgs ones.

Taking another coherent system of units, such as the MKS, it will be: $U'_Q=(U'_L)^{3/2}(U'_T)^{-1}(U'_M)^{1/2}$. On account of the two above relations we get $U'_Q/U_Q=(U'_L/U_L)^{3/2}\,(U'_T/U_T)^{-1}\,(U'_M/U_M)^{1/2}$, an *algebraic*, *operationally defined* equation[15], nowadays written in the form $[Q]=L^{3/2}\,T^{-1}\,M^{1/2}$. It is worthwhile to compare the last equation with equation (8).

References

[1] J.Guala Valverde, *Apeiron*, **6**, 202 (1999).

[2] J. Guala Valverde, *Physics Essays*, **12**, N 4, 785 (1999).

[3] E. Mach, *The Science of Mechanics...*, Open Court, La Salle (1960).

[4] E. Schrodinger,*Annalen der Physik* , **77**, 3325 (1925).

[5] J.B. Barbour & H. Pfister, *Mach's Principle: From Newton's Bucket to Quantum Gravity*. Birkhauser, Boston (1995).

[6] A.K.T. Assis, *Foundations of Physics Letters*, **2**, 301 (1989).

[7] A.K.T. Assis, *Apeiron*, **13**, 3 (1992)

[8] A.K.T. Assis, *Weber's Electrodynamics*, Kluwer Acad. Pub. Dordrecht, Boston, London (1994).

[9] A.K.T. Assis, *Relational Mechanics*, Apeiron, Montreal (1999).

[10] A.K.T. Assis and P. Graneau, *Foundations of Physics*, **26**, No 2, 271 (1996).

[11] T.E.Phipps Jr, *Speculations in Science & Technology*, **1**, 499 (1978).

[12] A.K.T. Assis & P. Graneau, *Hadronic Journal*, **18**, 271 (1995).

[13] A.K.T. Assis & J. Guala Valverde, *Apeiron*, **7**, N 3-4, 131 (2000).

[14] P. Graneau & N. Graneau, *Einstein vs. Einstein*, *How matter Interacts with Matter*, Carlton Press Inc. New York, (1993).

[15] J. Palacios, *Análisis Dimensional*, Espasa Calpe, Madrid (1956-64); Dimensional Analysis, McMillan, London (1964); *Analyse Dimensionnelle*,Gauthier Villar, Paris (1960).

[16] J. Guala Valverde, *Apeiron*, **3**, (2001).

[17] P.A.M. Dirac, *Proceedings of the Royal Society of London A*, **165** (1938).

[18] J.C. Maxwell, *A Treatise on Electricity & Magnetism*, Dover Pub. Inc. New York (1954). First Publication by the Clarendon Press (1891).

[19] J. A. Stratton, *Electromagnetic Theory*, McGraw-Hill Book Co.,Inc. New York and London (1941).

[20] J. Rey Pastor, P. Pi Calleja y C. A. Trejo., *Análisis Matemático*.Vol.3, Kapelusz, Buenos Aires, Argentina (1965).

[21] R.A. Rapacioli, *Apeiron*, **4**, (2001).